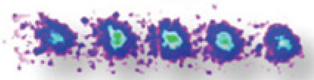
Error Resistant Quantum Gates with Trapped Ions

N.Timoney, V. Elman, C. Weiß, M. Johanning,
W. Neuhauser, Chr. Wunderlich



UNIVERSITÄT SIEGEN

1. Basics: Trapping Yb^+

2. Robust Gates for Single Qubits:
   Composite pulses and Optimal Control

3. Ion Spin Molecules
   - Concept
   - Experiment

Motivation

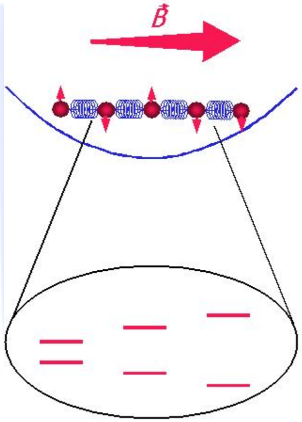
- Fault tolerant quantum computer requires high accuracy quantum gate operations.

- Quantum gate operation synthesized from a sequence of unitary operations.

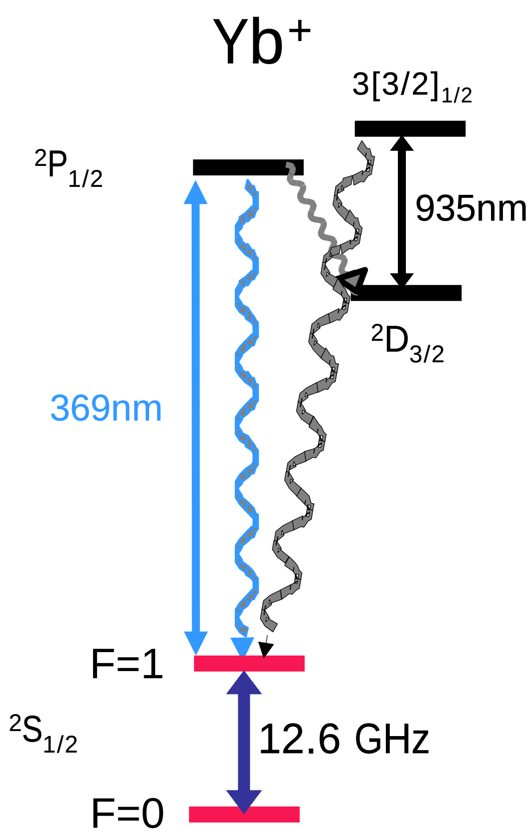
- For trapped ions, a unitary operation needs EM radiation of prescribed **frequency**, phase, **amplitude** and **duration**.

- Robust unitary operations presented here can be used for

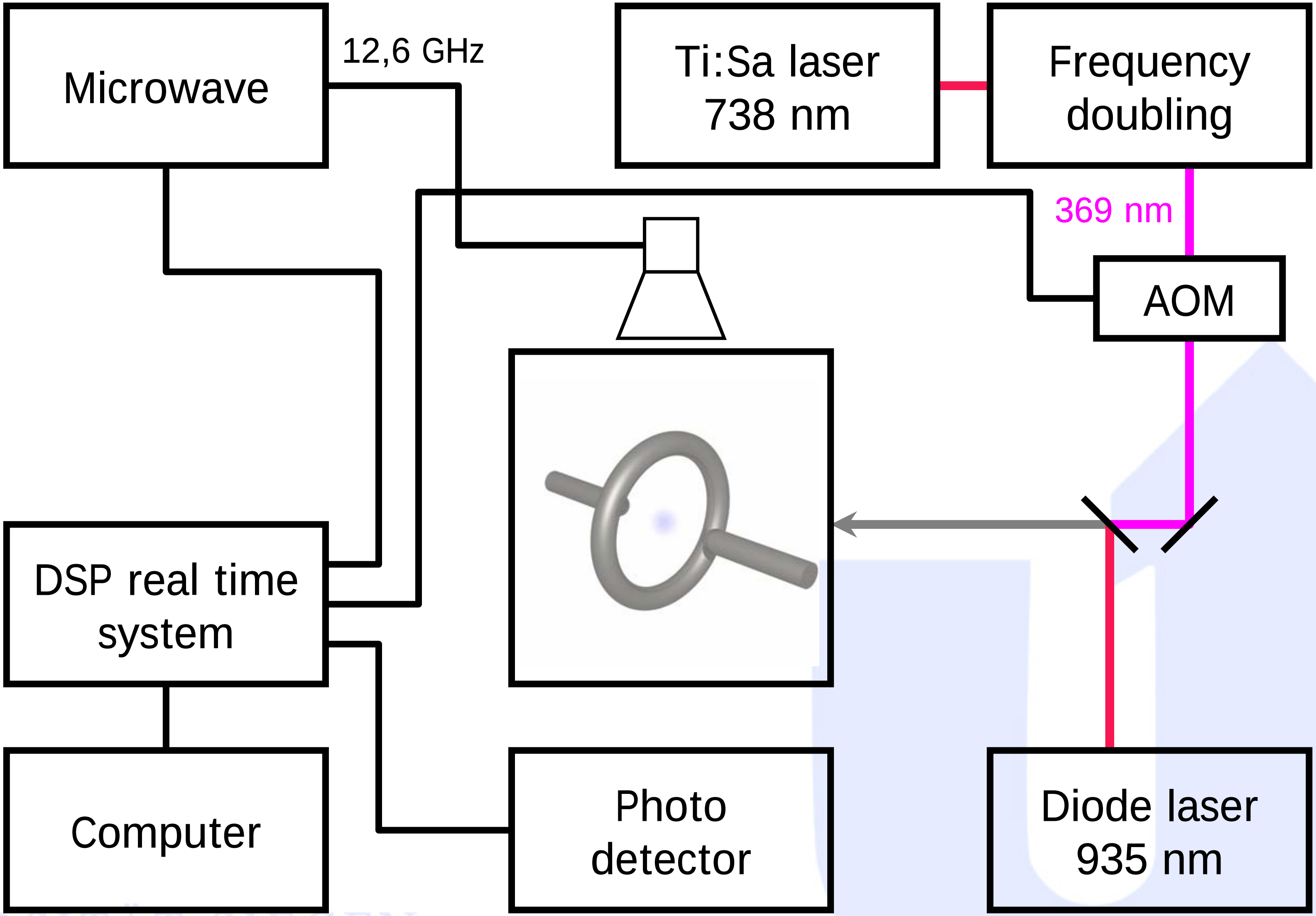
  - Optical transitions
  - „Ion spin molecule“.



Trapped Yb^+ ions

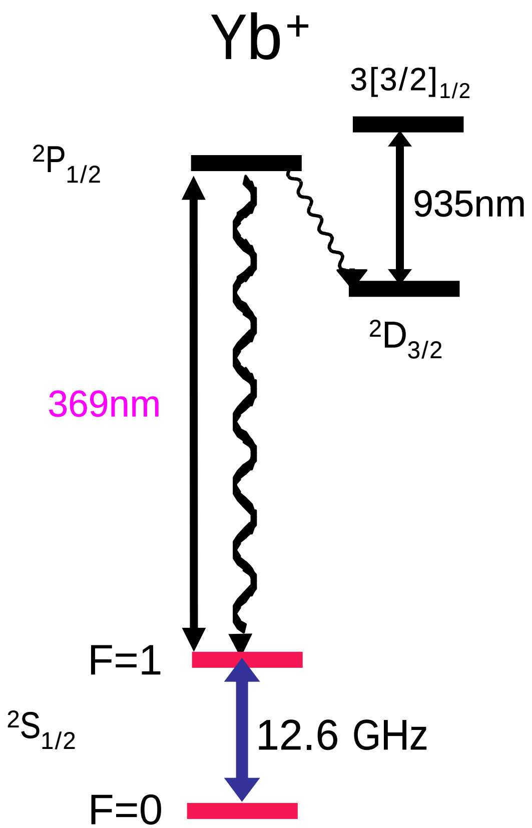


- $^{171}Yb^+$ ion
- Qubit: ground state hyperfine levels
- Preparation in F=0 by optical pumping
- Readout: optical dipole transition


Microwave
12,6 GHz
Ti:Sa laser
738 nm
Frequency
doubling
369 nm
AOM
DSP real time
system
Computer
Photo
detector
Diode laser
935 nm

Trapped Yb^+ ions

Hyperfine Qubit



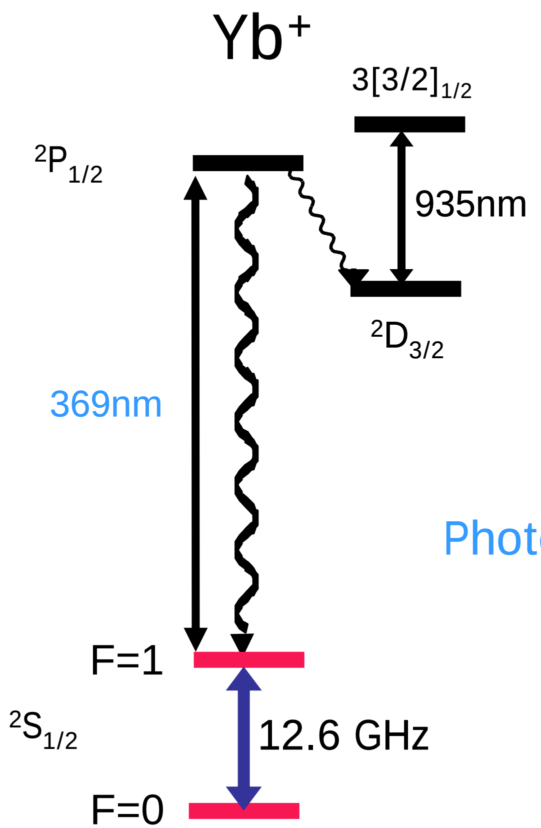
$$H = \frac{\hbar}{2}\delta\sigma_z + \frac{\hbar}{2}\Omega\left(\sigma_+ e^{-i\Phi} + \sigma_- e^{i\Phi}\right)$$

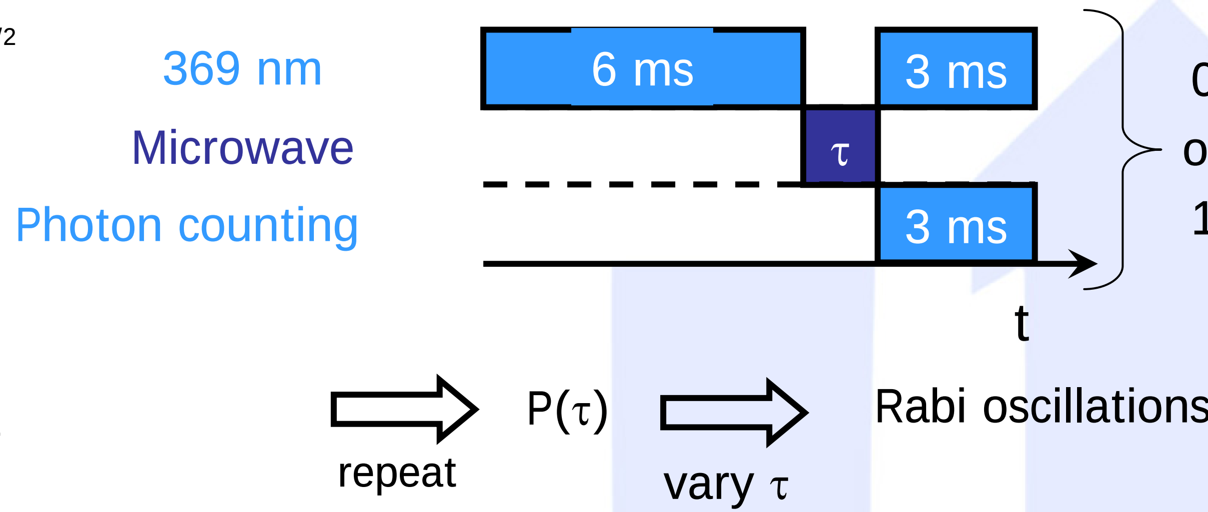
$$R\big(\theta(\Omega, t, \delta), \Phi\big) = e^{-iHt}$$

$$f \equiv \delta / \Omega \qquad g \equiv \Delta\theta / \theta$$

Trapped Yb$^+$ ions

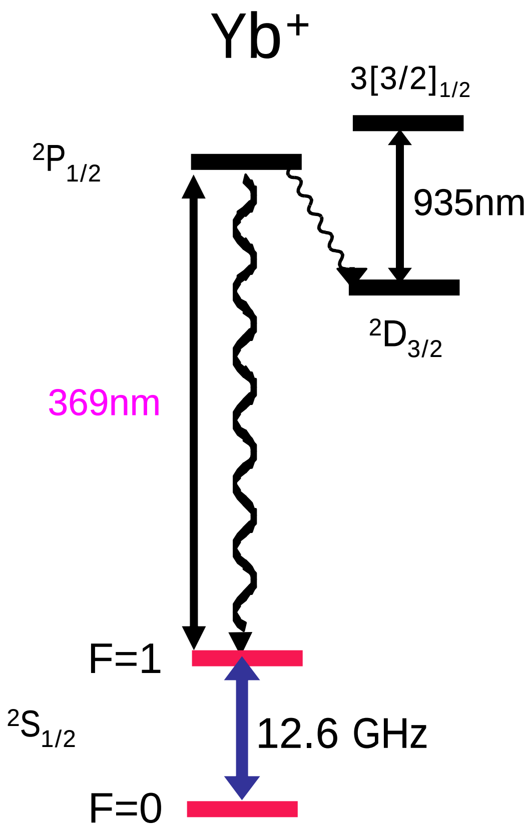
Coherent Excitation



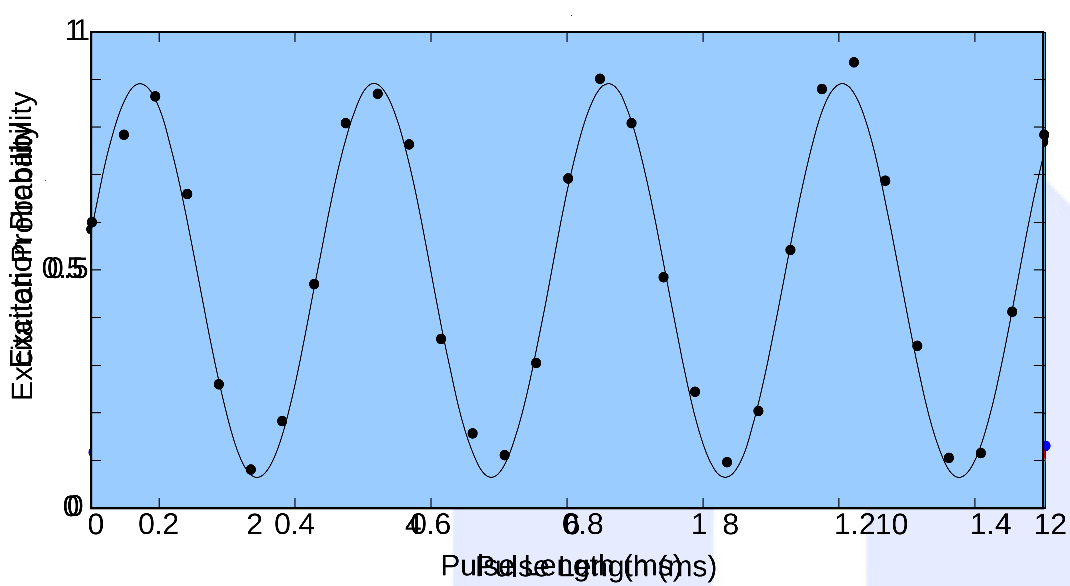


Trapped Yb^+ ions

Coherent Excitation



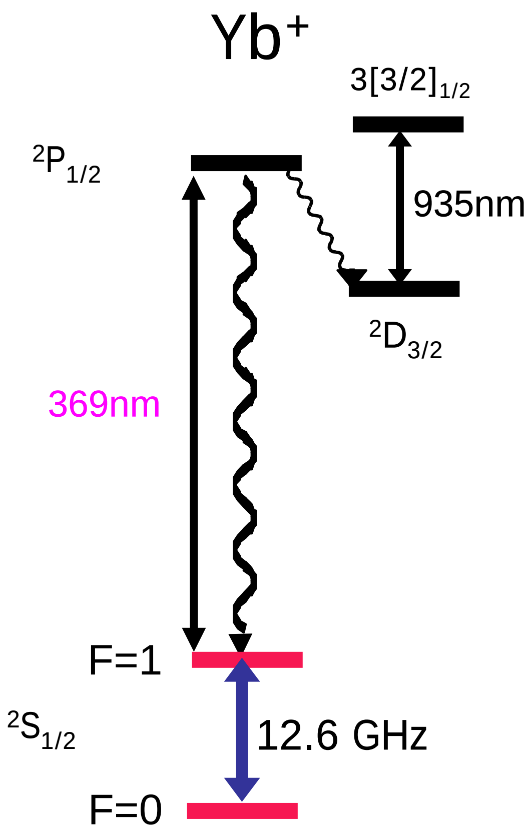
Individual Yb^+-Ion

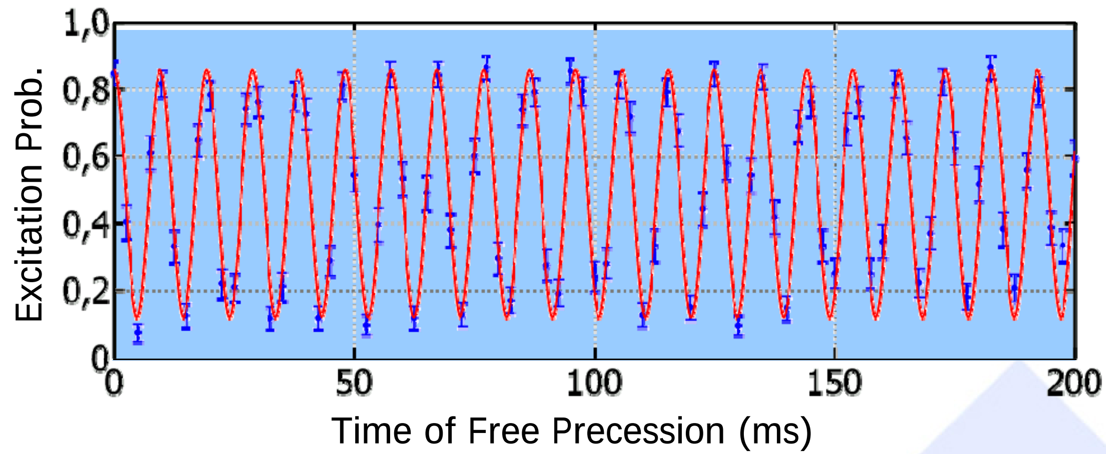


Rabi oscillations

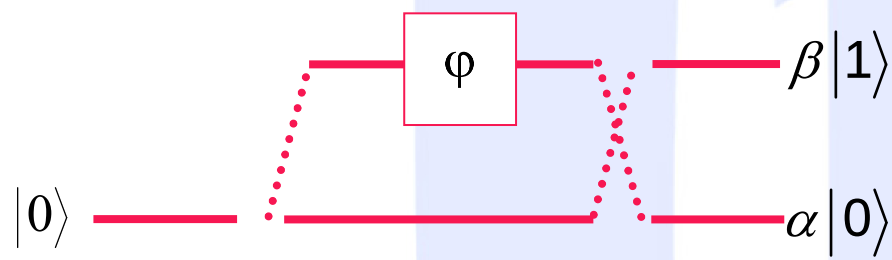
Trapped Yb^{+} ions

Interferometry





Single Atom Interferometer

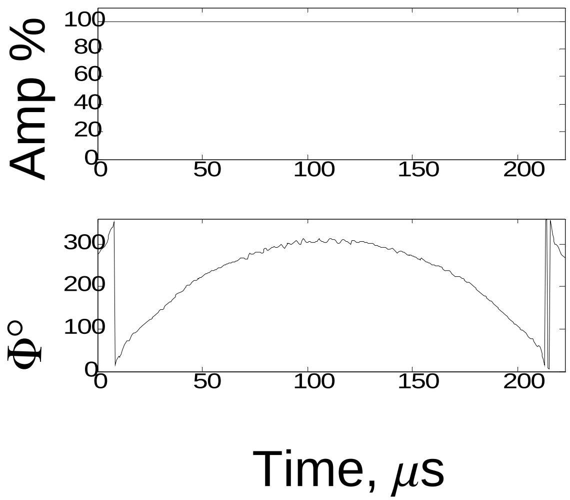


CW, Chr. Balzer, Adv.At.Mol.Opt.Phys. **49**, 295 (2003).

π pulses

OCT Pulse

→**445-step** pulse



Composite pulse

→**CORPSE**(Compensation for Off-Resonance with a Pulse Sequence)
π-pulse :

$$R(\theta_1 = 420°, \Phi_1)R(\theta_2=300°, \Phi_2=\Phi_1+\pi)R(\theta_3=60°, \Phi_1)$$

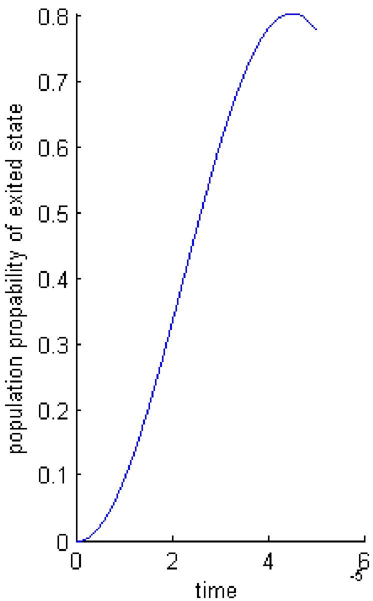
→**SCROFULOUS**(Short Composite Rotation For Undoing length Over and Under Shoot)
π-pulse:

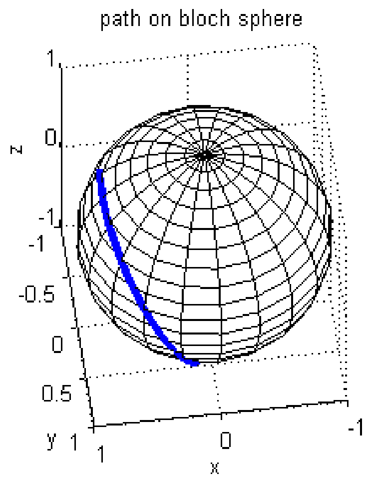
$$R(\theta_1 = \pi, \Phi_1=60°)R(\theta_2=\pi, \Phi_2=300°)R(\theta_3=\pi, \Phi_3=60°)$$

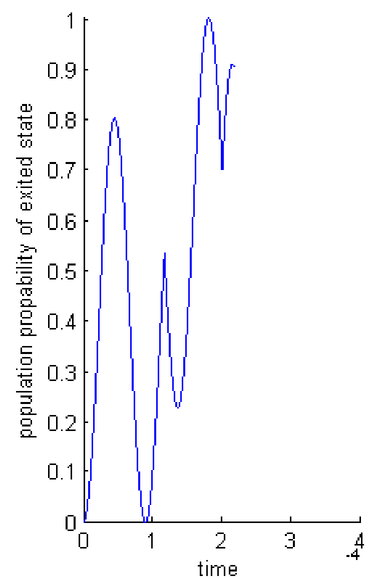
[1] H. Cummins , G. Llewellyn, and J. Jones,
Phys. Rev. A, **67**, 042308 (2003)

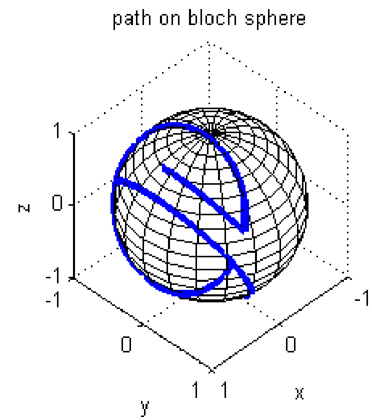
π pulses

CORPSE* (compensation for off-resonance with a pulse sequence)		
$R_1(\theta_1)_{\varphi 1}$	$R_2(\theta_2)_{\varphi 2}$	$R_3(\theta_3)_{\varphi 3}$
420_0	300_{180}	60_0





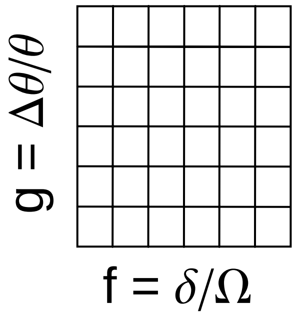




*H.Cummins, G.Llewellyn, and J.Jones, Phys Rev.A **67**, 043208 (2003).

Measurement Procedure

π–pulse



Repeat without step 2 to control initial preparation.

Repeat for f=0=g (π/2) to control state selective detection.

Repeat for **rectangular** pulse

Repeat *N* times. Vary *f* and *g*

D

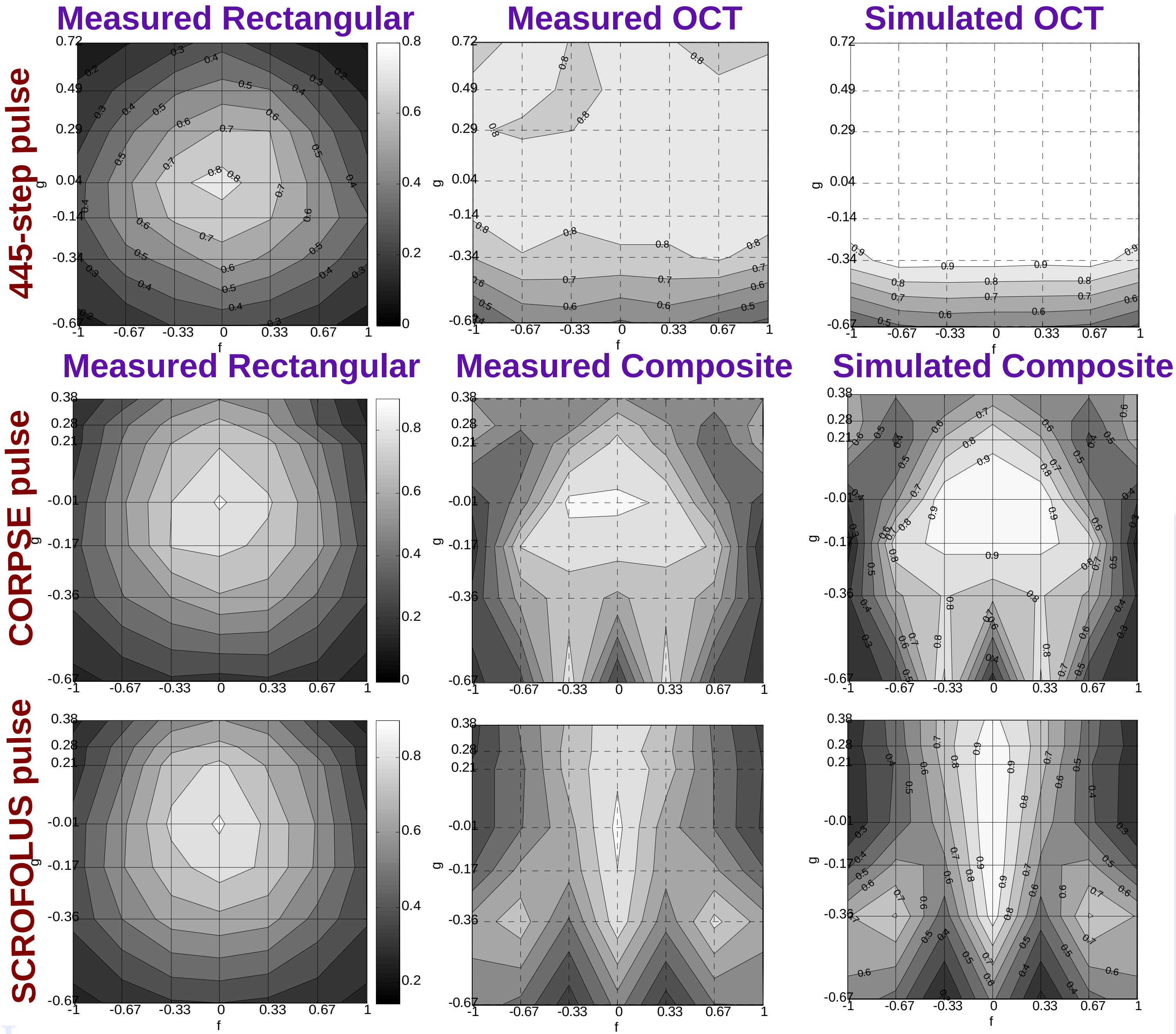
C

B

A

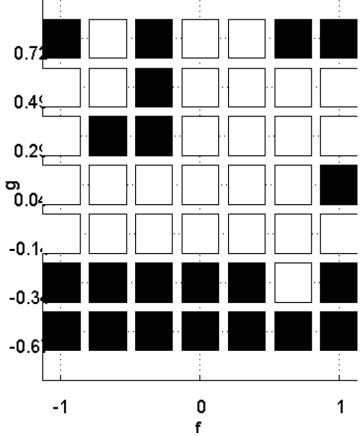
Individual Measurement for given *f* and *g*:

1. Prepare qubit
2. Apply **shaped pulse**.
3. State selective detection
4. Cool ion



$$F = |\langle \Psi_{desired} | \Psi_{achieved} \rangle|^2$$

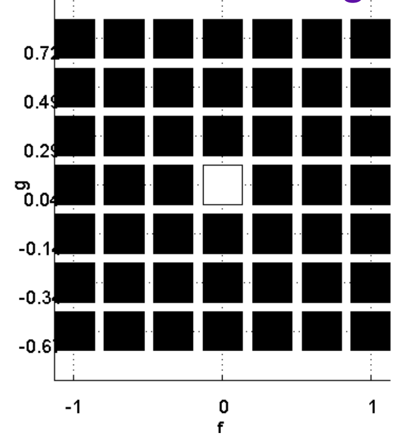
Measured OCT



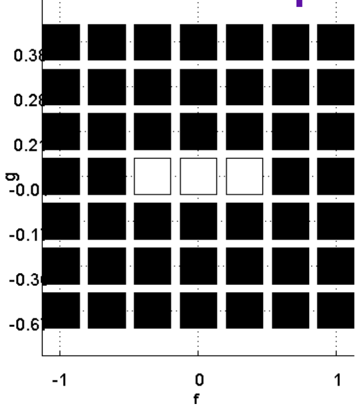
- **445-step** shaped pulse.
- ($\theta = \pi$) end state.
- True(white), false(black)

for $F/F_{max} > 0.96$, $F_{max} = 0.896$

Measured Rectangular



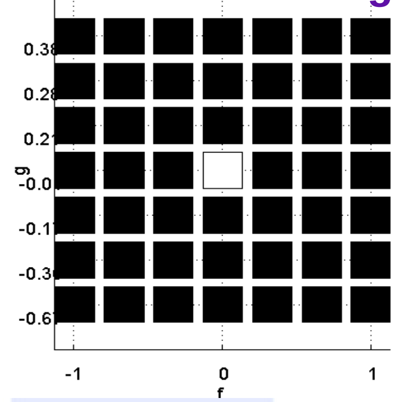
Measured Composite

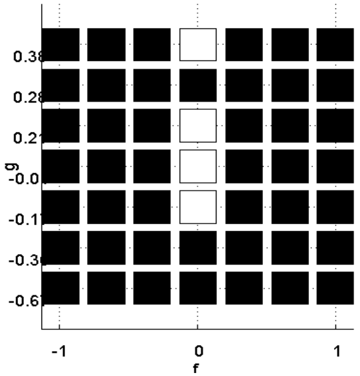


- **CORPSE** pulse.
- ($\theta = \pi$) end state.
- True(white), false(black)

for $F/F_{max} > 0.96$, $F_{max} = 0.930$

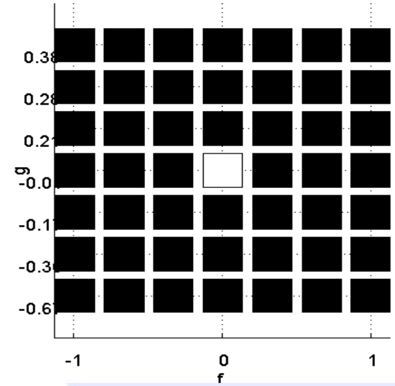
Measured Rectangular





- **SCROFOLUS** pulse.
- ($\theta = \pi$) end state.
- True(white), false(black)

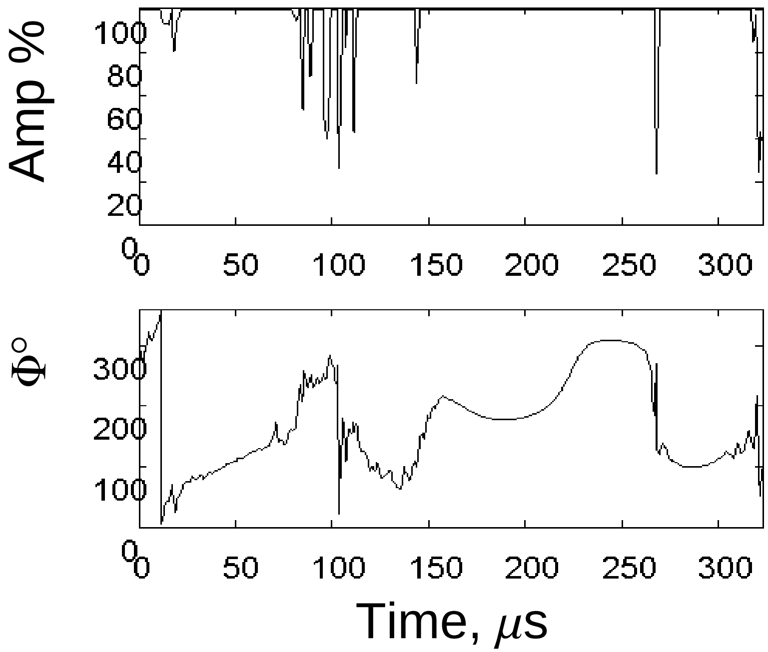
for $F/F_{max} > 0.96$, $F_{max} = 0.930$



$\pi/2$ pulses

Optimal Control Theory (OCT) pulse

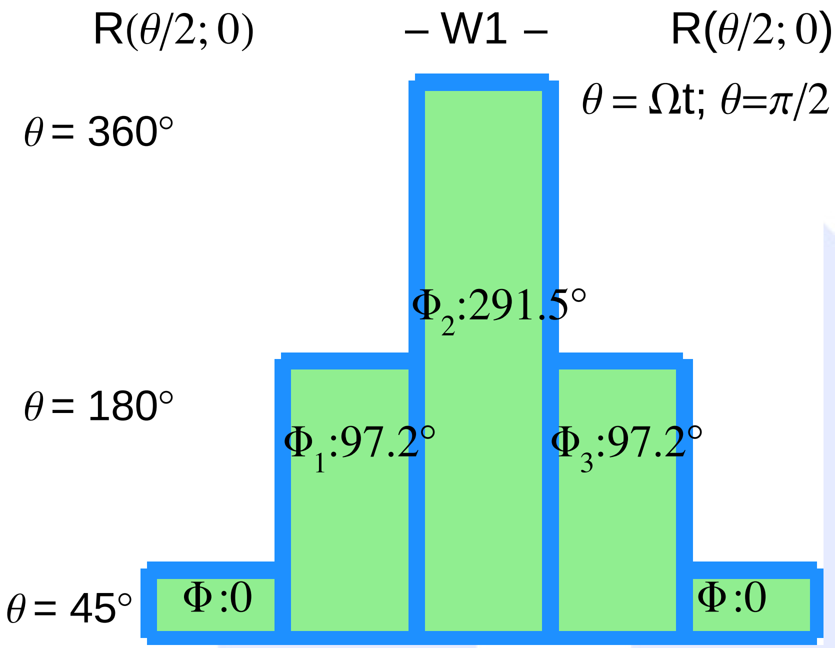
- **645-step** pulse
- Each step 0.5μs long



$$\left(\vartheta = \frac{\pi}{2}; \varphi = \frac{\pi}{2}\right)$$

Composite pulse

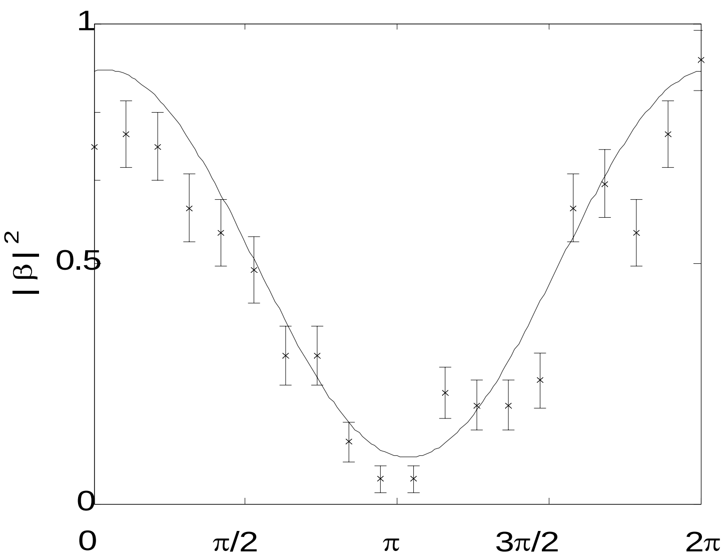
- **BB1 RWR** pulse



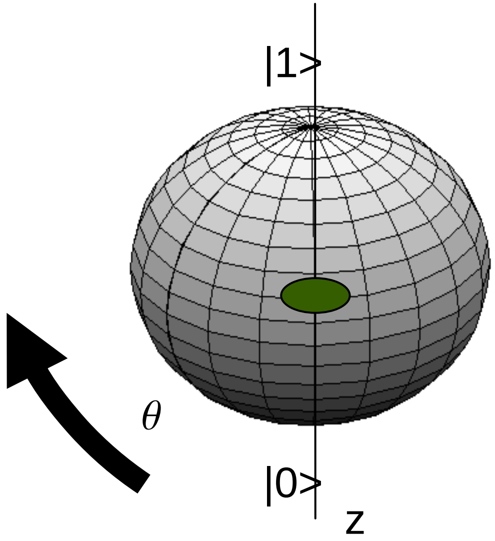
$$\left(\vartheta = \frac{\pi}{2}; \varphi = 0\right)$$

[1] H. Cummins , G. Llewellyn, and J. Jones, Phys. Rev. A, **67**, 042308 (2003)

Ramsey measurement, phase variation



$(\theta = \pi/2; \Phi = 0)$	$(\theta = \pi/2; \Phi)$



→ $|\beta|^2 = |\langle 1|R(\theta_2, \Phi_2)R(0, \delta T)R(\theta_1, \Phi_1)|0\rangle|^2$

Black box pulse

- →Unknown end result (θ, φ)
- →Use resonant $\pi/2$ of varying phase(Φ) to deduce (θ, φ)

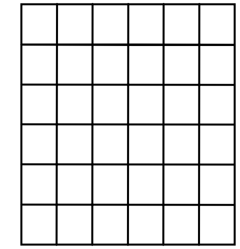
$(\theta ; \varphi)$	$(\theta = \pi/2; \Phi)$

→ $|\beta|^2 = 1/2\,[1 + \sin(\theta)\cos(\varphi + \Phi)]$

Measurement Procedure

$\pi/2$–pulse

$g = \Delta\theta/\theta$



$f = \delta/\Omega$

Add sequences C and D
Repeat 50 times

Repeat 20 times while incrementing Φ.

Replace 1st Ramsey pulse with **rectangular** pulse with given *f* and *g*.

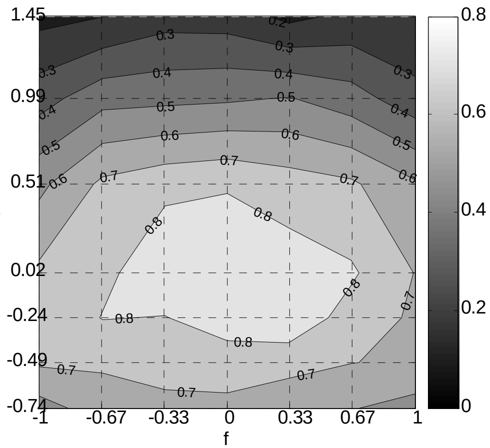
Replace 1st Ramsey pulse with **shaped** pulse with given *f* and *g*

Individual Reference Measurement:

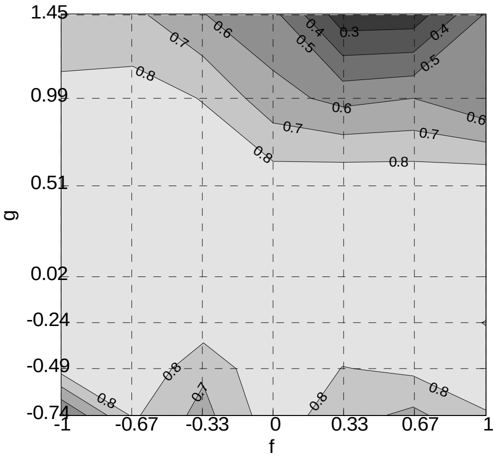
1. Prepare qubit
2. Two pi/2 Ramsey pulses *f*=0=*g* with given Φ.
3. State selective detection
4. Cool ion

645-step pulse

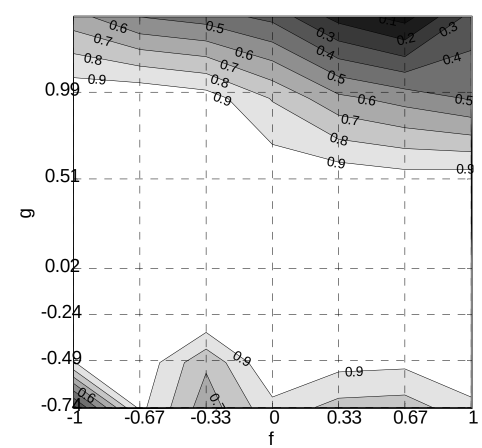
Measured Rectangular



Measured OCT

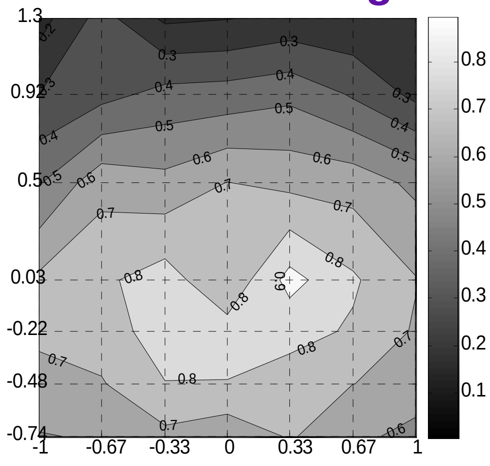


Simulated OCT

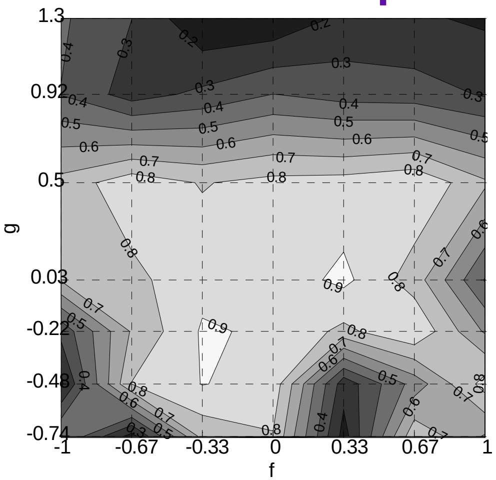


BB1 RWR pulse

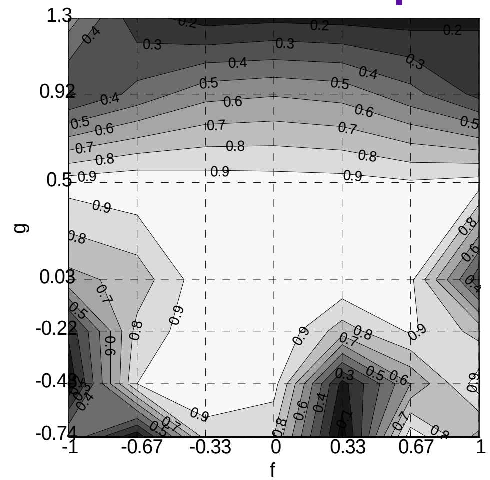
Measured Rectangular



Measured Composite



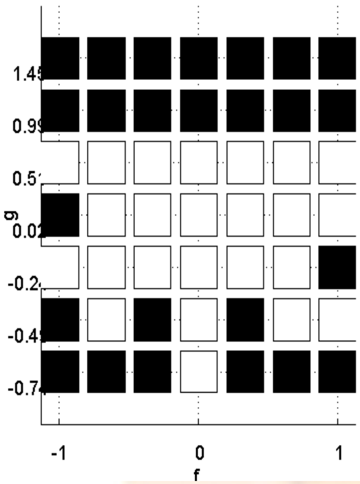
Simulated Composite



→Fidelity

$$F = |\langle \Psi_{desired} | \Psi_{achived} \rangle|^2$$

Meaured OCT

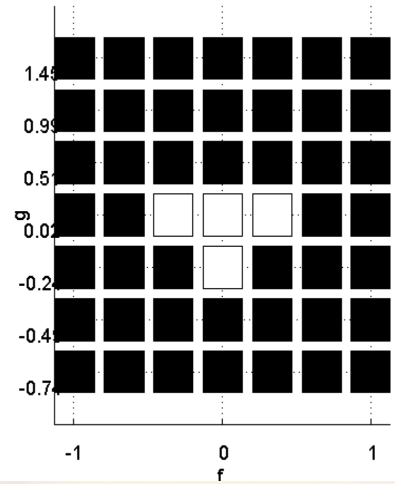


→**645-step** OCT pulse.

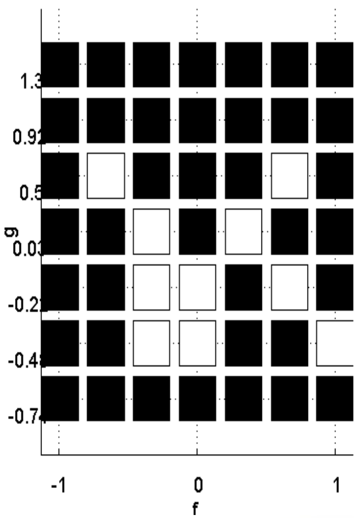
→ ($\theta = \pi/2$; $\phi = \pi/2$) end state.

→True(white), false(black) for $F/F_{max}>0.9$, $F_{max}=0.900$

Measured Rectangular



Meaured Composite

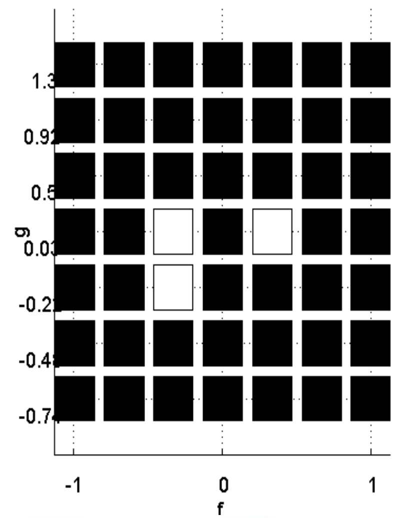


→**BB1 RWR** composite pulse.

→ ($\theta = \pi/2$, $\phi = 0$) end state.

→True(white), false(black) for $F/F_{max}>0.9$, $F_{max}= 0.936$

Measured Rectangular



Summary

- Compared shaped pulses, composite pulses and rectangular pulses.
- Wide tolerance to f(detuning) and g(amplitude) errors seen for shaped pulses.
- Robust unitary operations are building blocks of gate operations.

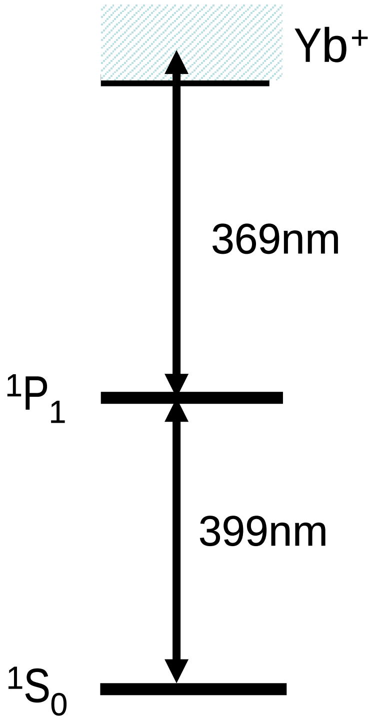
Ion Spin Molecules

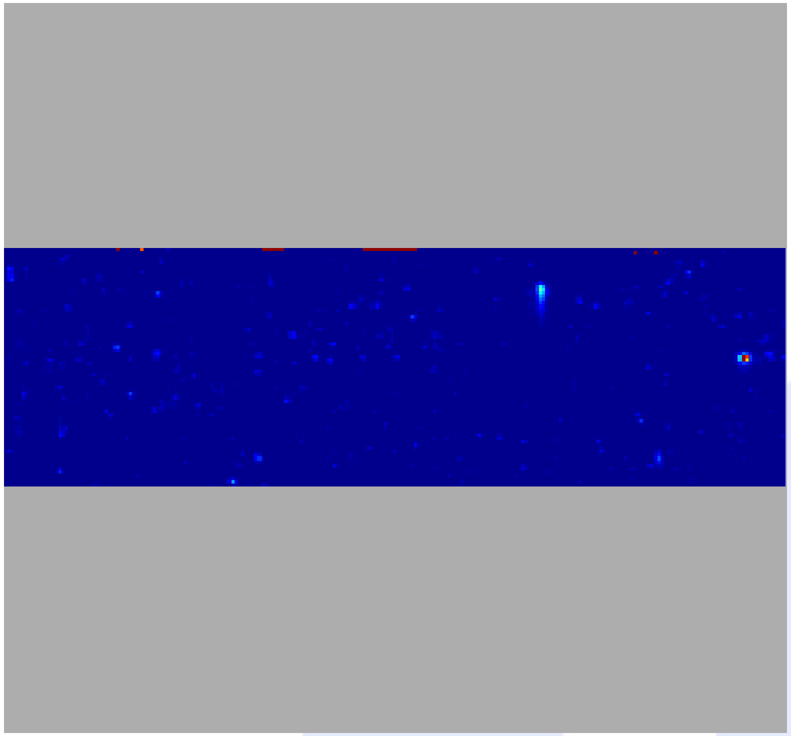
Nearly deterministic trap loading by photoionization

Concept

Experiment

Theory





Ion Spin Molecules

Nearly deterministic trap loading by photoionization

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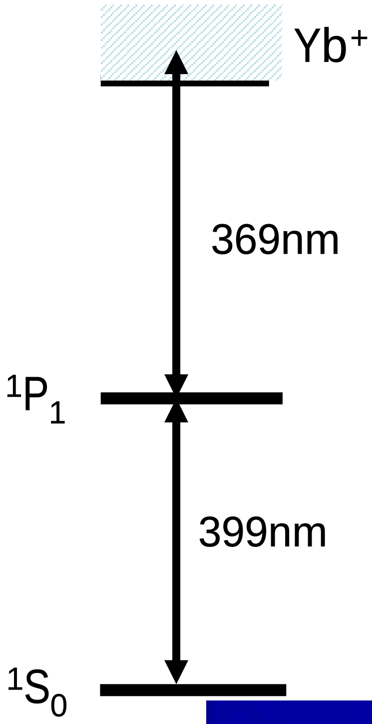
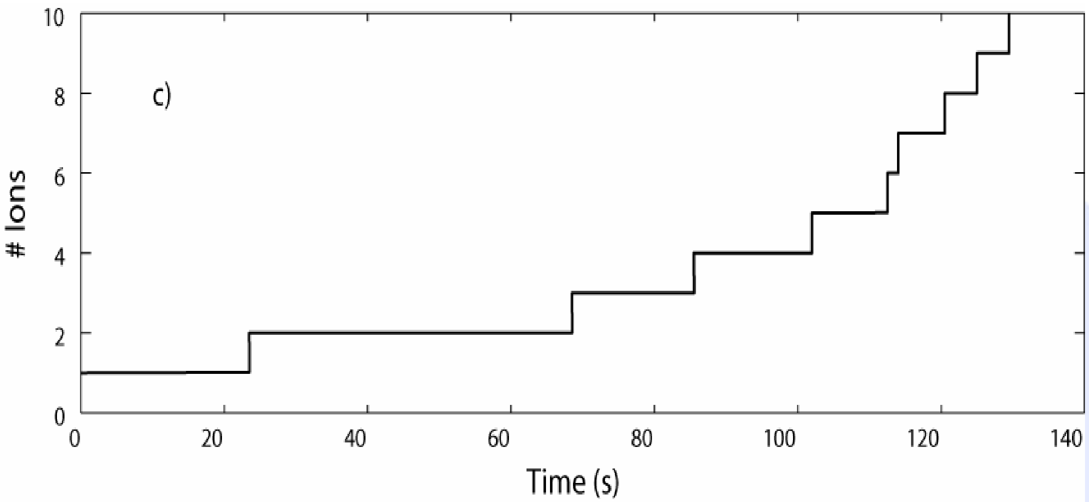
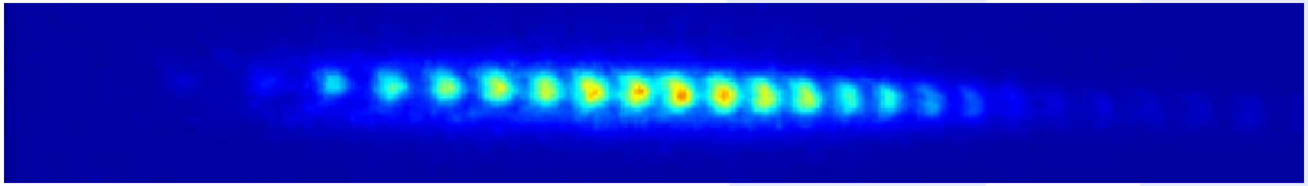


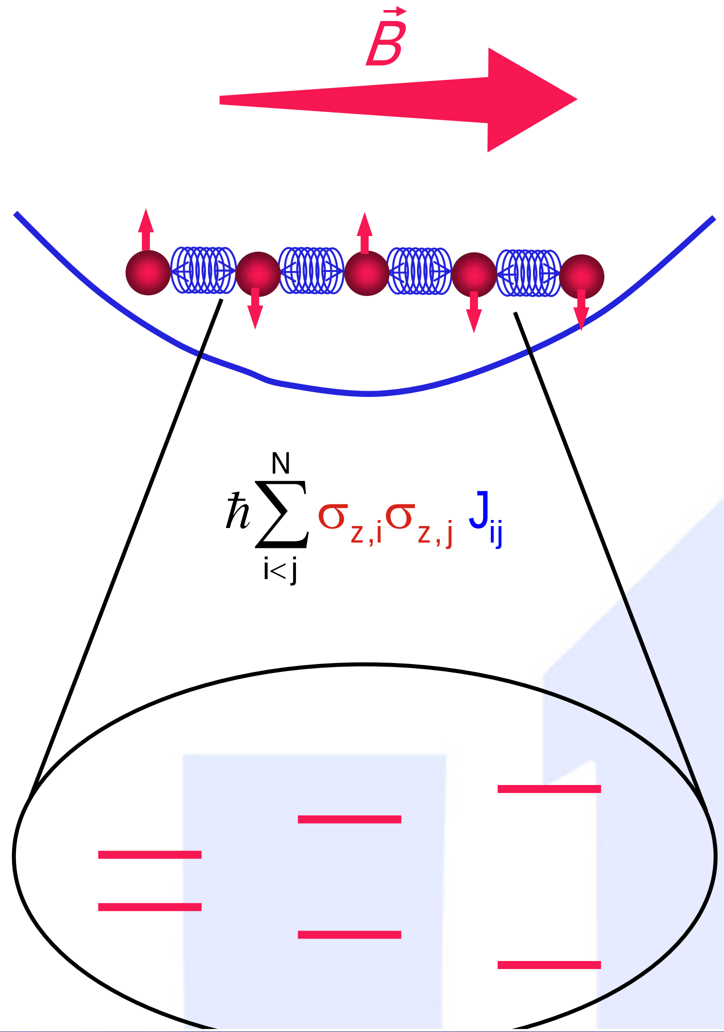




Ion Spin Molecule

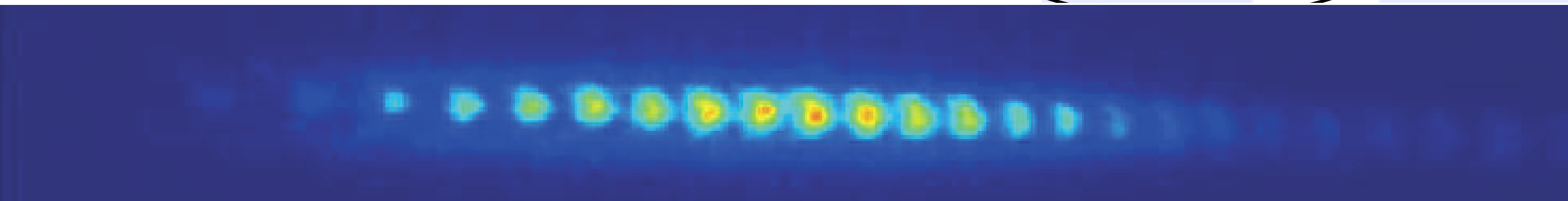
Novel concept:

- Qubit resonances shifted individually
- Spin-Spin coupling between individual qubits



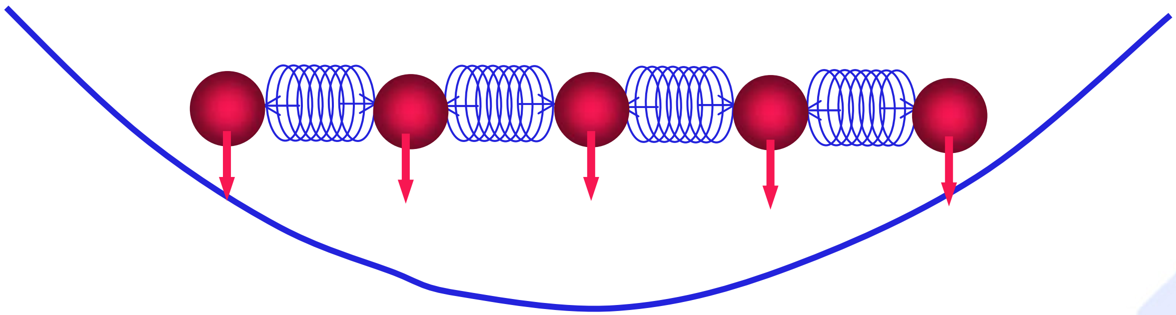
$$\hbar\sum_{i<j}^{N}\sigma_{z,i}\sigma_{z,j}\,J_{ij}$$

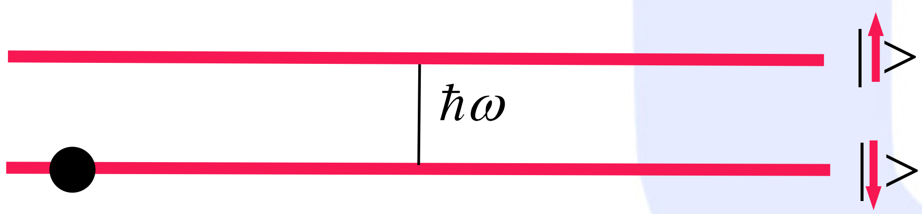
F. Mintert, CW, PRL **87**, 257904 (2001).
CW in *Laser Physics at the Limit,* Springer, 2002, p. 261.



Ion Spin Molecules

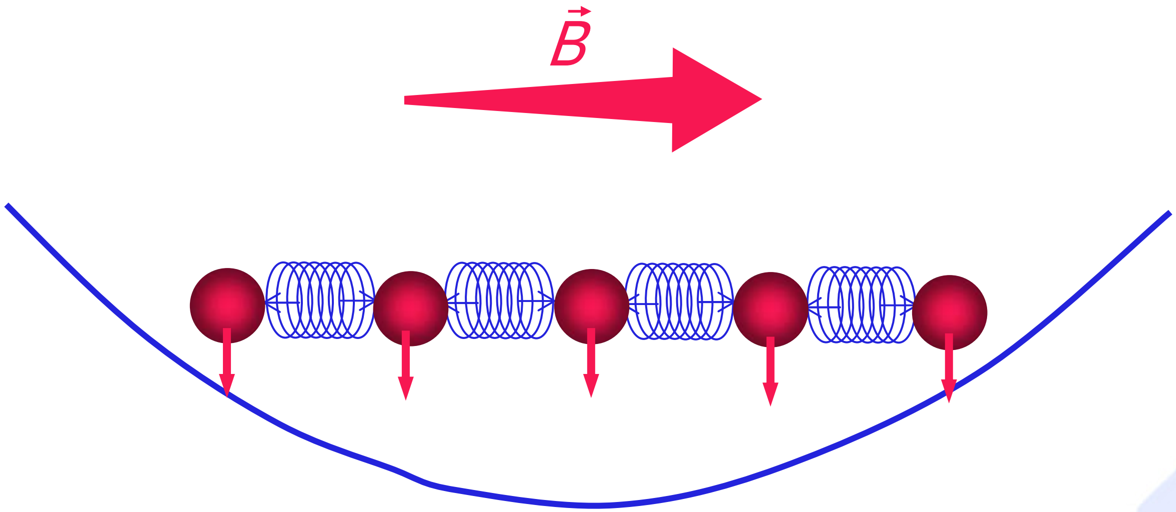
Long Range Spin-Spin coupling

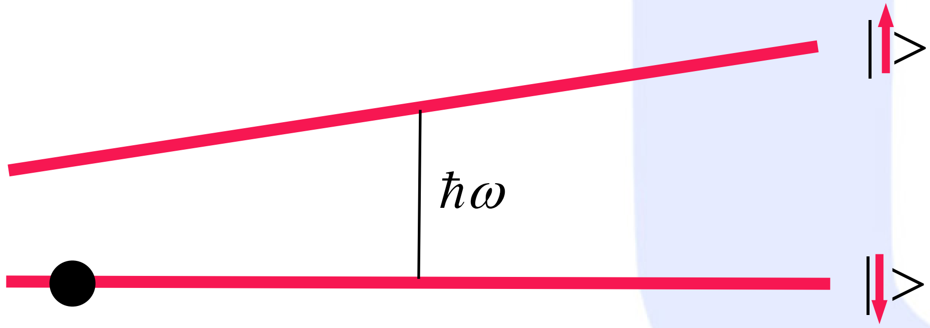




Ion Spin Molecules

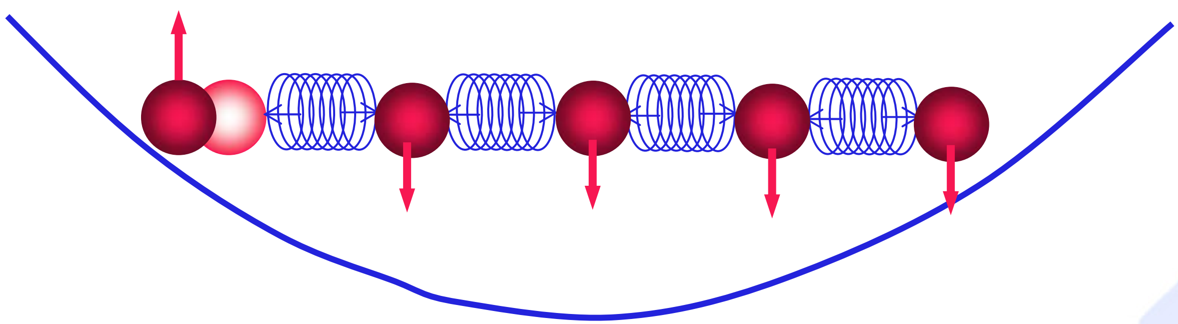
Long Range Spin-Spin coupling

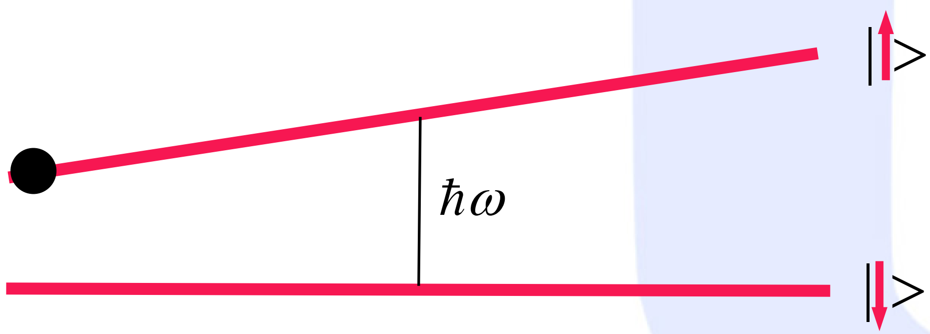




Ion Spin Molecules

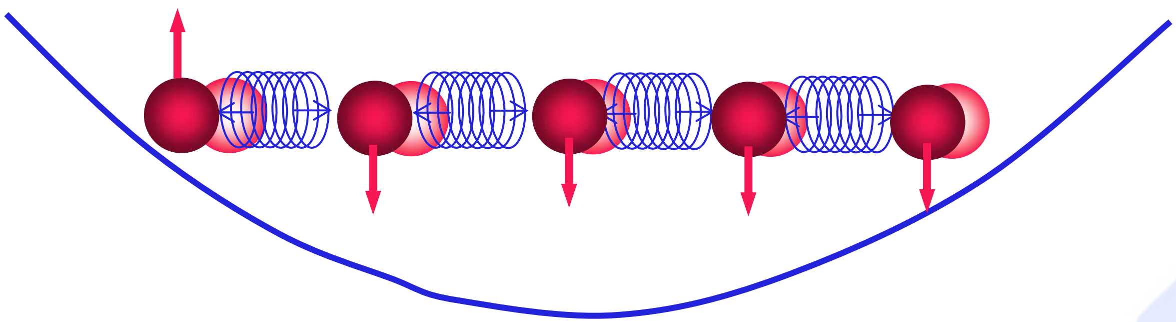
Long Range Spin-Spin coupling

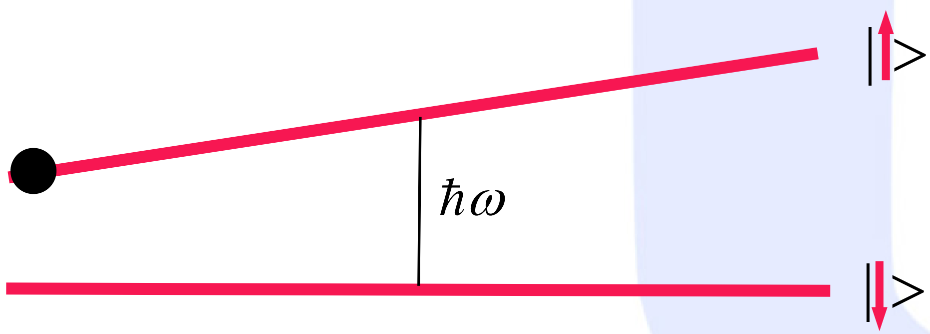




Ion Spin Molecules

Long Range Spin-Spin coupling

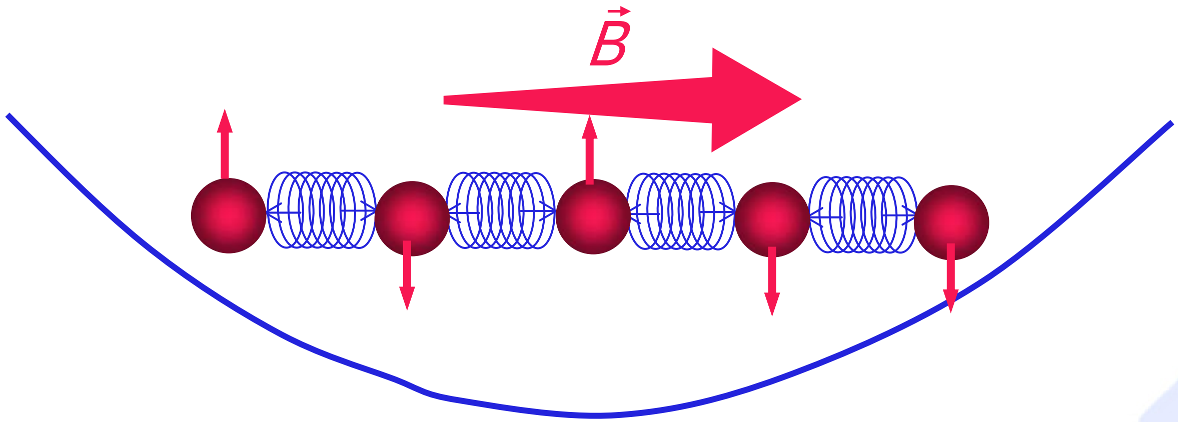




Ion Spin Molecule

$$\tilde{H} = H_{intern} + H_{extern} + \hbar \sum_{i<j}^{N} J_{ij}\, \sigma_{z,i}\sigma_{z,j}$$

Spin-Spin coupling



Individual N-qubit “designer molecule” with adjustable coupling constants

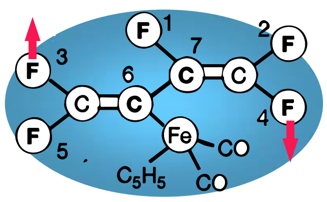
CW in *Laser Physics at the Limit*, Springer, 2002, p. 261. also: quant-ph/0111158.

F. Mintert, CW, PRL 87, 257904 (2001). D.Mc Hugh, J. Twamley PRA **71**, 012315 (2005), quant-ph/0310015

- Multi-qubit gates.
- Q.Simulations.
- Transport of Q.Information
- Entanglement and decoherence.

NMR, Trapped Ions, and Ion Molecules

- Coherent manipulation using rf and microwave radiation of long-lived spin states.
- Use sophisticated NMR concepts and techniques.



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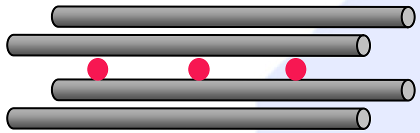
- Individual qubits.
- Efficient preparation and readout using projective measurements.



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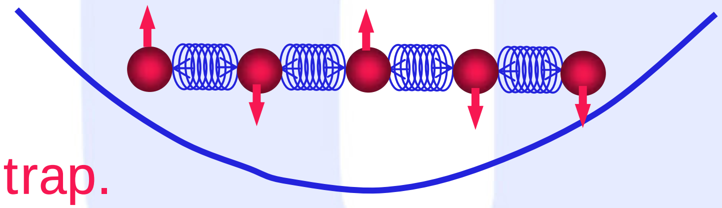
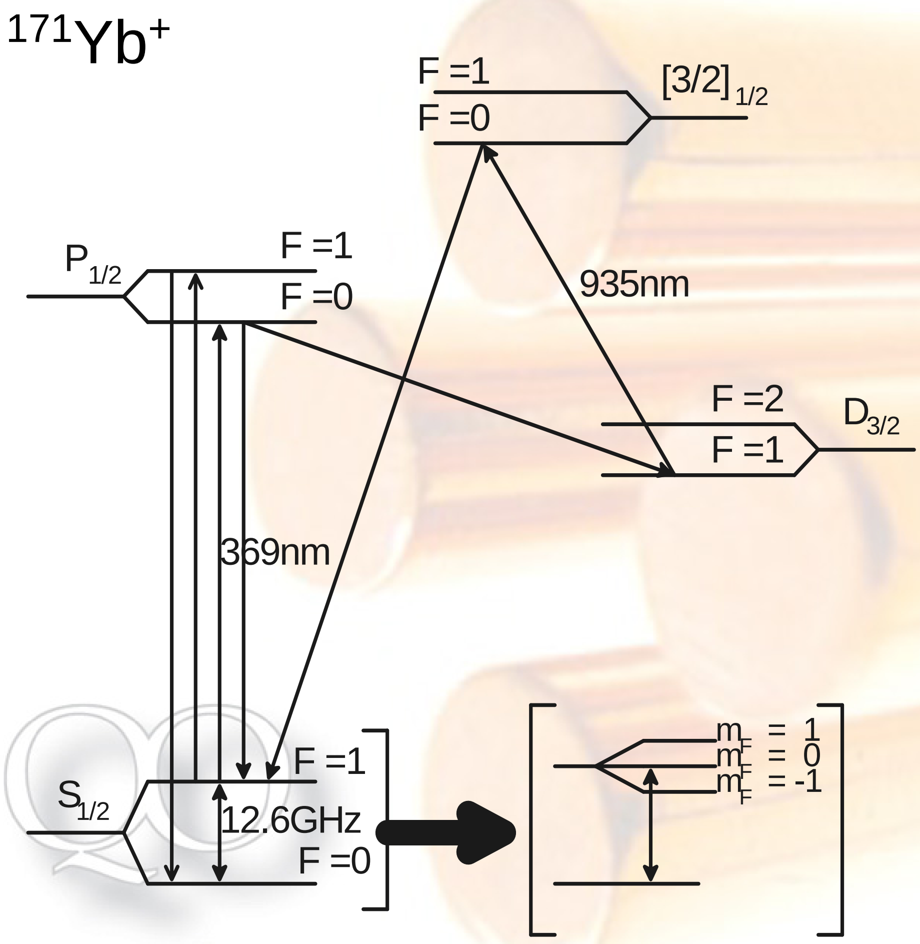


- Spin-spin coupling adjustable.
- (Nearly) insensitive to thermal excitation. ⇒ many ions in single trap.

M. Loewen, CW, Verhandl. DPG 2004 (VI) 39, 7/87 (2004).

Experimental System



- Miniature Paul trap
- Ions trapped using photoionization (398 nm)
- Stable magnetic field(7.5540±0.0047 G) corresponds to a splitting of 9 MHz. (stability of qubit transition 23 Hz).
- Frequency stability of microwave is $\sigma \sim 10^{-10}$, at 12.6 GHz, ~5 Hz.
- Vacuum to the order of 10^{-10} mbar
- Preparation efficiencies of >90% obtained.
- Increased preparation efficiencies possible.

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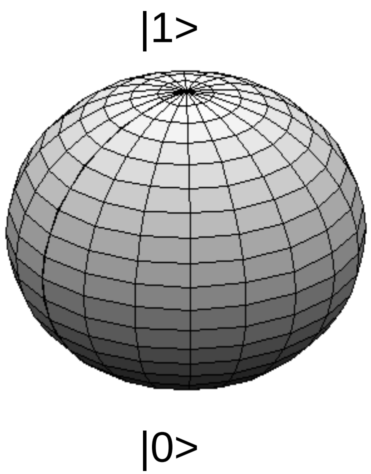
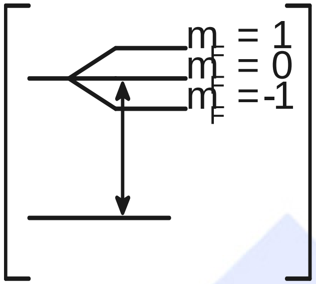
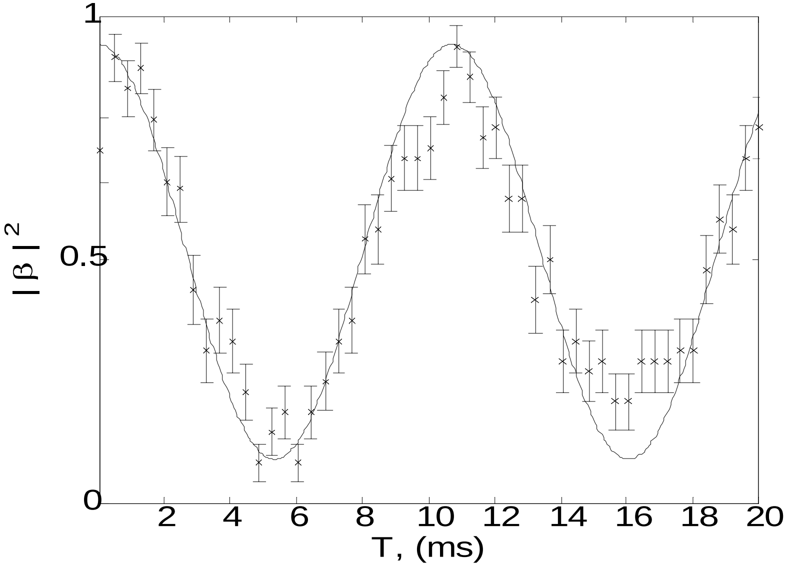
Ramsey measurement, time variation







$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$

for $\delta \ll \Omega$

$\mathbb{P}(|1\rangle) = |\beta|^2 \sim \cos^2(0.5\,\delta\,T)$

$\Omega_R^2 = \Omega^2 + \delta^2$

$\rightarrow \delta = 93\ [2\pi\text{Hz}],\ \Omega \sim 11[2\pi\text{kHz}]$