



Volkswirtschaftliche Diskussionsbeiträge  
Discussion Papers in Economics

No. 185-18

April 2018

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Dioxide Ceiling**

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<http://www.wiwi.uni-siegen.de/vwl/>

ISSN 1869-0211

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Discussion Papers in Economics of the University of Siegen are indexed in RePEc  
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# Demand versus Supply Side Climate Policies with a Carbon Dioxide Ceiling

Thomas Eichner, Gilbert Kollenbach, Mark Schopf\*

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## Abstract

Consider a dynamic model with two countries or coalitions that consume and trade fossil fuel. A non-abating country owns the entire fuel stock and is not concerned about climate change, represented by a ceiling on the carbon dioxide concentration. The government of the other country implements public policies against global warming, either by capping domestic fuel consumption or by buying deposits to postpone their extraction. The demand [supply] side policy is inefficient because the consumers [suppliers] in the non-abating country do not internalize the climate externality. In particular, at the demand side policy aggregated fuel consumption is inefficiently low [high] in the climate coalition [non-abating country]. If strategic price incentives are strong, the coalition further depresses its fuel consumption to reduce the fuel price and hence its fuel import bill. At the deposit policy, the fossil fuel consumption and price paths are discontinuous when the ceiling becomes binding and the coalition takes over complete fuel supply. If strategic price incentives are strong, the coalition decreases its deposit purchases to reduce the fuel and the deposit price. If the coalition is the sole fuel supplier, it reduces its extraction to raise the fuel price in a monopolistic fashion.

Keywords: Demand Side Policy, Supply Side Policy, Climate Change, Deposit, Fossil Fuel

JEL Classification: F55, H23, Q54, Q58

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## 1. Introduction

In recent years, climate change and its economic consequences received a lot of attention. There is a broad political consensus that the global temperature should not rise by more than two degrees Celsius (UN, 2015). However, even if the parties that ratified the Paris Agreement would fully implement their nationally determined contributions, the temperature would rise by about three degrees Celsius (UN, 2017). Thus, one can doubt whether voluntary contributions to a global climate agreements can guarantee the international climate goals. Efforts to mitigate climate change are very different across countries. While the European Union committed to reduce greenhouse gas emissions by at least 40% by 2030 compared to 1990 levels, other countries' submitted targets that are less ambitious. It is disturbing that worldwide carbon emissions are still increasing. If voluntary contributions to climate agreements cannot stabilize the temperature at safe levels, it is worth thinking of appropriate unilateral policies to fight against global warming at manageable cost.

This paper analyzes two different unilateral climate policies, demand and supply side climate policies, to ensure that the carbon dioxide concentration stays below a critical level. According to the IPCC (2013, chapter 8.5 and 10.3), it is very likely that more than half of the global temperature increase between 1951 and 2010 is due to the increase in greenhouse gas concentrations, and it is very likely that carbon dioxide accounted for more than half of the radiative forcing of greenhouse gases between 1750 and 2011 (and between 1980 and 2011). Thus, a ceiling on the carbon dioxide concentration is consistent with both the two degree target and the UN's (1992) objective to stabilize the greenhouse gas concentrations "at a level that would prevent dangerous anthropogenic interference with the climate system".

To account for the dynamic nature of fossil fuel depletion and carbon dioxide accumulation, we apply a Hotelling model of resource extraction. We assume constant marginal extraction costs to focus on the development of the scarcity rent and its change due to the different unilateral climate policies.<sup>1</sup> A perfect renewable substitute guarantees that energy consumption continues when the fossil fuel stock becomes exhausted. We consider

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<sup>1</sup>This is a common assumption in the ceiling literature. See, e.g., Amigues et al. (2011), Amigues et al. (2014), Chakravorty et al. (2006), Chakravorty et al. (2008), Henriot (2012), Lafforgue et al. (2008; 2009) and Smulders and Van der Werf (2008).

a world of two (groups of) countries that consume and trade fossil fuel. Country  $B$  owns the entire fuel stock and is not concerned about climate change. Country  $A$ , also denoted as climate coalition, implements public policies against global warming.

There is a large literature that analyzes optimal demand side policies to adhere the ceiling in dynamic one-country models. Chakravorty et al. (2006) analyze the implications of increasing or decreasing energy demand over time on optimal abatement and renewable energy utilization. Chakravorty et al. (2008) address the optimal extraction composition of two polluting nonrenewable resources and find that this composition can change several times until the cleaner resource is exhausted. Chakravorty et al. (2012) find that optimal energy prices can decline over time at the ceiling and in the long run if there is learning-by-doing in the renewable energy sector. Finally, Henriet (2012) analyzes the optimal date of backstop invention. Hoel (2011) is the only paper that considers carbon taxation in a dynamic two-country model without a ceiling on the carbon dioxide concentration.

The literature studying unilateral supply side policies is quite small. Harstad (2012) and Eichner and Pethig (2017a; 2017b) analyze the policy of purchasing deposits for preservation and extraction. Harstad (2012)'s deposit policy implements first-best by assuming Coasian bargaining on the deposit market, which removes trade and, thus, strategic incentives on the fuel market. To eliminate strategic incentives not only deposits for preservation but also deposits for extraction are traded. Efficiency is violated if the Coasean bargaining is replaced by deposit trade at a uniform price (Eichner and Pethig 2017b), and efficiency can be violated if deposits are only purchased for preservation but not for extraction (Eichner and Pethig 2017a). The analyses of supply side policies are carried out in static multi-country models (without any ceiling). To the best of our knowledge, our paper is the first that investigates demand side policies and supply side policies in a dynamic two-country Hotelling model with ceiling on the carbon dioxide concentration.

If the climate coalition applies a demand side policy by capping domestic fuel consumption, the climate externality is not internalized abroad. Consequently, aggregated fuel consumption of the non-abating country is higher and of the abating country lower than in the social optimum. If the coalition acts strategically on the fuel market, on the one hand it has a strategic incentive to reduce fuel consumption to depress the price, and on the other hand it has an incentive to increase fuel consumption to cope with emissions

leakage to the non-abating country. However, to adhere the ceiling the coalition's fuel consumption is lower than the other countries' fuel consumption. If the coalition behaves as price taker on the fuel market and the carbon dioxide regeneration rate is sufficiently small, then the coalition's fuel consumption is inefficiently low and the non-abating country's fuel consumption is inefficiently high until the ceiling becomes binding.

Next, we analyze the effects of buying deposits to postpone their extraction. In contrast to Harstad (2012) and Eichner and Pethig (2017a; 2017b), whose fuel deposits are heterogeneous and economically exhausted, we assume homogenous fuel deposits and physical exhaustion. Thus, buying deposits changes fuel supply by influencing the scarcity rent (and not by influencing the extraction cost structure). In Harstad (2012) and Eichner and Pethig (2017a; 2017b) the climate coalition purchases deposits for preservation to reduce the coalition's climate damage. In our model, the climate coalition must buy deposits to ensure that the carbon dioxide concentration stays below the critical level. However, at some point in time, the climate coalition owns the entire fuel stock that is left. Since the carbon dioxide concentration decays over time, it cannot be optimal to leave some of the deposits under the ground forever. Thus, the climate coalition becomes the only supplier on the fuel market from some moment on.

However, as long as the firms in the non-abating country are suppliers on the fuel market, the climate externality is not internalized in the fuel price, such that the supply side policy cannot implement the social optimum. In particular, the fossil fuel consumption and price paths exhibit a jump in the moment the ceiling becomes binding, if the coalition always acts as price taker. It turns out that the date of exhaustion coincides with the point of time at which the ceiling is binding. If the coalition behaves as price taker on the fuel and the deposit market, extraction is inefficiently high until the ceiling becomes binding and inefficiently low if the ceiling is not binding any more. If the coalition acts strategically on the fuel and deposit market, it faces opposing incentives. On the one hand, it faces strategic incentives to reduce its deposit purchases to lower the fuel and deposit price. On the other hand, the coalition can decelerate emission accumulation by increasing its deposit acquisition. When the former incentive overcompensates the latter, extraction is expanded at earlier points of time. After the non-abating country has sold its fuel stock, the coalition that is now the sole supplier has an incentive to raise the fuel price by reducing its extraction by analogy to the behavior of a monopolist.

The remainder of the paper is organized as follows: Section 2 outlines the model. Section 3 characterizes the social optimum. Section 4 analyzes the effects of the demand side policy. Section 5 investigates those of the supply side policy. Section 6 concludes.

## 2. The model

Consider an economy with two (groups of) countries,  $A$  and  $B$ . Country  $A$  is the climate coalition and country  $B$  a free rider. The representative consumer of country  $i = A, B$  derives instantaneous utility  $U(b_i(t) + x_i(t))$  from consuming  $b_i(t) + x_i(t)$  units of energy. The utility function is strictly increasing and strictly concave ( $U' > 0$ ,  $U'' < 0$ ). Energy is generated from fossil fuel and a renewable (backstop) such as solar energy. At each point in time, the consumption of fossil fuel and backstop in country  $i = A, B$  is denoted by  $x_i(t)$  and  $b_i(t)$ , respectively. Both kinds of energy are perfect substitutes.

The finite fossil fuel endowment is given by  $S(0)$  and is completely owned by a representative firm located in country  $B$ .<sup>2</sup> The evolution of the fossil fuel stock over time is given by<sup>3</sup>

$$\dot{S} = -s. \quad (1)$$

The production of energy from fossil fuel exhibits constant marginal extraction costs  $c > 0$ . Burning fossil fuels unleashes CO<sub>2</sub> emissions, which accumulate in the atmosphere according to

$$\dot{Z} = s - \gamma Z. \quad (2)$$

In (2),  $Z$  denotes the emission stock,  $\gamma > 0$  a natural regeneration rate and  $Z(0) \geq 0$  the emission stock endowment.<sup>4</sup> The CO<sub>2</sub> accumulation gives rise to global warming. In line with the ongoing climate protection discussion, in particular the Paris Agreement, we assume that the damages of climate change are controllable, if the global temperature does not increase by more than 2°C above the preindustrial level. This climate target is reflected by a ceiling  $\bar{Z}$  on the emission stock, so that

$$\bar{Z} - Z(t) \geq 0 \quad (3)$$

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<sup>2</sup>Endowing country  $A$  with a fossil fuel stock would considerably complicate the analysis. Our qualitative results do not change as long as country  $A$ 's fossil fuel endowment is too low to comply with the ceiling by postponing its extraction.

<sup>3</sup>We use the notation  $\dot{z}$  to indicate the derivation of an arbitrary variable  $z$  with respect to time  $t$ , i.e.  $\dot{z} = \frac{\partial z}{\partial t}$ . The growth rate  $\frac{1}{z} \frac{\partial z}{\partial t}$  is denoted by  $\hat{z}$ . For sake of simplicity, the time index  $t$  is omitted whenever this does not lead to confusion.

<sup>4</sup>This equation of motion is widely used in the literature, e.g. by Chakravorty et al. (2006), Kollenbach (2015a) and Tsur and Zemel (2009).

must hold at every point in time.<sup>5</sup> To sharpen our focus, we follow Chakravorty et al. (2008) and Lafforgue et al. (2009) and neglect the damages from emission stocks below the ceiling.<sup>6</sup> In the sequel we divide the planning period  $[0, \infty)$  into different time phases that belong to the following classes.

**Definition 1.**

- i) Phase I: The ceiling is non-binding but will bind in the future.*
- ii) Phase II: The ceiling is binding.*
- iii) Phase III: The ceiling is non-binding and will not bind in the future.*

Each country  $i = A, B$  hosts a representative firm that supplies renewable energy. Energy generation from the backstop exhibits constant marginal extraction costs  $m$ . We assume that  $m$  is sufficiently large such that the backstop does not become economically usable before Phase III.

**3. The social optimum**

In this section we characterize as a benchmark the (constrained) social optimum.<sup>7</sup> The social planner maximizes intertemporal utility net of energy costs  $\int_0^\infty e^{-\rho t} [U(x_A + b_A) + U(x_B + b_B) - mb - cs] dt$  subject to the limited fossil fuel stock and the CO<sub>2</sub> ceiling, with  $b := b_A + b_B$  and  $\rho > 0$  as the time preference rate. The corresponding current-value Lagrangian reads

$$L = \sum_i U(x_i + b_i) - mb - (c + \tau)s - (\mu - \theta)(s - \gamma Z), \quad (4)$$

where  $\tau$  is the shadow price of the fossil fuel stock,  $\mu$  is the Lagrange multiplier associated with the ceiling, and  $\theta$  is the costate variable of the emission stock. From the first-order conditions we obtain<sup>8</sup>

$$U'_A = U'_B = c + \tau + (\mu - \theta) = m, \quad (5)$$

$$\tau(t) = \tau(0)e^{\rho t}, \quad (6)$$

$$\dot{\theta} = (\rho + \gamma)\theta - \mu\gamma, \quad (7)$$

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<sup>5</sup>Chakravorty et al. (2006), Chakravorty et al. (2008), Chakravorty et al. (2012) and Eichner and Pethig (2013) also refer to a ceiling negotiated in an international climate agreement. In the following, we assume that the ceiling is exogenously given. Thus, as Chakravorty et al. (2006), Chakravorty et al. (2008), Chakravorty et al. (2012), Lafforgue et al. (2009), Kollenbach (2015a) and Kollenbach (2015b), we are not going to analyze whether the ceiling is optimal or not.

<sup>6</sup>Amigues et al. (2011) and Dullieux et al. (2011) assume a damage function that reflects manageable damages from emission stocks below the ceiling.

<sup>7</sup>The social optimum is constrained because the social planner takes the ceiling as exogenously given, see also footnote 4.

<sup>8</sup>We use  $U'_i$  as a shortcut for  $U'(x_i + b_i)$ ,  $i = A, B$ .



with  $\tau(0)$  as the initial scarcity rent. The complementary slackness conditions are

$$\begin{aligned} \frac{\partial L}{\partial \mu} = -s + \gamma Z \geq 0, & \quad \mu \geq 0, & \quad \mu \frac{\partial L}{\partial \mu} = 0, \\ \bar{Z} - Z \geq 0, & & \quad \mu[\bar{Z} - Z] = 0, \\ \rho\mu - \dot{\mu} \geq 0, & & \quad [= 0 \quad \text{if} \quad \bar{Z} - Z > 0]. \end{aligned} \quad (8)$$

Finally, the transversality conditions read<sup>9</sup>

$$(a) : \lim_{t \rightarrow \infty} e^{-\rho t} \tau(t)[S(t) - S^*(t)] \geq 0, \quad (b) : \lim_{t \rightarrow \infty} e^{-\rho t} \theta(t)[Z(t) - Z^*(t)] \geq 0. \quad (9)$$

In (9) and in what follows, variables marked with an asterisk (\*) denote socially optimal values, while unmarked variables refer to any possible path.

Equation (5) represents the rule for the socially optimal allocation of energy. It requires the marginal benefit of energy consumption in country  $i = A, B$ ,  $U'_i$ , and the social marginal cost of energy production to be equal. In case of energy generation from fossil fuel the social marginal costs consist of the marginal extraction costs  $c$ , the scarcity rent  $\tau^*$ , the Lagrange multiplier associated with the ceiling  $\mu^*$  and the costate variable of the emission stock  $\theta^*$ . While the scarcity rent grows with the constant rate  $\hat{\tau} = \rho$ , the growth rate of  $\theta^*$  depends on the time phase. During Phase I the ceiling is not binding, so that (8) connotes  $\mu^* = 0$ . Consequently, the costate variable of the emission stock evolves according to  $\hat{\theta} = \rho + \gamma$ , which allows us to write  $\theta_I^*(t) = \theta^*(0)e^{(\rho+\gamma)t}$ . In Phase II, the ceiling binds. According to (2) and (3), fossil fuel extraction is then fixed to  $\bar{s} := \gamma\bar{Z}$ . Due to  $U'_A = U'_B$ ,  $\bar{s}$  is divided over both countries such that  $\bar{x}_A = \bar{x}_B = \frac{\bar{s}}{2}$ .<sup>10</sup> As  $\bar{x}_A$  and  $\bar{x}_B$  are time-invariant, the sum  $c + \tau^* + (\mu^* - \theta^*)$  is constant during Phase II. In Phase III the ceiling never binds. Consequently, both  $\mu^*$  and  $\theta^*$  equal zero. Finally, note that a higher emission stock tightens the optimization problem of the social planner when the ceiling is not binding but will be in the future, implying  $\theta^* < 0$  in Phase I.<sup>11</sup>

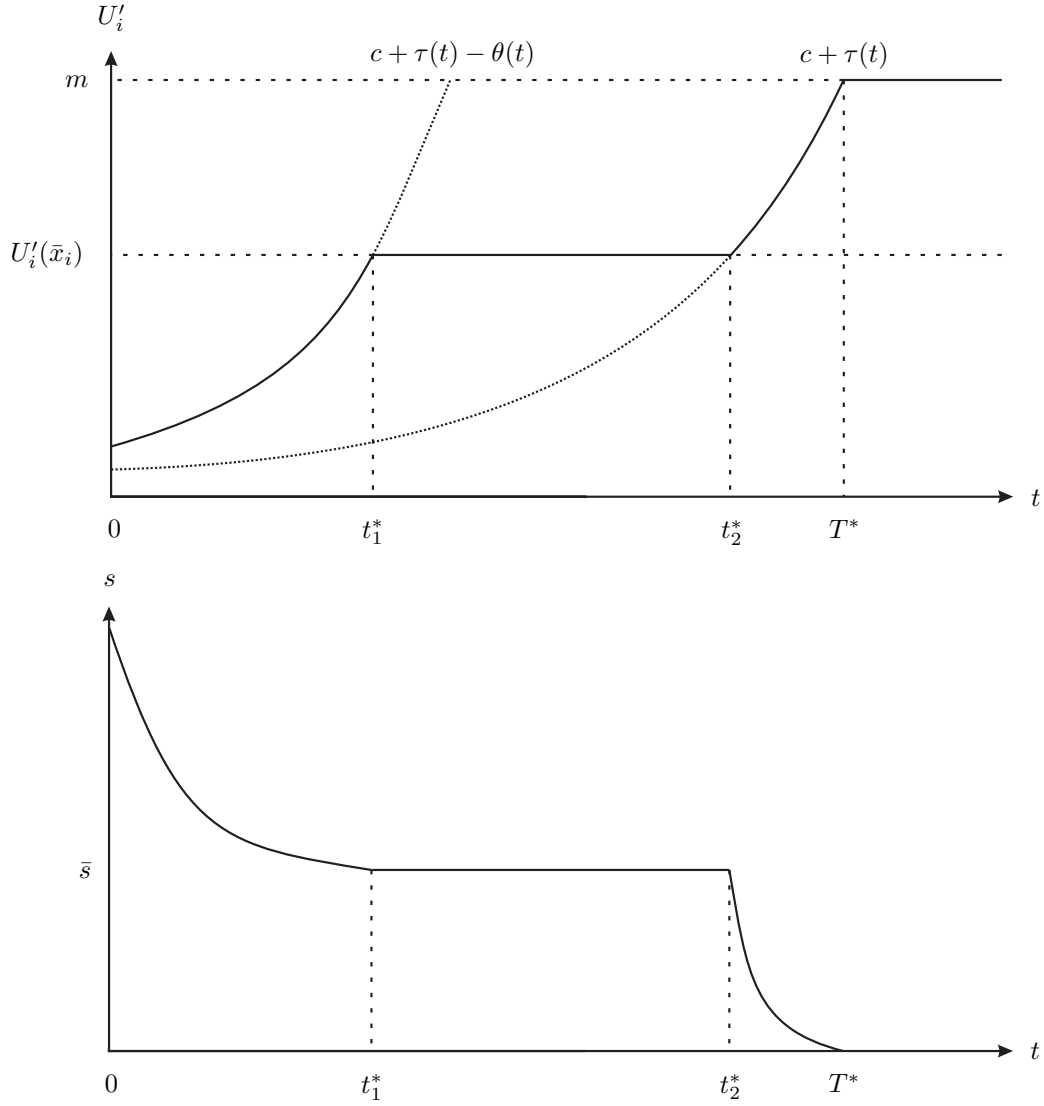
The socially optimal evolution of  $U'_i$  and the corresponding fossil fuel extraction are illustrated in Fig. 1.<sup>12</sup> The depicted sequence of phases; i.e. Phase I, Phase II, Phase III;

<sup>9</sup>Note that the transversality conditions belong to the sufficient conditions. We write the transversality conditions in the form used by Feichtinger and Hartl (1986, chapter 7.2).

<sup>10</sup>Recall that via assumption the backstop is not used before Phase III.

<sup>11</sup>During Phase I we can interpret  $\theta$  as the shadow price of emissions. Due to the chosen optimization approach this interpretation is not valid for Phase II. For details on  $\theta$  at the junction points and during Phase II, see Kollenbach (2015b, 621f.).

<sup>12</sup>Cf. Chakravorty et al. (2006).



**Figure 1:** Socially optimal evolution of  $U'_i$  in time and fossil fuel extraction path

is the only possible one.<sup>13</sup> Phase I lasts from  $t = 0$  to  $t_1^*$ , Phase II lasts from  $t_1^*$  to  $t_2^*$ , and Phase III begins at  $t_2^*$ . Consider Phase I. As the ceiling is not binding,  $U'_i$  equals  $c + \tau^* - \theta^*$ , where  $\tau^* - \theta^*$  monotonically grows in time. Fossil fuel extraction decreases in Phase I. At  $t = t_1^*$  the ceiling becomes binding and remains binding till  $t = t_2^*$ . Since fossil fuel extraction is fixed at  $\bar{s}$ ,  $U'(x_i(t)) = U'(\bar{x}_i)$  is constant for  $i = A, B$  and  $t \in [t_1^*, t_2^*]$ . From  $t = t_2^*$  the ceiling is non-binding and both  $\theta^*$  and  $\mu^*$  equal zero, so that  $U'_i$  equals the sum of marginal extraction costs and the monotonically increasing scarcity rent  $c + \tau^*$ . At  $t = T^*$  this sum reaches the marginal backstop costs. Consequently, at  $t = T^*$  fossil fuel

<sup>13</sup>The sequence of phases is proven in Kollenbach (2015b). According to Kollenbach (2015b), the term  $\tau + (\mu - \theta)$  switches smoothly from one phase to the next. Consequently, jumps in the fossil fuel extraction path are ruled out.

extraction expires and energy generation from the backstop begins. The transversality condition (9)(a) ensures that the fossil fuel stock becomes completely exhausted at  $t = T^*$ .

#### 4. Demand side policy

It is straightforward to show that the socially optimal solution is implemented if the countries  $A$  and  $B$  cooperate and maximize their joint welfare subject to the ceiling. One way to achieve the socially optimal solution is to appropriately reduce the countries' fossil fuel consumption (*demand side policy*). The other way is to appropriately reduce the countries' fossil fuel extraction (*supply side policy*). However, the international climate negotiations show that this is hardly the case. Rather, different countries or regions pursue their own climate policies. Therefore, we assume in the following analysis that only the government of country  $A$  adheres to the ceiling. In contrast, country  $B$  does not apply any climate policy, as it considers the ceiling to be wrong or shuffles off the responsibility to country  $A$ , that is, it applies a free riding policy. To ensure that the ceiling is not violated, the government of country  $A$  can apply a demand or supply side climate policy. The former is analyzed in this section and consists of levying a cap on fossil fuel consumption, fuel cap for short, in country  $A$ .

##### 4.1. Fossil fuel market

To determine the optimal fuel cap in country  $A$ ,  $x_A$ , consider the fossil fuel market. The fuel demand of country  $A$  is given by  $x_A$  and the fuel demand of country  $B$  by  $x_B = D_B(p) := U'^{-1}(p)$ , where  $p$  denotes the fuel price and  $U'^{-1}$  is the inverse of the marginal utility function  $U'$ . Recall that country  $A$  does not own any deposits. Hence, the representative firm of country  $B$  is the sole supplier of fossil fuel. It maximizes its intertemporal profits with respect to the fuel supply  $s_B$  subject to a limited fossil fuel stock. The corresponding first-order conditions yield<sup>14</sup>

$$p = c + \tau_B, \tag{10}$$

$$\hat{\tau}_B = \rho. \tag{11}$$

According to (10) the fossil fuel producer price  $p$  equals the sum of marginal extraction costs  $c$  and the scarcity rent  $\tau_B$ . (11) is the Hotelling-rule which requires that the scarcity

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<sup>14</sup>The Hamiltonian reads  $H = ps_B - cs_B - \tau_B s_B$ . The first-order conditions give (10) and (11). Groot et al. (2003) have shown that the optimal strategy of fossil fuel firms equals the strategy of a price taker if the number of fossil fuel firms approximates infinity.

rent grows in time with the time preference rate  $\rho$ . At every point in time, the representative fossil fuel firm is willing to sell any desired amount of fossil fuels if the market price satisfies (10).<sup>15</sup> The transversality condition

$$\tau_B(T)s_B(T) = 0 \quad (12)$$

determines the optimal time to cease fossil fuel extraction  $T$ .<sup>16</sup>

The intertemporal equilibrium on the fossil fuel market is characterized by two equations. First, total fuel demand must equal total fuel supply until the switch to the backstop:<sup>17</sup>

$$\int_0^T x_A(t) dt + \int_0^T D_B(c + \tau_B(0) e^{\rho t}) dt = S(0). \quad (13)$$

Second, at the point of time  $T$  when the fossil fuel stock becomes exhausted and the economy switches from fossil fuel to the backstop, the fuel price is equal to the backstop price:

$$c + \tau_B(0)e^{\rho T} = m \quad \Leftrightarrow \quad T = \frac{1}{\rho} \ln \left( \frac{m - c}{\tau_B(0)} \right). \quad (14)$$

Solving (13) and (14) for  $\tau_B(0)$  and  $T$  yields expressions for the initial scarcity rent and the exhaustion date as functions of the fossil fuel cap path of country  $A$   $\Phi := \{x_A(0), x_A(1), \dots\}$ , i.e.  $\tau_B(0, \Phi)$  and  $T(\Phi)$ . Making use of  $\tau_B = \tau_B(0, \Phi)e^{\rho t}$  in (10) we obtain the fuel price as function of the fossil fuel cap path, formally  $p(\Phi)$  with  $\frac{\partial p}{\partial x_A} > 0$ .<sup>18</sup> Relaxing the fuel cap  $x_A(t)$  at one point in time  $t$  increases country  $A$ 's fuel demand. To re-equilibrate the fuel market, both the fuel price and the fuel supply at  $t$  increase.

Next, we turn to the intratemporal equilibrium on the fossil fuel market. At every point of time until the switch to the backstop, fuel demand equals fuel supply:

$$x_A(t) + D_B[p(\Phi)] = s_B(t). \quad (15)$$

(15) determines the instant fuel supply of county  $B$  in dependence of the fuel cap path of country  $A$ ,  $s_B(\Phi)$ .

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<sup>15</sup>The supply function of the representative fossil fuel firm at time  $t$  is a horizontal at  $p(t) = c + \tau_B(t)$ .

<sup>16</sup>Cf. Feichtinger and Hartl (1986, Satz 7.6).  $T = 0$  and  $T = \infty$  are ruled out by a sufficiently large but finite  $m$ .

<sup>17</sup>(13) follows from  $x_B = D_B(p)$ ,  $p = c + \tau_B$  and  $\tau_B(t) = \tau_B(0) e^{\rho t}$ .

<sup>18</sup>Cf. Lemma A.3 of Appendix A.2.

## 4.2. Governmental policy

In this subsection the unilaterally optimal demand side policy is analyzed. For that purpose the government of country  $A$  maximizes its welfare  $\int_0^\infty e^{-\rho t} [U(x_A + b_A) - mb_A - px_A] dt$  with respect to its fuel cap  $x_A$  given the CO<sub>2</sub> ceiling. The government accounts for its influence on the instant fuel price  $p(\Phi)$  and on country  $B$ 's fuel demand  $D_B[p(\Phi)]$ .<sup>19</sup> The current-value Lagrangian reads<sup>20</sup>

$$L = U(x_A + b_A) - mb_A - p(\Phi) x_A - (\mu_A - \theta_A) [x_A + D_B[p(\Phi)] - \gamma Z]. \quad (16)$$

Restricting our attention to an interior solution of fossil fuel use, the first-order condition

$$U'_A = p + x_A \frac{\partial p}{\partial x_A} + (\mu_A - \theta_A) \left( 1 + \frac{\partial D_B}{\partial p} \cdot \frac{\partial p}{\partial x_A} \right) \quad (17)$$

characterizes country  $A$ 's optimal fuel cap. In (17) we denote  $x_A \frac{\partial p}{\partial x_A}$  with  $\frac{\partial p}{\partial x_A} > 0$  as *fuel price effect*, and  $(\mu_A - \theta_A) \frac{\partial D_B}{\partial p} \cdot \frac{\partial p}{\partial x_A} \leq 0$  with  $\frac{\partial D_B}{\partial p} \cdot \frac{\partial p}{\partial x_A} \in [-1, 0]$  as *emission effect*.<sup>21</sup> The evolution of the costate-variable  $\theta_A$  and the multiplier  $\mu_A$  are given by equations similar to (7) and (8), while a transversality condition like (9)(b) ensures that the value of the emission stock converges against zero for  $t \rightarrow \infty$ .

Comparing (17) with the socially optimal allocation rule  $U'_A = c + \tau^* + (\mu^* - \theta^*)$  reveals that country  $A$ 's fuel cap is inefficient. Due to  $p = c + \tau_B$  two different strategic effects explain the divergence from the social optimum. The first is the fuel price effect  $\left( x_A \frac{\partial p}{\partial x_A} > 0 \right)$ , i.e. country  $A$ 's incentive to decrease the fuel price in order to reduce its bill from importing fossil fuels.<sup>22</sup> The stronger the fuel price effect the lower is fuel demand in country  $A$ . The second effect is the emission effect<sup>23</sup>  $\left( (\mu_A - \theta_A) \frac{\partial D_B}{\partial p} \cdot \frac{\partial p}{\partial x_A} \leq 0 \right)$ . Tightening country  $A$ 's fuel cap reduces the fuel price and increases fuel consumption in country  $B$ .<sup>24</sup> Emissions leak to country  $B$ , but the leakage rate is less than 100%. If the social planner reduces country  $A$ 's fuel demand by one unit, total emissions exactly decrease by one unit. In contrast, if country  $A$  unilaterally tightens its fuel cap by one

<sup>19</sup>Note that the chosen optimization approach directly determines the optimal values of  $t_1$ ,  $t_2$  and  $T$ . For a more detailed discussion cf. Feichtinger and Hartl (1986, chapter 6).

<sup>20</sup>The variables  $\theta_A$  and  $\mu_A$  are interpreted in the same way as  $\theta$  and  $\mu$ . Therefore,  $\theta_A < 0$  in Phase I.

<sup>21</sup>See Lemma A.3 of Appendix A.2. Lemma A.1 of Appendix A.1 shows that  $\mu_A - \theta_A \geq 0$  in Phase II. In Phase I,  $\mu_A = 0$  and  $\theta_A < 0$ , while  $\mu_A = \theta_A = 0$  in Phase III. Consequently,  $\mu_A(t) - \theta_A(t) \geq 0$  for all  $t$ .

<sup>22</sup>The fuel price effect vanishes when energy supply switches to the backstop.

<sup>23</sup>The emission effect vanishes when the ceiling is not binding anymore.

<sup>24</sup>For a more detailed discussion on carbon leakage and the green paradox cf. Burniaux and Martins (2012), Copeland and Taylor (2005), Eichner and Pethig (2011), Hoel (1996; 2011) and Sinn (2008).

unit, total emissions decrease by less than one unit, which reduces the effectiveness of country  $A$ 's mitigation efforts. Hence, country  $A$ 's sacrifice of fuel consumption to adhere the ceiling is *ceteris paribus* larger at the unilateral demand side policy than in the social optimum. The stronger the emission effect the larger is the fuel cap in country  $A$ . The fuel price effect and the emission effect are opposite in sign. However, since the leakage rate is smaller than 100%, formally  $\left(1 + \frac{\partial D_B}{\partial p} \cdot \frac{\partial p}{\partial x_A}\right) > 0$ , and  $U'_B = p$ , we get  $U'_A > U'_B$ . Country  $A$  must drastically reduce its fuel consumption to adhere the ceiling at the benefit of country  $B$  that is able to increase its fuel consumption.  $x_A(t) < x_B(t)$  implies that country  $A$ 's ( $B$ 's) fuel consumption is inefficiently low (high) during Phase II, where  $x_A^*(t) = x_B^*(t) = \frac{\bar{s}}{2}$ . In addition, Lemma A.6 of Appendix A.2 proves that country  $B$  consumes more fuel than in the social optimum until the ceiling becomes binding. We summarize our results in Proposition 1.

**Proposition 1.** *Suppose that the government of country  $A$  applies a demand side climate policy. Then the demand side policy is inefficient.*

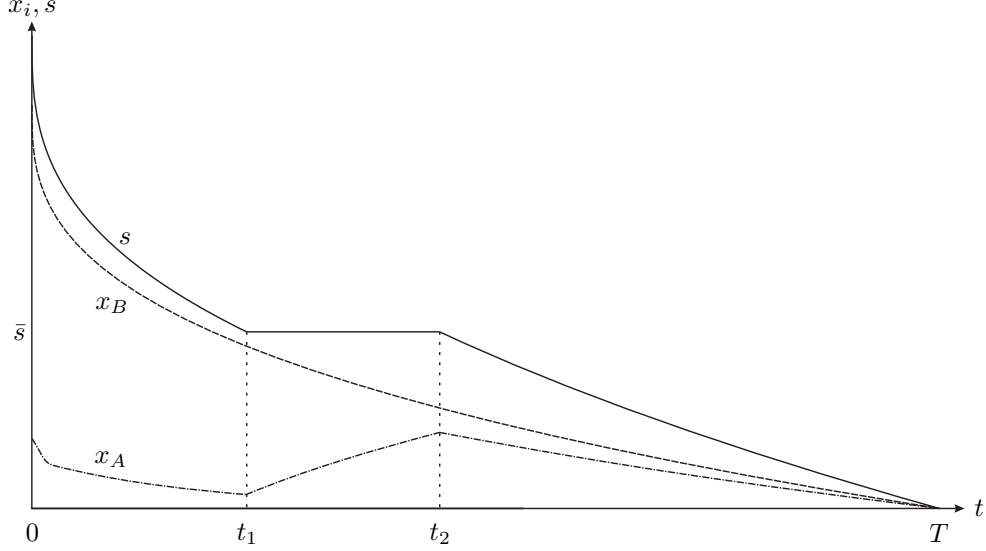
- (i) *In Phase I the fuel price path does not internalize the shadow price of the emission stock.*
- (ii) *Ceteris paribus, a strong fuel price effect (emission effect) reduces (increases) fuel demand in country  $A$ .*
- (iii) *At every point in time fuel consumption is larger in country  $B$  than in country  $A$ .*
- (iv) *In Phase I country  $B$ 's fuel consumption is inefficiently high.*
- (v) *In Phase II country  $A$ 's fuel consumption is inefficiently low, whereas country  $B$ 's fuel consumption is inefficiently high.*

Fig. 2 visualizes the evolution of fossil fuel extraction and consumption in both countries.<sup>25</sup> As  $x_B$  satisfies  $U'(x_B) = c + \tau_B(t)$  and  $\hat{\tau}_B = \rho$ , fossil fuel consumption in country  $B$  decreases continuously. In Phase II, fossil fuel extraction is constant at  $s(t) = \bar{s}$ , so that the decreasing consumption in country  $B$  implies an increasing utilization in country  $A$ . Finally,  $x_A(t)$  decreases continuously during Phase III.<sup>26</sup>

If strategic effects are absent and country  $A$  behaves as price-taker on the fuel market, i.e. if  $\frac{\partial p}{\partial x_A} \equiv 0$ , (17) corresponds to (5). In other words, the government would be able to implement the socially optimal fossil fuel consumption path in country  $A$  by setting  $(\mu_A - \theta_A) = c + \tau^* + (\mu^* - \theta^*) - p$ . However, the government of country  $A$  cannot

<sup>25</sup>Lemma A.4 of Appendix A.2 shows that marginal utility  $U'_A$  is continuous at the first junction point  $t_1$ . The continuity at  $t_2$  follows directly from the used optimization approach, cf. Feichtinger and Hartl (1986, p. 170).

<sup>26</sup>See Lemma A.5 of Appendix A.2.  $x_A(t)$  also decreases continuously during Phase I if  $D_B(p) = \alpha p^{-\beta}$ , where  $\alpha$  and  $\beta$  are positive parameters, or if the emission effect is sufficiently weak.



**Figure 2:** Fossil fuel consumption and extraction paths with demand side policy

control fossil fuel consumption in country  $B$ . Since the government of country  $B$  is inactive, the representative individual of country  $B$  consumes  $D_B[p(t)]$  at every point in time such that  $U'_B = p$  holds. Thus, the effect of fossil fuel use on the  $\text{CO}_2$  stock is not internalized in country  $B$ , which implies  $x_B > x_A$  until the ceiling is not binding anymore and  $x_B = x_A$  in Phase III. In addition, Lemma A.6 of Appendix A.2 shows that country  $A$ 's fuel consumption is inefficiently low in Phase I when the depreciation rate  $\gamma$  is sufficiently small.<sup>27</sup> Finally, the relation of the scarcity rents  $\tau_B(0)$  and  $\tau^*(0)$  provides information whether total extraction is inefficiently low or high in Phase III. If  $\tau_B(0) > \tau^*(0)$ , then the fuel price in Phase III is inefficiently high and total extraction is inefficiently low in Phase III. Since the fuel extraction at Phase II is  $\bar{s}$  both in the social optimum and at the demand side policy, total extraction is inefficiently high in Phase I, and Phase I is inefficiently short. We summarize our results in Proposition 2.

**Proposition 2.** *Suppose that the government of country  $A$  applies a demand side climate policy and country  $A$  behaves as price taker. Then the demand side policy is inefficient.*

- (i) *Proposition 1(i), (iv) and (v) continue to hold.*
- (ii) *In Phase I and II country  $B$ 's fuel consumption is larger than country  $A$ 's fuel consumption.*
- (iii) *In Phase III country  $A$ 's and  $B$ 's fuel consumption is identical.*
- (iv) *Suppose that  $\gamma \rightarrow 0$ . Then in Phase I country  $A$ 's fuel consumption is inefficiently low.*
- (v) *If  $\tau_B(0) > [\leq] \tau^*(0)$ , then total fuel extraction is inefficiently high [low] in Phase I*

<sup>27</sup>Country  $A$ 's cumulative fuel consumption is always inefficiently low until the ceiling is not binding anymore.

and inefficiently low [high] in Phase III. Phase I is inefficiently short [long].

## 5. Supply side policy

Having characterized the unilaterally optimal demand-side policy, we turn to the supply-side policy in this section. In that case, the government of country  $A$  purchases non-extracted fossil fuel reserves, i.e. deposits, and accumulates a state-owned fossil fuel stock  $S^A$  that evolves in time according to

$$\dot{S}^A = -s_A + y. \quad (18)$$

The extraction rate is denoted by  $s_A$ , while  $y$  refers to the reserves bought by the government. Hence, country  $A$ 's supply side policy consists of purchasing deposits,  $y$ , and supplying fossil fuel,  $s_A$ .

The costs and revenues of the state-owned resource are financed by lump-sum transfers  $\pi \gtrless 0$  to the individuals. We assume that funds  $\pi$  for the purchase of deposits are limited and cannot fall short of  $\bar{\pi} < 0$ . Consequently,  $y$  is constrained from above, as the government cannot buy more reserves than  $\frac{|\bar{\pi}|}{p_y(t)}$  at every point in time, where  $p_y$  denotes the price of deposits. However, we assume that  $|\bar{\pi}|$  is sufficiently high to allow the government to guarantee the adherence of the ceiling.

### 5.1. Fossil fuel and deposit market

Since the government of country  $A$  does not pursue a demand side policy, fuel demand in country  $A$  and  $B$  is given by  $D(p) := D_A(p) + D_B(p)$ , where  $D_i(p) := U'^{-1}(p)$  for  $i = A, B$ . The optimization problem of the representative fossil fuel firm is altered, as the firm not only sells extracted resources but also non-extracted ones. Hence, its Hamiltonian reads  $H = ps_B + p_y y - cs_B - \tau_B(s_B + y)$ , where  $s_B$  denotes the fossil fuel supply of the firm in country  $B$ , also denoted as *private* fuel supply. Solving the optimization problem and assuming an interior solution yield (10), (11) and

$$p_y = \tau_B. \quad (19)$$

At every point in time, the representative fossil fuel firm is willing to sell any desired amount of deposits if the price equals (or exceeds) country  $B$ 's scarcity rent. The transversality condition reads now

$$\tau_B(T_B) [s_B(T_B) + y(T_B)] = 0, \quad (20)$$



where  $T_B$  denotes the point in time when the privately owned fossil fuel stock becomes exhausted, so that supply  $s_B(t)$  and deposit sales  $y(t)$  vanish for all  $t \geq T_B$ .

In Appendix A.3, we prove Lemma 1.

**Lemma 1.** *Suppose the government of country A applies a supply side climate policy, then the private fossil fuel stock is exhausted before the ceiling becomes binding,  $T_B \leq t_1$ . There does [not] exist a market for deposits if  $t < T_B$  [ $t \geq T_B$ ].*

$t_1$  is the point of time at which the ceiling becomes binding. In view of Lemma 1, at  $t = t_1$  the firm of country B has completely sold its fuel deposits such that the private stock is exhausted. For  $t \geq T_B$  there does not exist any deposit trade, and country A is the sole supplier of fossil fuel.

Analogous to section 4, total fuel demand must equal total fuel supply until the exhaustion date of the private fossil fuel stock. The intertemporal equilibrium condition (until  $t = T_B$ ) is given by:

$$\int_0^{T_B} D(p(t)) dt = S(0) - S^A(T_B), \quad (21)$$

where  $S(0)$  is country B's fossil fuel stock at  $t = 0$ , and  $S^A(T_B) = \int_0^{T_B} [y(t) - s_A(t)] dt$  is country A's fossil fuel stock at  $t = T_B$ . Next, consider the evolution of emissions and the ceiling. Solving (2) for  $Z(t)$  and making use of  $s(t) = D(p(t))$ ,  $t = t_1$  and  $Z(t_1) = \bar{Z}$  yields

$$\bar{Z} = Z(0)e^{-\gamma t_1} + \int_0^{t_1} D(p(t))e^{-\gamma(t_1-t)} dt. \quad (22)$$

(21) and (22) determine the initial scarcity rent<sup>28</sup> and the exhaustion date of the private fossil fuel stock as functions of the growth path of country A's fossil fuel stock  $\Psi := \{[y(0) - s_A(0)], [y(1) - s_A(1)], \dots\}$ , formally  $\tau_B(0, \Psi)$  and  $T_B(\Psi)$ .<sup>29</sup>

Suppose that  $t < T_B$ . If country B supplies fossil fuel ( $s_B > 0$ ), then we use  $\tau_B(0, \Psi)$  in  $p = c + \tau_B(0)e^{\rho t}$  and  $p_y = \tau_B(0)e^{\rho t}$ , which follow from (10), (11) and (19), to obtain the fuel price function  $p(\Psi)$  and the deposit price function  $p_y(\Psi)$ . The intratemporal fossil fuel market equilibrium condition for  $t < T_B$

$$D[p(\Psi)] = s_A(t) + s_B(t) \quad (23)$$

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<sup>28</sup>Observe that  $D(p(t)) = D(c + \tau_B(0)e^{\rho t})$  if  $s_B > 0$ , and  $D(p(t)) = s_A(t)$  if  $s_B(t) = 0$  in (21) and (22).

<sup>29</sup>Strictly speaking,  $\tau_B(0, \Psi)$  and  $T_B(\Psi)$  also are functions of  $t_1$  which is implicitly determined by the optimization of government A. Since country A does not maximize with respect to  $t_1$  we suppress  $t_1$  as argument.

requires that fuel demand must equal fuel supply. (23) determines country  $B$ 's instant fuel supply in dependence of the growth path of country  $A$ 's fossil fuel stock,  $s_B(\Psi)$ .

If country  $B$  still owns some fossil fuel but does not supply it ( $s_B = 0$ ), then the intratemporal fuel market clearing condition  $D(p(t)) = s_A(t)$  and  $p_y = \tau_B(0)e^{\rho t}$  establish the price functions  $p(\Psi)$  and  $p_y(\Psi)$ .

In Appendix A.3, we prove Lemma 2.

**Lemma 2.** *For  $t < T_B$ , the price functions  $p(\Psi)$  and  $p_y(\Psi)$ , and country  $B$ 's instant fuel supply function  $s_B(\Psi)$  have the following properties:*

$$\frac{\partial p}{\partial s_A} = -\frac{\partial p}{\partial y} = \frac{\partial p_y}{\partial s_A} = -\frac{\partial p_y}{\partial y} < 0, \quad 1 + \frac{\partial s_B}{\partial s_A} = -\frac{\partial s_B}{\partial y} \in [0, 1], \quad \text{for } s_B > 0, \quad (24)$$

$$\frac{\partial p}{\partial s_A} = \frac{1}{D'(p)}, \quad -\frac{\partial p_y}{\partial y} < 0, \quad \frac{\partial p}{\partial y}, \frac{\partial p_y}{\partial s_A}, \frac{\partial s_B}{\partial s_A}, \frac{\partial s_B}{\partial y} = 0, \quad \text{for } s_B = 0. \quad (25)$$

If  $s_B > 0$ , a decrease of country  $A$ 's fuel supply,  $s_A$ , increases country  $B$ 's initial scarcity rent,  $\tau_B(0)$ , and with it the fuel price,  $p$ , and the deposit price  $p_y$ . While a higher fuel price implies a reduction of fuel demand, the supply of country  $B$ ,  $s_B$ , increases. In other words, the reduction of country  $A$ 's supply causes carbon leakage. The leakage effect in country  $B$  is smaller than the fuel supply reduction in country  $A$  implying a leakage rate of less than 100%. The effects of reducing deposit purchases are exactly reversed. If  $s_B = 0$ , then country  $B$ 's initial scarcity rent is invariant with respect to changes of  $s_A$ . An increase of country  $A$ 's fuel supply reduces the fuel price without any repercussions on country  $B$ 's fuel supply and on the deposit price. Conversely, an increase of deposit purchases  $y$  raises the deposit price, but causes no repercussions on the fuel price and on country  $B$ 's fuel supply.

Finally, suppose that  $t \geq T_B$ . According to Lemma 1 the private fossil fuel stock is exhausted such that there does not exist a deposit market ( $s_B = 0$ ). The intratemporal fuel equilibrium condition simplifies to  $D(p(t)) = s_A(t)$  and yields the fuel price function  $p(\Psi)$  with  $\frac{\partial p}{\partial s_A} < 0$ .

## 5.2. Governmental policy

In this subsection we turn to the unilaterally optimal supply side policy. The government of country  $A$  maximizes  $\int_0^T e^{-\rho t} [U(x_A + b_A) - mb_A - px_A - p_y y + (p - c)s_A] dt + \int_T^\infty e^{-\rho t} [U(b_A) - mb_A] dt$  given the CO<sub>2</sub> ceiling and its limited fossil fuel stock  $S^A(t)$ .<sup>30</sup>

<sup>30</sup>As  $b_A(t)$  for  $t \geq T$  is determined by  $U'(b_A) = m$  and, therefore, time-invariant, the latter term of the welfare function can be written as  $\int_T^\infty e^{-\rho t} [U(b_A) - mb_A] dt = e^{-\rho T} / \rho [U(b_A) - mb_A] > 0$ .

When doing so the government takes into account the information of Lemma 1 and 2. i.e. it chooses its supply-side policy  $(s_A(t), y(t))$  for  $t < T_B$  and  $s_A(t)$  for  $t \geq T_B$ .<sup>31</sup> Furthermore, it is aware of its influence on the instant fuel and deposit prices,  $p(\Psi)$  and  $p_y(\Psi)$ , and on the instant supply of country  $B$ ,  $s_B(\Psi)$ . The associated current-value Lagrangian reads<sup>32</sup>

$$L = U \left[ D_A [p(\Psi)] + b_A \right] - mb_A - cs_A - p(\Psi) \left[ D_A [p(\Psi)] - s_A \right] - p_y(\Psi) y \quad (26)$$

$$+ \tau_A(y - s_A) - (\mu_A - \theta_A) [s_A + s_B(\Psi) - \gamma Z] + \zeta_{\bar{\pi}} [|\bar{\pi}| - p_y(\Psi) y],$$

where  $\tau_A$  is the shadow price of the governmental fossil fuel stock, and  $\zeta_{\bar{\pi}}$  is the multiplier of the limited funds for deposit purchases. The governmental scarcity rent  $\tau_A$  reflects that even without climate concerns and strategic incentives, the government demands a fossil fuel price that exceeds the marginal extraction costs  $c$  due to the exhaustibility of the resource.

From the first-order conditions we obtain

$$\tau_A(t) = \tau_A(0)e^{\rho t}, \quad (27)$$

$$\dot{\theta}_A = (\rho + \gamma)\theta_A - \mu_A\gamma, \quad (28)$$

where  $\tau_A(0)$  is the initial scarcity rent of the government.

The complementary slackness conditions are

$$\frac{\partial L}{\partial \mu_A} = -s_A - s_B + \gamma Z \geq 0, \quad \mu_A \geq 0, \quad \mu_A \frac{\partial L}{\partial \mu_A} = 0,$$

$$\bar{Z} - Z \geq 0, \quad \mu_A[\bar{Z} - Z] = 0, \quad (29)$$

$$\rho\mu_A - \dot{\mu}_A \geq 0, \quad [= 0 \text{ if } \bar{Z} - Z > 0],$$

$$\zeta_{\bar{\pi}} \geq 0, \quad \zeta_{\bar{\pi}}[|\bar{\pi}| - p_y y] = 0. \quad (30)$$

Equivalent to section (3),  $\mu_A = 0$  and  $\theta_A < 0$  during Phase I, such that  $(\mu_A - \theta_A) > 0$ . Recall that Lemma A.1 of Appendix A.1 shows  $(\mu_A - \theta_A) > 0$  during Phase II. In Phase III, the ceiling is non-binding forever, such that  $\mu_A = \theta_A = 0$ .

To provide the transversality condition, which determines the optimal exhaustion time

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<sup>31</sup>An alternative procedure is to assume that the government of country  $A$  maximizes its welfare with respect to  $y$  when  $t \geq T_B$ , and to assume that the deposit price is prohibitively high  $p_y \rightarrow \infty$ . The solution yields  $y = 0$  for  $t \geq T_B$ .

<sup>32</sup>Observe that the equilibrium at supply-side policy is a kind of Stackelberg equilibrium and hence subgame perfect. It coincides with a feedback Nash equilibrium. For a more detailed discussion on the coincidence of these concepts cf. Rubio (2006).

$T$ , note that the emission stock converges against zero for  $t \rightarrow \infty$  but is positive for all  $t < \infty$ . Furthermore, the backstop does not become economically usable before Phase III. Therefore, we get

$$\tau_A(T)s_A(T) = 0. \quad (31)$$

Accounting for  $U' = p$ , the first derivatives of the Lagrangian (26) with respect to  $s_A$  and  $y$  are

$$L_{s_A} = p - c - \tau_A - (\mu_A - \theta_A) \left( 1 + \frac{\partial s_B}{\partial s_A} \right) + (s_A - x_A) \frac{\partial p}{\partial s_A} - (1 + \zeta_{\bar{\pi}})y \frac{\partial p_y}{\partial s_A}, \quad (32)$$

$$L_y = \tau_A - (1 + \zeta_{\bar{\pi}})p_y - (\mu_A - \theta_A) \frac{\partial s_B}{\partial y} + (s_A - x_A) \frac{\partial p}{\partial y} - (1 + \zeta_{\bar{\pi}})y \frac{\partial p_y}{\partial y}. \quad (33)$$

Similar to the demand-side policy, the expressions  $(\mu_A - \theta_A) \frac{\partial s_B}{\partial \chi}$  are *emission effects*,  $(s_A - x_A) \frac{\partial p}{\partial \chi}$  are *fuel price effects* and  $(1 + \zeta_{\bar{\pi}})y \frac{\partial p_y}{\partial \chi}$  are *deposit price effects* for  $\chi = s_A, y$ . Since corner solutions turn out to be relevant, the first-order conditions of maximizing the Lagrangian (26) are given by

$$L_{s_A} \leq 0, \quad s_A L_{s_A} = 0, \quad (34)$$

$$L_y \leq 0, \quad y L_y = 0. \quad (35)$$

Suppose first that  $t < T_B$ . In the sequel, we investigate whether each of the three equilibria  $(s_A \geq 0, s_B > 0, y > 0)$ ,  $(s_A \geq 0, s_B > 0, y = 0)$  and  $(s_A > 0, s_B = 0, y \geq 0)$  exists. To understand the following argumentation it is worth mentioning that for  $t < T_B$  in the economy there are two offer prices of fossil fuels, the offer price  $p_A$  of country  $A$  and the offer price  $p_B = c + \tau_B$  of country  $B$ . If  $p_A < [>] p_B$ , then only country  $A$  [ $B$ ] supplies fossil fuel and the equilibrium fuel price is  $p(t) = p_A(t)$  [ $p_B(t)$ ]. If  $p = p_A(t) = p_B(t)$ , then both countries simultaneously supply fossil fuel.

(i) We begin with the equilibria  $(s_A \geq 0, s_B > 0, y > 0)$ . In view of Lemma 2 it holds for  $s_B > 0$ :

$$\begin{aligned} \Gamma &:= -(\mu_A - \theta_A) \left( 1 + \frac{\partial s_B}{\partial s_A} \right) + (s_A - x_A) \frac{\partial p}{\partial s_A} - (1 + \zeta_{\bar{\pi}})y \frac{\partial p_y}{\partial s_A} \\ &= (\mu_A - \theta_A) \frac{\partial s_B}{\partial y} - (s_A - x_A) \frac{\partial p}{\partial y} + (1 + \zeta_{\bar{\pi}})y \frac{\partial p_y}{\partial y}, \end{aligned} \quad (36)$$

and the first-order conditions (34) and (35) can be written as

$$L_{s_A} = p - c - \tau_A + \Gamma \leq 0, \quad s_A L_{s_A} = 0, \quad (37)$$

$$L_y = \tau_A - (1 + \zeta_{\bar{\pi}})p_y - \Gamma \leq 0, \quad y L_y = 0. \quad (38)$$

Country  $A$  purchases deposits ( $y > 0$ ) and thus we infer from (38)  $L_y = 0$  or equivalently  $\tau_A = (1 + \zeta_{\bar{\pi}})p_y + \Gamma$ . Making use of this information and  $p_y = \tau_B$  (from (19)) in (37) yields  $p \leq c + (1 + \zeta_{\bar{\pi}})\tau_B =: p_A$ . The private fuel supply  $s_B > 0$  is characterized by  $p = c + \tau_B =: p_B$  in (10). Comparing the two offer prices  $p_A$  and  $p_B$  it is straightforward that  $p = p_B < p_A$  if funds for the purchase of deposits are limiting ( $\zeta_{\bar{\pi}} > 0$ ). In that case  $L_{s_A} \leq 0$  cannot hold as equality, we get the corner solution  $s_A = 0$  and only country  $B$  supplies fuel,  $s_B > 0$ . If funds for purchasing deposits are not limiting ( $\zeta_{\bar{\pi}} = 0$ ), it holds  $p = p_A = p_B$  and both countries simultaneously supply fossil fuel,  $s_A > 0$  and  $s_B > 0$ . Equilibria ( $s_A \geq 0, s_B > 0, y > 0$ ) do exist.

(ii) Next, consider equilibria ( $s_A \geq 0, s_B > 0, y = 0$ ). Since country  $A$  does not purchase deposits ( $y = 0$  and  $\zeta_{\bar{\pi}} = 0$ ), we have  $L_y < 0$  in (38) or equivalently  $\tau_A - \Gamma < p_y$ . The first-order condition (37) results in  $p \leq c + \tau_A - \Gamma =: p_A$ . Combining  $\tau_A - \Gamma < p_y$  and  $p_A = c + \tau_A - \Gamma$  establishes  $p_A < c + p_y$ . Via assumption country  $B$  supplies fuel ( $s_B > 0$ ), and hence  $p_B = c + \tau_B$  and  $p_y = \tau_B$ . However,  $p_A < c + \tau_B$ ,  $p_B = c + \tau_B$  and  $p = p_A = p_B$  cannot hold simultaneously which proves that equilibria ( $s_A \geq 0, s_B > 0, y = 0$ ) do not exist.

(iii) Finally, we consider equilibria ( $s_A > 0, s_B = 0, y \geq 0$ ). In view of Lemma 2, especially in view of the comparative static effects for  $s_B = 0$ , country  $A$ 's first order conditions (37) and (38) turn into  $p_A - c - \tau_A - (\mu_A - \theta_A) + (s_A - x_A) \frac{\partial p}{\partial s_A} = 0$  and  $\tau_A - (1 + \zeta_{\bar{\pi}})p_y - (1 + \zeta_{\bar{\pi}})y \frac{\partial p_y}{\partial y} \leq 0$ , which implies

$$L_y = p_A - c - (\mu_A - \theta_A) + (s_A - x_A) \frac{\partial p}{\partial s_A} - (1 + \zeta_{\bar{\pi}})p_y - (1 + \zeta_{\bar{\pi}})y \frac{\partial p_y}{\partial y} \leq 0, \quad (39)$$

$$yL_y = 0.$$

Since  $p_A < p_B = c + p_y$  and  $s_A - x_A = D_B(p)$ , (39) implies  $L_y < 0$  and, therefore,  $y = 0$ . Thus, if  $s_A(t) > 0$  and  $s_B(t) = 0$ , then  $y(t) = 0$  holds. Hence, the equilibria ( $s_A > 0, s_B = 0, y > 0$ ) do not exist. It remains to consider equilibria ( $s_A > 0, s_B = 0, y = 0$ ). These equilibria do not exist at  $t = 0$  and directly before  $T_B$ . In the former case,  $S^A(0) = 0$  rules out  $s_A(0) > 0$ . In the latter case,  $s_B(T_B^-) = y(T_B^-) = 0$  and  $S(T_B^-) > 0$  contradict the exhaustion of the privately owned fossil fuel stock at  $T_B$ .<sup>33</sup>

In Lemma A.7 of Appendix A.3 we investigate the transition from equilibria ( $s_A \geq 0, s_B > 0, y > 0$ ) to equilibria ( $s_A > 0, s_B = 0, y = 0$ ) and prove that such transitions are

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<sup>33</sup>The superscript "−" indicates the value directly before  $T_B$ .

infeasible for  $t \in [0, t_1)$ . In that proof we make use of the

**Assumption 1.** *The (absolute) price elasticity of demand,  $\epsilon(p) := -\frac{D'(p)p}{D(p)} > 0$ , is not declining in the price, i.e.  $\epsilon'(p) = -\frac{[D''(p)p + D'(p)]D(p) - [D'(p)]^2 p}{[D(p)]^2} \geq 0$ .*

The sign of  $\epsilon'(p)$  also plays an important role in the literature on monopolistic competition. Dixit and Stiglitz (1977) expect and Krugman (1979) assumes  $\epsilon'(p)$  to be positive. Bertoletti and Etro (2017) use  $\epsilon'(p) > 0$  as standard assumption and Mrázová and Neary (2017) find empirical evidence for  $\epsilon'(p) > 0$ .

In the proof of Lemma A.7 Assumption 1 ensures that the offer price  $p_A$  at equilibria  $(s_A \geq 0, s_B > 0, y > 0)$  is lower than the offer price  $p_A$  at equilibria  $(s_A > 0, s_B = 0, y = 0)$ . Hence, the offer price  $p_A$  at equilibria  $(s_A > 0, s_B = 0, y = 0)$  is larger than  $p_B$  which proves that these equilibria do not exist for  $t < T_B$ . Now, suppose  $t \in [T_B, t_1]$ . In view of Lemma 1 the associated equilibria are characterized by  $(s_A > 0, s_B = 0, y = 0)$ . Since, the offer price  $p_A$  at equilibria  $(s_A > 0, s_B = 0, y = 0)$  does not depend on  $S(t)$ , it is also higher than  $p_B$  at equilibria  $(s_A \geq 0, s_B > 0, y > 0)$  when the private stock becomes exhausted at  $t = T_B < t_1$ . Since the representative fossil fuel firm could exploit the corresponding price jump by keeping some deposits and selling them for  $p > c + \tau_B$ ,  $T_B < t_1$  cannot hold. At  $t = t_1$ , positive supplies from country  $B$  would violate the ceiling, such that the price can in general jump upwards. In conjunction with Lemma 1, this implies  $T_B = t_1$ .

To sum up, we have shown that the exhaustion date coincides with the date at which the ceiling becomes binding ( $t_1 = T_B$ ), and for  $t < T_B$  there exist only one type of equilibria, namely equilibria satisfying  $s_B > 0, y > 0$  and  $s_A \geq 0$ . The allocation rule that guides these equilibria is given by

$$U'_A = U'_B = p = c + \tau_B \quad \text{for } t < t_1. \quad (40)$$

Other equilibria do not exist. Strategic effects are not present in the allocation rule (40), since the private offer price prevails.

Now, suppose that  $t \geq T_B = t_1$ . According to Lemma 1, the private stock of fossil fuel is exhausted at  $t = t_1$  and there is no deposit trade anymore. Country  $A$  is the sole supplier of fossil fuel. Recall that at  $t_1$  the ceiling is binding and it remains binding for the period of time  $[t_1, t_2)$ . In that period of time country  $A$  has no degree of freedom to vary its fuel supply  $s_A$  that is determined by the ceiling. Formally, it holds  $U'_i\left(\frac{\bar{s}}{2}\right) =: \bar{p}$  for  $t \in [t_1, t_2)$ .

For  $t \geq t_2$  the ceiling binds no more, and country  $A$ 's optimal fuel supply  $s_A$  is given by

$$U'_A = U'_B = p = c + \tau_A + (\mu_A - \theta_A) - (s_A - x_A) \frac{\partial p}{\partial s_A} \quad \text{for } t \geq t_2. \quad (41)$$

In (41) there emerges a fuel price effect. Since country  $A$  exports fossil fuel, it has a strategic incentive to increase the fuel price.<sup>34</sup>

As shown above, private fuel supply is positive until the ceiling becomes binding at  $t = t_1$ . According to the Hotelling-rule (11), the private scarcity rent  $\tau_B(t)$  and, therefore, the fuel price  $p(t) = c + \tau_B(t)$  continuously increase over time for  $t \in [0, t_1)$ . Consequently, fossil fuel consumption in both countries decreases during Phase I. Furthermore, Lemma A.7 of Appendix A.3 shows that the fuel price path jumps upwards when the ceiling becomes binding at  $t = t_1$  if  $\tau_B(0) \leq \tau^*(0) - \theta^*(0)$ , which leads to a downward jump of fossil fuel consumption. The condition  $\tau_B(0) \leq \tau^*(0) - \theta^*(0)$  ensures that the fossil fuel price path under supply side policy lies below the social optimal one if  $t < t_1$ . Observe that the socially optimal fuel extraction and the fuel extraction of country  $A$  under supply side policy are equal ( $U'_i(\frac{\bar{s}}{2}) = \bar{p}$ ), if the ceiling is binding in Phase II. In addition, the evolution of the socially optimal fuel price  $p = U'_i$  is continuous at  $t = t_1^*$  as illustrated in Figure 1. Hence, the finding that the fossil fuel price path under supply side policy lies below the social optimal one implies that the ceiling becomes earlier binding under supply side policy  $t_1 < t_1^*$ , and that the fuel price path is discontinuous at  $t = t_1$ .

If  $\tau_B(0) > \tau^*(0) - \theta^*(0)$ , the efficient fossil fuel price path intersects once the fuel price path under supply side policy, and the latter can be continuous or jump upwards at  $t = t_1$ . By contrast, the price path under fuel supply policy is definitely continuous at  $t = t_2$ , where the economy switches from Phase II to Phase III.<sup>35</sup> During Phase III, assumption 1 ensures that fossil fuel consumption in both countries decreases,<sup>36</sup> while the binding ceiling implies  $D_A(\bar{p}) + D_B(\bar{p}) = \bar{s}$  in Phase II, where  $\bar{p} = c + \tau^*(t) + (\mu^*(t) - \theta^*(t))$  for  $t \in [t_1^*, t_2^*)$ . We visualize our results in Fig. 3 and summarize them in Proposition 3.

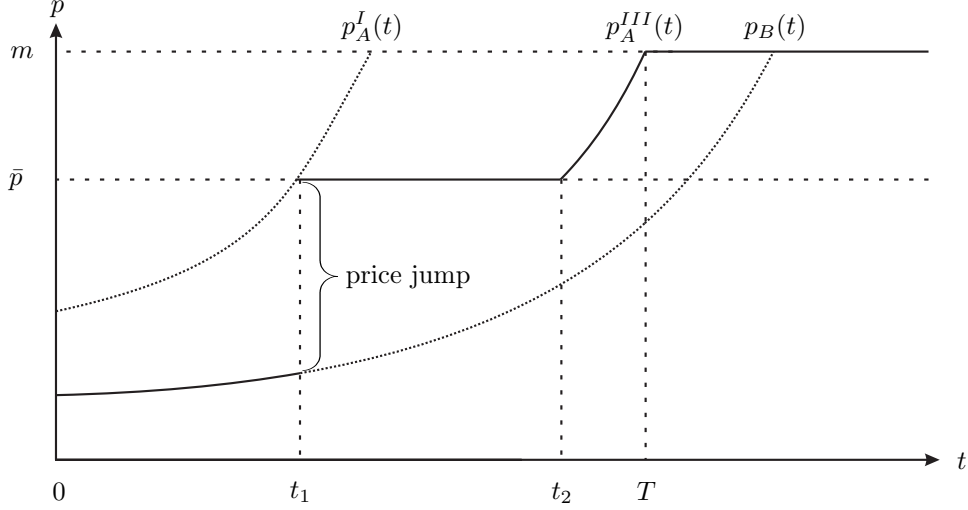
**Proposition 3.** *Assume that the government of country  $A$  applies a supply side climate policy, that the ceiling is initially non-binding and suppose that assumption 1 holds.*

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<sup>34</sup>Note that  $s_A - x_A > 0$  implies that country  $A$  uses less fuel than it extracts from it stock. The corresponding additional extraction is sold to country  $B$ , so that country  $A$  economically exports fuel. However, the fuel stock is still located in country  $B$ . Consequently, in physical terms country  $A$  imports fuel.

<sup>35</sup>See Lemma A.9 of Appendix A.3.

<sup>36</sup>See Lemma A.8 of Appendix A.3.



**Figure 3:** Private price path  $p_B(t) = c + \tau_B(t)$  and governmental price paths for Phase I  $p_A^I(t) = c + \tau_A - \theta_A - \Gamma$  and Phase III  $p_A^{III}(t) = c + \tau_A - (s_A - x_A) \frac{\partial p}{\partial s_A}$

- (i) The only possible sequence of phases which includes all three phases is Phase I, Phase II, Phase III.
- (ii) The private fossil fuel stock becomes exhausted at  $t = t_1$ . Private fuel supply is always positive for  $t < t_1$  and governmental fuel supply is always positive for  $t_1 \leq t < T$ .
- (iii) Fuel consumption in both countries jumps downwards at  $t = t_1$  if  $\tau_B(0) \leq \tau^*(0) - \theta^*(0)$ , can be continuous or jump downwards at  $t = t_1$  if  $\tau_B(0) > \tau^*(0) - \theta^*(0)$ , and is continuous at  $t = t_2$ .
- (iv) Fuel consumption in both countries declines over time during Phase I and Phase III and is constant during Phase II.

Comparing the allocation rules (40) and (41) with the efficient rule  $U'_A = U'_B = c + \tau^* + (\mu^* - \theta^*)$  shows that the supply side policy is inefficient. In Phase I the effect of country A's and country B's extraction on the  $CO_2$  stock and hence on the ceiling is not internalized. In Phase III the inefficiency of the supply side policy is driven by strategic incentives. Due to the fuel price effect  $\left( (s_A - x_A) \frac{\partial p}{\partial s_A} < 0 \right)$ , the government reduces fuel supply during Phase III to increase the fuel price and, therefore, to raise export revenues. Lower fuel utilization during Phase III implies higher fuel utilization and, therefore, a lower fuel price during Phase I.

Although, in the allocation rule (40) of Phase I, there do not emerge strategic effects, in case of  $s_A = 0$  country A influences the scarcity rent of country B and, therefore, the fuel and deposit price. Making use of  $L_y = 0$ ,  $p_y = \tau_B$  and Lemma 2 in (33) we get

$$\tau_A - (1 + \zeta_{\bar{\pi}})\tau_B - \left[ (\mu_A - \theta_A) \frac{\partial s_B}{\partial y} + x_A \frac{\partial p}{\partial y} + (1 + \zeta_{\bar{\pi}})y \frac{\partial p_y}{\partial y} \right] = 0. \quad (42)$$

The sum of fuel price and deposit price effects  $\left( x_A \frac{\partial p}{\partial y} + (1 + \zeta_{\bar{\pi}})y \frac{\partial p_y}{\partial y} \right)$  is positive. Country



$A$  has a strategic incentive to decrease its deposit demand to reduce  $p$  and  $p_y$ , and, therefore, its expenditures for fossil fuel and deposits. The emission effect  $\left(\frac{\partial s_B}{\partial y}\right)$  is negative and countervailing. The emission effect induces country  $A$  to increase its deposit purchases to raise  $p$  and  $p_y$ , with the consequence to slow down pollution accumulation. If the fuel price and deposit price effects overcompensate the emission effect, the fuel price decreases and extraction increases at early points of time, which implies that Phase I is shortened as Lemma A.2 of Appendix A.1 shows. By contrast, a strong emission effect can increase the initial private price above the initial social optimal price,  $c + \tau_B(0) > c + \tau^*(0) - \theta^*(0)$ . However, if the depreciation rate  $\gamma$  is sufficiently low, then  $c + \tau_B(0) \leq c + \tau^*(0) - \theta^*(0)$  must hold to guarantee that the ceiling is reached at all. By Proposition 3(iii), this implies that fuel consumption jumps downwards when the strategic effects vanish at  $t = t_1$ . We summarize our results in Proposition 4.

**Proposition 4.** *Suppose that the government of country  $A$  applies a supply side climate policy. Then the supply side policy is inefficient.*

- (i) *In Phase I the fuel price path does not internalize the shadow price of the emission stock.*
- (ii) *Since  $s_A = 0$  for some  $t \in [0, t_1)$  and  $s_A > 0$  for  $t \in [t_2, T)$ , strong fuel price and deposit price effects (a strong emission effect), ceteris paribus, increase (reduces) extraction at early points of time and decrease (increases) extraction at late points of time. Phase I is shortened (extended).*
- (iii) *Suppose that  $\gamma \rightarrow 0$ . Then fuel consumption in both countries jumps downwards at  $t = t_1$ .*

If country  $A$  behaves as price taker on the fuel market and refrains from strategic action ( $\frac{\partial p}{\partial s_A} = \frac{\partial p}{\partial y} = \frac{\partial p_y}{\partial s_A} = \frac{\partial p_y}{\partial y} = 0$  which implies  $\frac{\partial s_B}{\partial s_A} = \frac{\partial s_B}{\partial y} = 0$ ), (34) and (35) turn into

$$L_{s_A} = p - c - \tau_A - (\mu_A - \theta_A) \leq 0, \quad s_A L_{s_A} = 0 \quad \text{for } t < t_1, \quad (43)$$

$$L_y = \tau_A - (1 + \zeta_{\bar{\pi}})p_y \leq 0, \quad yL_y = 0 \quad \text{for } t < t_1. \quad (44)$$

and (41) simplifies to

$$U'_A = U'_B = p = c + \tau_A + (\mu_A - \theta_A) \quad \text{for } t \geq t_1. \quad (45)$$

At some point in time,  $y(t)$  must be positive to ensure that the ceiling is not violated. Then, (11), (27) and (44) imply  $\tau_A(0)e^{\rho t} = [1 + \zeta_{\bar{\pi}}(t)]\tau_B(0)e^{\rho t}$ . We have to distinguish between two cases concerning  $y(t)$ . If deposit purchases are maximal in some instant of time, then they are maximal for all  $t \in [0, t_1)$ . Then,  $\tau_A(0) > \tau_B(0)$  and  $\zeta_{\bar{\pi}}(t) = \frac{\tau_A(0) - \tau_B(0)}{\tau_B(0)} =: \bar{\zeta}_{\bar{\pi}} > 0$  is time-invariant. Otherwise, if deposit purchases are positive but

not maximal in some instant of time, then  $\tau_A(0) = \tau_B(0)$ . In both cases,  $\tau_A \geq \tau_B$ ,  $(\mu_A - \theta_A) > 0$  for all  $t \in [0, t_2)$ , (43) and (45) imply  $p_A > p_B$  for all  $t \in [0, T_B)$ . Thus,  $s_A = 0$  in Phase I, the private fossil fuel stock becomes exhausted at  $t = t_1$  and the price jumps upwards when the ceiling becomes binding.

If we compare the supply side policy without strategic action with the social optimum, the supply-side policy is characterized by  $s_B = D(c + \tau_B)$  in Phase I, such that the effect of fossil fuel extraction on the CO<sub>2</sub> stock is not internalized until  $t = t_1$ . Furthermore, we find that  $\tau_A(0) > \tau^*(0)$ . The proof follows by contradiction. Suppose that  $\tau_A(0) \leq \tau^*(0)$ . Then cumulative extraction would be the same during Phase II,  $D(\bar{p}) = D(U'(\frac{\bar{s}}{2}))$ , and not lower during Phase III,  $D(c + \tau_A) \leq D(c + \tau^*)$ , which implies that cumulative extraction is inefficiently low in Phase I. However,  $\tau_A(0) \leq \tau^*(0)$  also yields  $c + \tau_B(0) \leq c + \tau_A(0) < c + \tau^*(0) - \theta^*(0)$ , and implies a higher cumulative extraction during Phase I due to a higher initial extraction (see Lemma A.2 of Appendix A.1) which contradicts the first implication. Thus,  $\tau_A(0) > \tau^*(0)$  holds and cumulative extraction at Phase III [Phase I] is lower [higher] than in the social optimum.

In Phase I, the private price path,  $c + \tau_B$ , is flatter than the optimal price path,  $c + \tau^* - \theta^*$ . Now we have to distinguish two cases. If the initial private price is smaller than or equal to the initial optimal price,  $\tau_B(0) \leq \tau^*(0) - \theta^*(0)$ , then the ceiling is reached earlier and the price jumps upwards at  $t_1 < t_1^*$ . If  $\tau_B(0) > \tau^*(0) - \theta^*(0)$ , then the private price path cuts the optimal price path before  $t = t_1^{37}$  and the price jumps upwards at  $t_1 < t_1^*$ .<sup>38</sup> We summarize our results in Proposition 5.

**Proposition 5.** *Suppose that the government of country A applies a supply side climate policy and behaves as price taker on the fuel and the deposit market. Then the fuel supply policy is inefficient.*

- (i) *In Phase I the fuel price path does not internalize the shadow price of the emission stock.*
- (ii) *Fuel consumption in both countries jumps downwards at  $t = t_1$ .*
- (iii) *Fuel extraction is inefficiently high in Phase I ( $\int_0^{t_1^*} s(t) - s^*(t) dt > 0$ ) and inefficiently low in Phase III ( $\int_{t_2}^T s(t) - s^*(t) dt < 0$ ). Consequently, Phase I is inefficiently short, and the switch to the backstop is inefficiently early.*

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<sup>37</sup>Otherwise the ceiling would never bind.

<sup>38</sup>This follows from contradiction. Suppose the price jumps upwards at  $t_1 > t_1^*$  and the price paths would cut only once during Phase I. Then by  $\tau_B(0) > \tau^*(0) - \theta^*(0)$  and Lemma A.2 of Appendix A.1 cumulative extraction is inefficiently low in Phase I which contradicts the conclusion of the previous paragraph.

## 6. Conclusion

This paper compares unilateral demand and supply side climate policies in terms of both their fuel price and consumption paths. In our dynamic model, the climate coalition ensures that a ceiling on the carbon dioxide concentration is not violated by either limiting domestic fuel consumption or by buying deposits to postpone their extraction. Even if the climate coalition ignores its influence on the fuel and deposit prices, the social optimum cannot be implemented. In case of the demand side policy, the consumers in the non-abating country do not internalize the climate externality. The climate coalition must reduce domestic fuel consumption below its socially optimal level to ensure that the ceiling is not violated. Consequently, the climate coalition faces a higher consumer price path than the non-abating country. If the climate coalition acts strategically on the fuel market, it decreases the fuel price to reduce its import bill of fossil fuels.

The supply side policy is inefficient, too, since the non-abating country's extraction firm does not internalize the climate externality. The fuel consumption and the price path exhibit a jump at the date when the ceiling becomes binding. The climate coalition makes use of its market power on both the fossil fuel and the deposit market to manipulate the fuel and deposit prices. If only the extraction firm of the non-abating country supplies fossil fuel, the climate coalition has an incentive to reduce its deposit purchases to lower both the fuel and deposit price, which leads to more (less) fossil fuel consumption at early (late) points of time. Additionally, there is a countervailing incentive to increase its deposit purchase to reduce fuel extraction and to slow down the accumulation of emissions in the atmosphere. If the coalition owns the complete fossil fuel stock and is the sole supplier of fossil fuels, strategic price incentives prevail. As monopolist the coalition reduces its fuel extraction to raise the fuel price.

Our analysis can be extended into other directions. First, one could replace the CO<sub>2</sub> ceiling by a climate damage function to check the robustness of the results. Second, one could analyze how extraction, consumption and welfare would change if the climate coalition could use demand and supply side policies simultaneously. Third, a comprehensive comparison between the policies needs a computable general equilibrium model that is empirically calibrated. These issues are beyond the scope of the present paper but may be interesting and important tasks for future research.

## A. Appendix

### A.1. General

**Lemma A.1.** *Consider Phase II. The term  $\mu_A - \theta_A$  is non-negative.*

*Proof of Lemma A.1.* Consider Phase II, i.e. a time phase characterized by a binding ceiling, such that  $\mu_A \geq 0$ . Assume  $\mu_A - \theta_A < 0$ . Since  $\mu_A \geq 0$ ,  $\theta_A > \mu_A \geq 0$  needs to hold. From (7) we get

$$\dot{\theta}_A = \gamma(\theta_A - \mu_A) + \rho\theta_A > \rho\theta_A > 0. \quad (\text{A.1})$$

As Phase II cannot last forever, there is a junction point  $j = t_2, t_3$  to Phase I or Phase III, respectively. We use the indexes "−" and "+" to refer to the values directly before and directly after the junction point. Then,  $\theta_A^+(j) \leq 0$ . According to Feichtinger and Hartl (1986),

$$\theta_A^-(j) = \theta_A^+(j) \quad (\text{A.2})$$

holds at exit points. Since  $\mu_A - \theta_A < 0$  and (A.1) connote  $\theta_A^-(j) > 0$ , the assumption  $\mu_A - \theta_A < 0$  cannot hold.  $\square$

**Lemma A.2.** *Suppose that  $s(0) > s'(0)$ , that the extraction paths intersect only once until  $t = \max[t_1, t'_1]$ , and that  $\dot{Z}(t), \dot{Z}'(t) \geq 0$  for all  $t \in [0, \max[t_1, t'_1]]$ . Then,  $t_1 < t'_1$  and  $\int_0^{t'_1} s(t) dt > \int_0^{t'_1} s'(t) dt$ .*

*Proof of Lemma A.2.* Suppose that  $s(0) > s'(0)$ , that the extraction paths intersect only once until  $t = \max[t_1, t'_1]$  at  $t = t_I$ , and that  $\dot{Z}(t), \dot{Z}'(t) \geq 0$  for all  $t \in [0, \max[t_1, t'_1]]$ . Then,  $Z(t) > Z'(t)$  for all  $t \in [0, t_I]$ , since  $z(t) > z'(t)$  for all  $t \in [0, t_I]$ . Furthermore,  $Z(t) > Z'(t)$  for all  $t \in [t_I, t'_1]$ , since  $s(t) < s'(t)$  for all  $t \in [t_I, t'_1]$ , so that  $Z(t) < Z'(t)$  for some  $t \in [t_I, t'_1]$  would imply  $Z(t'_1) < Z'(t'_1) = \bar{Z}$  and, thus,  $s(t) > s'(t)$  for some  $t \in [t'_1, t_1]$ , so that the extraction paths would intersect twice. Thus, we obtain  $Z(t) \geq Z'(t)$  for all  $t \in [0, t'_1]$  and, therefore,  $t_1 < t'_1$ . Consequently,  $\int_0^{t'_1} [Z(t) - Z'(t)] dt > 0$  holds. From (2) we get

$$\int_0^{t'_1} s(t) - s'(t) dt = \int_0^{t'_1} \dot{Z}(t) - \dot{Z}'(t) dt + \gamma \int_0^{t'_1} Z(t) - Z'(t) dt. \quad (\text{A.3})$$

Since  $\int_0^{t'_1} \dot{Z}(t) dt = \int_0^{t'_1} \dot{Z}'(t) dt = \bar{Z} - Z(0)$  for  $\dot{Z}(t), \dot{Z}'(t) \geq 0$  and  $Z(t'_1) = Z'(t'_1) = \bar{Z}$ , we rewrite (A.3) as

$$\int_0^{t'_1} s(t) - s'(t) dt = \gamma \int_0^{t'_1} Z(t) - Z'(t) dt > 0. \quad (\text{A.4})$$

$\square$

A.2. Demand side policy

**Lemma A.3.** *The price function  $p(\Psi)$  and country B's instant fuel demand function  $D_B(\Psi)$  have the following properties:*

$$\frac{\partial p}{\partial x_A} = \frac{\partial \tau_B}{\partial x_A} > 0, \quad \frac{\partial D_B}{\partial x_A} \in [-1, 0]. \quad (\text{A.5})$$

*Proof of Lemma A.3.* Differentiating (13) with respect to  $x_A(t)$ , substituting  $x_A(T) + D_B(c + \tau_B(0, \Phi)e^{\rho T}) = s_B(T)$ , taking note of (12) with  $\tau_B(T) > 0$  and rearranging yields<sup>39</sup>

$$\frac{\partial \tau_B(0, \Phi)}{\partial x_A(t)} = -\frac{1}{\int_0^T D'_B(c + \tau_B(0, \Phi)e^{\rho t})e^{\rho t} dt} = \frac{\rho \tau_B(0, \Phi)}{D_B(c + \tau_B(0, \Phi))} > 0. \quad (\text{A.6})$$

Differentiating  $D_B(c + \tau_B(0, \Phi)e^{\rho t})$  with respect to  $x_A(t)$  and substituting (A.6) yields:

$$\begin{aligned} \frac{\partial D_B(c + \tau_B(0, \Phi)e^{\rho t})}{\partial x_A(t)} &= D'_B(c + \tau_B(0, \Phi)e^{\rho t})e^{\rho t} \frac{\partial \tau_B(0, \Phi)}{\partial x_A(t)} \\ &= -\frac{D'_B(c + \tau_B(0, \Phi)e^{\rho t})e^{\rho t}}{\int_0^T D'_B(c + \tau_B(0, \Phi)e^{\rho t})e^{\rho t} dt} \in [-1, 0]. \end{aligned} \quad (\text{A.7})$$

□

**Lemma A.4.** *Suppose the government of country A applies a demand side climate policy. Then, fuel consumption is continuous at  $t = t_1$ .*

*Proof of Lemma A.4.* Our proof follows the one of the Appendix of Kollenbach (2015b). Therefore, we give here a sketch and refer to Kollenbach (2015b) and Feichtinger and Hartl (1986, 166ff.) for a more detailed discussion.

At an entry point (where the ceiling becomes binding) the costate variable  $\theta_A$  may jump according to the condition

$$\theta_A^+(j) = \theta_A^-(j) + Y \frac{\partial [Z - \bar{Z}]}{\partial Z}, \quad Y \geq 0, \quad (\text{A.8})$$

with the superscripts  $+$  and  $-$  denoting the value just after and just before the junction point. We find

$$\theta_A^+(j) = \theta_A^-(j) + \mu_A^+(j) - \mu_A^-(j) + Y_\theta, \quad (\text{A.9})$$

with  $Y_\theta \geq 0$  as jump parameter.<sup>40</sup> At an entry point fossil fuel extraction is either constant or decreases. By applying (A.8) to  $\tau_B$ , we find that  $\tau_B$  is continuous, which implies the

<sup>39</sup> $\tau_B(t) = 0$  cannot be the equilibrium scarcity rent, as a fossil fuel firm can increase its profit by setting  $m > p > c$  at  $t = T$ .

<sup>40</sup>Note that we use the indirect approach with respect to optimization problems with state-space constraints.

continuity of  $D_B$ , so that  $x_A^+ \leq x_A^-$ . In case of a jump

$$\begin{aligned} U_A^{'+} &= c + \tau_B^+ + (\mu_A^+ - \theta_A^+) \left( 1 + \frac{\partial D_B^+}{\partial x_A} \right) + x_A^+ \frac{\partial \tau_B^+}{\partial x_A} \\ &> U_A'^- = c + \tau_B^- + (\mu_A^- - \theta_A^-) \left( 1 + \frac{\partial D_B^-}{\partial x_A} \right) + x_A^- \frac{\partial \tau_B^-}{\partial x_A} \end{aligned} \quad (\text{A.10})$$

holds. Substituting  $\tau_B^+ = \tau_B^-$ ,  $D_B^+ = D_B^-$  and (A.9) yields

$$(x_A^+ - x_A^-) \frac{\partial \tau_B}{\partial x_A} > Y_\theta \left( 1 + \frac{\partial D_B}{\partial x_A} \right). \quad (\text{A.11})$$

As  $\frac{\partial \tau_B}{\partial x_A} > 0$ ,  $x_A^+ \leq x_A^-$ ,  $Y_\theta \geq 0$  and  $\frac{\partial D_B}{\partial x_A} \in [-1, 0]$ , inequality (A.11) cannot hold, which rules out a jump of  $x_A$ .  $\square$

**Lemma A.5.** *Suppose the government of country A applies a demand side climate policy. Then, fuel consumption in country A declines over time during Phase I if  $D_B(p(t)) = \alpha(c + \tau_B(t))^{-\beta}$ , it increases over time during Phase II, and it declines over time during Phase III.*

*Proof of Lemma A.5.* Substituting (A.6), (A.7),  $\tau_B(t) = \tau_B(0)e^{\rho t}$ ,  $|\theta_A(t)| = |\theta_A(0)|e^{(\rho+\gamma)t}$  and  $\mu_A(t) = 0$  into (17), marginal utility of country A in Phase I reads

$$U_A'(t) = c + \tau_B(0)e^{\rho t} + |\theta_A(0)|e^{(\rho+\gamma)t} \left[ 1 + \frac{\dot{D}_B(p(t))}{D_B(p(0))} \right] + x_A(t) \frac{\partial \tau_B(0)}{\partial x_A(t)} e^{\rho t}. \quad (\text{A.12})$$

Differentiating with respect to  $t$  and rearranging yields

$$\begin{aligned} \left[ U_A''(t) - \frac{\partial \tau_B(t)}{\partial x_A(t)} \right] \frac{\partial x_A(t)}{\partial t} &= \rho \left[ \tau_B(t) + x_A(t) \frac{\partial \tau_B(t)}{\partial x_A(t)} \right] \\ &+ |\theta_A(t)| \left[ \rho + \gamma - \frac{\rho D_B(p(t)) - \gamma \dot{D}_B(p(t))}{D_B(p(0))} \right] \\ &+ |\theta_A(t)| \left[ \frac{\rho D_B(p(t)) + \rho \dot{D}_B(p(t)) + \ddot{D}_B(p(t))}{D_B(p(0))} \right]. \end{aligned} \quad (\text{A.13})$$

The bracketed term on the left-hand side is negative and first two lines on the right-hand side are positive, such that  $\frac{\partial x_A(t)}{\partial t}$  is negative in Phase III where  $|\theta_A(t)| = 0$ . If  $D_B(p(t)) = \alpha(c + \tau_B(t))^{-\beta}$ , then

$$\begin{aligned} &\rho D_B(p(t)) + \rho \dot{D}_B(p(t)) + \ddot{D}_B(p(t)) \\ &= \rho \alpha (c + \tau_B(t))^{-\beta-2} \left[ (1 + \beta^2 \rho - \beta \rho) (\tau_B(t))^2 + 2(1 - \beta \rho) c \tau_B(t) + c^2 \right], \end{aligned} \quad (\text{A.14})$$

which is positive for all  $\rho \in [0, 1]$ , such that  $\frac{\partial x_A(t)}{\partial t}$  is then also negative in Phase I. In Phase II,  $\frac{\partial x_A(t)}{\partial t} + \dot{D}_B(p(t)) = 0$ , such that  $\frac{\partial x_A(t)}{\partial t} = -\dot{D}_B(p(t)) > 0$ .  $\square$

**Lemma A.6.** *Suppose the government of country A applies a demand side climate policy. Then, fuel consumption in country A [B] falls short of [exceeds] its socially optimal value at every point in time during Phase II [Phase I and Phase II]. If the government ignores its strategic influence, cumulative fuel consumption in country A falls short of its socially optimal value in Phase I. Furthermore, fuel consumption in country A falls short of its socially optimal value at every point in time during Phase I if  $\tau_B(0) > \tau^*(0)$  or if  $\tau_B(0) \leq \tau^*(0)$  and  $\gamma$  is sufficiently low.*

*Proof of Lemma A.6.* First, consider the case in which the government ignores its strategic influence. Suppose  $\tau_B(0) < \tau^*(0)$ . Then,  $x_B(t) > x_B^*(t)$  for all  $t < T$ ,  $x_A(t) > x_A^*(t)$  during Phase III and  $s(t) = s^*(t) = \bar{s}$  during Phase II, such that  $\int_0^{\max[t_1, t_1^*]} x_A(t) dt < \int_0^{\max[t_1, t_1^*]} x_A^*(t) dt$  and  $\int_0^{\max[t_1, t_1^*]} s(t) dt < \int_0^{\max[t_1, t_1^*]} s^*(t) dt$  must hold.  $t_1 < t_1^*$  would imply that the extraction paths intersect only once and that  $s(0) > s^*(0)$ , such that  $\int_0^{t_1^*} s(t) dt > \int_0^{t_1^*} s^*(t) dt$  would hold. Consequently,  $t_1 > t_1^*$ . Finally, if  $\gamma$  is sufficiently low, then  $x_A(0) > x_A^*(0) \iff \tau_B(0) + |\theta_A(0)| < \tau^*(0) + |\theta^*(0)|$  would imply that  $\tau_B(t) + |\theta_A(t)|$  and  $\tau^*(t) + |\theta^*(t)|$  would not cut during Phase I and that the ceiling would be violated. Then,  $x_A(0) < x_A^*(0) \iff \tau_B(0) + |\theta_A(0)| > \tau^*(0) + |\theta^*(0)|$  must hold, which in turn implies  $x_A(t) < x_A^*(t)$  during Phase I.

Now suppose  $\tau_B(0) > \tau^*(0)$ . Then,  $x_i(t) < x_i^*(t)$  during Phase III and  $s(t) = s^*(t) = \bar{s}$  during Phase II, such that  $\int_0^{\max[t_1, t_1^*]} s(t) dt > \int_0^{\max[t_1, t_1^*]} s^*(t) dt$  must hold. This implies  $s(0) > s^*(0)$  by Lemma A.2, such that  $x_B(0) > x_B^*(0) \iff \tau_B(0) < \tau^*(0) + |\theta^*(0)|$  must hold, which in turn implies  $x_B(t) > x_B^*(t)$  during Phase I. Furthermore,  $x_A(0) < x_A^*(0) \iff \tau_B(0) + |\theta_A(0)| > \tau^*(0) + |\theta^*(0)|$  must hold to guarantee the ceiling, which in turn implies  $x_A(t) < x_A^*(t)$  during Phase I.

Now consider the case in which the government is aware of its strategic influence. Suppose  $\tau_B(0) < \tau^*(0)$  and  $\int_{t_2^*}^{\infty} s(t) dt > \int_{t_2^*}^{\infty} s^*(t) dt$ . This case is equivalent to the corresponding case without strategic effects. Now suppose  $\tau_B(0) < \tau^*(0)$  and  $\int_{t_2^*}^{\infty} s(t) dt < \int_{t_2^*}^{\infty} s^*(t) dt$  due to the fuel price effect, such that  $\int_0^{\max[t_1, t_1^*]} s(t) dt > \int_0^{\max[t_1, t_1^*]} s^*(t) dt$  must hold. Equivalent to the case of  $\tau_B(0) > \tau^*(0)$  without strategic effects,  $s(0) > s^*(0)$ . However,  $x_A(0) > x_A^*(0)$  and  $\int_0^{\max[t_1, t_1^*]} x_A(t) dt > \int_0^{\max[t_1, t_1^*]} x_A^*(t) dt$  can now hold without violating the ceiling.

Now suppose  $\tau_B(0) > \tau^*(0)$ . Equivalent to the corresponding case without strategic effects,  $s(0) > s^*(0)$  and  $x_B(t) > x_B^*(t)$  during Phase I. Furthermore,  $x_A(0) > x_A^*(0)$  and  $\int_0^{\max[t_1, t_1^*]} x_A(t) dt > \int_0^{\max[t_1, t_1^*]} x_A^*(t) dt$  can now hold if the strategic effects increase over

time, such that

$$\begin{aligned} & \tau_B(0) - \theta_A(0) \left( 1 + \frac{\partial x_B(0)}{\partial x_A} \right) + x_A(0) \frac{\partial \tau_B(0)}{\partial x_A} < \tau^*(0) - \theta^*(0) \\ \text{and } & \tau_B(t_1) - \theta_A(t_1) \left( 1 + \frac{\partial x_B(t_1)}{\partial x_A} \right) + x_A(t_1) \frac{\partial \tau_B(t_1)}{\partial x_A} > \tau^*(t_1) - \theta^*(t_1) \end{aligned}$$

can hold simultaneously for  $\tau_B(0) > \tau^*(0)$ .  $\square$

### A.3. Supply side policy

**Proof of Lemma 1.** During Phase II,  $s_B(t) > 0$  cannot hold because then  $\dot{s} = D'(c + \tau_B(0, \Psi)e^{\rho t})\rho\tau_B(0, \Psi)e^{\rho t} < 0$ , which contradicts  $\dot{s} = 0$  at the ceiling. Thus, there are only governmental supplies during Phase II. Furthermore,  $\bar{s} > D(c + \tau_B(0, \Psi)e^{\rho t_1})$  cannot hold because then the ceiling would not be binding. Thus, the governmental fuel price exceeds the private fuel price at  $t = t_1$ . Consequently,  $s_B(t_1) > 0$  if  $S(t_1) > 0$ . Therefore,  $S(t_1) = 0$  to guarantee the ceiling.  $\square$

### Proof of Lemma 2.

At every point in time, the price of the representative fossil fuel firm is given by (10), which describes a continuous function in time for  $0 \leq t < t_1$ . Assume that the governmental fuel price  $p_A(t)$  is also continuous for all  $t \in [0, t_1)$ . Only if  $p_A(t)$  equals or falls short of  $c + \tau_B(t)$ , governmental supplies  $s_A(t)$  can be positive. Thus, if  $s_A(t) > 0$  and  $s_B(t) = 0$  for some  $0 \leq t < T_B \leq t_1$ , the governmental price path is either located below  $c + \tau_B(t)$  for all  $t \in [0, T_B)$  or the price paths intersect at least once.

Suppose that  $s_A(t) > 0$  and  $s_B(t) = 0$  for  $t \in [t_a, t_b)$ , with  $0 \leq t_a < t_b < T_B$ . Then, the market clearing condition for Phase I is given by<sup>41</sup>

$$\begin{aligned} & \int_0^{t_a} D(c + \tau_B(0, \Psi)e^{\rho t}) dt + \int_{t_a}^{t_b} s_A(t) dt + \int_{t_b}^{T_B} D(c + \tau_B(0, \Psi)e^{\rho t}) dt \\ & = S(0) - \int_0^{T_B} [y(t) - s_A(t)] dt. \end{aligned} \tag{A.15}$$

Differentiating with respect to  $s_A(t)$  for  $t < T_B$  and using (20) yields

$$\left[ \int_0^{t_a} D'(p(t))e^{\rho t} dt + \int_{t_b}^{T_B} D'(p(t))e^{\rho t} dt \right] \frac{\partial \tau_B(0, \Psi)}{\partial s_A(t)} + \begin{cases} 0 & \text{if } t \notin [t_a, t_b) \\ 1 & \text{if } t \in [t_a, t_b) \end{cases} = 1. \tag{A.16}$$

<sup>41</sup>Note that the qualitative results would be the same if there were more than one time interval characterized by  $s_A(t) > 0$  and  $s_B(t) = 0$ .



If  $t \in [t_a, t_b)$ , rearranging gives  $\frac{\partial \tau_B(0, \Psi)}{\partial s_A(t)} e^{\rho t} = \frac{\partial p_y(t)}{\partial s_A(t)} = 0$ . Furthermore, from  $s_A(t) = D(p(t))$  and  $s_B(t) = 0$  we get  $\frac{\partial p(t)}{\partial s_A(t)} = \frac{1}{D'(p(t))} < 0$  and  $\frac{\partial s_B(t)}{\partial s_A(t)} = 0$ . If  $t \notin [t_a, t_b)$ , (A.16) gives

$$\frac{\partial \tau_B(0, \Psi)}{\partial s_A(t)} = \frac{1}{\int_0^{t_a} D'(p(t)) e^{\rho t} dt + \int_{t_b}^{T_B} D'(p(t)) e^{\rho t} dt} < 0, \quad (\text{A.17})$$

and, therefore,  $\frac{\partial p_y(t)}{\partial s_A(t)} = \frac{\partial p(t)}{\partial s_A(t)} = \frac{\partial \tau_B(0, \Psi)}{\partial s_A(t)} e^{\rho t} < 0$ . Using  $s_A(t) + s_B(t) = D(c + \tau_B(0, \Psi) e^{\rho t})$  yields

$$\frac{\partial s_B(t)}{\partial s_A(t)} = \frac{D'(p(t)) e^{\rho t}}{\int_0^{t_a} D'(p(t)) e^{\rho t} dt + \int_{t_b}^{T_B} D'(p(t)) e^{\rho t} dt} - 1 \in [-1, 0]. \quad (\text{A.18})$$

Differentiating (A.15) with respect to  $y(t)$  for  $t < T_B$  gives

$$\frac{\partial \tau_B(0, \Psi)}{\partial y(t)} = -\frac{1}{\int_0^{t_a} D'(p(t)) e^{\rho t} dt + \int_{t_b}^{T_B} D'(p(t)) e^{\rho t} dt} > 0, \quad (\text{A.19})$$

If  $t \in [t_a, t_b)$ , then  $\frac{\partial p_y(t)}{\partial y(t)} = \frac{\partial \tau_B(0, \Psi)}{\partial y(t)} e^{\rho t} > 0$  and  $\frac{\partial p(t)}{\partial y(t)} = \frac{\partial s_B(t)}{\partial y(t)} = 0$ . If  $t \notin [t_a, t_b)$ , then  $\frac{\partial p_y(t)}{\partial y(t)} = \frac{\partial p(t)}{\partial y(t)} = \frac{\partial \tau_B(0, \Psi)}{\partial y(t)} e^{\rho t} > 0$ . Using  $s_A(t) + s_B(t) = D(c + \tau_B(0, \Psi) e^{\rho t})$  yields

$$\frac{\partial s_B(t)}{\partial y(t)} = -\frac{D'(p(t)) e^{\rho t}}{\int_0^{t_a} D'(p(t)) e^{\rho t} dt + \int_{t_b}^{T_B} D'(p(t)) e^{\rho t} dt} \in [-1, 0]. \quad (\text{A.20})$$

□

**Lemma A.7.** *Suppose the government of country A applies a supply side climate policy and suppose that assumption 1 holds, then*

- (-)  $s_B(t) > 0$  for all  $t \in [0, t_1)$ ,
- (-) if  $\tau_B(0) \leq \tau^*(0) - \theta^*(0)$ , the fossil fuel price jumps upwards at  $t = t_1$ ,
- (-) if  $\tau_B(0) > \tau^*(0) - \theta^*(0)$ , the fossil fuel price is either continuous or jumps upwards at  $t = t_1$ .

*Proof of Lemma A.7.* Using Lemma 2 and  $s_A - x_A = D_B(p) - s_B$  in (37), we get

$$p_A = c + \tau_A + (\mu_A - \theta_A) - D_B(p_A) \frac{\partial p}{\partial s_A} \quad \text{for } s_B, y = 0, \quad (\text{A.21})$$

$$p_A = c + \tau_A + (\mu_A - \theta_A) \left( 1 + \frac{\partial s_B}{\partial s_A} \right) - [D_B(p_B) - s_B - (1 + \zeta_{\bar{\pi}})y] \frac{\partial p}{\partial s_A} \quad \text{for } s_B > 0. \quad (\text{A.22})$$

From  $s_A + s_B = D(p)$ , we infer  $\frac{\partial p}{\partial s_A} = \frac{1}{D'(p)} \left( 1 + \frac{\partial s_B}{\partial s_A} \right)$ . Using this in (A.21) and (A.22) yields

$$p_A = c + \tau_A + (\mu_A - \theta_A) - \frac{D_B(p_A)}{D'(p_A)} \quad \text{for } s_B, y = 0, \quad (\text{A.23})$$

$$p_A = c + \tau_A + (\mu_A - \theta_A) - \frac{D_B(p_B)}{D'(p_B)} - \Theta \quad \text{for } s_B > 0, \quad (\text{A.24})$$

where

$$\Theta = - \left[ (\mu_A - \theta_A) - \frac{D_B(p_B)}{D'(p_B)} \right] \frac{\partial s_B}{\partial s_A} - \frac{s_B + (1 + \zeta_{\bar{\pi}})y}{D'(p_B)} \left( 1 + \frac{\partial s_B}{\partial s_A} \right) > 0. \quad (\text{A.25})$$

Using  $\epsilon(p) = -\frac{D'(p)p}{D(p)}$  and  $D_B(p) = \frac{D(p)}{2}$  in (A.23) and (A.24) and rearranging yields

$$p_A = \frac{1}{1 - \frac{1}{2\epsilon(p_A)}} [c + \tau_A + (\mu_A - \theta_A)] \quad \text{for } s_B, y = 0, \quad (\text{A.26})$$

$$p_A = \frac{1}{1 - \frac{p_B}{p_A} \frac{1}{2\epsilon(p_B)}} [c + \tau_A + (\mu_A - \theta_A) - \Theta] \quad \text{for } s_B > 0, \quad (\text{A.27})$$

Due to Lemma 1 and  $s_B(T_B^-) > 0$ , there is a point in time  $t_0 \leq t_1$  where fossil fuel supply switches from private supplies to pure governmental supplies, i.e.  $s_B(t_0^-) > 0$  and  $s_A(t_0^+) > 0, s_B(t_0^+), y(t_0^+) = 0$ . Suppose that the price path does not jump upwards at  $t_0 < t_1$ . From (10), (A.23) and (A.24), we obtain

$$\frac{1}{1 - \frac{1}{2\epsilon(p_A^+)}} (c + \tau_A^+ - \theta_A^+) \leq c + \tau_B^- \leq \frac{1}{1 - \frac{p_B^-}{p_A^+} \frac{1}{2\epsilon(p_B^-)}} (c + \tau_A^- - \theta_A^- - \Theta^-). \quad (\text{A.28})$$

Since  $\tau_A^+ = \tau_A^-, \theta_A^+ = \theta_A^-$  and  $\Theta^- > 0$ , the first bracketed term is greater than the second bracketed term. Furthermore,  $p_B^- \leq p_A^-$  and assumption 1 imply that the first fraction is greater than or equal to the second fraction if  $p_A^+ \leq p_B^-$ . Thus, there is an upward jump of  $p$  at  $t = t_0$ . As long as  $t_0 < t_1$ , the representative fossil fuel firm could exploit this jump by setting  $p > c + \tau_B$  at  $t = t_0$  such that  $\tau_B$  could not be the equilibrium scarcity rent path. Consequently, a switch to pure governmental supplies before  $t_1$  is not possible.

At  $t = t_1$ ,  $s_B(t) > 0$  would violate the ceiling. Consequently, the price path is either continuous or jumps upwards at  $t = t_1$ . If it continuous, we obtain from (10), (A.9), (A.23) and (A.24)

$$\begin{aligned} & \frac{1}{1 - \frac{1}{2\epsilon(p_A^+)}} (c + \tau_A^+ + \mu_A^+ - \theta_A^+) = c + \tau_B^- \\ & \leq \frac{1}{1 - \frac{p_B^-}{p_A^+} \frac{1}{2\epsilon(p_B^-)}} (c + \tau_A^- + \mu_A^+ - \theta_A^+ + Y_\theta - \Theta^-). \end{aligned} \quad (\text{A.29})$$

The inequality holds if the jump parameter  $Y_\theta$  is sufficiently high. Thus, an upward jump can but need not occur. It definitely occurs if  $\tau_B(0) \leq \tau^*(0) - \theta^*(0)$ . Else, the ceiling would be violated.  $\square$

**Lemma A.8.** *Suppose the government of country A applies a supply side climate policy and suppose that assumption 1 holds. Then, fuel production declines over time during Phase I for  $s_B(t), y(t) = 0$  and during Phase III.*

*Proof of Lemma A.8.* Substituting  $U'_i = p_A$ ,  $\tau_A(t) = \tau_A(0)e^{\rho t}$ ,  $\mu_A(t) = 0$  and  $|\theta_A(t)| = |\theta_A(0)|e^{(\rho+\gamma)t}$  into (A.23) yields

$$U'_i = c + \tau_A(0)e^{\rho t} + |\theta_A(0)|e^{(\rho+\gamma)t} - \frac{D_B(p(t))}{D'(p(t))}. \quad (\text{A.30})$$

Differentiating with respect to  $t$  and substituting  $U''_i = \frac{1}{D'_i}$ ,  $\frac{\partial p}{\partial s_A} = \frac{1}{D'}$  and  $\frac{\partial s_A(t)}{\partial t} = \frac{\partial x_A(t)}{\partial t} + \frac{\partial x_B(t)}{\partial t}$  yields

$$\frac{1}{D'_i} \frac{\partial x_i(t)}{\partial t} = \rho \tau_A(0)e^{\rho t} + (\rho + \gamma)|\theta_A(0)|e^{(\rho+\gamma)t} - \frac{D'_B D' - D_B D''}{(D')^2} \frac{1}{D'} \left( \frac{\partial x_A(t)}{\partial t} + \frac{\partial x_B(t)}{\partial t} \right). \quad (\text{A.31})$$

Solving for  $\frac{\partial x_i(t)}{\partial t}$  and using  $\epsilon = -\frac{D'p}{D}$ ,  $D_B = \frac{D}{2}$  and  $D'_B = \frac{D'}{2}$  yields

$$\frac{\partial x_i(t)}{\partial t} = \frac{D'_i [\rho \tau_A + (\rho + \gamma)|\theta_A|]}{1 + \frac{D'_B D' - D_B D''}{(D')^2}} = \frac{D'_i [\rho \tau_A + (\rho + \gamma)|\theta_A|]}{1 - \frac{1}{2\epsilon} \left( 1 - \frac{\partial \epsilon}{\partial p} \frac{p}{\epsilon} \right)}. \quad (\text{A.32})$$

The denominator is positive by  $1 - \frac{1}{2\epsilon} > 0$  and assumption 1. The numerator is negative. Thus, fuel consumption declines over time during Phase I for  $s_B(t), y(t) = 0$  and during Phase III where  $\theta_A(t) = 0$ .  $\square$

**Lemma A.9.** *Suppose the government of country A applies a supply side climate policy and suppose that assumption 1 holds, then*

- (-) *the only possible phase-sequence which includes all three phases reads Phase I, Phase II, Phase III,*
- (-) *fuel production is continuous at  $t = t_2$ .*

*Proof of Lemma A.9.* At the end of Phase II the economy switches either to Phase III or to Phase I at a  $t_2$  or  $t_3$  junction point. Assume that the price path jumps upwards at  $t = t_2$  or  $t = t_3$ . From (A.23),  $\tau_A^+ = \tau_A^-$ ,  $\theta_A^+ = \theta_A^-$  and  $\mu_A^+ = 0$  we obtain

$$\frac{1}{1 - \frac{1}{2\epsilon(p_A^+)}} (c + \tau_A^- - \theta_A^-) > \frac{1}{1 - \frac{1}{2\epsilon(p_A^-)}} (c + \tau_A^- + \mu_A^- - \theta_A^-). \quad (\text{A.33})$$

By (29), the first bracketed term is smaller than or equal to the second bracketed term. Furthermore, assumption 1 implies that the first fraction is smaller than or equal to the second fraction if  $p_A^+ > p_A^-$ . Thus, there is no upward jump of  $p$  at  $t_2$  or  $t_3$ . Since a downward jump would violate the ceiling, fuel production is continuous at  $t_2$  or  $t_3$ . By Lemma A.8, fuel production declines over time during Phase I for  $s_B(t), y(t) = 0$  and during Phase III. Since the ceiling was binding until the end of Phase II, it will not bind in the future. Consequently, the economy switches to Phase III.  $\square$

## References

- Amigues, J.-P., Lafforgue, G., Moreaux, M., 2014. Optimal timing of CCS policies with heterogeneous energy consumption sectors. *Environmental and Resource Economics* 57 (3), 345–366.
- Amigues, J.-P., Moreaux, M., Schubert, K., 2011. Optimal use of a polluting non-renewable resource generating both manageable and catastrophic damages. *Annals of Economics and Statistics/Annales d'Économie et de Statistique* 103/104, 107–141.
- Bertoletti, P., Etro, F., 2017. Monopolistic competition when income matters. *The Economic Journal* 127 (603), 1217–1243.
- Burniaux, J.-M., Martins, J. O., 2012. Carbon leakages: a general equilibrium view. *Economic Theory* 49 (2), 473–495.
- Chakravorty, U., Leach, A., Moreaux, M., 2012. Cycles in nonrenewable resource prices with pollution and learning-by-doing. *Journal of Economic Dynamics and Control* 36 (10), 1448–1461.
- Chakravorty, U., Magné, B., Moreaux, M., 2006. A Hotelling model with a ceiling on the stock of pollution. *Journal of Economic Dynamics and Control* 30 (12), 2875–2904.
- Chakravorty, U., Moreaux, M., Tidball, M., 2008. Ordering the extraction of polluting non-renewable resources. *The American Economic Review* 98 (3), 1128–1144.
- Copeland, B. R., Taylor, M. S., 2005. Free trade and global warming: a trade theory view of the Kyoto protocol. *Journal of Environmental Economics and Management* 49 (2), 205–234.
- Dixit, A. K., Stiglitz, J. E., 1977. Monopolistic competition and optimum product diversity. *American Economic Review* 67 (3), 297–308.
- Dullieux, R., Ragot, L., Schubert, K., 2011. Carbon tax and OPEC's rents under a ceiling constraint. *The Scandinavian Journal of Economics* 113 (4), 798–824.
- Eichner, T., Pethig, R., 2011. Carbon leakage, the green paradox, and perfect future markets. *International Economic Review* 52 (3), 767–805.
- Eichner, T., Pethig, R., 2013. Flattening the carbon extraction path in unilateral cost-effective action. *Journal of Environmental Economics and Management* 66 (2), 185–201.
- Eichner, T., Pethig, R., 2017a. Buy coal and act strategically on the fuel market. *European Economic Review* 99, 77–92.
- Eichner, T., Pethig, R., 2017b. Trade in fossil fuel deposits for preservation and strategic action. *Journal of Public Economics* 147, 50–61.
- Feichtinger, G., Hartl, R., 1986. *Optimale Kontrolle ökonomischer Prozesse: Anwendungen des Maximumprinzips in den Wirtschaftswissenschaften*. Walter de Gruyter.
- Groot, F., Withagen, C., De Zeeuw, A., 2003. Strong time-consistency in the cartel-versus-fringe model. *Journal of Economic Dynamics and Control* 28 (2), 287–306.
- Harstad, B., 2012. Buy coal! a case for supply-side environmental policy. *Journal of Political Economy* 120 (1), 77–115.

- Henriet, F., 2012. Optimal extraction of a polluting nonrenewable resource with R&D toward a clean backstop technology. *Journal of Public Economic Theory* 14 (2), 311–347.
- Hoel, M., 1996. Should a carbon tax be differentiated across sectors? *Journal of Public Economics* 59 (1), 17–32.
- Hoel, M., 2011. The supply side of CO<sub>2</sub> with country heterogeneity. *The Scandinavian Journal of Economics* 113 (4), 846–865.
- IPCC, 2013. Climate change 2013: The physical science basis. Online at: <https://www.ipcc.ch/report/ar5/wg1>.
- Kollenbach, G., 2015a. Abatement, r&d and growth with a pollution ceiling. *Journal of Economic Dynamics and Control* 54, 1–16.
- Kollenbach, G., 2015b. Endogenous growth with a ceiling on the stock of pollution. *Environmental and Resource Economics* 62 (3), 615–635.
- Krugman, P. R., 1979. Increasing returns, monopolistic competition, and international trade. *Journal of International Economics* 9 (4), 469–479.
- Lafforgue, G., Magné, B., Moreaux, M., 2008. Energy substitutions, climate change and carbon sinks. *Ecological Economics* 67 (4), 589–597.
- Lafforgue, G., Magné, B., Moreaux, M., 2009. Optimal sequestration policy with a ceiling on the stock of carbon in the atmosphere. In: Guesnerie, R., Tulkens, H. (Eds.), *The Design of Climate Policy*. MIT Press, pp. 273–306.
- Mrázová, M., Neary, J. P., 2017. Not so demanding: Demand structure and firm behavior. *American Economic Review* 107 (12), 3835–3874.
- Rubio, S. J., 2006. On coincidence of feedback nash equilibria and stackelberg equilibria in economic applications of differential games. *Journal of Optimization Theory and Applications* 128 (1), 203–221.
- Sinn, H., 2008. Public policies against global warming: a supply side approach. *International Tax and Public Finance* 15 (4), 360–394.
- Smulders, S., Van der Werf, E., 2008. Climate policy and the optimal extraction of high-and low-carbon fossil fuels. *Canadian Journal of Economics/Revue canadienne d'économie* 41 (4), 1421–1444.
- Tsur, Y., Zemel, A., 2009. Endogenous discounting and climate policy. *Environmental and Resource Economics* 44 (4), 507–520.
- UN, 1992. United nations framework convention on climate change. Online at: <https://unfccc.int/resource/docs/convkp/conveng.pdf>.
- UN, 2015. Paris agreement. Online at: [http://unfccc.int/files/essential\\_background/convention/application/pdf/english\\_paris\\_agreement.pdf](http://unfccc.int/files/essential_background/convention/application/pdf/english_paris_agreement.pdf).
- UN, 2017. The emissions gap report. Online at: [https://wedocs.unep.org/bitstream/handle/20.500.11822/22070/EGR\\_2017.pdf](https://wedocs.unep.org/bitstream/handle/20.500.11822/22070/EGR_2017.pdf).