Weyl's Laplacian eigenvalue asymptotics for the measurable Riemannian structure on the Sierpiński gasket

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On the Sierpiński gasket K, Kigami [3] introduced the notion of the measurable Riemannian structure, with which the "gradient vector field" $\widetilde{\nabla} u$ of a function u, the "Riemannian volume measure" μ and the "geodesic metric" $\rho_{\mathcal{H}}$ are naturally associated. Kigami also proved in [3] the two-sided Gaussian bound for the corresponding heat kernel $p_t^{\mathcal{H}}(x,y)$, and I showed in [1] further several detailed heat kernel asymptotics, such as Varadhan's asymptotic relation

$$\lim_{t\downarrow 0} 4t \log p_t^{\mathcal{H}}(x,y) = -\rho_{\mathcal{H}}(x,y).$$

Furthermore Koskela and Zhou proved in [4] that for any Lipschitz function u on $(K,\rho_{\mathcal{H}})$,

$$|\widetilde{\nabla} u(x)| = \limsup_{y\to x} \frac{|u(y)-u(x)|}{\rho_{\mathcal{H}}(x,y)} =: (\mathrm{Lip}_{\rho_{\mathcal{H}}} u)(x) \qquad \text{for } \mu\text{-a.e. } x\in K,$$

which means that the canonical Dirichlet form $\mathcal{E}(u,u) := \int_K |\widetilde{\nabla} u|^2 d\mu$ associated with the measurable Riemannian structure on K coincides with Cheeger type $H_{1,2}$-seminorm in $(K,\rho_{\mathcal{H}},\mu)$.

In the talk, Weyl's Laplacian eigenvalue asymptotics is presented for this case. Specifically, let d be the Hausdorff dimension of K and $\mathcal{H}^d$ the d-dimensional Hausdorff measure on K, both with respect to the "geodesic metric" $\rho_{\mathcal{H}}$. Then for some $c_{\mathcal{N}} > 0$ and for any non-empty open subset U of K with $\mathcal{H}^d(\partial U) = 0$,

$$\lim_{\lambda\to\infty} \frac{\mathcal{N}_U(\lambda)}{\lambda^{d/2}} = c_{\mathcal{N}} \mathcal{H}^d(U),$$

where $\mathcal{N}_U(\lambda)$ is the number of the eigenvalues, less than or equal to λ, of the Dirichlet Laplacian on U. Moreover, we will also see that the Hausdorff measure $\mathcal{H}^d$ is Ahlfors regular with respect to $\rho_{\mathcal{H}}$ but that it is singular to the "Riemannian volume measure" μ. A renewal theorem for functionals of Markov chains due to Kesten [2] plays a crucial role in the proof of the above asymptotic behavior of $\mathcal{N}_U(\lambda)$.

References

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