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„COST OF CAPITAL AND VALUATION WITH IMPERFECT DIVERSIFICATION AND UNSYSTEMATIC RISKS"

COST OF CAPITAL AND VALUATION WITH IMPERFECT DIVERSIFICATION AND UNSYSTEMATIC RISKS

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I Introduction and Statement of the Problem

An essential challenge in determining enterprise values lies in defining appropriate (risk adequate) discount rates, which in the case of an unlevered firm can be interpreted as an approximation to equity capital cost. In spite of numerous restrictive assumptions and unsatisfactory empirical results the Capital Asset Pricing Model (CAPM) will continue to be the dominating method for determining the cost of equity capital used in practice for the foreseeable future. According to the (unconditional) CAPM the following equation holds:

(1) $$r_E = r_f + \beta_A \cdot \left(r_M^e - r_f \right)$$

whereby

(2) $$\beta_A = \frac{Cov(r_A, r_M)}{\sigma_M^2} = \frac{\rho_{AM} \cdot \sigma_A}{\sigma_M}$$

with

r_E	Expected return from a risk-carrying company (cost of equity capital)
r_f	Return from a risk-free investment (government bond)
r_M^e	Expected value of market return, i.e. of a portfolio of all risk-carrying investment possibilities ("market portfolio")
β_A	Systematic risk of equity capital of investment A
$Cov(r_A, r_M)$	Covariance between return from investment A and market portfolio
σ_M	Standard deviation of market return
σ_A	Standard deviation of return from investment A
ρ_{AM}	Correlation between return from investment A and market portfolio.

Especially with the evaluation of non-listed (medium-sized) companies, the following problems and significant restrictions pertaining to the applicability of the CAPM must be taken into account when determining cost of capital.

1. Homogeneity of expectations and planning consistency. Given the reality that information is distributed asymmetrically, how should the individual state of information be taken into account when determining (subjective) decision values? [3] And how should capital cost be kept consistent with the risks that are explicitly or implicitly taken into account when revenues (cash flows) are being planned?
2. Diversification. How should non-diversified (idiosyncratic) risks in capital cost and evaluation be taken into account when the evaluator does not have a perfectly diversified portfolio (and also cannot obtain one)?
3. Risk measures. When determining cost of capital and company value, what are the consequences of applying measures of risk other than the beta factor and standard deviations of the CAPM because in an imperfect capital market (a) creditors are subject to financing restrictions and/or (b) the evaluator is applying a safety first approach and thus wants to limit the extent of downside risks such as the probability of insolvency[4]?

[3] see *Matschke/Brösel*, Unternehmensbewertung: Funktionen – Methoden – Grundsätze, 2005. (Company Evaluation: Functions – Methods – Principles)

[4] This can definitely be economically rational with an imperfectly diversified portfolio (see 2), such as is generally found at medium-sized companies, even in the sense of expected utility theory. For example, cf *Ker-*

This paper deals with all of these aspects, whereby, for the sake of simplicity, only a 1-period-model (without taxes) is examined. Chapter II shows how the expected values of returns and the risk measure are derived consistent to planning from the company's (i.e. the evaluator's) information regarding the uncertain returns to be evaluated. Chapter III then shows how a replication model can be used to take imperfect diversification into account. The equity capital cost rate is calculated for an arbitrary degree of diversification and it is shown how a company's capital cost can be derived from the probability distribution of its uncertain returns (earnings), rather than from its returns expressed as a relative change in value. It is then shown how this approach can be generalised by utilizing risk measures other than standard deviations and the beta factor based on them. Such risk measures are helpful with non-normal distributions of returns.

II Subjective State of Information and Capital Cost consistent with Planning

The value $V\left(\widetilde{CF}\right)$, as also the subjective use of a returns series $\widetilde{CF}$ (company value), is dependent on the probability distribution and the risk that this distribution implicitly describes. One can take risks into account by means of an interest surcharge on the interest of a risk-free investment $\left(r_f\right)$ in the discount rate, or by means of a deduction for risk $\left(\pi = \lambda_{CE} \times R\left(\widetilde{CF}\right)\right)$ from the expected value of the returns series $\mathrm{E}\left(\widetilde{\mathrm{CF}}\right)$.[5]

With the deduction for risk, certainty equivalents $CE\left(\widetilde{CF}\right)$ are calculated. Certainty equivalents are to be discounted with the risk-free interest rate (base rate).

(3) $$V\left(\widetilde{CF}\right) = \frac{E\left(\widetilde{CF}\right)}{1+r_f+r_s} = \frac{E\left(\widetilde{CF}\right)}{1+r_f+\lambda_{RS}\cdot R\left(\widetilde{CF}'\right)} = \frac{CE\left(\widetilde{CF}\right)}{1+r_f} = \frac{E\left(\widetilde{CF}\right)-\lambda_{CE}\cdot R\left(\widetilde{CF}\right)}{1+r_f}$$

With the so-called "risk surcharge method", which dominates in practice, the risk-free interest rate (r_f) is increased by a risk surcharge (r_s) when the value of the returns series $(\widetilde{CF})$ is determined. The risk surcharge can be described as a product of the risk set, measured by a suitable risk measure $R\left(\widetilde{CF}'\right)$[6], and the price per unit risk λ_{RS}.

ins/Smith/Smith, Opportunity Cost of Capital for Venture Capital Investors and Entrepreneurs, JoFQA 2/2004 pp. 385-405.

[5] $CE\left(\widetilde{CF}\right) = E\left(\widetilde{CF}\right) - \pi$ and further to such risk value models *Sarin/Weber*, Risk-Value Models, EJoOR 70/1993 pp. 135-149.

[6] $R(\widetilde{CF}')$ is a risk measure that is standardised to the amounts of the returns, for example as operationalised by the expected value or the value. It is to be interpreted as a risk measure for a returns distribution.

$R(\widetilde{CF}') = \dfrac{R\left(\widetilde{CF}\right)}{V\left(\widetilde{CF}\right)}$ implies that $\lambda_{RS} = \lambda_{CE}$.

With the CAPM as in equation (1), it follows that

(4) $r_s = \beta_A \cdot \left(r_M^e - r_f \right)$

and β can therefore be interpreted in the CAPM[7] as the risk set $R\left(\widetilde{CF}'\right)$ and the difference between the market return and the risk-free interest as the market price of the risk λ_{RS}.

In principle, any evaluation that takes account of risk, i.e. determination of a value higher than that yielded by the certainty equivalents principle in dependence on the individual utility functions, is possible. In practice, however, most of the time λ_{CE} is taken as a market price of the risk (risk premium) of capital market data. The risk measure $R\left(\widetilde{CF}\right)$ can likewise be determined on the basis of (historical) capital market data. This, for example, is the route most frequently taken in practice with CAPM, with the beta factor as the risk measure. However, when determining (subjective) decision values it is advantageous to use the superior data of a risk analysis (e.g. with a due diligence), which leads to a description of the returns to be evaluated by means of a suitable stochastic process. The evaluators have an information advantage ("insider information") with respect to the capital market, which should consequently – and consistent to planning - be utilized by rational evaluators in the course of the evaluation. Specifically, this means that capital cost rates have to be derived on the basis of internal planning- and risk information (e.g. over the aggregated overall risk exposure). This is mandatory when there is no capital market data at all, as is the case with non-listed companies.

Risk information that shows the causes and extent of possible deviations from plan plays a key role as private information of the planner or evaluator. Here, building up on the identified and evaluated risks, the evaluation relevant "overall risk exposure", which is captured by the risk measure, is determined with the aid of aggregation, in the context of the planning[8]. When this is done, the risks, which can be systematic or non-diversified, unsystematic risks, and their stochastic interdependencies, such as, for example, correlations, are integrated into the company planning based on the evaluation and, by means of a simulation, a representative random sample of risk-related possible future scenarios of the company is calculated.

The established realisations of the target variable $\widetilde{CF}$ (e.g. profit) give rise to aggregated frequency distributions[9] that allow conclusions pertaining to the extent of risk-related losses. In this manner, arbitrary risk measures $R\left(\widetilde{CF}\right)$ can be calculated and, for example, the equity

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[7] The CAPM certainty equivalent is $V(\widetilde{CF}) = \dfrac{E(\widetilde{CF}) - \dfrac{\rho_{AM}\sigma_A}{\sigma_M}\left(r_M^e - r_f\right)}{1 + r_f}$, see *Spremann*, Valuation: Grundlagen moderner Unternehmensbewertung, 2004 (Valuation: Fundamentals of Modern Company Evaluation).

[8] Regarding methodology, for the utilization of Monte-Carlo simulations for evaluations with capital market imperfections see: *Gleissner*, Kapitalkosten - der Schwachpunkt bei der Unternehmensbewertung, (Capital Cost – the Weak Spot with Evaluating Companies) FB 4/2005 pp. 217-229 as well as *Gleissner*, Grundlagen des Risikomanagements im Unternehmen, Vahlen, 2008 (Fundamentals of Risk Management in a Company), see also *Gleissner*, Value-Based Corporate Risk Management, in: Frenkel/Hommel/Rudolf (Ed.) Risk Management: Challenge and Opportunity, Axel Springer, 2004, pp. 479-494.

[9] These are described by a large number of calculated individual scenarios. Here, in contrast to capital market equilibrium models for perfect markets (e.g. CAPM), it is systematic risks and not diversified unsystematic risks that are relevant. This is to be explained, for example, by the cost of bankruptcy. Cf *Froot/Scharfstein/Stein*, A Framework for Risk Management, Harvard Business Review, Nov.-Dec. 1994, pp. 91-102.

capital requirements for covering risk so that a predefined, goal rating dependent insolvency probability (p) is not exceeded is derived. $E\left(\widetilde{CF}\right)$ and $R\left(\widetilde{CF}\right)$ are therefore evidently consistently derived from the probability distribution $\widetilde{CF}$, whereby diversification advantages and other capital assets or the market portfolio can be taken into account (cf chapter III.1).

The possibility of modelling almost all probability distributions and inter-temporal dependencies of multi-periodic returns series (e.g. autoregressive processes or even GARCH-models) is the big advantage when one carries out such a simulation-based evaluation. In this case, the typical restriction to martingale processes is not required.[10]

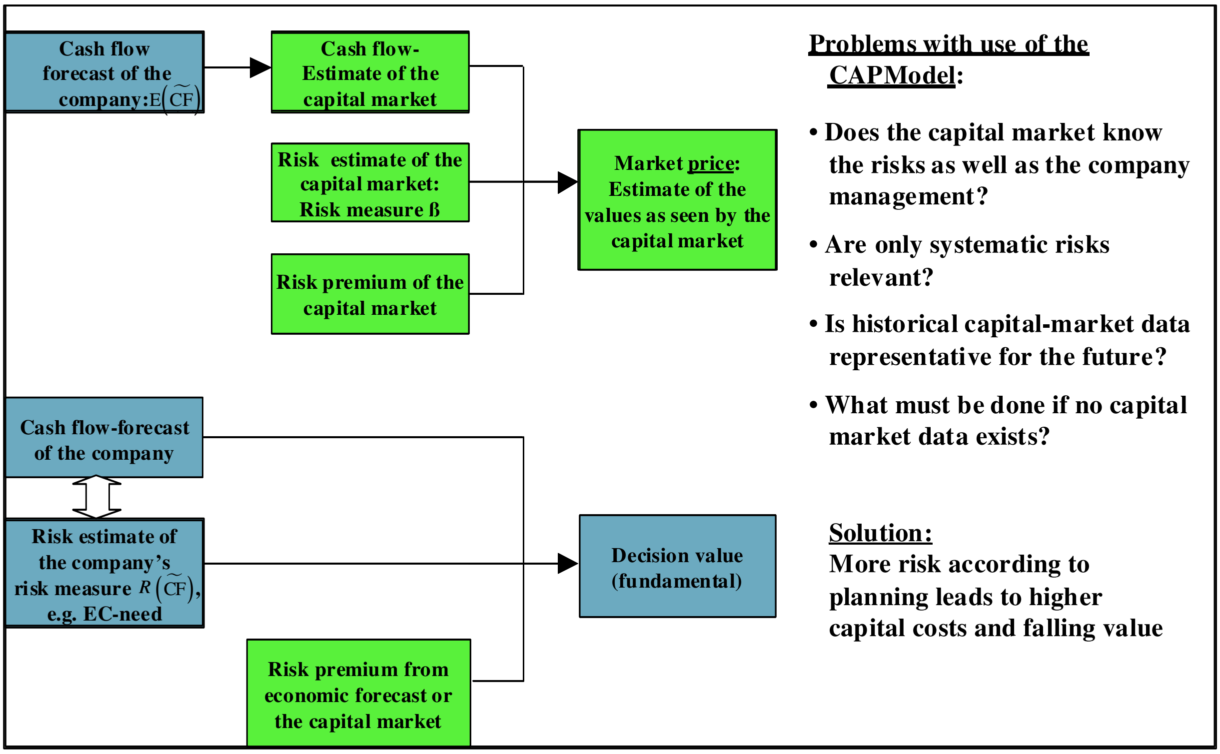

Figure 1: Capital market-oriented evaluation vs. simulation-based evaluation consistent with planning

[10] Cf *Gleissner/Wolfrum*, Simulationsbasierte Bewertung und Exit Preis-Schätzung bei PE-Gesellschaften, M&A Review, FACHVERLAG der Verlagsgruppe Handelsblatt, Düsseldorf, 7/2008, pp. 343-350 (Simulation-based Evaluation and Exit Price Estimation with PE-companies, M&A Review, PUBLISHING HOUSE of the publishing group Handelsblatt, Düsseldorf, 7/2008), as well as *Gleissner/Kamaras/Wolfrum*, Simulationsbasierte Bewertung von Akquisitionszielen und Beteiligungen, in: Gleissner/Schaller (Hrsg.), Private Equity – Beurteilungs- und Bewertungsverfahren von Kapitalbeteiligungsgesellschaften, Weinheim, 2008. (Simulation-based Evaluation of Acquisition Objectives and Participations, in: Gleissner/Schaller (editors), Private Equity – Appraisal and Evaluation Procedures of Equity Investment Companies.)

III Calculation of Cost of Capital with Imperfect Diversification

This section shows the way in which imperfect diversification is relevant for the cost of capital and value and also explains how these can be determined.

III.1 A General Replication Approach for Determining Risk-Adjusted Cost of Equity Capital

III.1.a Uncertain Returns as the Starting Point for Calculating Cost of Capital

The starting point for the one-period observation[11] is uncertain returns that are expected in the future $\widetilde{CF}_A$.[12] In order to determine the value of these returns, a replication that preserves expected values and is adequate for the risks is carried out. For this purpose, two investment possibilities have to be present: an investment in the market portfolio with an uncertain return $\tilde{r}_M$ and a risk-free investment with interest rate r_f. Now select the amounts x and y of investment capital so that the risk of investing x in the market portfolio and investing y risk-free with interest rate r_f equals the risk of the uncertain return $\widetilde{CF}_A$, whereby risk is measured by an appropriate risk measure $R(\widetilde{CF}_A)$. For example, such a risk measure could be the standard deviation, the value at risk, the conditional value at risk, or even an LPM-measure.

$$R\left(\widetilde{CF}_A\right) = R\left(x\cdot\left(1+\tilde{r}_M\right) + y\cdot\left(1+r_f\right)\right) \tag{5}$$

The replication should preserve expected values, which means that the expected value of the repayments of the investment into the market portfolio and into the risk-free investment should equal the expected value $E\left(\widetilde{CF}_A\right)$ of the returns series $\widetilde{CF}_A$. This is achieved by investing capital amount y in the risk-free investment.

[11] For an extension of a simple replication approach to several periods, see *Spremann*, Valuation: Grundlagen moderner Unternehmensbewertung, 2004, pp. 277 ff. (Valuation: Fundamentals of Modern Company Evaluation).

[12] This returns series not only characterises the return flows from the operational business in the period $\widetilde{CF}_{A,t=1}$ under consideration, it also contains the value (or attainable price) of the company at the end of period $V_1\left(\widetilde{CF}_{A,t=1}\right)$. If only one period is actually being considered, then this value will be zero. If, however, the investment being considered is still recoverable after the period being considered, then one assumes that it will, or can, be sold at the end of the period. This can make it necessary to distinguish between the fundamental value, for the determination of which a recursive evaluation is necessary so as to guarantee consistency with planning, and the attainable sales price. The latter can, for example, be estimated with the aid of a multiplier model in that a perpetual annuity is determined on the basis of the (uncertain) value of the operational returns and has discounting interest $\tilde{i}_{t=1}$ that is derived from capital market data and is thus likewise uncertain from today's point of view.

$$V_1\left(\widetilde{CF}_{A,t=1}\right) = \frac{\widetilde{CF}_{A,t=1}}{\tilde{i}_{t=1}} = \tilde{m}\cdot\widetilde{CF}_{A,t=1}, \qquad \text{so}$$

$$\widetilde{CF}_A = \widetilde{CF}_{A,t=1} + W_1\left(\widetilde{CF}_{A,t=1}\right) = \widetilde{CF}_{A,t=1} + \tilde{m}\cdot\widetilde{CF}_{A,t=1} = \widetilde{CF}_{A,t=1}\cdot\left(1+\tilde{m}\right).$$

(6) $E\left(\widetilde{CF}_A\right)=E\left(x\cdot\left(1+\tilde{r}_M\right)\right)+E\left(y\cdot\left(1+r_f\right)\right)=x\cdot\left(1+E\left(\tilde{r}_M\right)\right)+y\cdot\left(1+r_f\right)$

With arbitrage freedom, the value of the uncertain returns series $\widetilde{CF}_A$ is the sum of the investments x and y.

(7) $V\left(\widetilde{CF}_A\right)=x+y$

The next steps require additional knowledge of the risk measure in use. If the standard deviation is taken as the risk measure[13], then the value of the returns series $\widetilde{CF}_A$ [14] is

(8) $$V_0\left(\tilde{Z}_A\right)=x+y=\frac{E\left(\widetilde{CF}_A\right)-\frac{\sigma\left(\widetilde{CF}_A\right)}{\sigma\left(\tilde{r}_M\right)}\cdot\left(E\left(\tilde{r}_M\right)-r_f\right)}{\left(1+r_f\right)}$$

Then the weighted average cost of capital expressed as expected return from the uncertain investment A turns out to be[15]

(9) $$r_E=r_A^e=\frac{E(\widetilde{CF}_A)}{W_0(\widetilde{CF}_A)}-1=r_f+\frac{\frac{\sigma\left(\widetilde{CF}_A\right)}{W_0\left(\widetilde{CF}_A\right)}}{\sigma\left(\tilde{r}_M\right)}\left(E\left(\tilde{r}_M\right)-r_f\right)=$$
$$=r_f+\frac{\sigma\left(\widetilde{CF}_A\right)\left(1+r_f\right)}{E\left(\widetilde{CF}_A\right)\sigma\left(\tilde{r}_M\right)-\sigma\left(\widetilde{CF}_A\right)\cdot\left(E\left(\tilde{r}_M\right)-r_f\right)}\left(E\left(\tilde{r}_M\right)-r_f\right)$$

Here, the return of investment A and the market portfolio are assumed to be completely correlated, so that $\rho_{AM}=1$, which says that the company in question is its only asset, whence diversification effects can be neglected.

III.1.b Extension: Consideration of Correlations

In general, however, the assumption $\rho_{AM}=1$ will not apply and thus diversification options will be available. This is to be taken into account when evaluating the uncertain returns. In this regard, though, only the non-diversifiable share of the risk (the systematic risks) of the returns is relevant for the evaluation inasmuch as unsystematic risks are assumed to be diversifiable within the context of the (sufficiently large) total portfolio.[16]

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[13] The following equation holds in general for standard deviation: $\sigma\left(a\cdot\widetilde{CF}_A+b\right)=a\cdot\sigma\left(\widetilde{CF}_A\right)$.

[14] For the derivation see *Gleissner/Wolfrum*, Assessing the Equity Capital Cost of Non-Listed Companies, in: Finanz Betrieb 9/2008, pp. 602-614.

[15] If the value of the returns series for capital asset A is zero, then the expected return is infinite.

[16] Cf *Spremann*, Valuation: Grundlagen moderner Unternehmensbewertung, 2004 (Valuation: Fundamentals of Modern Company Assessment)

Normally, the correlation coefficient (of Bravais and Peason) is used to measure the stochastic dependence between two risks (or the underlying distributions)[17]. Let ρ_{AM} denote the correlation coefficient between the returns $\widetilde{CF}_A$ from the capital asset A and the market portfolio M. Then the standard deviation of the returns series $\widetilde{CF}_A$ is reduced in its relevance for the evaluation by being multiplied by ρ_{AM} [18] so that the resulting value of the returns series $\widetilde{CF}_A$ is

$$(10)\quad V_0\left(\widetilde{CF}_A\right)=\frac{E\left(\widetilde{CF}_A\right)-\frac{\rho_{AM}\sigma(\widetilde{CF}_A)}{\sigma(\tilde{r}_M)}\cdot\left(E(\tilde{r}_M)-r_f\right)}{\left(1+r_f\right)}$$

Accordingly, the weighted average cost of capital is

$$(11)\quad \begin{aligned} r_E = r_A^e = \frac{E(\widetilde{CF}_A)}{V_0(\widetilde{CF}_A)}-1 &= r_f+\frac{\frac{\rho_{AM}\sigma\left(\widetilde{CF}_A\right)}{V\left(\widetilde{CF}_A\right)}}{\sigma\left(\tilde{r}_M\right)}\left(E\left(\tilde{r}_M\right)-r_f\right)= \\ &= r_f+\frac{\rho_{AM}\sigma\left(\widetilde{CF}_A\right)\left(1+r_f\right)}{E\left(\widetilde{CF}_A\right)\sigma\left(\tilde{r}_M\right)-\rho_{AM}\sigma(\widetilde{CF}_A)\cdot\left(E\left(\tilde{r}_M\right)-r_f\right)}\left(E\left(\tilde{r}_M\right)-r_f\right)\end{aligned}$$

If the returns from the investment A and the market return (stochastic) are independent of each other, so that $\rho_{AM} = 0$, then the equity capital cost rate is, as expected, identical with the risk-free interest rate, inasmuch as all risks are then diversifiable and thus assumed not to be relevant for the evaluation.

Comparison of the evaluation equation (10) with the CAPM's certainty equivalent equation (see footnote 7) shows that these are only in agreement under the following conditions:

- The risk measure used is the standard deviation
- Only the non-diversified risks are evaluated.
- The state of information is symmetric, i.e. the returns evaluated are assumed to be assessed by the capital market in the same way as by company planning ($\sigma(\widetilde{CF}_A)=\sigma_A$).

III.1.c Extension: Portfolio Examination and Imperfect Diversification

Evaluation of the uncertain returns series $\widetilde{CF}_A$ was undertaken in Chapter III.1a "for its own sake". Thus it was assumed that the investor holds no other assets in his portfolio or that the amount of risk relevant to evaluating the returns in question is not influenced by the remainder of his portfolio. On the contrary, chapter III.1.b assumes that the remaining portfolio (market portfolio investment) is so large that only systematic risks are relevant for evaluation.

[17] The problem with using this correlation coefficient is that it only captures linear stochastic dependencies. The rank correlation coefficient, by comparison, only captures monotonic dependencies. Theoretically, copulae, on the other hand, capture nearly all dependencies.

[18] This is evident from the equation of the beta factor $\beta_A = \frac{\rho_{AM}\cdot\sigma_A}{\sigma_M}$.

Here, we assume that an investor has just invested a part of his own assets in his own company and that he also has an arbitrarily large portfolio with value P_0. For the sake of simplicity, this portfolio is assumed to be composed of one investment in the market portfolio (share in portfolio a_M) and one risk-free investment (share in portfolio $1-a_M$).

Thus the expected value of the investor's portfolio (P') at the end of the period in question is

$$E(P') = E\left(P_0 a_M (1+\tilde{r}_M) + P_0(1-a_M)(1+r_f) + \widetilde{CF}_A\right) = P_0 a_M E(1+\tilde{r}_M) + P_0(1-a_M)(1+r_f) + E\left(\widetilde{CF}_A\right) \tag{12}$$

Taking the standard deviation as the risk measure leads to

$$\sigma(P') = \sigma\left(P_0 a_M (1+\tilde{r}_M) + P_0(1-a_M)(1+r_f) + \widetilde{CF}_A\right) = \sigma\left(P_0 a_M (1+\tilde{r}_M) + \widetilde{CF}_A\right) \tag{13}$$

Let ρ_{AM} denote the correlation coefficient between the returns series $\tilde{Z}_A$ and the market portfolio M. Then the resulting standard deviation is

$$\sigma(P') = \sqrt{P_0^2 a_M^2 \sigma^2(\tilde{r}_M) + 2\rho_{AM} P_0 a_M \sigma(\tilde{r}_M)\sigma\left(\widetilde{CF}_A\right) + \sigma^2\left(\widetilde{CF}_A\right)} \tag{14}$$

Now let the portfolio be evaluated by means of replication once again. The value of the complete portfolio results in[19]

$$\begin{aligned} V_0(P') &= \frac{E(P') - \dfrac{\sigma(P')}{\sigma(\tilde{r}_M)}\left(E(\tilde{r}_M) - r_f\right)}{1+r_f} = \\ &= \frac{E(P') - \dfrac{\sqrt{P_0^2 a_M^2 \sigma^2(\tilde{r}_M) + 2\rho_{AM} P_0 a_M \sigma(\tilde{r}_M)\sigma\left(\widetilde{CF}_A\right) + \sigma^2\left(\widetilde{CF}_A\right)}}{\sigma(\tilde{r}_M)}\left(E(\tilde{r}_M) - r_f\right)}{1+r_f} \end{aligned} \tag{15}$$

The diversification advantage in this portfolio is now treated as part of the returns from the company. Diversification has a risk-lowering effect and thus increases value; therefore, value additivity does not apply. Thus the value of the company turns out to be:

$$V_0\left(\widetilde{CF}_A\right) = \frac{E\left(\widetilde{CF}_A\right) - \left(\dfrac{\sqrt{P_0^2 a_M^2 \sigma^2(\tilde{r}_M) + 2\rho_{AM} P_0 a_M \sigma(\tilde{r}_M)\sigma\left(\widetilde{CF}_A\right) + \sigma^2\left(\widetilde{CF}_A\right)}}{\sigma(\tilde{r}_M)} - P_0 a_M\right)\left(E(\tilde{r}_M) - r_f\right)}{1+r_f} \tag{16}$$

Accordingly, the weighted average cost of capital is

$$\begin{aligned} r_E = r_A^e &= \frac{E(\widetilde{CF}_A)}{V_0(\widetilde{CF}_A)} - 1 = \\ &= r_f + \frac{\left(\sqrt{P_0^2 a_M^2 \sigma^2(\tilde{r}_M) + 2\rho_{AM} P_0 a_M \sigma(\tilde{r}_M)\sigma\left(\widetilde{CF}_A\right) + \sigma^2\left(\widetilde{CF}_A\right)} - P_0 a_M \sigma(\tilde{r}_M)\right)(1+r_f)}{E\left(\widetilde{CF}_A\right)\sigma(\tilde{r}_M) - \left(\sqrt{P_0^2 a_M^2 \sigma^2(\tilde{r}_M) + 2\rho_{AM} P_0 a_M \sigma(\tilde{r}_M)\sigma\left(\widetilde{CF}_A\right) + \sigma^2\left(\widetilde{CF}_A\right)} - P_0 a_M \sigma(\tilde{r}_M)\right)\left(E(\tilde{r}_M) - r_f\right)}\left(E(\tilde{r}_M) - r_f\right) \end{aligned} \tag{17}$$

Let us now consider an investor who has invested capital P_0 of 1.000 altogether, placing one half of it ($a_M = 0.5$) in the market portfolio and the other half in a risk-free investment. Let the market return have the normal distribution with an expected value $E(\tilde{r}_M)$ of 9% and a stan-

[19] Cf *Gleissner/Wolfrum*, Die Ermittlung von Eigenkapitalkosten nicht börsenorientierter Unternehmen (Assessing the Equity Capital Cost of Non-Listed Companies), Finanz Betrieb 9/2008, pp. 602-614.

dard deviation $\sigma\left(\tilde{r}_M\right)$ of 30%. Assume further that the risk-free interest r_f is 5%. In addition, suppose this investor has a company of his own with an expected return of $E(\widetilde{CF}_A)$ = 1.000 at time t = 1[20] and a standard deviation $\sigma(\widetilde{CF}_A)$ of 300. Finally, let the correlation ρ_{AM} between return from the investor's company and market return be 0.75. Then the standard deviation of the investor's complete portfolio is calculated in accordance with (14) as

$$\sigma\left(P_0 a_M\left(1+\tilde{r}_M\right)+\widetilde{CF}_A\right)=\sqrt{1{,}000^2\cdot 0{,}75^2\cdot 0{,}3^2+2\cdot 0{,}5\cdot 1{,}000\cdot 0{,}75\cdot 0{,}3\cdot 300+300^2}=456$$

Thus the value of the returns series for the capital asset "company" is calculated at

$$W_0\left(\widetilde{CF}_A\right)=\frac{1{,}000-\left(\frac{456}{0{,}3}-1{,}000\cdot 0{,}75\right)(0{,}09-0{,}05)}{1+0{,}05}=923$$

Hence the weighted average cost of capital in this example is

$$r_E=r_A^e=\frac{1{,}000}{923}-1=8{,}3\%$$

Finally, the next section shows how this approach can be generalised to accommodate risk measures other than the beta factor and the standard deviation which it is based on here. The beta factor is only an appropriate measure of risk if a normal distribution can be assumed. In particular, standard deviation is not an appropriate measure of risk when returns series distributions are asymmetric or strongly arched because then the extent of risk may well, under certain circumstances, be significantly underestimated. For a risk to be evaluated, both possible positive as well as possible negative impacts have to be captured – whereby the latter have an even stronger influence on evaluation, according to psychological research, than is shown by the utility theory (see the prospect theory). Due to the special importance of possible losses, so-called "down-side risk measures", which are designed to capture the possible scope of negative deviations, are used in addition when overall risk exposure is being described. Two such risk measures are value at risk[21] and conditional value at risk[22]. Use of them is sensible when the risks are not symmetric and losses require special attention.

III.1.d Extension of the Model to Arbitrary Risk Measures

The replication equation for general risk measures $R\left(\widetilde{CF}_A\right)$ is given in Chapter III.3.a (equation (5)) as follows:

(18) $$R\left(\widetilde{CF}_A\right)=R\left(x\cdot\left(\tilde{r}_M-E\left(\tilde{r}_M\right)\right)+E\left(\widetilde{CF}_A\right)\right)$$

[20] This expected return can, for example, be estimated by means of a multiplier model (cf footnote 12). With $\widetilde{CF}_{A,t=1}$=100 and $\tilde{m}$=9 the following arises as a result: $\widetilde{CF}_A=\widetilde{CF}_{A,t=1}\cdot\left(1+\tilde{m}\right)$=1,000.

[21] The value at risk at confidence level α is defined as $P\left(\widetilde{CF}_A\geq VaR_\alpha(\widetilde{CF}_A)\right)=\alpha$, with $0<\alpha<1$.

[22] The conditional value at risk at confidence level α is defined as $CVaR_a(\widetilde{CF}_A)=-\mathrm{E}[\widetilde{CF}_A \mid \widetilde{CF}_A < -\mathrm{VaR}_a(\widetilde{CF}_A)]$.

If the risk measure is known, this equation can be solved for x and thus be evaluated. Here, however, one has to distinguish between position-dependent measures of risk, such as the value at risk and the conditional value at risk, and position-independent measures of risk, such as the standard deviation and the deviation value at risk. Since real world returns can frequently not be described by means of a normal- or lognormal distribution (e.g. due to fat tails), position-dependent risk measures are increasing in importance.

In the following, only position-independent risk measures (such as standard deviation and DVaR[23]) are treated in greater detail. Accordingly, let [24]

(19) $$R\left(a+b\widetilde{CF}_A\right)=bR\left(\widetilde{CF}_A\right).$$

Then equation (18) simplifies to

(20) $$R\left(\widetilde{CF}_A\right)=x\cdot R\left(\tilde{r}_M\right)$$

Hence this risk measure is position-independent and can be regarded as a measure of planning certainty or, equivalently, of the extent of possible plan deviations (from the expected value).

The value obtained after suitable transformations is

(21) $$W_0\left(\widetilde{CF}_A\right)=\frac{E\left(\widetilde{CF}_A\right)-\frac{R\left(\widetilde{CF}_A\right)}{R\left(\tilde{r}_M\right)}\left(E\left(\tilde{r}_M\right)-r_f\right)}{\left(1+r_f\right)}$$

Thus the weighted average cost of equity capital, expressed as the expected return of the entrepreneur, is

(22) $$\begin{aligned} r_E=r_A^e &= \frac{E(\widetilde{CF}_A)}{W_0(\widetilde{CF}_A)}-1=r_f+\frac{\frac{R\left(\widetilde{CF}_A\right)}{W(\widetilde{CF}_A)}}{R\left(\tilde{r}_M\right)}\left(E\left(\tilde{r}_M\right)-r_f\right)= \\ &= r_f+\frac{R\left(\widetilde{CF}_A\right)\left(1+r_f\right)}{E\left(\widetilde{CF}_A\right)R\left(\tilde{r}_M\right)-R\left(\widetilde{CF}_A\right)\left(E\left(\tilde{r}_M\right)-r_f\right)}\left(E\left(\tilde{r}_M\right)-r_f\right)\end{aligned}$$

IV Summary and Outlook

Altogether, it is shown that in an imperfect, incomplete market the cost of equity capital and values, especially of non-listed companies (or of individual business areas of listed companies) can be ascertained with the specific restrictions and the information state of the evaluator being taken into account. With the aid of a simple and robust replication model, we have shown how the company value (decision values) and the cost of capital (discount rate) can be computed. The computation starts with the probability distribution of the uncertain returns

[23] The deviation value at risk (or relative value at risk) is defined as $DVaR_\alpha\left(\widetilde{CF}_A\right)=E\left(\widetilde{CF}_A\right)+VaR_\alpha\left(\widetilde{CF}_A\right)$.

[24] See the risk measures axiom system of *Rockafellar/Uryasev/Zabarankin*, Deviation Measure in Risk Analysis and Optimization, Research Report, 2002.

series $\widetilde{CF}_A$ to be evaluated, which is consistently transformed to the expected value E($\widetilde{CF}_A$) and a risk measure R($\widetilde{CF}_A$) that need not be the standard deviation (but can also, for example, be the value at risk). This consistency between the expected value of returns (as derived from planning) and the risk measure is not given if the latter (as for example in practical utilization of the CAPM) is determined from capital market data but strict information efficiency in the sense of Fama (1970) is not assumed at the same time.[25]
This paper has also shown which consequences for value and cost of capital result from correlation of the cash flows to be evaluated with the evaluator's remaining asset positions (e.g. its investment of the market portfolio) and an imperfect diversification. Imperfect diversification is characteristic for the proprietors of numerous non-listed companies (especially in small and medium-sized businesses) and implies higher cost of capital and hence a higher risk-adjusted return as a result of the greater risks. These higher cost of the evaluator's capital are to be taken into account during evaluation, especially when determining subjective decision values, inasmuch as capital market imperfections generally make it impossible to realise perfectly diversified portfolios.

All in all, the approach proposed here shows how evaluation of companies that reflects risks appropriately is directly possible on the basis of company planning that creates transparency of risks while taking account of capital market imperfections.

[25] Cf *Fama*, Efficient Capital Markets: A Review of Theory and Empirical Work, JoF 25/1970 pp. 383-417.