Mini-Workshop on Stochastic Processes and their Limit Theorems

Universität Siegen, August 22nd-23rd

Program:

Thursday, 22nd

14:00 - 15:00 **Yuguang Fan** (ANU, Canberra)
Distributional representations and small time asymptotics for a trimmed Lévy Process and its quadratic and maximal jump process

15:15 - 16:15 **Ross Maller** (ANU, Canberra)
Regularity and extensions for Lévy Processes

16:15 - 16:45 *coffee break*

16:45 - 17:15 **Katharina Hees** (Universität Siegen)
Scaling Limits for Coupled Continuous Time Random Maxima

19:00 *Dinner*

Friday, 23rd

9:15 - 10:15 **Alexander Schnurr** (Universität Dortmund & Siegen)
How to make generalized Levy Processes analytically tractable

10:30 - 11:30 **Anita Winter** (Universität Duisburg-Essen)
Invariance Principle for variable speed random walks on trees

11:30 - 12:00 *coffee break*

12:00 - 12:30 **Anissa Bouk Ali** (Universität Siegen)

Abstracts

Distributional Representations and Small Time Asymptotics for a Trimmed Lévy Process and its Quadratic and Maximal Jump Process

Yuguang Fan, the Australian National University, Canberra, Australia

August 22nd, 2013, 14:00-15:00.

Abstract

As part of a study of the way a Lévy process interacts with its maximal jump process, representational identities for the distribution of a Lévy process X_t subtracting off its maximal or maximal modulus jump processes are derived with the corresponding quadratic variation process. We have observed that the distribution of the trimmed Lévy process can be recovered from the maximal (modulus) jump process through the randomization of an exponential random variable and a Poisson number of ties at the maximum. This representation enables us to investigate the asymptotic behaviour of the trimmed process when $t \downarrow 0$. For example, we can obtain the convergence of the trimmed Lévy process joint with the quadratic variation process when X_t is in the domain of attraction of a stable law. Furthermore the limit distribution is identified by the representational identity as a trimmed stable random variable. Functional laws of the trimmed process, joint with the maximal processes are derived too. A necessary and sufficient condition for the trimmed Lévy process to be asymptotically normal is that the untrimmed process is asymptotically normal.

This talk is based on joint works with Boris Buchmann (ANU) and Ross Maller (ANU).

References:

[1] Mori, Toshio (1984) On the limit distributions of lightly trimmed sums. *Mathematical Proceedings of the Cambridge Philosophical Society*. Volume **96**. Issue 03. pp 507-516.

[2] Maller, R.A. and Mason, D.M. (2008) Convergence in distribution of Lévy processes at small times with self-normalization. Acta. Sci. Math. (Szeged) **74**, 315–347.

[3] Sato, K. (1973) A note on infinitely divisible distributions and their Lévy measures, Sci. Rep. Tokyo Kyoiku Daigaku, Sect. A, 12, 101–109.

Regularity and Extensions for Lévy Processes

ROSS MALLER, MATH. SCIENCES INST., ANU, CANBERRA, AUSTRALIA

August 22nd, 2013, 15:15-16:15.

Abstract

There is a long and distinguished history concerning regularity of a Lévy process, $(X_t)_{t\geq 0}$, with significant contributions by Rogozin, Shtatland and Bertoin, among others. Let $T_0^- := \inf\{t > 0 : X_t < 0\}$. We say 0 is regular for $(-\infty, 0)$ at 0 if $P(T_0^- = 0) = 1$, otherwise, irregular. Rogozin (1966) showed:

$$0 \text{ is irregular for } (-\infty, 0) \text{ iff } \int_0^1 t^{-1} P\left(X_t \leq 0\right) dt < \infty.$$

Rogozin (1966) established: $X \notin bv$ or $X \in bv$ with $d_X < 0$ implies 0 is regular for $(-\infty, 0)$. Also, $X \in bv$ with $d_X > 0$ implies 0 is irregular for $(-\infty, 0)$.

Bertoin (1997) showed: suppose $X \in bv$ with drift $d_X = 0$. Then

$$0 \text{ is irregular for } (-\infty, 0) \text{ iff } \int_0^1 \Pi\{(-\infty, -x]\} d\left(\frac{x}{\int_0^x \Pi\{(y,\infty)\} dy}\right) < \infty$$

iff

$$\lim_{t\to 0} \frac{X_t^{(-)}}{X_t^{(+)}} = 0, \text{ a.s.},$$

where Π is the Lévy measure of X and $X_t^{(\pm)} = \sum_{0<s\leq t} \Delta X_s^{\pm}$.

Rogozin's proof is by characteristic functions and appealing to random walk renewal results of Spitzer. Our aim is to give a direct proof by direct estimation of probabilities which additionally allows substantial generalisation along lines of Doney and Maller (2004).

This talk is based on joint work with Yuguang Fan, RSFAS, ANU

References:

[1] Bertoin, J. (1997) Regularity of the half-line for Lévy processes, Bulletin des Sciences Mathématiques, 121, 345–354.

[2] Doney R. and Maller R. (2004) Moments of passage times for Lévy processes, Ann. Inst. Henri Poincaré, 40, 279-297.

[3] Rogozin, B.A., (1966) On the distribution of functionals related to boundary problems for processes with independent increments, Theor. Prob. Appl., 11, 580–591.

Scaling limits of Coupled Continuous Time Random Maxima

Katharina Hees, Universität Siegen
August 22nd, 2013, 16:45-17:15.

Abstract

Continuous Time Random Walks are a generalization of classical random walks: after a random waiting time W_i appears a particle jump with random jump size J_i. The Continuous Time Random Maxima is the process which tracks the largest jump that appears in a series of jumps seperated by the random waiting times. We consider the coupled case where the waiting times and the jump sizes are not assumed to be independent. To analyze this we use harmonic analysis on the semigroup $(\mathbb{R}_+ \times \mathbb{R}, \overset{+}{\vee})$, where the operation $\overset{+}{\vee}$ means "$+$" in the first argument and "$\max$" in the second.

How to make generalized Lévy processes analytically tractable

Alexander Schnurr, Universität Dortmund & Siegen
August 23rd, 2013, 9:15-10:15.

Abstract

We start with a short historical introduction on various classes of stochastic processes which contain Lévy processes as special cases. Afterwards we introduce the probabilistic symbol of a homogeneous diffusion with jumps, i.e., a generalization of the well-known characteristic exponent of a Lévy process. Using the symbol we introduce 8 indices which generalize the Blumenthal-Getoor index. These indices are used to derive growth and Hölder-conditions of the paths. As examples we consider Lévy driven SDEs as well as the COGARCH process. In the near future the technical main results will be used to analyze further properties of the paths: strong variation, Hausdorff dimension and Besov spaces.

Invariance Principle for variable speed random walks on trees

Anita Winter, Universität Duisburg-Essen

August 23rd, 2013, 10:30-11:30.

Abstract

In Athreya, Eckhoff and Winter 2013 the authors constructed Brownian motion on any locally compact R-tree (T, r) equipped with a (speed) Radon measure ν. In the present paper we show that this ν-Brownian motion on (T, r) is the scaling limit of variable ν_n-speed random walks on discrete metric graphtrees (T_n, r_n) which jump from a vertex $x \in T_n$ to a neighboring vertex x' with rate

$$\frac{1}{\nu_n(x) \cdot r_n(x, x')}$$

provided that (T_n, r_n, ν_n) converges to (T, r, ν) in a Gromov-weak type of topology. The corresponding topology will be developed as well.

Tempered operator stable laws

Anissa Bouk Ali, Universität Siegen

August 23rd, 2013, 12:00-12:30

Abstract

Tempered operator stable laws are operator stable laws without normal component, for which we modify their Lévy measure to reduce the expected number of large jumps. We will introduce a characterisation of the obtained Lévy measure. Tempered operator stable distributions may have all moments finite. We prove short and long time behavior of the tempered operator stable Lévy process: In a short time frame it is close to an operator stable process while in a long time frame it approximates a Brownian motion.