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The problem of transition from school to university mathematics.

A) Short Description

A survey amongst preservice teachers in Germany shows that the transition from school to university mathematics is experienced in the context of a major revolution regarding their views about the *nature of mathematics*. Motivated by the survey the author presents a concept for an undergraduate course helping to bridge the gap.

1.) Motivation: a survey

Around 250 preservice secondary school teachers from German universities have been asked for their retrospective view on their way from school to university mathematics. Surprisingly, a systematic qualitative content analysis based on an independent coding process of the data (Mayring 2002; Huberman & Miles 1994) shows that to a substantial extent, students have more problems with a feeling of "differentness" of school and university mathematics than with the abrupt rise in content-specific requirements. This "differentness" is specified by the students in the open parts of the questionnaire with terms as *vividness, references to everyday life, applicability to the real world, ways of argumentation, mathematical rigor, axiomatic design etc.*.

Using additionally results of studies with a similar interest (e.g. Gruenwald et al. 2004; Hoyles et al. 2001) the author comes to the preliminary conclusion that preservice teachers clearly distinguish between school and university mathematics regarding the *nature of mathematics*. This sets the framework for further research concerning the problem of transition: following the idea of constructivism in mathematics education, students construct their own picture of mathematics with the material, problems and stimulations teachers provide in the classroom respectively lecture hall (Anderson, et al. 2000; Bauersfeld 1992). Thus it is helpful to reconstruct the *nature of mathematics* communicated explicitly and implicitly in highschool and university textbooks, lecture notes, standards etc. with a special focus on differences.

2.) Different views about the nature of mathematics in school and university

Authors of prominent textbooks (in Germany as well as in the U.S.) for beginners at university level often depict mathematics in quite a formal rigorous way oriented towards the formalistic concept of mathematics of Hilbert. For example in the preface of Abbott's popular book for undergraduate students "Understanding Analysis" it becomes very clear where mathematicians see a major difference between school and university mathematics: "Having seen mainly graphical, numerical, or intuitive arguments, students need to learn what constitutes a rigorous proof and how to write one" (Abbott 2000, p. vi).

In fact if we look at the current NCTM standards and prominent school books we see that for good reasons, mathematics is taught differently at school: The "process standards" of the NCTM and in particular "connections" and "representations" (which are comparable to similar mathematics standards in Germany) focus on different aspects. At school it is important that students "recognize and apply mathematics in contexts outside of mathematics" or "use representations to model and interpret physical, social, and mathematical" (NCTM 2013).

The prominent place of illustrative material and visual representations in the mathematics classroom has important consequences for the students' views about the *nature of mathematics*. Schoenfeld proposes that students acquire an *empiricist belief system* of mathematics at school (Schoenfeld 1985 & 2011). This is caused by the fact that mathematics in modern classrooms does not describe abstract entities but a universe of discourse ontologically bounded to "real objects": *Probability Theory* is bounded to random experiments from everyday life, *Fractional Arithmetic* to "pizza models", *Geometry* to straightedge and compass constructions, *Analytical Geometry* to vectors as arrows, *Calculus* to functions as curves (graphs) ...). If we contrast this *empiricist belief system* students acquire in classroom with the *formalist belief system* students are supposed to learn at universities we see that major differences stand in their way: For example the notion of proof differs substantially in school and university mathematics. Whereas at universities (especially in pure mathematics) only formal deductive reasoning is acceptable, non-rigorous proofs relying on "graphical, numerical and intuitive arguments" are an essential part of proofs in school mathematics where we explain phenomena of the "real world". In the terminology of Sierpinska (1992), students at this point have to overcome a variety of "epistemological obstacles", requiring a big change in their understanding of what mathematics is about.

3.) Seminar conception helping to bridge the gap

The findings of 1.) & 2.) are essential for the author's design of a course for preservice teachers which will be taught for the first time in spring 2014. The aim is to make students aware of crucial changes regarding the nature of mathematics from school to university (by discussing transcripts, textbooks, standards, historical sources etc.) caused by different ways of teaching. The conceptual design of the course draws upon positive experience with explicit approaches regarding changes in the *belief system* of students from science education (esp. “Nature of Science”, cp. Abd-El-Khalick & Lederman 2001). A central theme of the seminar will be the historical and philosophical development of mathematics especially at the beginning of the last century. The plan is to take advantage of the idea that students on an individual level have to overcome similar problems as mankind had in history: the author is convinced that on an epistemological level, students can learn e.g. from aspects of the foundational crisis of mathematics at the beginning of the 20thcentury (cp. Davis et. al. 1995).

Before this crisis, mathematics was to a great extent developed in the sense of empirical sciences (e.g. Euclidean Geometry describes our physical space) – which is for good reasons close to how students experience and learn mathematics at school today. In contrast, with the emergence of Non-Euclidean Geometries, questions about the *nature of mathematics*arose which paved the way for Hilbert’s formalistic point of view – forming for comprehensible reasons a framework for pure mathematics at university level today.

The discussion of selected examples of historical examples taken from Euclid’s Geometry, Leibniz’ Differential Calculus or Hilbert’s Fundations of Geometry thereby serves two purposes. On the one hand it helps to understand why the *nature of mathematics* changed dramatically in history from empiricism towards formalism giving undergraduate students crucial insights regarding their further individual mathematics’ biography. On the other hand history provides us with a variety of beautiful pieces of substantial mathematics which was developed in an empirical manner (describing the physical world around us) providing prospective teachers with material which shows how to do it right in school (cp. Ernest 1998, Struik 1986).

Taking all these aspects into consideration it is not the author’s aim to solve the problem of transition by equalizing school mathematics and university mathematics. In contrast the focus lies on raising awareness regarding the epistemological obstacles which are caused by different ways of teaching at school and university. This is connected with the hope that the knowledge about these obstacles will help more students to bridge the gap and supports them to develop an adequate perspective regarding the nature of mathematics in classroom and lecture hall.

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