The moments of the eigenvalue distributions of generalised Cantor Chains

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Based upon earlier work on the vibrations of drums with fractal boundaries, C. Pomerance and M.L. Lapidus proved the "modified Weyl-Berry conjecture" for the asymptotics of the eigenvalues of the Laplace-operator for a bounded open subset of the real line in 1993 [1]. In this context some new and intriguing relations between spectral geometry, fractal geometry and the Riemann Zeta function were discovered. A new characterisation of the Riemann-conjecture as an inverse spectral problem was thereby obtained and a new light was shed on some aspects of the theory of the Riemann Zeta function.

Generalised Cantor Strings are one-dimensional drums with Cantor set type fractal boundaries. These strings will be modelled by linear chains of masses coupled by harmonic springs, which in turn may be described by their dynamic matrices. This model, introduced as Cantor Chains by the author in [2], provides a first hint at a connection between random matrix theory and the modified Weyl-Berry conjecture.

In the case of self-similar chains such as those here studied, oscillations in the spectrum of the eigenvalues occur so that the spectral counting function does not asymptotically converge, a fact that can also be investigated through the moments of the eigenvalue distribution. Exact results as well as upper and lower bounds for the moments will be given. If time permits, some preliminary results on the relation between the moments of the eigenvalue distributions of fractal chains and the Minkowski-measurability of their boundaries will be presented.

[1] M. L. Lapidus and C. Pomerance, Proc. London Math. Soc. (3) 66, pp.41-69 (1993).

[2] R. J. Etienne, 2nd Conference on Analysis and Probability on Fractals, Cornell University, Ithaca NY, 2005, (unpublished).