Workshop

Geomathematics Meets Medical Imaging

(GEMMI 2017)

5–8 September 2017

Hotel Speyer am Technik Museum

Speyer, Germany

Conference Guide

The workshop is organized by the Geomathematics Group Siegen

`http://www.gemmi2017.de/`   `gemmi2017@uni-siegen.de`

About GEMMI 2017

The occurring problems and challenges in geomathematics and in medical imaging as well as the methodologies for solving them have a lot of common aspects. It is not only the almost spherical shape of the objects under investigation which interconnects these fields of research. Also inverse problems play a central role in both areas with all its consequences like the need for regularization methods and the analysis of big data sets with spatial and temporal multiscale features.

The purpose of this workshop is to bring together researchers working in geomathematics or medical imaging. It is expected to inspire new research projects and to encourage new collaborations. It addresses geoscientists and applied scientists from medical imaging with interests in research on mathematical methodologies. In the same manner, mathematicians working in any of these applied areas are invited to participate in the workshop.

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1. Programme

Invited speakers are marked with an asterisk ($*$) next to their name.

For multiple authors, the speaker's name is underlined.

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Recent progress in shearlet theory

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Coffee break

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Efficient minimization of Tikhonov functionals with discrete TV and TGV penalties

16:30–17:00 *Jakob Alexander Geppert* *(p. 18)*
Improving phase retrieval via compression in Fresnel domain

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Parameter choice methods for fast multipole accelerated spline approximation

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Study on parameter choice methods for the RFMP with respect to downward continuation

11:45–12:15 *Sina Bittens* *(p. 15)*
A deterministic sparse FFT for functions with structured Fourier sparsity

12:30
Lunch at the conference site

13:30 *Social programme* *(p. 11)*
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16:00–16:30
Coffee break

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17:15–17:45 *Sarah Orzlowski and Volker Michel* *(p. 26)*
Inversion of electric, gravitational, and magnetic data in medical imaging or Earth sciences—What can we learn from these closely related problems?

17:45–18:15 *Naomi Schneider* *(p. 29)*
Vectorial Slepian functions on the ball

18:15–18:45 *Katrin Seibert* *(p. 30)*
Spin-weighted spherical harmonics and their application for the construction of tensor Slepian functions on the spherical cap

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Coffee break

Chair: Willi Freeden

10:45–11:15 *Amna Ishtiaq* *(p. 20)*
Construction and analysis of well-distributed quadrature points on the ball

11:15–11:45 *Christian Gerhards* *(p. 18)*
On the effect of localization constraints in some inverse problems in geomagnetism

11:45–12:15 *Roger Telschow* *(p. 32)*
Extraction of tidal signals from the geomagnetic field with adapted basis functions

12:30
Lunch at the conference site

Chair: Otmar Scherzer

13:30–14:15 *Peter Maaß* (∗) *(p. 24)*
Machine learning for inverse problems

14:15–14:45 *Max Kontak* *(p. 22)*
Two modifications of the RFMP algorithm for ill-posed inverse problems

14:45–15:15 *Anne Wald* *(p. 33)*
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15:15–15:45
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15:45 *Social programme* *(p. 11)*
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19:00 *Social programme* *(p. 12)*
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09:30–10:15 *Karin Sigloch* (∗) *(p. 31)*
Seismic tomography on a planetary scale: sensitivity modelling, parametrization and regularization.

10:15–10:45
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10:45–11:15 *Michael Quellmalz* *(p. 27)*
An SVD in spherical surface wave tomography

11:15–11:45 *Isabel Michel, Jörg Kuhnert, Fabian Nick, and Bram Metsch* *(p. 25)*
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11:45–12:00 *Volker Michel*
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2. Information about the social programme

Tour of the *Technik Museum Speyer*

> "At the *Technik Museum Speyer* you can wander around inside an original jumbo jet, as well as inspect the inner workings of a submarine and a sea rescue cruiser. In the halls you will find the largest space flight exhibition in Europe, offering the Russian space shuttle BURAN, an original Moonstone, space suits, a Soyuz landing capsule, as well as locomotives, vintage cars, fire trucks, motorcycles, and thousands of other exhibits. In the Museum Wilhelmsbau you will experience mechanical musical instruments, fashions, and dolls. Unique in Germany, the IMAX®-DOME movie theatre presents exclusive films on a giant dome screen." (`https://speyer.technik-museum.de/en/`)

We will have a guided tour of the museum. The admission fee will be approximately 20 € per person and has to be paid separately.

Tour of the city of Speyer

> "Travel through time in a city with more than 2000 years of history. Unique historic attractions, romantic lanes, lively squares and promenades offer new discoveries at every turn." (`http://www.speyer.de/sv_speyer/de/Tourismus/Service/Urlaubsplaner_2017.pdf`)

We will have a guided tour of the old town with visits to the most significant historic attractions. The tour will cost around 10 € and has to be paid separately.

Conference dinner

The conference dinner will be held on

Thursday, 7 September 2017 at 19:00

at the restaurant *Il Rustico* (Rheintorstraße 5, 67346 Speyer).

Order of courses

starter	Variation of Italian antipasti	
main dish	meat:	braised beef in chianti-red-wine sauce with rosemary potatoes
	fish:	grilled tuna steak with green lentil salad in a lemon-balsamico dressing
	vegetarian:	potato gnocchi stuffed with porcini and tossed in tomato-cream sauce
dessert	Tiramisu	

The price depends on the chosen main dish:

meat:	34.50 €
fish:	35.50 €
vegetarian:	25.80 €

Participation is only possible, if you have already registered for the conference dinner. Drinks are not included in the prices. The dinner will have to be paid on site.

Note that you have already picked an option for the main dish with your registration.

Directions

Whether you join the guided tour of the city of Speyer or not, you are welcome to meet us in front of the cathedral at 18:45 to walk to the restaurant together.

If you prefer to go by yourself, here are some instructions on how to get to the restaurant.

From the cathedral:

- start in front of the main entrance of the cathedral, stand near the big bowl such that the cathedral is to your right
- walk straight ahead for a bit more than 100m, then turn right into the “Tränkgasse” (it’s the next opportunity after you left the cathedral)
- turn left at the next opportunity (Sonnengasse) and walk straight ahead on the bridge to cross the beck/creek (Speyerbach)
- turn right at the next opportunity (Rheintorstraße)
- the last house on the left-hand side is the restaurant

From the conference site:

- starting from the large car park between the museum and the hotel, walk towards the street “Geibstraße” (you see the museum on the left-hand side and the cathedral is approximately in front of you)
- turn left to enter the street and soon after that turn right again to walk below the bridge
- walk along the street for a while (you pass by the cathedral which is on the left-hand side)
- at the last opportunity before you would have to cross the rail track, turn left (there is also a sign “Hotel 1735”)
- you head directly towards the restaurant

3. Abstracts

A deterministic sparse FFT for functions with structured Fourier sparsity

Sina Bittens
Institute for Numerical and Applied Mathematics, University of Göttingen

In [4, 5], a deterministic combinatorial Fourier algorithm for estimating the best k-term Fourier representation for a given frequency sparse signal, relying heavily on the Chinese Remainder Theorem and combinatorial concepts, was introduced. Its runtime is sublinear in the input length and scales quadratically in the sparsity k. In [2], the algorithm from [4, 5] was adapted for input functions with short frequency support, achieving a runtime that is sub-quadratic in the sparsity. A similar setting was also considered in [3], where a different deterministic sublinear Fourier algorithm employing periodizations of the signal was proposed.

Inspired by these approaches, and using ideas from [4, 5], we develop a deterministic sparse Fourier algorithm for input functions where we generalize the setting of a short frequency support to a *structured frequency support*, especially allowing a support consisting of multiple long intervals. Our method then achieves a runtime that is sublinear in the input length and scales sub-quadratically in the sparsity, which, for the class of functions considered, has so far only been achieved by randomized techniques.

This talk is based on [1], which is joint work with Mark A. Iwen and Ruochuan Zhang, Department of Mathematics, Michigan State University.

References

[1] S. Bittens, R. Zhang and M. A. Iwen. A Deterministic Sparse FFT for Functions with Structured Fourier Sparsity.

http://arxiv.org/abs/1705.05256, 2017.

[2] S. Bittens. Sparse FFT for Functions with Short Frequency Support. *Dolomites Research Notes on Approximation*, 2017. To appear.

[3] G. Plonka and K. Wannenwetsch. A deterministic sparse FFT algorithm for vectors with small support. *Numerical Algorithms*, 71(4):889-905, 2016.

[4] M. A. Iwen. Combinatorial Sublinear-Time Fourier Algorithms. *Found. Comput. Math.*, 10(3):303-338, 2010.

[5] M. A. Iwen. Improved Approximation Guarantees for Sublinear-Time Fourier Algorithms. *Appl. and Comp. Harmonic Analysis*, 34(1):57-82, 2013.

A region based easy path wavelet transform for sparse image representation

Renato Budinich
Institute for Numerical and Applied Mathematics, University of Göttingen

In [1] G. Plonka proposed an innovative method for sparse image representation, consisting of successively finding a path in the image with a greedy procedure and applying one level of a one dimensional wavelet transform. This method applied to natural images gives better results than the typical tensor product wavelet transform. However, when interested in image compression, there are additional adaptivity costs: for each level one has to store the path, i.e. a permutation of the pixel points.

This is the point we wish to improve on, by first applying a segmentation method to the image which partitions it into regions of low gray value variation. In each region we then find a path according to some geometrical rules (i.e. not depending on the values of the pixels) and then apply a one-dimensional wavelet transform to the image vectorized along the glued paths. This method is particularly well suited to encode a Region of Interest in the image with different quality than the rest of the image.

References

[1] Gerlind Plonka. The easy path wavelet transform: A new adaptive wavelet transform for sparse representation of two-dimensional data. *Multiscale Modeling & Simulation*, 7(3):1474–1496, 2009.

Recent progress in shearlet theory

Stephan Dahlke
Philipps-University of Marburg, Germany

Shearlets are recently developed affine representation systems that can very efficiently be used to resolve directional information in signals. Among other directional representation systems, the shearlet approach stands out since it is related with a square–integrable representation of a specific groups, the full shearlet group. This property paves the way to the combination of the famous coorbit space theory derived by Feichtinger and Gröchenig with the shearlet approach. By proceeding this way, new families of smoothness spaces, the shearlet coorbit spaces, can be established. In this talk, we will give an overview concerning the construction and the structural properties of these new regularity spaces. Moreover, we will discuss the applications of the shearlet transform to geophysical problems. In particular, we will explain how the shearlet transform can be used in seismic channel boundary detection. Applications of shearlets to image registration problems in medical image processing will also briefly be sketched.

Gravimetry and exploration: geomathematical background and geothermal applications

Willi Freeden
University of Kaiserslautern

This lecture deals with the characteristic ill-posed features of transferring input gravitational information in the form of Newtonian volume integral values to geological output characteristics of the density contrast function. Some properties of the Newton volume integral are recapitulated. Different methodologies of the resolution of the (inverse) gravimetry problem and their numerical implementations are examined dependent on the data source. Three cases of input information may be distinguished, namely internal (borehole), terrestrial (surface), and/or external (spaceborne) gravitational data sets. Singular integral theory based inversion of the Newtonian integral equation such as Haar-type solutions are proposed in a multi-scale framework to decorrelate specific geological signal signatures with respect to inherently available features. Reproducing kernel Hilbert space regularization techniques are studied (together with their transition to mollified variants) to provide geological contrast density distributions by downward continuation from terrestrial and/or spaceborne data. Finally, reproducing kernel Hilbert space solutions are formulated for use of gravimeter data, independent of a specifically chosen input area, i.e., in whole Euclidean space.

Improving phase retrieval via compression in Fresnel domain

Jakob Alexander Geppert
Institute of Numerical and Applied Mathematics, Universität Göttingen

We consider the problem of phase retrieval of two-dimensional objects that occurs in X-ray microscopy, where we can only measure the intensities of the image diffraction which is approximated by the Fresnel transform in the near field. In order to solve this phase reconstruction problem, we need to employ further a-priori knowledge about the object image. Using ideas from image processing, we can assume that the desired image is governed by anisotropic features and can be sparsely represented by a shearlet transform. However, this is no longer true for the holographic image in the Fresnel domain. Therefore, we need to modify the discrete shearlet transform suitably. Our new approach is based on the Fast Discrete Shearlet Transform (FDST, [Kutyniok, Lim, Zhuang 2012]) using shearlet elements with compact support in frequency domain. The new modification is then obtained by applying a convolution of the underlying Meyer scaling function and wavelet with the Fresnel kernel.

Numerical experiments with single-step algorithms based on the Transport of Intensity Equation show that the new Fresnel shearlet soft thresholding procedure improves the image recovery remarkably.

On the effect of localization constraints in some inverse problems in geomagnetism

Christian Gerhards
University of Vienna, Computational Science Center

We start by an overview on some inverse problems that arise with the processing of geomagnetic satellite data: the reconstruction of magnetizations and the separation of the overall magnetic field into core and crustal contributions. These problems are typically non-unique. We show what can be gained by the additional assumption that the underlying magnetization is localized in some subregion of the spherical Earth's surface. Namely, many of the introduced inverse problems then yield uniqueness and the additional assumption of localization allows the derivation of some alternative reconstruction procedures.

Parameter choice methods for fast multipole accelerated spline approximation

Martin Gutting
Geomathematics Group, University of Siegen

Solutions of boundary value problems, e.g. in geoscience, can be constructed by using localizing harmonic trial functions. When dealing with discrete data the problem can be interpreted as a generalized interpolation problem on a regular surface. Harmonic splines in our approach are based on a Runge sphere, but can deal with complicated regular surfaces like the Earth. Due to their localizing properties regional modeling is possible and global models can be improved in a part of the Earth's surface.

Fast multipole methods have been developed for certain cases of the occurring kernels to obtain a fast matrix-vector multiplication. This reduces the numerical effort of the matrix-vector multiplication from quadratic to linear in reference to the number of points for a prescribed accuracy of the kernel approximation while inducing a small error due to the approximation of the kernel.

The approach also allows the treatment of noisy data, but this requires the choice of the smoothing parameter of a functional similar to the Tikhonov functional in the regularization of ill-posed problems. We investigate different methods to (ideally automatically) choose this parameter with and without prior knowledge of the noise level. The performance of these methods is considered for different noise scenarios in a simulation study with regard to applications in gravitational field modeling and boundary value problems where the boundary is the Earth's surface. An essential requirement for the parameter choice method is that it has to be compatible with the fast matrix vector multiplication, i.e. that the numerical effort stays of linear order.

Further applications can be ill-posed problems arising in geoscience (downward continuation of satellite data) or other applications where (generalized) interpolation problems on sphere-like surfaces come into play.

Validation of inversion techniques in EEG and MEG imaging using realistic head models and real data

A.S. Fokas and Parham Hashemzadeh
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, CB3 0WA, UK

The inverse problems for EEG and MEG involve the estimation of the neuronal current from the knowledge of the above electric potential and magnetic flux, respectively. These inverse problems are highly non-unique, namely *different* electric currents can produce *identical* electric potentials, and magnetic flux. The fundamental questions are which part of the current affects the electric potential on the scalp and which part of the current affects the magnetic flux outside the head? The definitive answer to this question was finally obtained in [1]. It is well known that these inverse problems are linear. Here, we present an algorithmic procedure for constructing the relevant inversion matrices on a finite elmenent mesh (FE) of realistic head models. This step involves the accurate solution of a boundary value problem to obtain the function values of the geometry dependent auxiliary functions at the nodes of the FE mesh. The validation step involves constructing a linear Bayesian estimator to estimate missing data at locations where there are no physical sensors. This estimator has *only* two ingredients, namely: (i) The relevant elements of the inversion matrix and (ii) partially observed data. In a cross-validation strategy, the estimator proves that the inversion formulation presented in [1] is an accurate model of the underlying physics.

A common practice in EEG imaging is to assume that the neuronal current is supported on a two dimensional surface, namely the cortex. To this end, we present an analogue formulae to the one derived in [1], for computing the potential on the scalp. Regarding this formula we note that in addition to the irrotational component of the current, there also exists an explicit dependency of the normal component of the current. The numerical results show an excellent agreement between the formulae derived in [1] and the one derived for the $2D$ surface, for the case when the current is supported on a very thin shell.

The inverse problem formulation for MEG in [1] is extended for magnetometers and gradiometers which may not be strictly oriented radially. Through a rigorous mathematical formulation, it is shown that components of the neuronal current to be reconstructed can be expressed in terms of a single scalar function.

References

[1] Fokas A.S 2009. Electro-magneto-encephalography for a three-shell model: distributed current in arbitrary, spherical and ellipsoidal geometries. *J.R.Soc. Interface* **6**: 479-488.

Construction and analysis of well-distributed quadrature points on the ball

Amna Ishtiaq
Geomathematics Group, University of Siegen

The distribution of a point set and the numerical integration on a given domain are well connected. A well-distributed set of points on the domain of interest is supposed to assure the best possible outcomes for the numerical integration on that domain. Using this approach for distributing points on the 3-dimensional ball, we target two aspects: Firstly, finding the most suitable and efficient way of distributing points on the ball and secondly, measuring how uniform the resulting distribution is. Given a configuration on the ball, the Gaussian quadrature rule helps us answering the second question by giving us a quantifying criterion for the given points. These targets, at the same time, aim to reduce the integral error for the given distribution on the ball. By virtue of this connection, the results cannot only be used in the computational fields (for problems with numerical integration) but also in the fields of geosciences and medicine (for nice configurations on the ball). A set of nicely distributed points on domains like a ball has applications in inverse MEG/EEG and other inverse problems, also from the geosciences.

References

[1] A. Ishtiaq, V. Michel: *Pseudodifferential operators, cubature and equidistribution on the 3D-ball – an approach based on orthonormal basis systems.* Numerical Functional Analysis and Optimization, 38:891-910, 2017.

Two modifications of the RFMP algorithm for ill-posed inverse problems

Max Kontak
Geomathematics Group, University of Siegen

In recent years, the Geomathematics Group at the University of Siegen has developed the *Regularized Functional Matching Pursuit (RFMP)* and *Regularized Orthogonal Functional Matching Pursuit (ROFMP)* algorithms, which are greedy algorithms for (linear) ill-posed inverse problems. The idea of these algorithms is to iteratively add trial functions to the current approximation, which are taken from a so-called *dictionary*. The latter is a predefined subset of functions from the underlying Hilbert space. In every iteration of the algorithm, the next dictionary element is chosen such that it yields the minimal value of a Tikhonov type functional.

In this talk, we present two new modifications of the original RFMP algorithm:

1. The *Regularized Weak Functional Matching Pursuit (RWFMP)*, which is achieved by applying a so-called *weak strategy* to the selection of the next dictionary element. This means that it is no longer needed to make an optimal choice in the selection process. Instead, it suffices to select a function from the dictionary which is near to the optimum in a certain sense.

   The theoretical analysis of this algorithm has an additional beneficial side effect: We will present convergence results for the algorithm for data which come from an arbitrary Hilbert space. For the original RFMP, convergence was only proven for finite-dimensional data. Additionally, we will present convergence rates for the algorithm.

2. The *RFMP for nonlinear inverse problems*, which is derived by replacing the Tikhonov functional, which is used for linear problems, by a linearized Tikhonov functional. For this modification, we will show numerical results for the nonlinear inverse gravimetric problem, which deals with the determination of the shape of a body from its gravitational field.

Study on parameter choice methods for the RFMP with respect to downward continuation

Bianca Kretz
Geomathematics Group, University of Siegen

The regularized functional matching pursuit (RFMP) is a greedy algorithm for linear ill-posed inverse problems. This algorithm incorporates the Tikhonov-Phillips regularization which implies the necessity of a parameter choice. We present some known parameter choice methods and evaluate them with respect to their performance in the RFMP. Furthermore, we discuss these methods with regard to an enhancement of this algorithm, the regularized orthogonal functional matching pursuit (ROFMP). As an example of a linear inverse problem, the downward continuation of gravitational field data from the satellite orbit to the Earth's surface is chosen, because it is exponentially ill-posed. For the test scenarios, we combine different satellite heights with several noise-to-signal ratios, kinds of noise and data grids. The performances of the parameter choice strategies in these scenarios are analyzed to obtain a first orientation which methods could be most appropriate for the RFMP and ROFMP.

References

[1] Gutting M, Kretz B, Michel V, Telschow R (2017) Study on Parameter Choice Methods for the RFMP with Respect to Downward Continuation. Front. Appl. Math. Stat. 3:10. doi: 10.3389/fams.2017.00010

Machine learning for inverse problems

Peter Maaß
University of Bremen

The classical approach to inverse problems starts with an analytical description $F : X \to Y$ of the forward operator in some function spaces X, Y. The field of inverse problems addresses the task of reconstructing an unknown x^* from noisy data $y^\delta \sim F(x^*)$ with the further complication that F^{-1} or any type of generalized inverse is unbounded. The mathematical analysis stays within this framework and provides a regularization theory for optimal analytical convergence rates, stability estimates and convergence of numerical schemes.

This model driven approach has at least two shortcomings. First of all, the mathematical model is never complete. Extending the model might be challenging due to an only partial understanding of the underlying physical or technical setting. Secondly, most applications will have inputs which do not cover the full space X but stem from an unknown subset or obey an unknown stochastic distribution. E.g. there is no satisfactory mathematical model, which characterizes tomographic images or other image classes amongst all L_2-functions.

Machine learning offers several approaches for amending such analytical models by a data driven approach. Based on sets of training data either a specific problem adapted operator update is constructed and an established inversion process is used for regularizing the updated operator or the inverse problems is addressed by a machine learning method directly.

We present an overview on machine learning approaches for inverse problems. We include some first numerical experiments on how to apply deep learning concepts to inverse problems and we finsih by showing some real life applications of a model reduction/ basis learning approach in imaging mass spectrometry.

MESHFREE simulation in continuum and fluid mechanics: from geomechanical to medical applications

Isabel Michel[1], Jörg Kuhnert[1], Fabian Nick[2], and Bram Metsch[2]
[1]Fraunhofer Institute for Industrial Mathematics ITWM, Germany (isabel.michel@itwm.fraunhofer.de); [2]Fraunhofer Institute for Algorithms and Scientific Computing SCAI, Germany

Meshfree methods show their strengths best in case of applications which are characterized by strong deformations, free surfaces, or phase boundaries. During the last 15 years the meshfree Finite Pointset Method (FPM) developed by Fraunhofer ITWM has been successfully applied to problems in computational fluid dynamics such as water crossing of cars, water turbines, and hydraulic valves. Currently, it is further developed to an integrated tool called MESHFREE which combines the advantages of FPM as well as SAMG (a fast solver for large sparse linear systems developed by Fraunhofer SCAI). This synergy drastically increases the applicability of the method in medical problems since SAMG provides a robust and scalable linear solver for a wide class of problems – especially in case of large relative information paths.

In MESHFREE there is no limitation to Newtonian fluids. This is due to the generalized finite difference formulation on a scattered cloud of numerical points which allows for an easy implementation of various material models, e.g. a general Drucker-Prager yield stress criterion. In this regard, numerical simulations of granular flows (soil interaction of cars, avalanches) as well as medical pumps (blood) have been tackled most recently.

The key features of realistic simulations in these scientific fields are: (1) an adequate material model, (2) the choice of the boundary conditions between the continuum and the geometry, as well as, if applicable, also (3) the interaction with flexible structures. We will discuss their influence for examples in geomechanics and medical sciences.

Inversion of electric, gravitational, and magnetic data in medical imaging or Earth sciences—What can we learn from these closely related problems?

Sarah Orzlowski and Volker Michel
Geomathematics Group, University of Siegen

Electroencephalography (EEG) yields the electric potential at the scalp, whereas magnetoencephalography (MEG) provides (some components of) the magnetic field outside the head. Both observables depend on the neural currents inside the brain. This leads to inverse problems, where active brain regions shall be detected from time-dependent EEG and MEG data.

Inverse gravimetry is concerned with the inversion of gravitational data for mass density anomalies. Moreover, also magnetic data can be used to conclude on structures at the Earth's surface and inside the Earth. The applications are manifold, like the observation of climate-based mass transports or the exploration of potential geothermal energy plants.

These inverse problems can be linked to some general classes of Fredholm integral equations, where the integral kernel can be represented by an expansion in Legendre polynomials. In [1], we showed for the scalar case that a general theory for such a class of inverse problems can be developed. This includes a characterization of the null space of the forward operator and a discussion of possible modelling assumptions for its reduction.

In this talk, we summarize the results that can be proved for the general vectorial class of problems. We have elaborated new theoretical results for the choice of appropriate basis functions in the singular value decompositions of the forward operators. This also provides us with further insight into the non-uniqueness of the solution. Then, we focus on the specific details of the inversion of EEG-MEG data, which is connected to similar, but vectorial integral equations. Moreover, we have adapted greedy algorithms (Regularized Functional Matching Pursuit (RFMP) and Regularized Orthogonal Functional Matching Pursuit (ROFMP)), which were developed by the Geomathematics Group Siegen for some geophysical inverse problems (like inverse gravimetry), and applied them to the inversion of EEG and MEG data.

We also show numerical results for EEG-MEG-data inversion, where we evaluate the method for a synthetic example with a known solution and for real data, where a certain behaviour of the result can be expected due to neuroscientific and physical knowledge.

The talk is based on a joint project with A.S. Fokas (University of Cambridge) and O. Hauk (MRC Cognition and Brain Sciences Unit, Cambridge).

References

[1] V. Michel, S. Orzlowski: *On the null space of a class of Fredholm integral equations of the first kind*, Journal of Inverse and Ill-Posed Problems, 24 (2016), 687-710.

An SVD in spherical surface wave tomography

Michael Quellmalz
Faculty of Mathematics, Chemnitz University of Technology

The Funk-Radon transform, also known as the spherical Radon transform, assigns to a function on the sphere its mean values along all great circles. Since its invention by Paul Funk in 1911, the Funk-Radon transform has been generalized to other families of circles as well as to higher dimensions.

We consider a generalization to circle arcs that is motivated by a problem from geomathematics. In spherical surface wave tomography, one measures the traveltimes of surface waves on the Earth from an epicenter to a detector. Then one wants to recover the phase velocity. A common approach is the great circle ray approximation: we assume that a wave travels along a great circle and the traveltime equals the mean of the local phase velocity along the great circle arc connecting the epicenter and the receiver. Mathematically, we want to recover the local phase velocity, which is a real-valued function defined on the sphere, from its integrals along certain arcs of great circles. In this talk, we provide a singular value decomposition of the surface wave tomography with full data and for the case of great circle arcs whose length is fixed.

This is joint work with Ralf Hielscher and Daniel Potts.

Efficient minimization of Tikhonov functionals with discrete TV and TGV penalties

Thorsten Raasch
Johannes Gutenberg University Mainz

We are concerned with the efficient numerical minimization of Tikhonov-type functionals with discrete total variation (TV) and total generalized variation (TGV) penalty terms which arise, e.g., from medical imaging. It is well-known that even in a one-dimensional setting, the proximity operators of TV and TGV penalties do not have a closed-form expression. As an alternative to the widely-used dual or primal-dual strategies to compute the Tikhonov minimizer, we investigate a purely primal reformulation of the optimality conditions as a piecewise smooth zero-finding problem which can then be efficiently solved with semismooth Newton techniques. As a variation of the theme, the perspective allows to draw a connection to taut string and tube methods.

Photoacoustics imaging

Otmar Scherzer
University of Vienna and Radon Institute of Computational and Applied Mathematics (Linz), Austria

In this talk we review the method of photoacoustic imaging, which is promising new imaging modality for diagnosis of small biological probes or human tissue.

The mathematical problem is an inverse source problem for the wave-equation. We discuss reconstruction formulas and computational techniques for photoacoustic imaging. Moreover, we discuss recent advances in that field, which are based on more detailed mathematical models. Particular recent research topics are on quantitative photoacoustics, combination with focusing techniques, such as single plane illumination microscopy, etc.

This is joint work with A. Beigl, Z. Belhachmi, P.Elbau, T. Glatz, A. Kirsch, K. Sadiq and C. Shi.

Vectorial Slepian functions on the ball

Naomi Schneider
Geomathematics Group, University of Siegen

Due to Heisenberg's uncertainty principle, a function cannot be simultaneously limited in space as well as in frequency. The idea of Slepian functions in general is to find functions that are at least optimally spatio-spectrally localised. Here, we are looking for Slepian functions which are suitable for the representation of real-valued vector fields on a three-dimensional ball. We intend to use the functions in a project of our group dealing with the imaging of neural currents in the brain.

We work with diverse vectorial bases on the ball which all consist of Jacobi polynomials and vector spherical harmonics. Such basis functions occur in the singular value decomposition of some tomographic inverse problems in geophysics and medical imaging. Our aim is to find bandlimited vector fields that are well-localised in a part of a cone whose apex is situated in the origin. Following the original approach towards Slepian functions by Simons, Sneeuw and others, the optimisation problem can be transformed into a finite-dimensional algebraic eigenvalue problem. This decouples into a normal and a tangential problem similar to the scalar case. The entries of the corresponding matrix are treated analytically as far as possible. For the remaining integrals, numerical quadrature formulae have to be applied.

The number of well-localised vector fields can be estimated by a Shannon number which mainly depends on the maximal radial and angular degree of the basis functions as well as the size of the localisation region.

In this talk, the theoretical approach and numerical results of the localisation problem are presented.

References

[1] V. Michel, S. Orzlowski and N. Schneider. Vectorial Slepian Functions on the Ball. **arXiv:** 1707.00245.

Spin-weighted spherical harmonics and their application for the construction of tensor Slepian functions on the spherical cap

Katrin Seibert
Geomathematics Group, University of Siegen

The spin-weighted spherical harmonics (by Newman and Penrose) form an orthonormal basis of $L^2(\Omega)$ on the unit sphere Ω and have a huge field of applications. Mainly, they are used in quantum mechanics and geophysics for the theory of gravitation and in early universe and classical cosmology. The quantity of formulations conditioned this huge spectrum of versatility.
We present a unified mathematical theory which implies the collection of already known properties of the spin-weighted spherical harmonics. We recapitulate this in a mathematical way and connect it to the notation of the theory of spherical harmonics. Furthermore, we use the spin-weighted spherical harmonics to construct tensor Slepian functions on the sphere. Slepian functions are spatially concentrated and spectrally limited, or spatially limited and spectrally concentrated functions. For scalar and vectorial data on the sphere they have proven to be a viable and versatile tool. They have been applied in a variety of fields including geodesy, planetary magnetism, cosmology, and biomedical imaging. Their concentration within a chosen region on the planet allows for local inversions, when only regional data are available or are of desired quality, or they enable us to extract regional information. We focus on the analysis of tensorial fields, as they occur e.g. for the GRACE mission, by means of Slepian functions. Furthermore, we present a method for an efficient construction of tensor Slepian functions for symmetric regions such as spherical caps. In this context, we are able to construct a localized basis on the spherical cap for the cosmic microwave background (CMB) polarization. As an additional feature, this enables us to separate the polarization into an electric and a magnetic component.

Seismic tomography on a planetary scale: sensitivity modelling, parametrization and regularization.

Karin Sigloch
Earth Sciences Department, University of Oxford, South Parks Road, Oxford OX1 3AN, UK

Seismic tomography is a high-dimensional inverse problem in geophysics that uses observations of seismic waves travelling through the earth's interior, permitting to estimate material properties of the subsurface as a function of latitude, longitude and depth. When practiced on a planetary scale, the primary challenge is the very uneven sampling of the interior because wave sources (earthquakes) cannot be influenced, and receivers (seismometers) are chiefly deployed on continents of the wealthier northern hemisphere, whereas poor countries and the oceans are much less sampled. Hence source-receiver geometries as much less engineerable than in medical tomography. This implies a significant reliance on adaptive parametrization and regularization of the inverse problem.

A second challenge is that seismic wavelengths are typically not short compared to the imaging targets of scientific interest. This means that wave scattering phenomena need to be modelled computationally, both in the forward problem (wave propagation), and to compute sensitivity kernels for the inverse problem. This issue could occur similarly in medical tomography using ultrasonic waves.

I will present these challenges as well as some of the approaches we have been pursuing.

Filtering GRACE gravity fields: anisotropy and other annoyances

Nico Sneeuw and Balaji Devaraju
Institute of Geodesy, University of Stuttgart & Institute of Geodesy, Leibniz University Hannover

Before the launch of the spaceborne gravimetry mission GRACE in 2002 filter theory on the sphere was still in its infancy in the geodetic literature. At the most the spherical versions of some familiar isotropic filters like the box-car filter or the Gaussian filter were used as a common tool. The satellite pair GRACE changed the field drastically. The thoroughly anisotropic error pattern of GRACE gravity field solutions, characterized by North-South stripes and a steep ascent of the spherical harmonic error spectra, necessitated and accelerated the development of new filter theory, particularly in the spherical harmonic domain.

We here provide a short overview of the different filter classes, divided into deterministic, stochastic and heuristic filters. Starting with the general principle of a two-point anisotropic filter, we develop filter concepts like filter weights, leakage, attenuation and, particularly, resolution. Examples with real GRACE data elucidate these concepts. Moreover we examine the effect of filtering on the phase spectrum, an issue that is mostly neglected in geodesy. Future gravimetry missions will continue to drive filter theory development. For instance, future constellations of two or more GRACE-type satellite pairs in different orbits will create inhomogeneous spatio-temporal error patterns that require dedicated filters.

Extraction of tidal signals from the geomagnetic field with adapted basis functions

Roger Telschow
Computational Science Center, University of Vienna

The ocean tides generated by the lunar gravity cause a periodic motion of conducting sea water in the ambient main magnetic field. Thus, the tides eventually induce an additional magnetic field which is contained in the measurements of recent satellite missons. Due to the well known periodicity of the tidal signal, one is able to distinguish it from other already modelled contributions of the field.

We use the Regularized Functional Matching Pursuit to extract the dominant semidiurnal lunar tidal signal from geomagnetic field measurements. For that purpose, we use a dictionary containing particularly tailored spatially localized basis functions which satisfy the underlying Maxwell equations.

Sequential subspace optimization for nonlinear inverse problems with an application in terahertz tomography

Anne Wald
Saarland University, Saarbrücken

We introduce a sequential subspace optimization (SESOP) method for the iterative solution of nonlinear inverse problems in Hilbert spaces, based on the well-known methods for linear problems. The key idea is to use multiple search directions per iteration. Their length is determined by the nonlinearity and the local character of the forward operator. This choice admits a geometric interpretation after which the method is originally named: The current iterate is projected sequentially onto (intersections of) stripes, which emerge from affine hyperplanes whose respective normal vectors are given by the search directions and contain the solution set of the unperturbed inverse problem. We prove convergence and regularization properties. Furthermore, we extend our methods for complex Hilbert spaces and apply it to solve the inverse problem of terahertz tomography, a nonlinear parameter identification problem based on the Helmholtz equation, which consists in the nondestructive testing of dielectric media. The tested object is illuminated by an electromagnetic Gaussian beam and the goal is the reconstruction of the complex refractive index from measurements of the electric field. We conclude with some numerical reconstructions from synthetic data.

Fast SGL Fourier transforms for scattered data

Christian Wülker
Institute of Mathematics, University of Lübeck

Spherical Gauss-Laguerre (SGL) basis functions constitute an orthonormal polynomial basis of the space L^2 on $\mathbb{R}^3$ with Gaussian weight $\exp(-r^2)$. We have previously described fast $\mathcal{O}(B^3 \log^2 B)$ algorithms for the discrete SGL Fourier synthesis and decomposition of functions with bandwidth B. In these algorithms, the function of interest must be sampled at $\mathcal{O}(B^3)$ fixed grid points in $\mathbb{R}^3$. However, some applications call for fast SGL Fourier transforms that can work with scattered data. In this work, we translate a given function with bandwidth B into a three-dimensional trigonometric polynomial, using a radial cutoff and fast cosine and Legendre transforms. Subsequent to these $\mathcal{O}(B^3 \log^2 B)$ steps, the trigonometric polynomial can be evaluated at M scattered points in another $\mathcal{O}(B^3 \log B + M)$ steps, using the non-equispaced fast Fourier transform (NFFT) of Potts et al. This $\mathcal{O}(B^3 \log^2 B + M)$ algorithm improves the naive bound of $\mathcal{O}(MB^3)$ for the discrete SGL Fourier synthesis. With the corresponding adjoint algorithm, we also obtain a fast $\mathcal{O}(B^3 \log^2 B + M)$ algorithm for the discrete SGL Fourier decomposition. We demonstrate the usability of our fast algorithms by a numerical experiment.

4. List of participants

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Anne Wald
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Christian Wülker
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Timetable

Tuesday, 5/9	Wednesday, 6/9	Thursday, 7/9	Friday, 8/9
	09:30 N. Sneeuw (∗)	09:30 W. Freeden (∗)	09:30 K. Sigloch (∗)
	10:15 **Coffee break**	10:15 **Coffee break**	10:15 **Coffee break**
	10:45 M. Gutting 11:15 B. Kretz 11:45 S. Bittens	10:45 A. Ishtiaq 11:15 C. Gerhards 11:45 R. Telschow	10:45 M. Quellmalz 11:15 I. Michel 11:45 Closing
	12:30 **Lunch**	12:30 **Lunch**	
14:00 Registration 14:30 Opening 14:45 S. Dahlke (∗)	13:30 *Museum tour*	13:30 P. Maaß (∗) 14:15 M. Kontak 14:45 A. Wald	
15:30 **Coffee break**	16:00 **Coffee break**	15:15 **Coffee break**	
16:00 T. Raasch 16:30 J. A. Geppert 17:00 R. Budinich 17:30 C. Wülker 18:00 O. Scherzer (∗)	16:30 P. Hashemzadeh (∗) 17:15 V. Michel 17:45 N. Schneider 18:15 K. Seibert	16:00 *City tour*	
19:00 **Dinner**		19:00 *Conference dinner*	

Invited speakers are marked with an asterisk (∗) next to their name.
Meals that are included in the registration fee are typeset in a **bold font**.
Events from the social programme (to be paid seperately) are typeset in an *italic font*.