Evolution of Heat during Bacterial growth Processes

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1. Bacteria

2. Phenomenology of Bacterial Heat Production
   Kleiber‘s Law, Temperature Dependence
   Autometabolism

3. Heat Production in Metabolic Reactions

4. Heat Production in Bacterial Growth Processes

1. Bacteria

A. van Leeuwenhoek 1676
Microscop

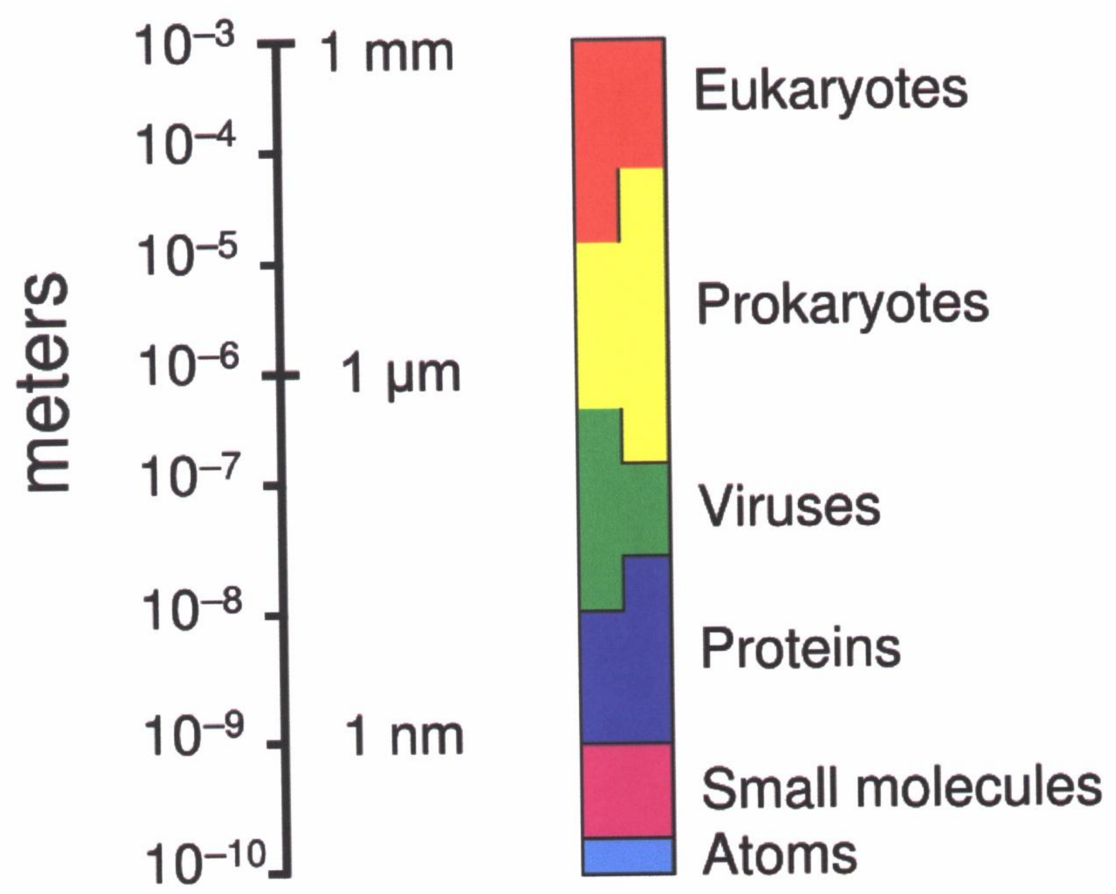
Water
Human spittle

10*9 – 10*12 / liter

Known bacteria 1 – 5 %

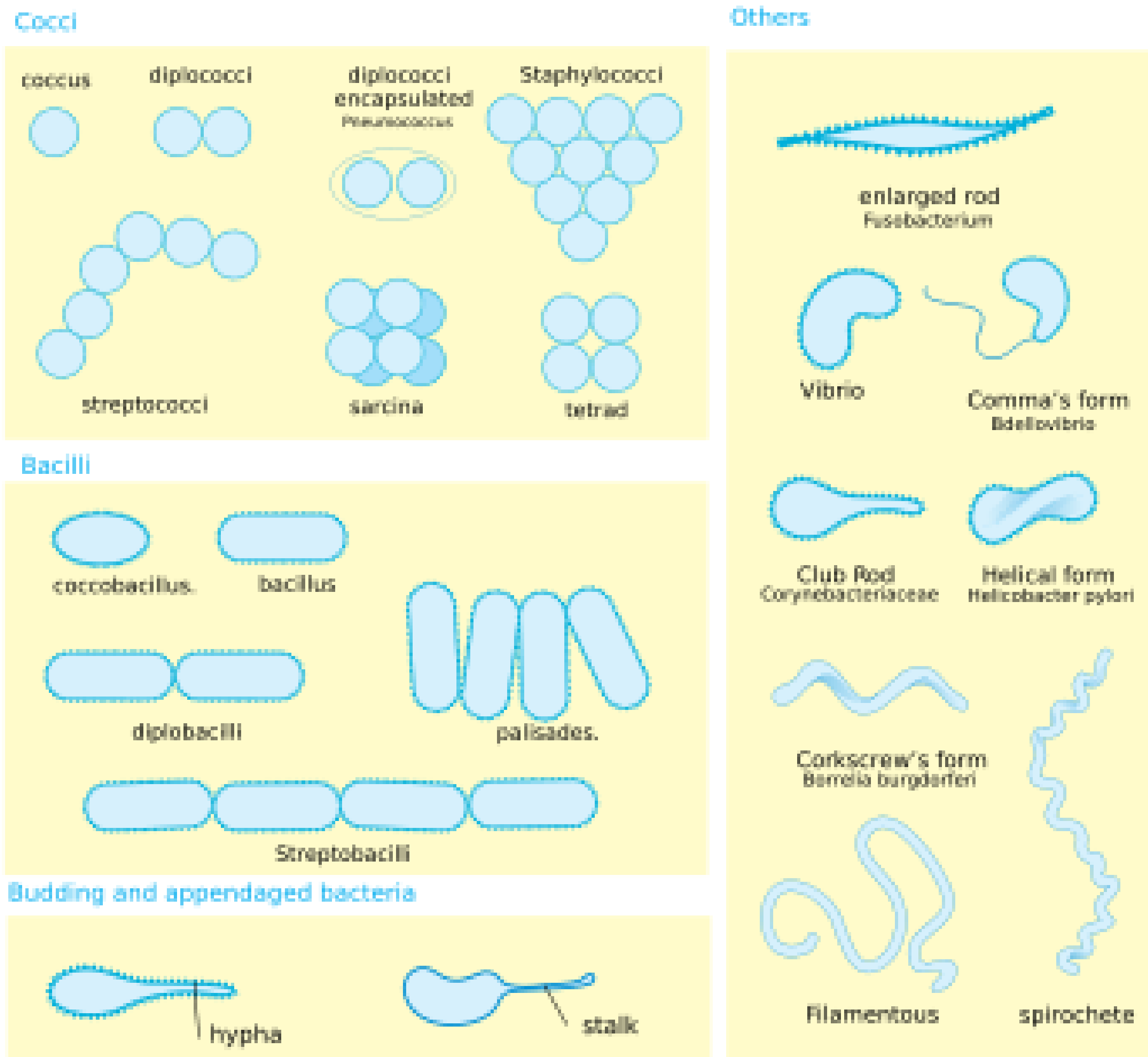
Unknown bacteria 99 – 95 %

Mass: 1 – 10 p g, p = 10*-12



Relative_scale.svg (SVG file, nominally 936 × 762 pixels, file size: 31 KB)

Bacteria, Morphology

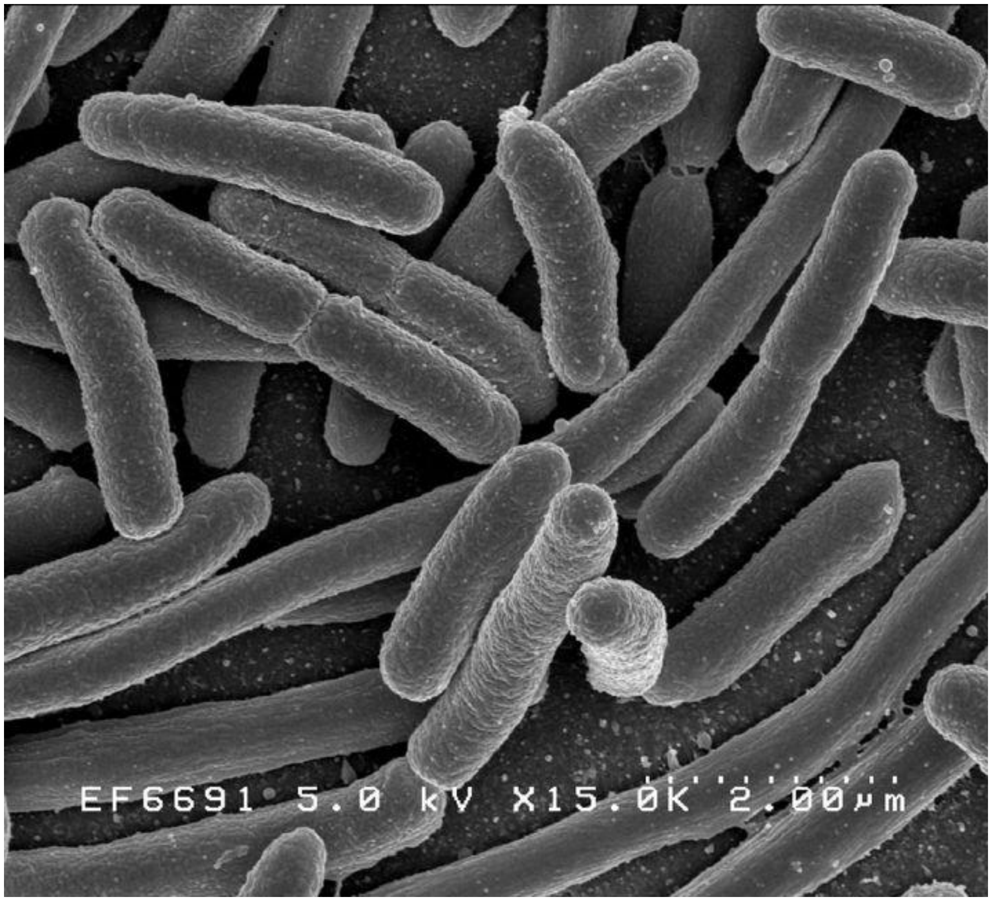


Coccus (Kokken)

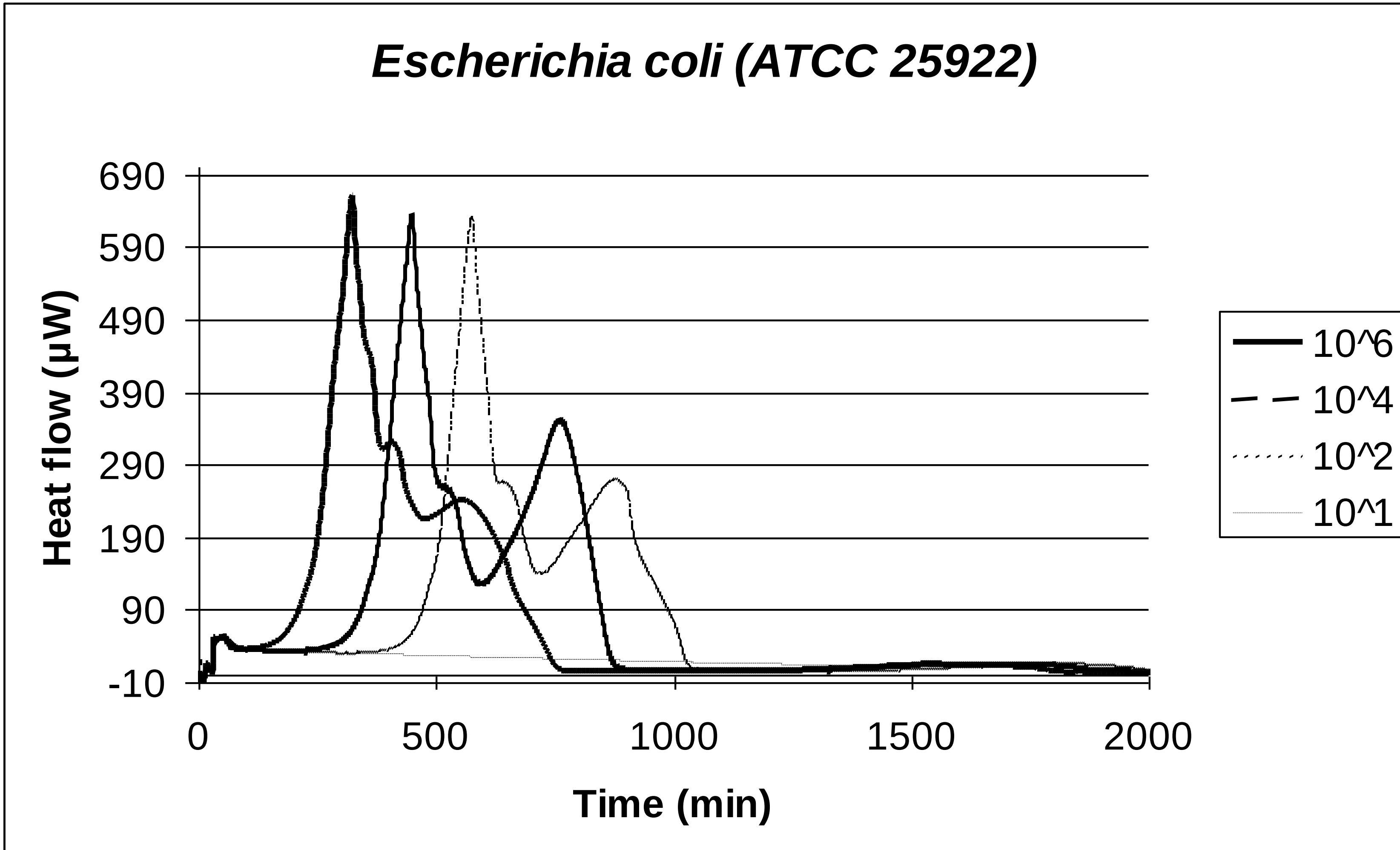
Bacili (Stäbchen)

Spirochetes (Spiralen)

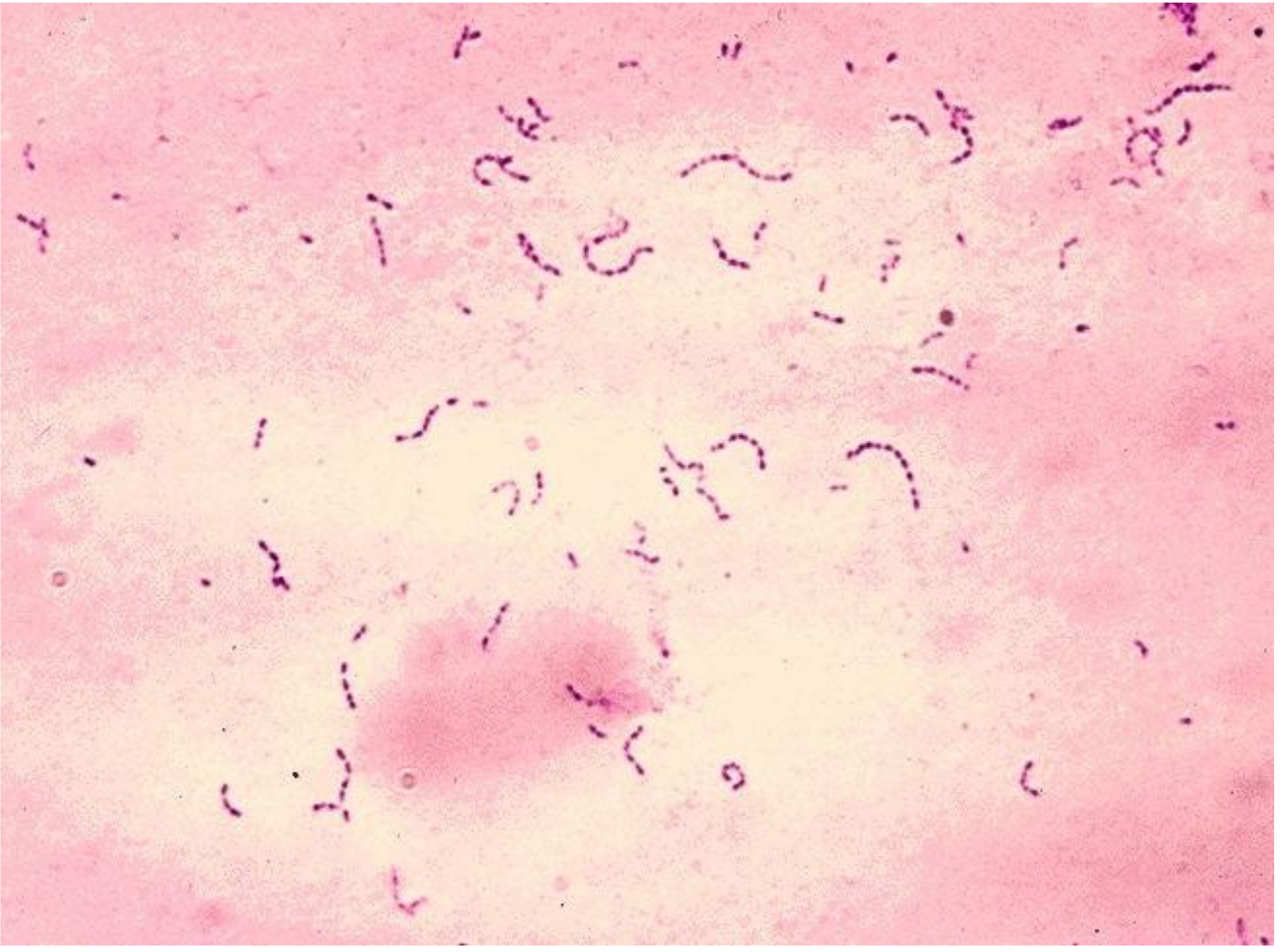
CFU= Colony forming units



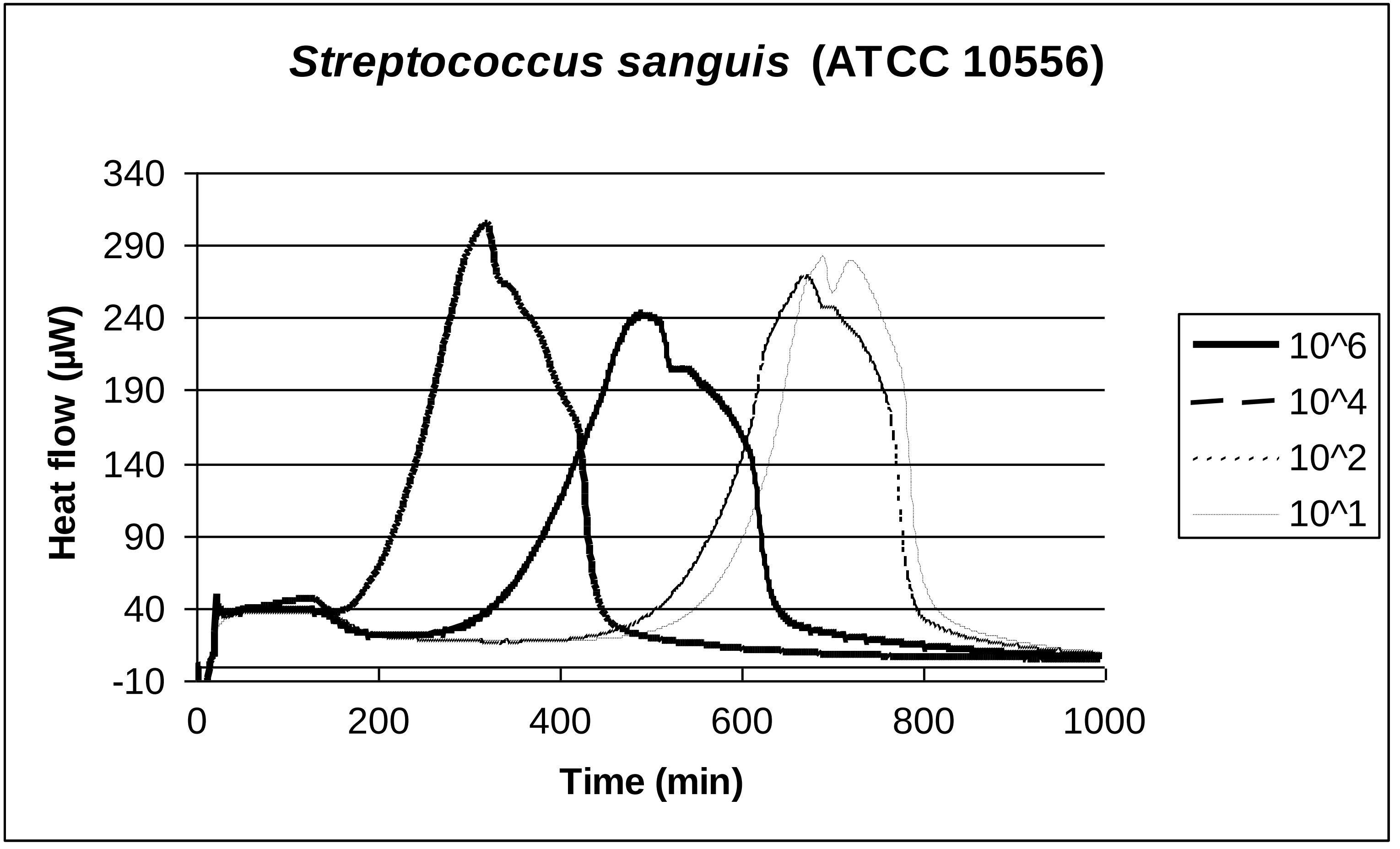
Bacteria Escherichia coli, Th. Escherich (1919)



Ref. A.Trampuz et al., Biocalorimetry..., Trans-2007-0018.R2



Bacteria Streptococcus Mutans (Karies), Clarke (1924)



Ref. A.Trampuz et al., Biocalorimetry..., Trans-2007-0018.R2

2. Phenomenology of Bacterial Heat Production (Allometry)

Kleiber´s Law (1932)

Basic metabolic rate of aerobic living systems at T=298 K :

$$J_0 = aM^{\alpha}$$

$$a = 3{,}5 W / kg^{\alpha}$$

$$\frac{2}{3} < \alpha < 1$$

$$\alpha \cong \frac{3}{4}$$

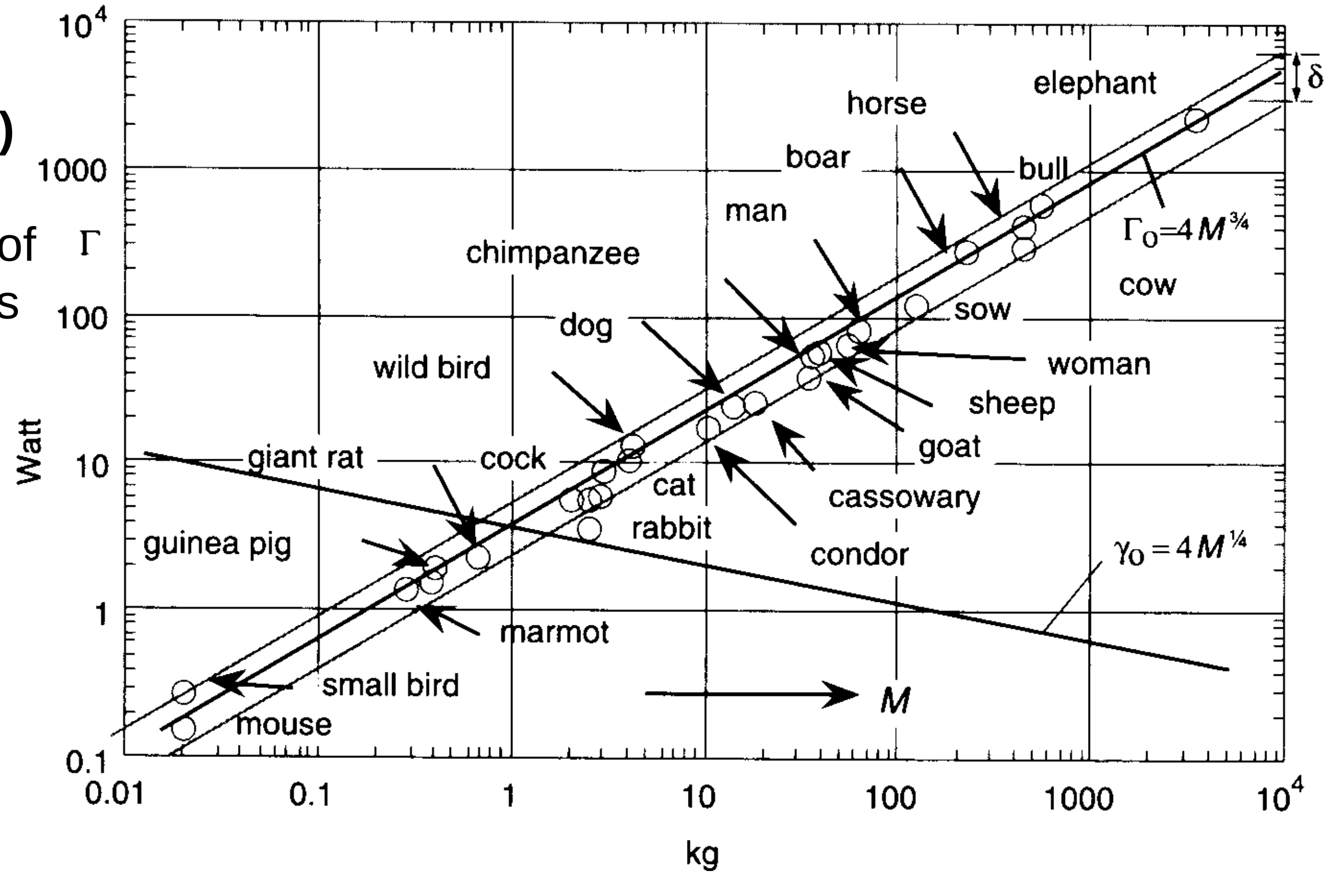


Figure A3
Metabolic rate of oxygen consumption based living systems. Mouse-Elephant-curve, Kleiber,1932. This curve also holds for bacteria (M $\cong$ 10* (-4) – 10*(-12) g).

Basic Metabolic Rate ≃ Heat Production of Aerobics

Creature	Mass/kg	Metabolic Rate J0 / W	Food Substrate	Heating Value MJ/kg	Consump-tion kg/day
Bacteria Staphylococcus aureus	$0{,}5 \bullet 10^{-15}$	120 nW	glucose	15,6	0,665. 10*(-9)
Men	80 kg	94 W	various	20	0,40
Lion	120 kg	127 W	meat	30	0,37
Elephant	3000 kg	1,418 W	grass	10	12,3

Activation factor : $J_0 \rightarrow (2\text{-}5)J_0$

Kleiber's Constant : Temperature Dependence

Bacteria growth processes, sterilisation.

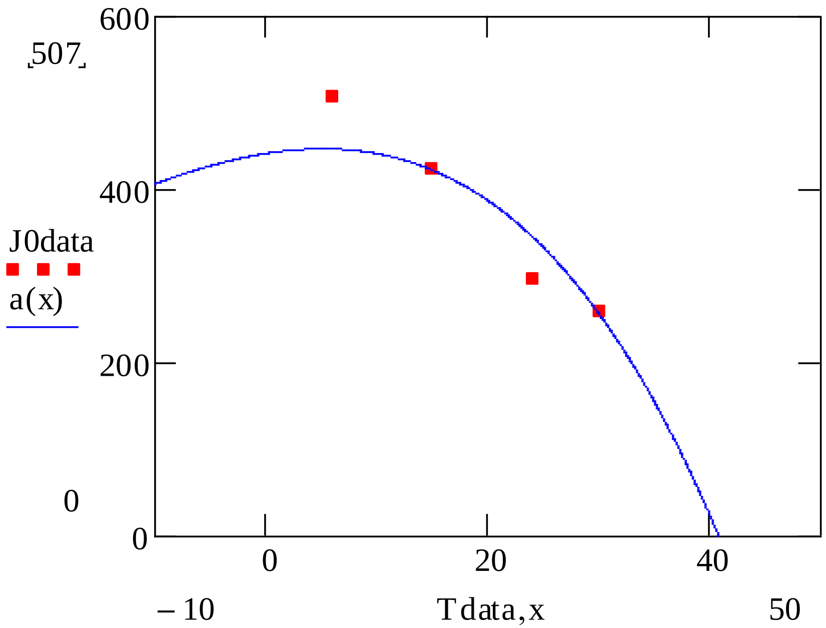
$$a = a(T_b, T^*) = A.(T_B - T^*).e^{-q^*/RT^*}$$

Example : Dogs,
hair cut,TB=41 C
Data : Jeroch et.al.(1999)

T_b....Maximum temperature
of living system

T^*....Environmental temperature

q^*....Energy (metabolism, heat transfer)



Enviromental temperature
for maximum metabolism

$$T^*_{\max} = \frac{q^*}{2R}\left(-1 + \left(1 + 4RT_b / q^*\right)^{1/2}\right)$$

Autometabolism of Bacteria

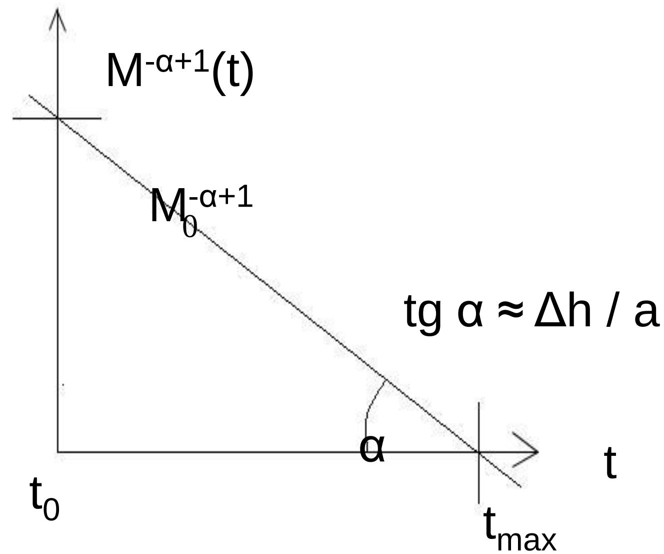
Lack of substrate: Living period ? Heat production ?

$$J_0 = -\Delta h \bullet \dot{M}$$

$$J_0 = aM^{\alpha}$$

$$\dot{M} + \frac{a}{\Delta h} M^{\alpha} = 0$$

$$M(t)^{-\alpha+1} = M_0^{-\alpha+1} - \frac{a}{\Delta h}\left(t - t_0\right)$$



Living period: $M \geq 0$

$$t_{max} - t_0 = \frac{\Delta h}{a} M_0^{-\alpha+1} \geq 0$$

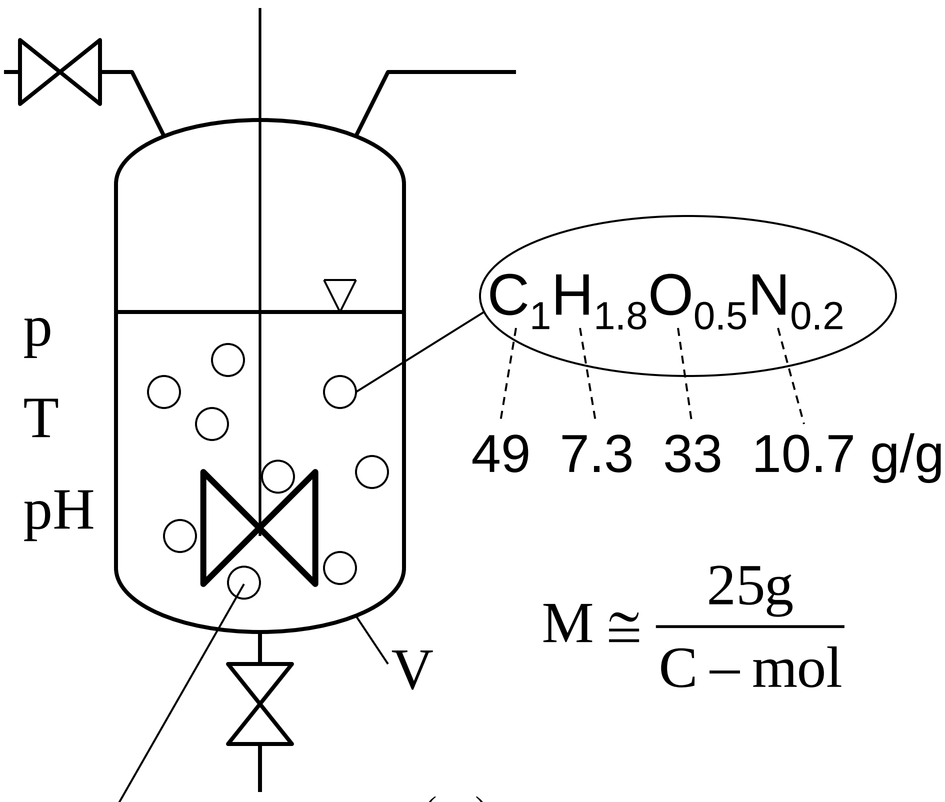
Example : Staph. Aureus

M_0=0,5 pg; Δh=18 MJ/kg;

$t_{max} \simeq$ 4h

3. Heat Production in Metabolic Reactions*

Fermenter



$$M \cong \frac{25g}{C-mol}$$

$$[C_x] = \frac{C-mol(x)}{e}$$ ... Biomass (x)

<u>Example (Yeast)</u>

Genes	5000
Metabolites	1000-5000
Concentration*)	$(0.1-10)\,mmol$

Turn over time

$$\frac{\text{Concentration}}{\text{Reaction rate}} = (1-10)s$$

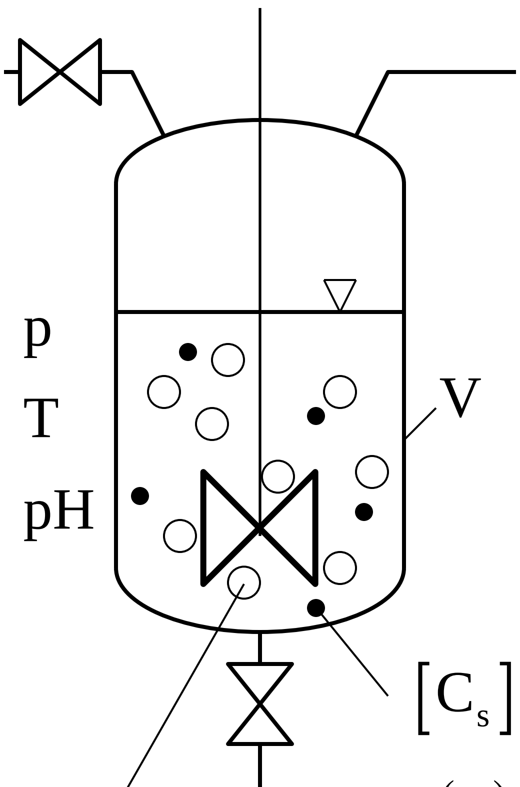
*) Osmotic pressure limited. Avoiding byproducts and byreactions.

*Microbiothermodynamic system, Microbioreactor

Bioreactors

Microbial Growth at Constant Substrate Concentration

Fermenter



Rate equation

$$dC_x = \mu_x C_x dt$$

Growth rate*)

$$[\mu_x] = \frac{C-mol(x)}{C-mol(x)\cdot h}$$

$(0.001-2)$

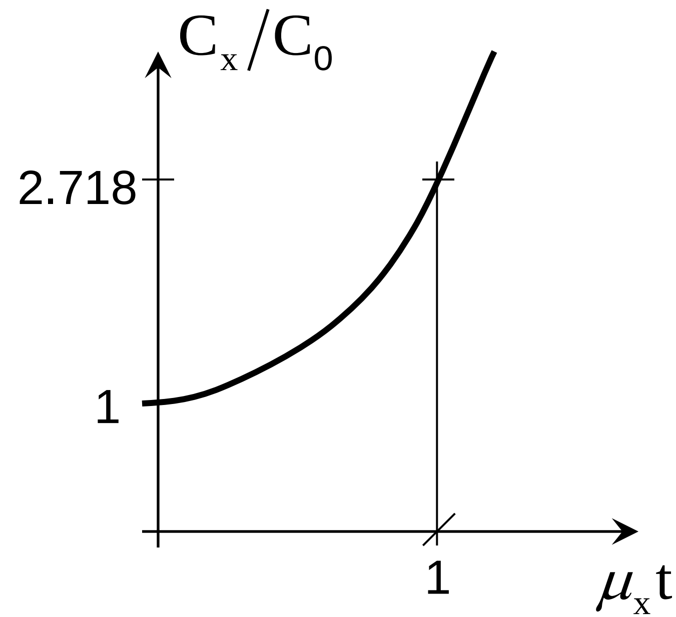
$$[C_s] = \frac{mol(s)}{e} \quad \text{... Substrate}(s)$$

$$[C_x] = \frac{C-mol(x)}{e} \quad \text{... Biomass}(x)$$

Microbial growth

$$C_x(t) = C_0\, e^{\mu_x t}$$

$$\mu_x = \text{const} = \frac{1}{\tau}$$

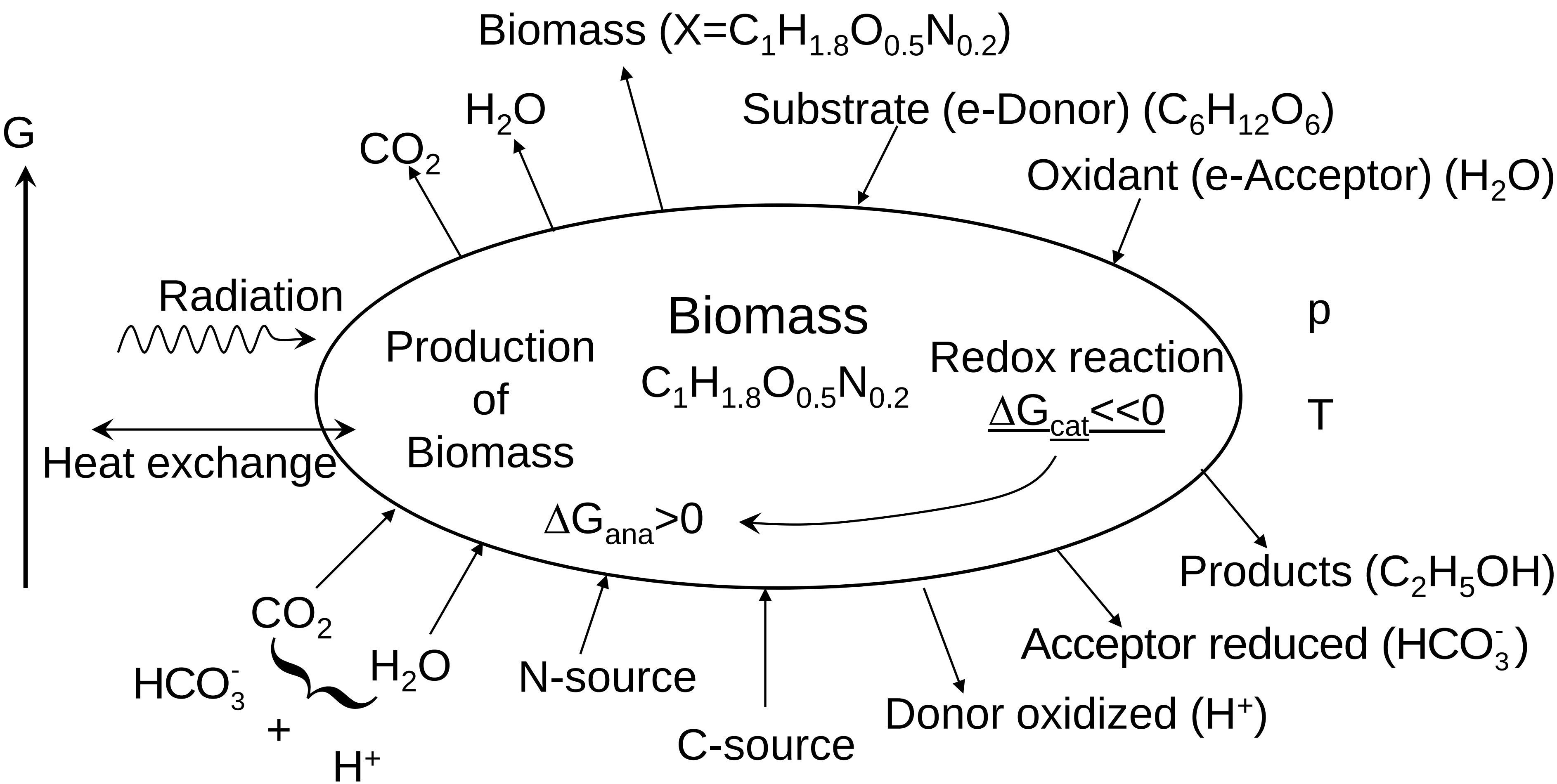


*) Limited by $\bar{e}$-transport

capacity in cell membranes: $3\,mol(\bar{e})/C-mol(x)h\ (298\,K)$

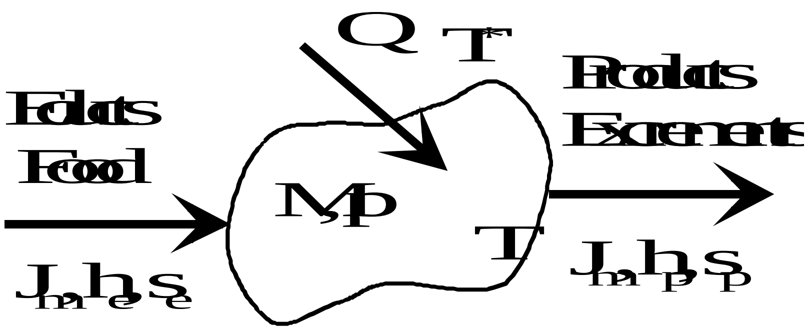
Bacteria Stylonychia (Wimpertierchen / Eyelash bacteria)

Microbial Growth System



Anabolism + Catabolism (Free Entalpy)

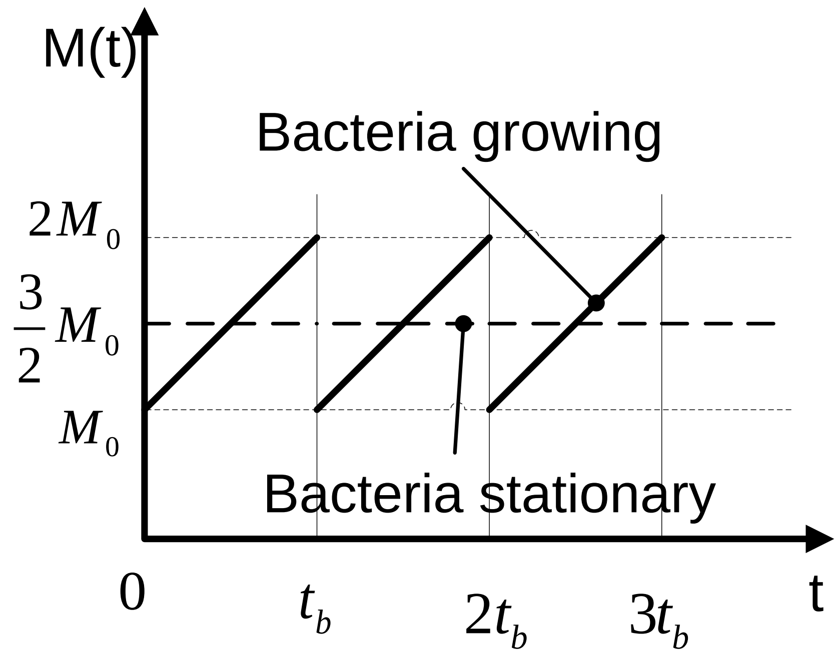
Bacterial Growth Process and Heat Production (I)
Thermodynamic Analysis



Mass balance

$$\dot{M} = J_e - J_p \qquad (1)$$

$$\dot{M} = \frac{M_0}{t_B} \qquad (2)$$



Population dynamics

$$n(t) = n_0 \exp(t/\tau) \quad \tau = \frac{t_B}{\ln 2}$$

Bacterial Growth Process and Heat Production (I)

Thermodynamic Analysis

Energy balance

$$\dot{U} = h_e J_e - h_p J_p + \dot{Q}_B \qquad (3),$$

Extensivity, CEOS

$$\dot{U} = u\dot{M} \qquad (3A)$$

Entropy balance

$$\dot{S} = s_e J_e - s_p J_p + \frac{\dot{Q}_B}{T} + P_s \qquad (4),$$

Extensivity, EEOS

$$\dot{S} = s\dot{M} \qquad (4A)$$

$$P_s \geq 0 \qquad (5)$$

Bacterial Growth Process and Heat Production (II)

Thermodynamic Analysis, Eqs. (1-4A)

Substrate flow

$$J_e = \underbrace{\frac{g - g_e}{g_e - g_p}}_{\geq 0} \bullet \frac{M_0}{t_B} + \frac{TP_s}{\underbrace{g_e - g_p}_{\geq 0}}$$

Irrev. Process needs more substrate

Product flow

$$J_p = \underbrace{\frac{g - g_e}{g_e - g_p}}_{\leq 0} \bullet \frac{M_0}{t_B} + \frac{\overbrace{TP_s}^{\geq 0}}{g_e - g_p}$$

Delivers less product

Bacterial Growth Process and Heat Production (IIa)

Heat flow Irreversible process

$$Q_B = \left(u + \frac{(g_p - g)h_e - (g_e - g)h_p}{g_e - g_p} \right) - \frac{h_e - h_p}{g_e - g_p} \bullet TP_s < \dot{Q}_{B\,rev}$$

needs less heat $(\dot{Q}_{B\,rev} > 0)$

produces more heat $(\dot{Q}_{B\,rev} < 0)$

Population dynamics

$$J_{e\,tot} = n_0 \exp(t/\tau) J_e \;;$$

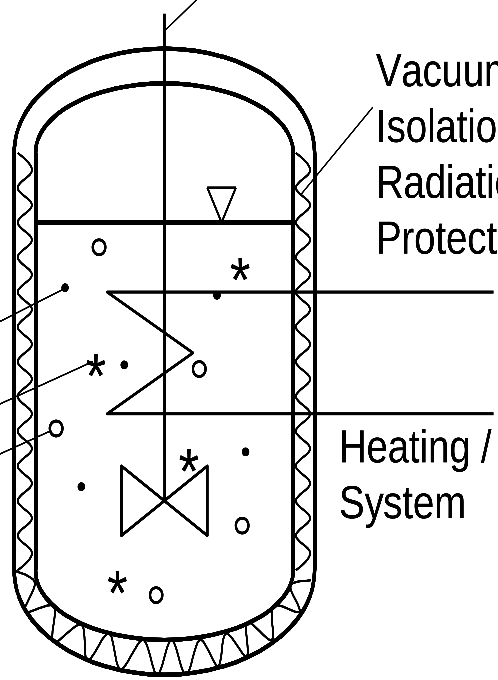
$$J_{p\,tot} = n_0 \exp(t/\tau) J_p;$$

$$\dot{Q} = n_0 \exp(t/\tau) \dot{Q}_B$$

Problems:
Metabolic reactions ?
Thermodynamic data ?

4. **Heat Production in Bacterial Growth Processes**

Isothermal Calorimeter



Metabolic generation of heat

$$dQ \simeq -dm_s \quad (1)$$

$$\dot{Q} = -K_s \dot{m}_s \quad (2)$$

$$Q(t) = K_s (m_{s0} - m_{s(t)}) \quad (3)$$

$$Q(\infty) = K_s m_{s0} \quad (3a)$$

Mass / molar balance grouth process

$$dn \simeq -dn_s \quad (4)$$

$$\dot{n} = -C\dot{n} \quad (4a)$$

$$C \geq 0$$

Bacterial Growth Prozess Model I (Monod)

$$n(t) = n_0 + (n_\infty - n_0)\frac{(bt)^\alpha}{1+(bt)^\alpha};$$

(5)

$$n(0) = n_0 \qquad \alpha \geq 1;$$

$$n(\infty) = n_\infty \qquad b > 0.$$

Substrate

(4)→ $$\dot{n} = -C\dot{n}_s;$$

(5)→ $$n_s = \frac{n_{s0}}{1+(bt)^\alpha}.$$ (6)

Heat Production during Bacterial Growth Processes

(2)→

$$\dot{Q} = -K_s \dot{n}_s = \frac{K_s}{C} \dot{n}; \qquad (7)$$

$$\dot{Q} = K_\alpha b(n_\infty - n_0) \frac{(bt)^{\alpha-1}}{(1+(bt)^\alpha)^2}. \qquad (8)$$

Maximum value

$$bt_{\max} = (\alpha - 1)^{\frac{1}{\alpha}}; \qquad (9)$$

$$\dot{Q}_{\max} = \frac{Kb(n_\infty - n_0)(\alpha - 1)^{1-\frac{1}{\alpha}}}{\alpha}. \qquad (10)$$

Bacterial Population Growth Prozess Model II

$$dn \simeq n n_s dt \qquad (12)$$

$$\dot{n} = A n_s n \qquad (13)$$

$$n(t) = n_0 \exp\left\{ A \int_0^t n_s(t')dt' \right\} \qquad (15)$$

(3)→
$$n(t) = n_0 \exp\left\{ A \int_0^t \left(n_{so} - \frac{Q(t')}{(K_s)}\right)dt' \right\} \qquad (16)$$

Bacterial Population

Heat generated in broth

Growth of Population and Depletion of Substrate

Substrate

$$dn_s \simeq -n_s n dt \qquad (17)$$

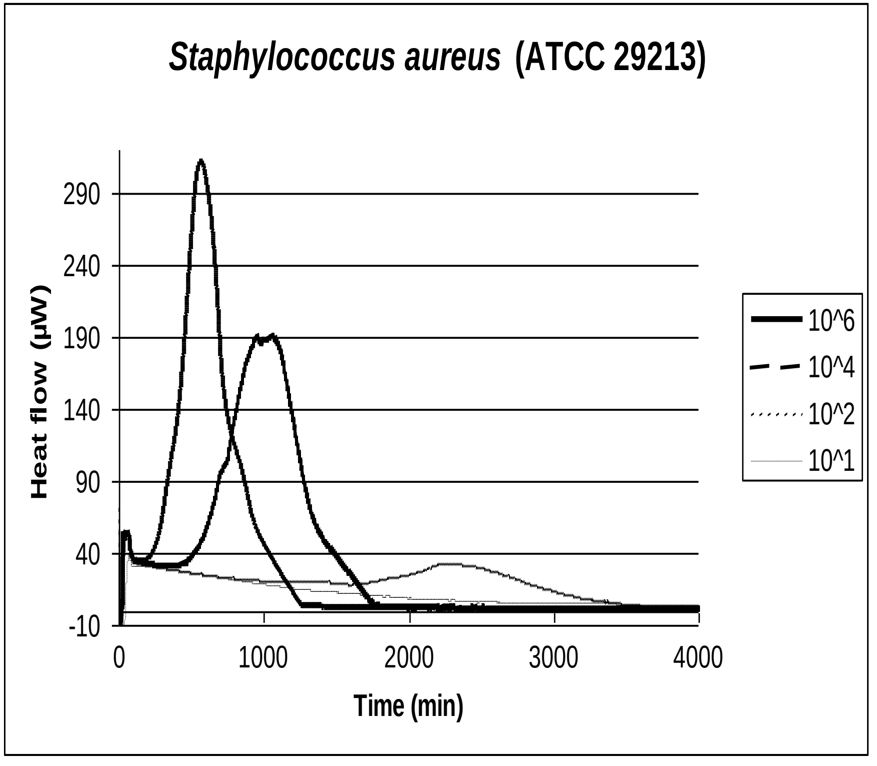
$$\dot{n} = -Bnn_s \qquad (18)$$

(13,18)→
$$\ddot{n} - \frac{\dot{n}^2}{n} + Bn\dot{n} = 0$$

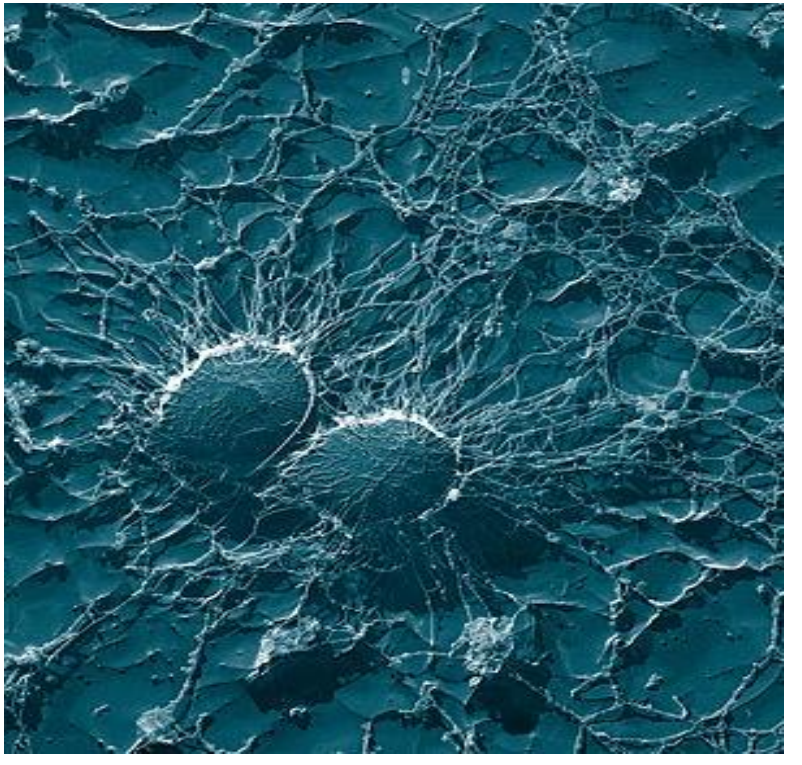
$$\ddot{n} - \frac{\dot{n}_s{}^2}{n_s} - An_s\dot{n}_s = 0$$

$$ODEs : n = n(t),\ n_s = n_s(t)$$

Bacterial Growth Prozess Model I (Monod)



Staphylococcus aureus
(ATCC29213)



$$Size: \quad 50.000:1$$

$$Diameter: \quad (0,8-1,2)\mu m$$

$$Density: \simeq 0,8 \; g/cm^3$$

Bacterial Growth Prozess Model I (Monod)

Parameter determination from heat power-curves: Staphylococcus aureus

$$z_1 : \ t_1, \dot{Q}_1 \qquad (bt_1)^{\alpha} << 1$$

$$z_2 : \ t_2, \dot{Q}_2 \qquad (bt_2)^{\alpha} << 1$$

(5)→ $$\alpha = 1 + \ln\left(\frac{\dot{Q}_1}{\dot{Q}_2}\right) / \ln\left(\frac{t_1}{t_2}\right)$$

Maximum heat power: $\left(t_{\max}, \dot{Q}_{\max}\right)$

(9)→ $$b = \frac{1}{t_{\max}}(\alpha - 1)^{\frac{1}{\alpha}} = \ldots$$

(10)→ $$\dot{Q}_{\max} = K\left(n_{\infty} - n_0\right) b (\alpha - 1)^{1-\frac{1}{\alpha}} / \alpha$$

$$Q_{\infty} = K\left(n_{\infty} - n_0\right) = \ldots$$

CFU	$n_0^1=10^6$	$n_0^2=10^4$
α/1	3,789	2,905
b/min^{-1}	$2{,}63\cdot10^{-3}$	$1{,}24\cdot10^{-3}$
b^{-1}/min	380	806
Q_∞/J	12,63	17,45
n_∞	$3{,}6\cdot10^{6}$	$3{,}6\cdot10^{6}$

CFU= Colony forming units / bacterien

Staphylococcus aureus

Population Dynamiks, Heat Production*)

$$n(t) = n_0 + (n_\infty - n_0)\frac{(bt)^\alpha}{1+(bt)^\alpha}$$

$$\dot{Q} = K\dot{n}$$

$$\dot{Q} = K\left(n_\infty - n_0\right)\alpha b\frac{(bt)^{\alpha-1}}{\left(1+(bt)^\alpha\right)^2}$$

*) Caloric measurements: Trampuz et.al., Basel

	Unit	1	2
Initial number of bacteria	CFU	10^6	10^4
Max. heat production	µW	310	190
Time at max. production	min	500	1000
Total heat generated	J	12,6	17,5
Final number bacteria	CFU	3,6• 10^6	3,6• 10^6
Time at half prod.	min.	400	800

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