**Example**

**Photosynthesis of an Oak Tree**

An oak tree (fagus sylvatica) at age 100y is 20 m height and is limited by a sphere with diameter d = 12 m. Its leafs cover an area of 1,600 $m^2$. They produce during a summer day $p_{GLU}$ = 7,5 g / $m^2$ d glucose ($C_6H_{12}O_6$) which during night time mainly is converted to starch and cellulose. The direct solar radiation during daytime of 10 hours is assumed to be $\sigma$ = 0.8 kW / $m^2$. The leafs are in thermal equilibrium with ambient air and also in radiation equilibrium with the indirect radiation from sky,

1. How much glucose is produced by the oak tree per summer day?
2. How much carbon dioxide is extracted from the air and how much water is needed for the daily glucose production?
3. How much oxygen is produced by the oak tree per day?
4. What percentage of the daily solar radiation ($E^*_{sd}$) is needed for the photosynthesis reaction assuming that it is reversible and the entropy production is leveled by evaporation of (additional) water only.
5. What percentage of the daily solar radiation ($E^*_{sd}$) is included in the enthalpy of the glucose produced per day?
6. How many liters of water could be evaporated (at 25 °C) per day by the direct solar radiation ($E^*_{sd}$) neglecting reflection and the indirect radiation from sky? Compare this amount to the statement of a botanist saying that the oak tree may evaporate during a summer day as much as 400 l of water.

**Solution**

1. Daily production of glucose ( $P_{GLU}$ )

   Production per m²

   $p_{GLU} = 7,5 \ \ g/m^2 d$

   Area of leafs

   $A_L = 1600 \ \ m^2$

   $$P_{GLU} = p_{GLU} \cdot A_L = 7,5 \frac{g}{m^2 d} \cdot 1600 m^2 = 12\, kg/d$$

2. Supply of carbon dioxide and water for glucose production:

   Stochiometric equation of photosynthesis reaction

   $$6\ CO_2 + 6\ H_2O - C_6H_{12}O_6 - 6\ O_2 = 0 \qquad (1)$$

   educts products

   Molar glucose production per day:

   $$P_{GLU,m} = P_{GLU}/M_{GLU} = \frac{12 \ \ kg \ \ mol}{180 \ \ g \ \ d} = 66,7\, mol/d$$

   Molar masses ( g/mol )

| | |
|---|---|
| $CO_2$ | 44 |
| $H_2O$ | 18 |
| $C_6H_{12}O_6$ | 180 |
| $O_2$ | 32 |

(1) $$m_{CO_2} = 6 \ \ M_{CO_2} \cdot P_{GLU,m} = 6 \cdot 44 \frac{g}{mol} \cdot 66,7 \frac{mol}{d}$$

$$m_{CO_2} = 17,6\,kg/d$$

$$m_{H_2O} = 6 \ \ M_{H_2O} \cdot P_{GLU,m} = 6 \cdot 18 \frac{g}{mol} \cdot 66,7 \frac{mol}{d}$$

$$m_{H_2O} = 7,2\,kg/d$$

3. Oxygen production per day, $c_p$ Eq. (1)

$$m_{O_2} = 6 \ \ M_{O_2} \cdot P_{GLU,m} = 6 \cdot 32 \frac{g}{mol} \cdot 66,7 \frac{mol}{d}$$

$$m_{O_2} = 12,8\,kg/d$$

4. Radiation needed for photosynthesis reaction (1):

   Total radiation energy available per day

$$E^*_{sd} = \sigma \cdot d \cdot A$$

$\sigma = 0,8 \ \ kw/m^2$ … solar radiation

$d = 10 \ \ h/d$ … sunshine

$A = \pi \cdot r^2_{OAK} = 113,1 \ \ m^2$ … cross section of “oak sphere “

$r_{OAK} = 6 \ \ m$

$$E^*_{sd} = 904,8 \ \ kwh/d = 3257,3 \ \ MJ/d$$

Radiation energy needed for photosynthesis of / mol glucose / ( GLU):

(PH 19): $E_{S,HX} = 2 \cdot 816 + 267 \quad kJ/mol, GLU = 3083 \;\; kJ/mol, GLU$

energy balance Eq. ( PH 8a ) ← $2 \cdot 816$; $267$ → entropy balance Eq. ( PH 24,24a )

Radiation energy needed for photosynthesis per day:

$$E_{sd} = E_{s,1+x} \cdot P_{GLU,m}$$

$$E_{sd} = 3083 \frac{kJ}{mol, GLU} \cdot 66,7 \frac{mol, GLU}{d}$$

$$E_{sd} = 205,6 \;\; MJ/d$$

Percentage of $E_{sd}$ compared to $E^*_{sd}$ :

$$E_{sd} = \frac{205,6}{3257,3} E^*_{sd} = 6,31\% \; E^*_{sd}$$

5. Glucose as storage system for solar radiation

Enthalpy of glucose produced per day

$$\Delta H_{GLU,d} = \Delta H_{GLU} \cdot P_{GLU,m}$$

$$\Delta H_{GLU,d} = 1264 \frac{kJ}{mol, GLU} \cdot 66,7 \frac{mol, GLU}{d}$$

$$\Delta H_{GLU,d} = 84,31 \;\; MJ/d$$

Percentage of $\Delta H_{GLU,d}$ compared to $E^*_{sd}$ :

$$\Delta H_{GLU,d} = \frac{84,31}{3257,3} E^*_{sd} = 2,6\% \; E^*_{sd}$$

6. Evaporation of water by solar radiation

Evaporation enthalpy of water at 25 °C: $r_{H_2O} = 2400$ kJ/kg

Amount of water evaporated by $E^*_{sd} = 3257,3 \ \ \mathrm{MJ/d}$:

$$m_{H_2O} = \frac{E^*_{sd}}{r_{H_2O}} = \frac{3257,3}{2400\mathrm{kJ}} \frac{\mathrm{MJ}}{\mathrm{d}} = 1360\,\mathrm{kg/d} \simeq 1360\,\ell/\mathrm{d}$$

Water being sorbed in the leaves of a tree may require a much higher energy to be evaporated as it first has to be transported from micro-channels within the leaf to its surface and only then can be transferred to the gaseous state. Hence $r_{H_2O}$ may be substituted by the adsorption / evaporation energy $e_{H_2O} \simeq 2r_{H_2O}$ thus reducing the mass of water actually evaporated to ca. $700\,\mathrm{kg/d}$. Taking also reflection of sun light on the surface of the leaves into account by reducing $E^*_{sd}$ or –equivalently - $m_{H_2O}$ to – say – 70 % of its value we get 490 $\mathrm{kg/d}$ of water evaporated [PH 10], which is in the range of the botanist's value of $400\,\mathrm{kg/d}$.