Cavities in a visco-plastic material: A mesoscopic concept

Kerstin Weinberg* and **Thomas Böhme**

Institut für Mechanik (MS-2), Technische Universität Berlin, Einsteinufer 5, 10587 Berlin, Germany

In the present work we investigate the temporal development of arbitrarily distributed voids in a visco-plastic material under different loading regimes. A mesoscopic continuum model is used in order to take the microstructure of the material into account. In particular, we introduce a mesoscopic space representing an extension of the space-time domain of the continuum mechanical fields. This extended domain requires a reformulation of the classical balance equations as well as the consideration of additional constitutive quantities. Furthermore a mesoscopic distribution function can be introduced which follows an own balance. Assuming a special model of porous composites, the spherical shell model, all required steps are elaborated in order to describe load-induced void-growth in a metal-like matrix. We conclude with some exemplary results which show astonishing similarities with co-called LSW-theories.

1 Spherical Shell Model

Most materials contain a certain amount of cavities (voids). Usually the voids are small compared to the size of the surrounding structure. In order to analyze the growth of voids in a general deforming material let us assume that each void i is surrounded by a spherical material shell as illustrated in Fig. 1. Furthermore the porous material is modeled as an ensemble of isotropic *spherical shells* (void plus spherically surrounded matrix) with a certain given initial void volume fraction and an initial distribution. In detail holds:

$$\frac{V_{void}^{(i)}}{V} = f_V^{(i)} \quad , \quad \sum_i V_{shell}^{(i)} = V \quad , \quad \sum_i f_V^{(i)} = f_V \, .$$

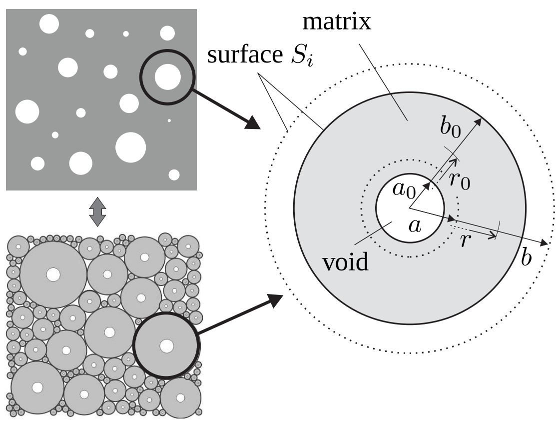

Fig. 1 From arbitrarily distributed voids to the spherical shell model.

Due to this construction the remaining volume between the spheres can be made infinitesimally small and the deformation energy density of the composite can be approximated as the sum of the deformation energies densities stored in all spherical shells. Consider now a spherical shell under deformation. The initial geometry a_0, b_0 changes to $a(t)$ and $b(t)$. Presume an incompressible deformation and denote the *velocity of void expansion* with $\dot{a}$ it follows for $r \in [a, b]$:

$$\frac{d}{dt}\frac{4\pi}{3}(r^3 - a^3) = 0 \quad , \quad b = \left(b_0^3 - a_0^3 + a^3\right)^{1/3} \quad , \quad v_r \overset{(\mathrm{def})}{=} \frac{dr}{dt} = \frac{a^2}{r^2}\dot{a} \quad . \tag{2}$$

2 Mesoscopic Space, Distribution Function and Balance Equations

In classical (five-field) continuum mechanics the *wanted fields* are the mass density $\rho(\mathbf{x}, t)$, the material velocity $\mathbf{v}(\mathbf{x}, t)$ and the (mass-specific) internal energy $u(\mathbf{x}, t)$. These five fields are defined on the *space-time* domain $(\mathbf{x}, t)$ and follow universal balance equations. In order to solve the balances one must specify the material through constitutive equations, which are defined on the *state space*. Now the question arise, how to introduce the additional information about the microstructure of the material into the continuum mechanical framework. The central idea of the mesoscopic concept is the Extension of the space time domain $(\mathbf{x}, t) \in \mathbb{R}^3 \times \mathbb{R} \rightarrow (\mathbf{m}, \mathbf{x}, t) \in \mathcal{M} \times \mathbb{R}^3 \times \mathbb{R}$ to the *mesoscopic space*, where $\mathbf{m}$ are 'suitable' variables of arbitrary tensorial order describing the microstructure and $\mathcal{M}$ are the manifold according to $\mathbf{m}$, [1]. For our problem we can identify: $\mathcal{M} \subset \mathbb{R}$ and $\mathbf{m} \equiv a$.

Thus, a (normalized) mesoscopic distribution function $\tilde{d}(a, \mathbf{x}, t)$ can be found representing the number of the voids $\tilde{N}_V$ with radius a relatively to the total number of voids N_V at position $\mathbf{x}$ and time t (The tilde in the variables refers to the mesoscopic space):

$$\tilde{d} \overset{(\mathrm{def})}{=} \frac{\tilde{N}_V}{N_V} \quad \text{and} \quad N_V = \int_{a_0}^{\infty} \tilde{N}_V \, \mathrm{d}a \quad . \tag{3}$$

* Corresponding author: e-mail: kerstin.weinberg@tu-berlin.de, Phone: +49 30 314 24214, Fax: +49 30 314 24499

On the other hand a reformulation of the balances is necessary, in which occur now additional derivatives and fluxes due to $\mathbf{m}$ or $\mathcal{M}$, respectively. In particular the (local) mass balance and the balance of the distribution function read:

$$\frac{\partial \tilde{\rho}}{\partial t} + \nabla \cdot (\tilde{\rho} \mathbf{v}) + \frac{\partial}{\partial a} (\tilde{\rho} \dot{a}) = 0 \qquad \text{and} \qquad \frac{\partial \tilde{d}}{\partial t} + \nabla \cdot \left(\tilde{d} \mathbf{v} \right) + \frac{\partial}{\partial a} \left(\tilde{d} \dot{a} \right) = 0, \tag{4}$$

where we presume that the voids can not move independly with respect to the surrounding material, i.e. $\mathbf{v} = \tilde{\mathbf{v}}$ and neglect any void production, i.e. $\dot{N}_V = 0$. Eq. $(4)_2$ determines the temporal development of arbitrary distributed voids in a material. For its (numerical) solution one must insert a material law for $\dot{a}$.

3 Constitutive Model for $\dot{a}$

In order to derive a material law $\dot{a}(a, \mathbf{x}, t)$ we firstly ask, which energies are necessary to deform *one* spherical shell, [2]. Here *three* contributions are considered: **(a)** plastic energy $\tilde{W}$, **(b)** kinetic energy $\tilde{K}$ and **(c)** surface energy $\tilde{S}$. Neglecting elastic effects and taking into account effective von Mises strains $|\dot{\varepsilon}| = \dot{\varepsilon}^p = 2a^2\dot{a}/r^3$ and hardening effects results in expressions for $\tilde{W}$, $\tilde{K}$ and $\tilde{S}$ as well as for the external power $\tilde{P}$ as functions of $a(t)$ and $\dot{a}(t)$. Setting the external power equal the "sum of the internal powers", $\tilde{P} = d(\tilde{W}^p + \tilde{S} + \tilde{K})/dt$, yields an ODE for $a(t)$ as follows:

$$pa^2\dot{a} + \dot{p}\frac{b^3(a)}{3} = \varepsilon_0\sigma_0 \frac{n}{n+1}\frac{a_0^3}{3}\dot{g}(a;n) + 2a\gamma\dot{a} \; + \frac{\rho_0}{2}\left[2\ddot{a}\dot{a}a^3 + 3\dot{a}^3a^2 - \frac{a^3}{b(a)}\left(2\ddot{a}\dot{a}a + 4\dot{a}^3 - \frac{a^3\dot{a}^3}{b^3(a)}\right)\right]. \tag{5}$$

By means of an initial geometry (a_0, b_0) a (numerical) solution of Eq (5) can be perform providing a constitutive relation for $\dot{a}(t)$ for *one* chosen spherical shell. Exploiting this ODE for different spherical shells, i.e., for different initial values a_0 and b_0, yields $\dot{a}(a, t)$.

4 Results

In order to investigate *dynamical void expansion* in visco-plastic material we apply to the spherical shells a high pressure impulse $p(t)$ as illustrated in Fig. 2 (left). Furthermore all required material were chosen exemplarily for typical aluminum.

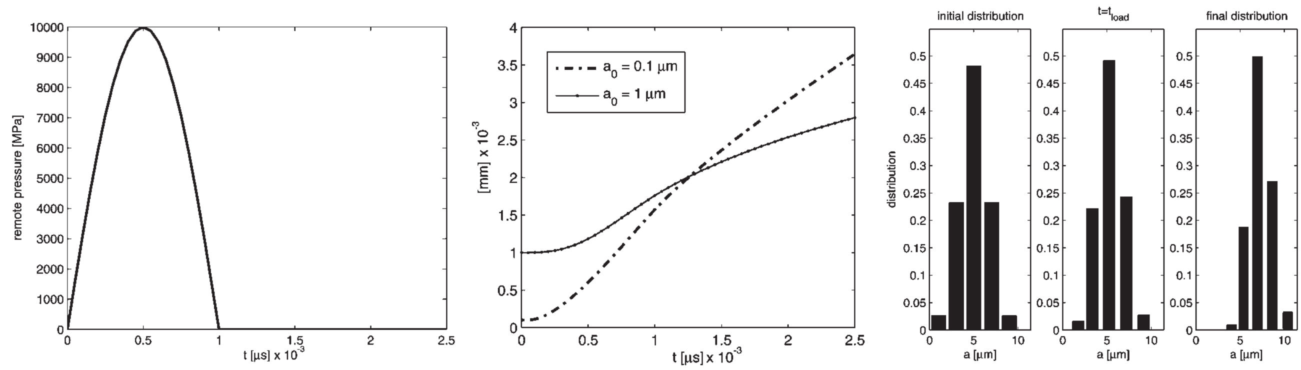

Fig. 2 The external load, the expansion of two different voids and the temporal development of five different void radii.

In the centered picture two different voids with the initial radii of $a_0 = 0.1$ μm, 1 μm and the initial void volume fraction $f_{v0} = a_0^3/b_0^3 = 10^{-4}$ were considered. Obviously the void growth continues after $p(t)$ finished. This fact is caused by inertia effects due to the kinetic energy $\tilde{K}$. Furthermore smaller voids grow faster than bigger voids, they can even "overtake" the bigger voids.

Finally we investigate the temporal development of a discrete (Gaussian) void distribution function with five different initial void radii, right £gure. Due to the fact that smaller voids grow faster than bigger voids the initial symmetric distribution change to an asymmetrical distribution in such a manner that the fraction of smaller voids decrease. Such results can be also found in so-called LSW-theories (Ostwald ripening), although the driving forces are completely different.

References

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[2] M. Ortiz, A. Molinari, Effect of strain hardening and rate sensitivity on the dynamic growth of a void in a plastic material, Trans. *ASME*, **59**, 48 (1992).