Non-regularly varying and non-periodic oscillation of the on-diagonal heat kernels on self-similar fractals

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The purpose of this talk is to present the author's result in a recent preprint [2] on on-diagonal oscillatory behavior of the canonical heat kernels on self-similar fractals.

Let K be either a nested fractal or a generalized Sierpiński carpet, which is a compact subset of the Euclidean space, and let $p_t(x,y)$ be the transition density of the Brownian motion on K. Then it is well-known that there exist $c_1, c_2 \in (0,\infty)$ and $d_{\mathrm{s}} \in [1,\infty)$ such that for any $x \in K$,

$$c_1 \leq t^{d_{\mathrm{s}}/2} p_t(x,x) \leq c_2, \qquad t \in (0,1].$$

The exponent d_s is called the *spectral dimension of* K. Then it is natural to ask how $t^{d_{\mathrm{s}}/2} p_t(x,x)$ behaves as $t \downarrow 0$ and in particular whether the limit

$$\lim_{t\downarrow 0} t^{d_s/2} p_t(x,x) \tag{1}$$

exists. When K is a nested fractal, the author has proved in [1] that the limit (1) does not exist for "generic" (hence almost every) $x \in K$ under very weak assumptions on K. The proof of this fact, however, heavily relied on the two important features of nested fractals — they are *finitely ramified* (i.e. can be made disconnected by removing finitely many points) and *highly symmetric*. In particular, the result of [1] is not applicable to generalized Sierpiński carpets, which are *infinitely ramified*.

The main results of [2] have overcome this difficulty by a completely different method, thereby establishing the non-existence of the limit (1) for "generic" $x \in K$ when K is an *arbitrary* generalized Sierpiński carpet. More strongly, we have the following assertion under a quite general setting of a self-similar Dirichlet form on a self-similar set K. Let V_0 be the set of *"boundary points" of* K (whose meaning will be explained in the talk).

(NRV) $p_{(\cdot)}(x,x)$ *does **not** vary regularly at* 0 *for "generic"* $x \in K$, *if*

$$\limsup_{t\downarrow 0} \frac{p_t(y,y)}{p_t(z,z)} > 1 \qquad \textit{for some } y,z \in K \setminus \overline{V_0}. \tag{2}$$

(NP) *"Generic"* $x \in K$ *does **not** admit a periodic function* $G : \mathbb{R} \to \mathbb{R}$ *such that*

$$p_t(x,x) = t^{-d_{\mathrm{s}}/2} G(-\log t) + o(t^{-d_{\mathrm{s}}/2}) \qquad \textit{as } t \downarrow 0, \textit{ if} \tag{3}$$

$$\liminf_{t\downarrow 0} \frac{p_t(y,y)}{p_t(z,z)} > 1 \qquad \textit{for some } y,z \in K \setminus \overline{V_0}. \tag{4}$$

We will also see that the conditions (2) and (4) can be easily verified for most (though not all) typical self-similar fractals.

References

[1] N. Kajino, On-diagonal oscillation of the heat kernels on post-critically finite self-similar fractals, *Probab. Theory Related Fields*, 2012, in press. doi:10.1007/s00440-012-0420-9

[2] N. Kajino, *Non-regularly varying and non-periodic oscillation of the on-diagonal heat kernels on self-similar fractals*, 2012, preprint.