



Resetting uncontrolled quantum systems

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MN, arXiv:1710.02470



I invented a time-warping device,
ask me how!

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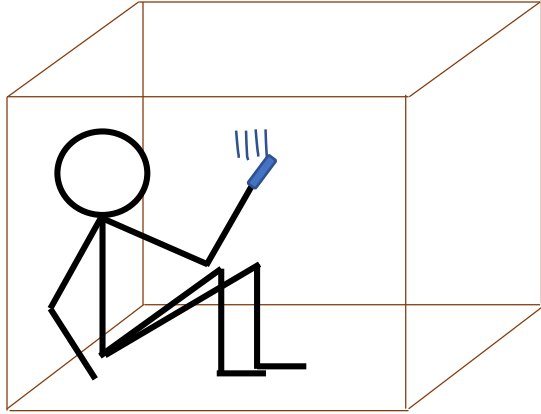


Time-warp

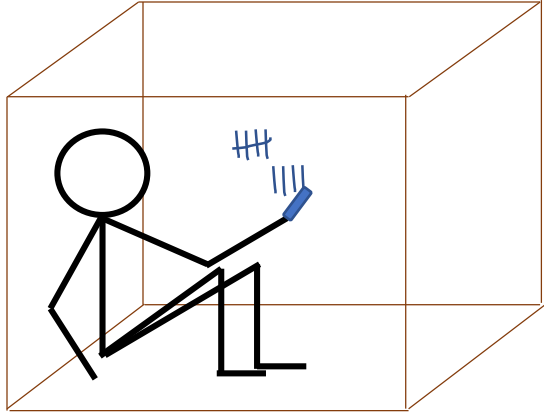
noun | \ 'tīm 'wɔrp \

Definition of TIME-WARP

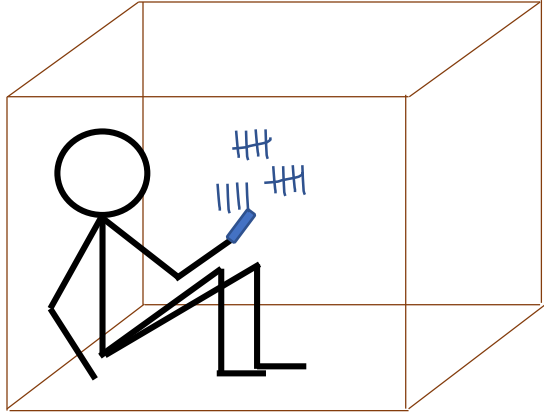
1. an anomaly, discontinuity, or suspension held to occur in the progress of time



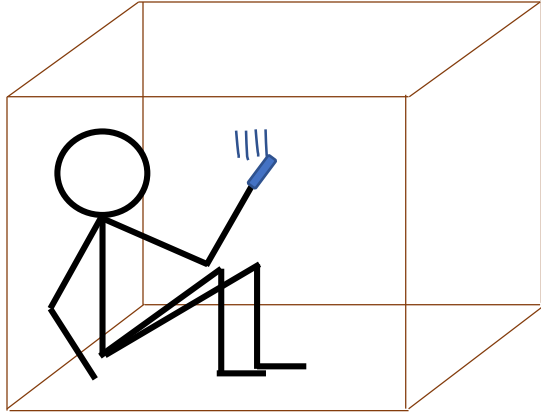
$t = 0$



$t = 1$ hour



$t = 2$ hours

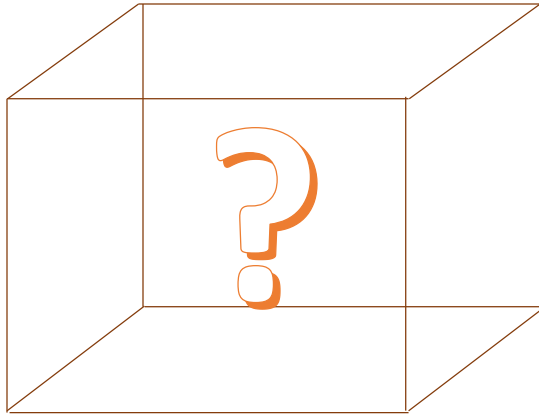


$t = 0$



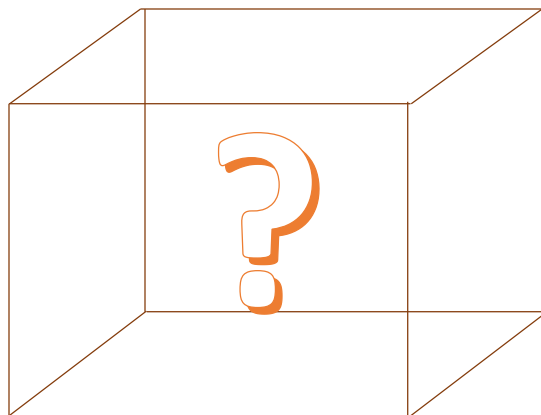
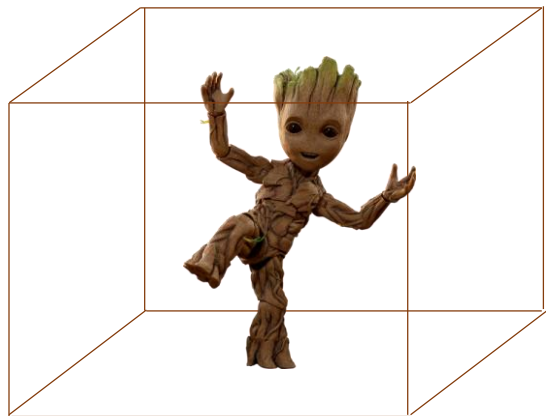
Time warp for two hours

What should we find in the box if we are to claim a time warp experience?



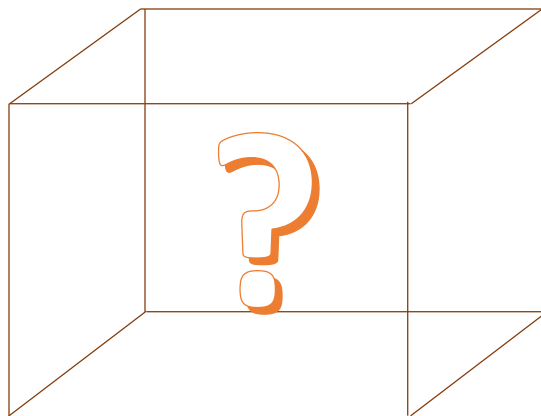
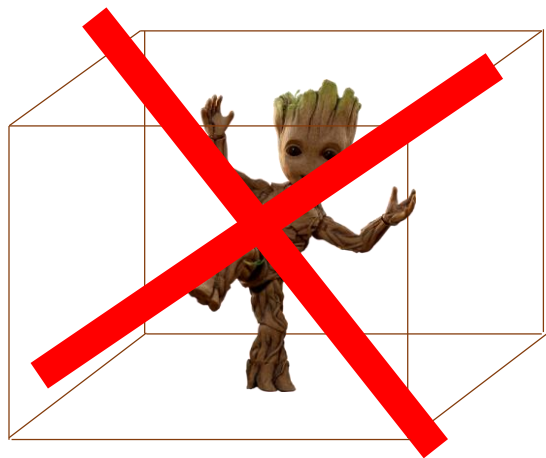
$t = 2$ hours

Not expected



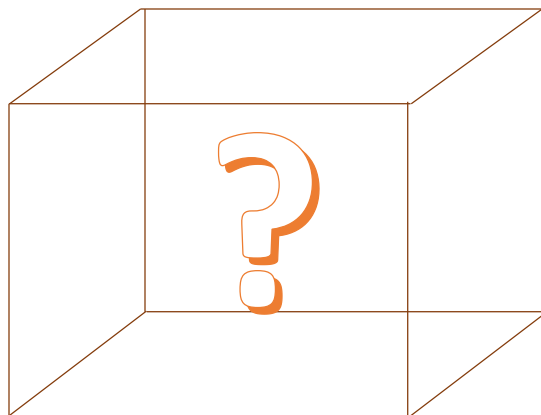
$t = 2$ hours

Not expected

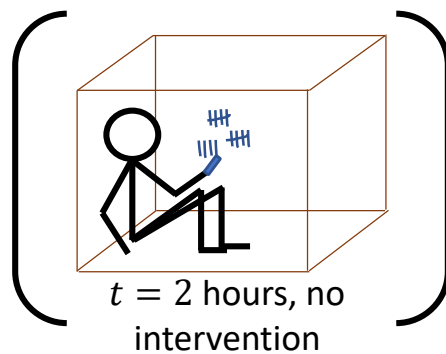
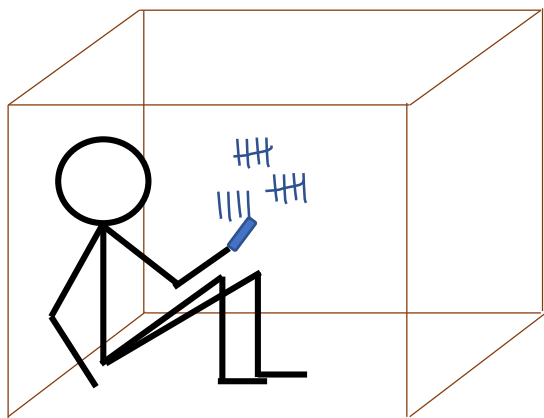


$t = 2$ hours

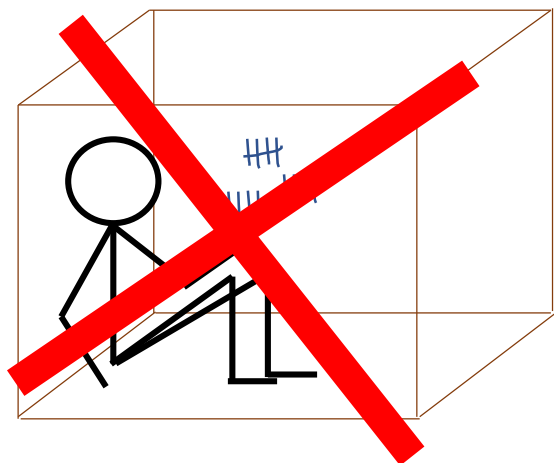
Not expected



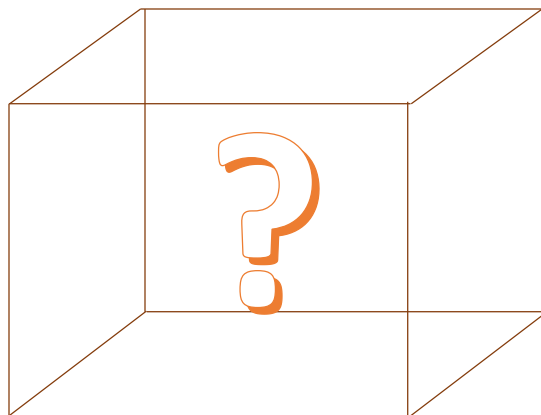
$t = 2$ hours



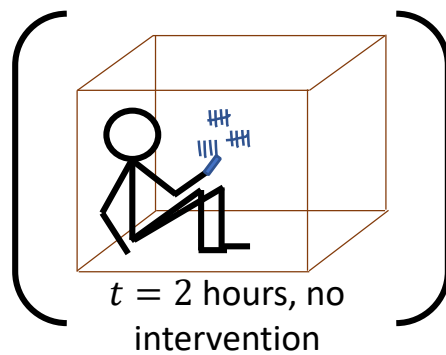
Not expected



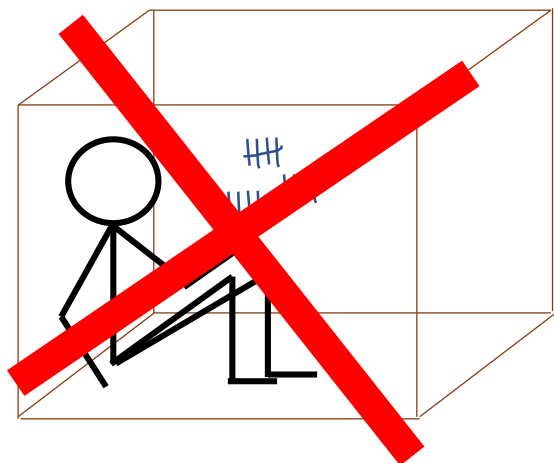
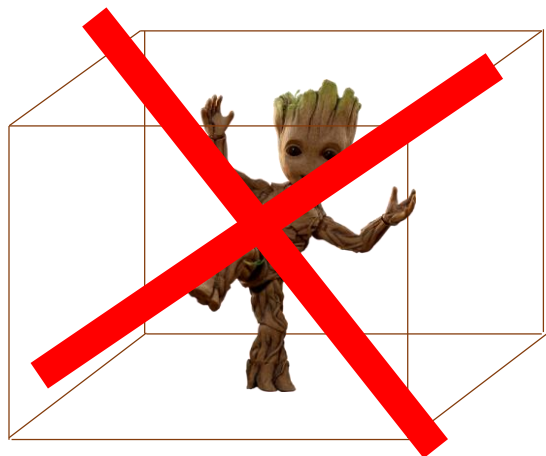
Expected



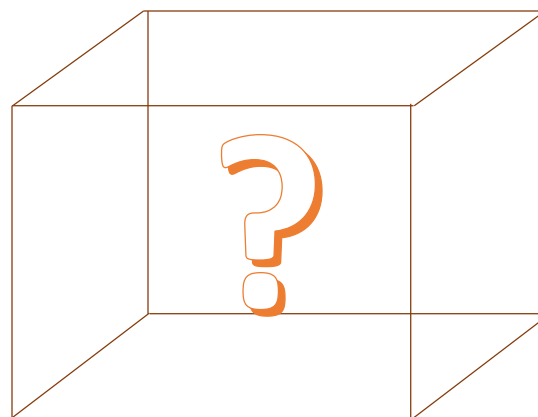
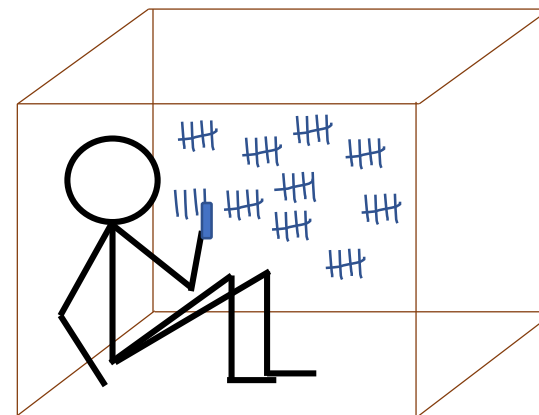
$t = 2$ hours



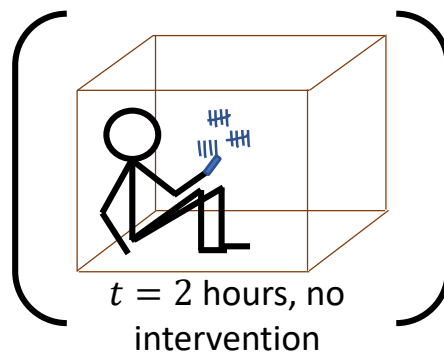
Not expected



Expected

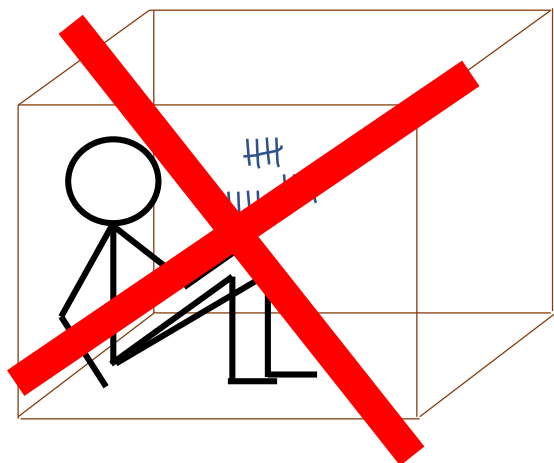


$t = 2$ hours

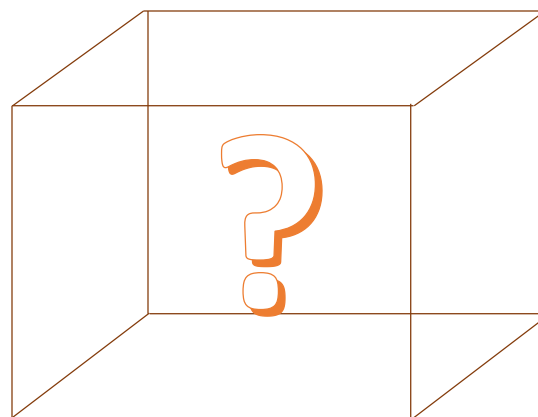
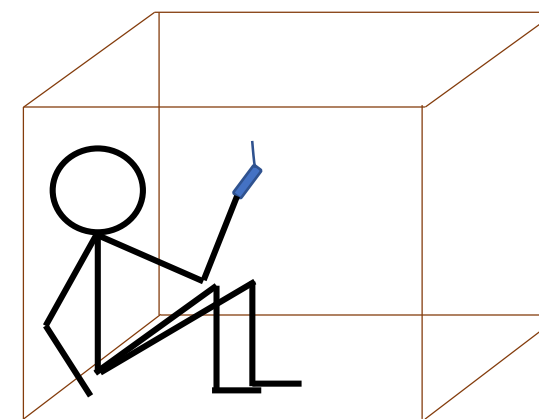
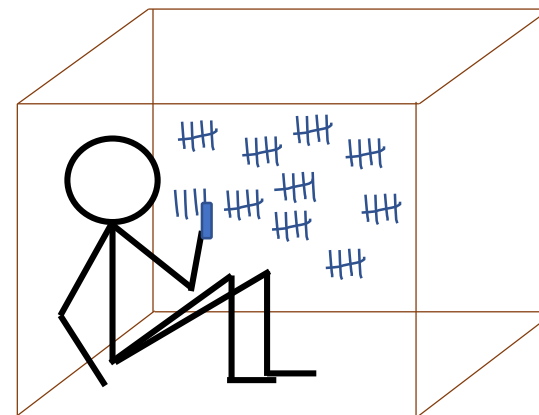


$t = 2$ hours, no
intervention

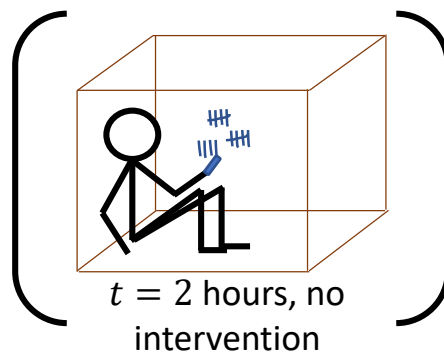
Not expected



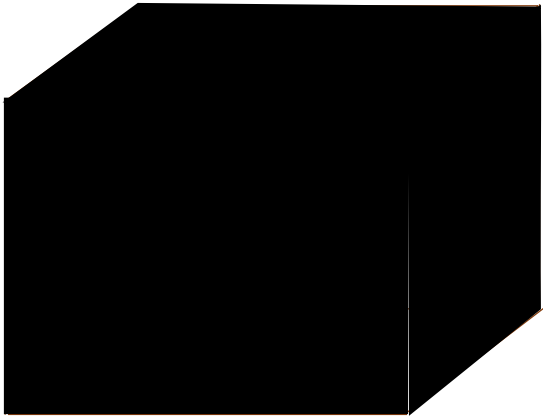
Expected



$t = 2$ hours

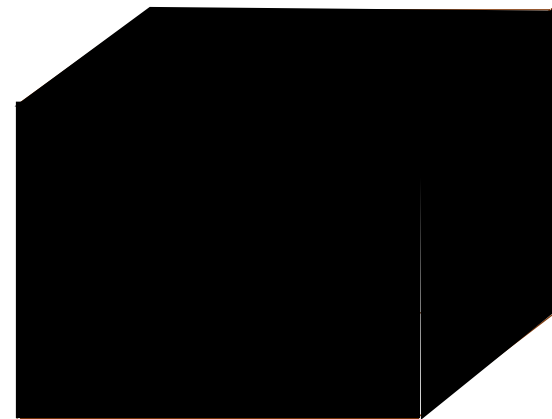


Time warp: operational definition

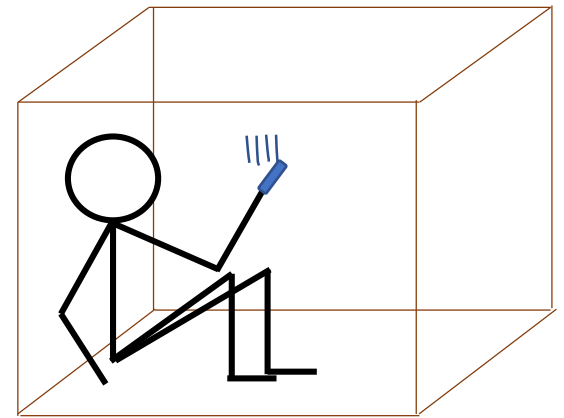
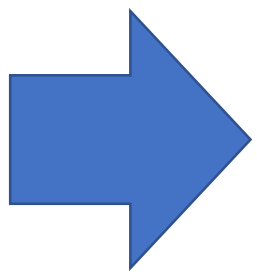


$\{\psi(t): t\}$

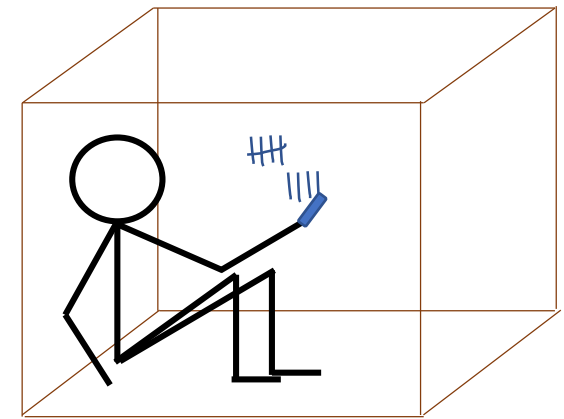
Time warp: operational definition



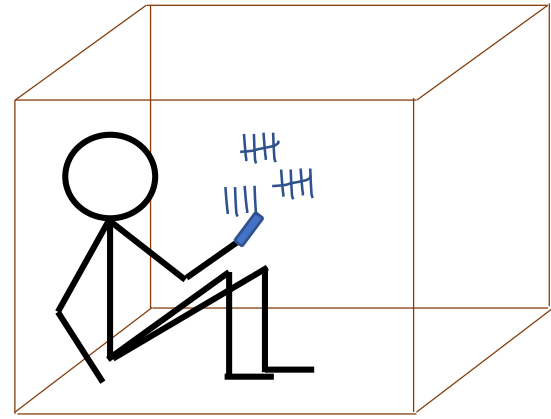
$\{\psi(t): t\}$



$\psi(0)$



$\psi(1)$



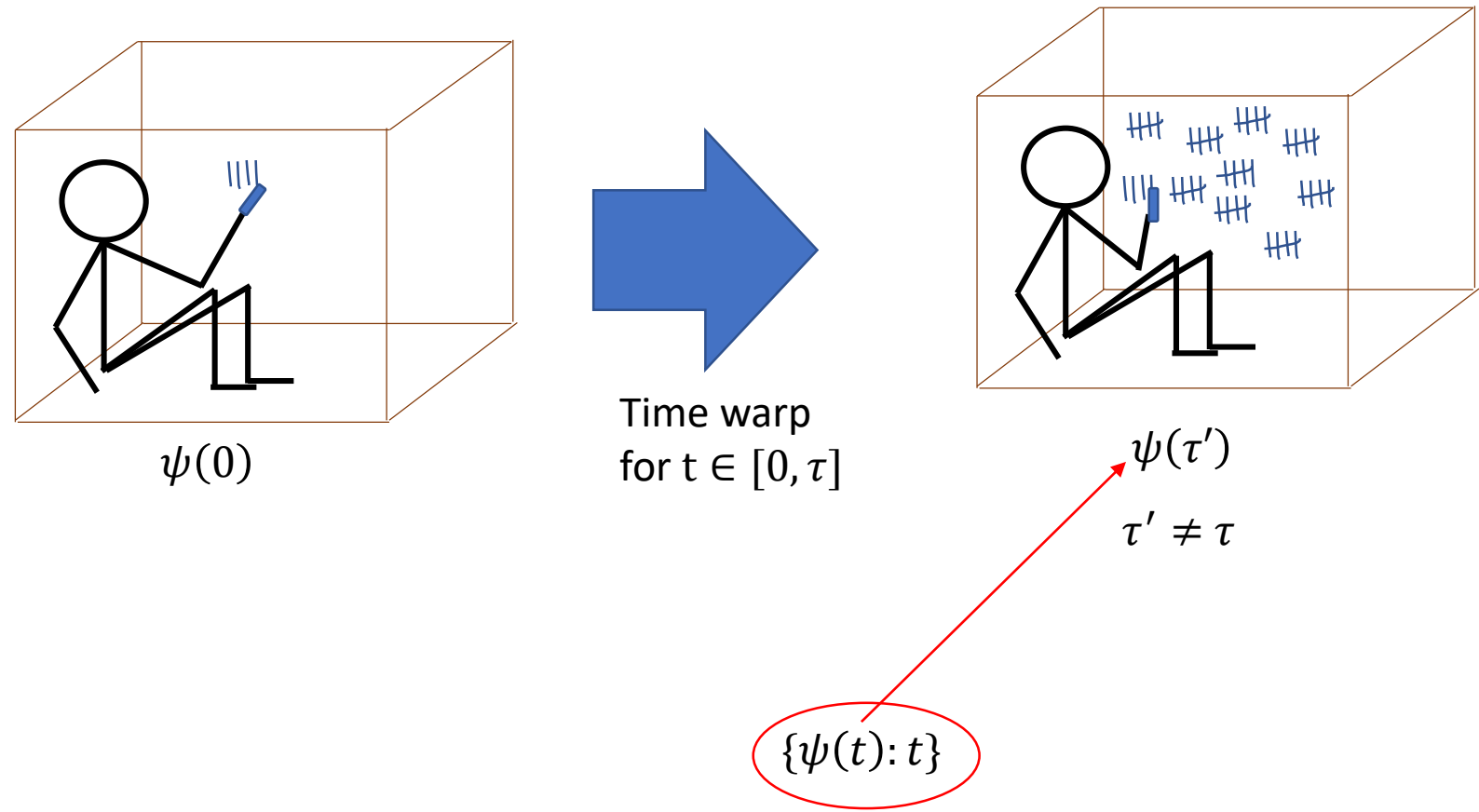
$\psi(2)$

Time warp: operational definition



Time warp protocol for $t \in [0, \tau]$

Time warp: operational definition





A brief history of time warp

King Raivata and
princess Revati
(400BC?)





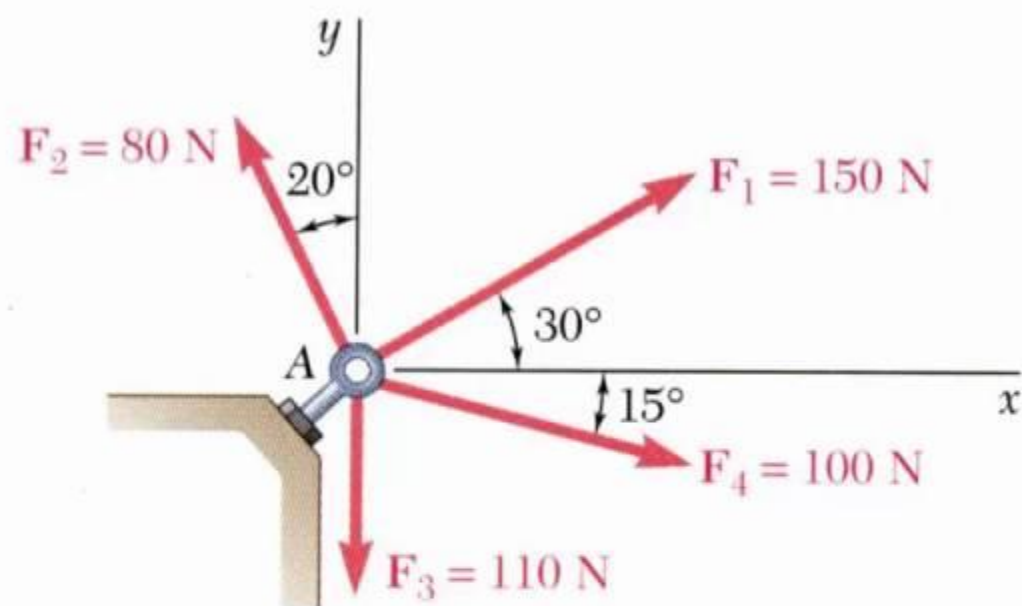
Peter Damian
(1007-1072)

The Time Machine



H. G. Wells

(first edition in 1895)



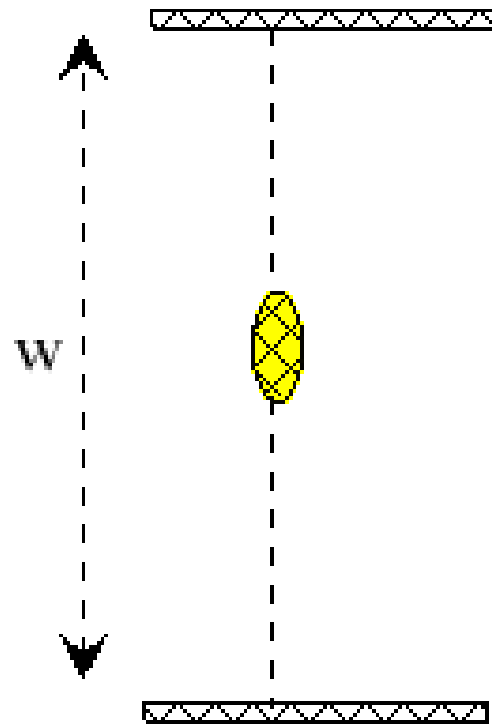
Four forces act on bolt A as shown. Determine the resultant of the force on the bolt.

- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

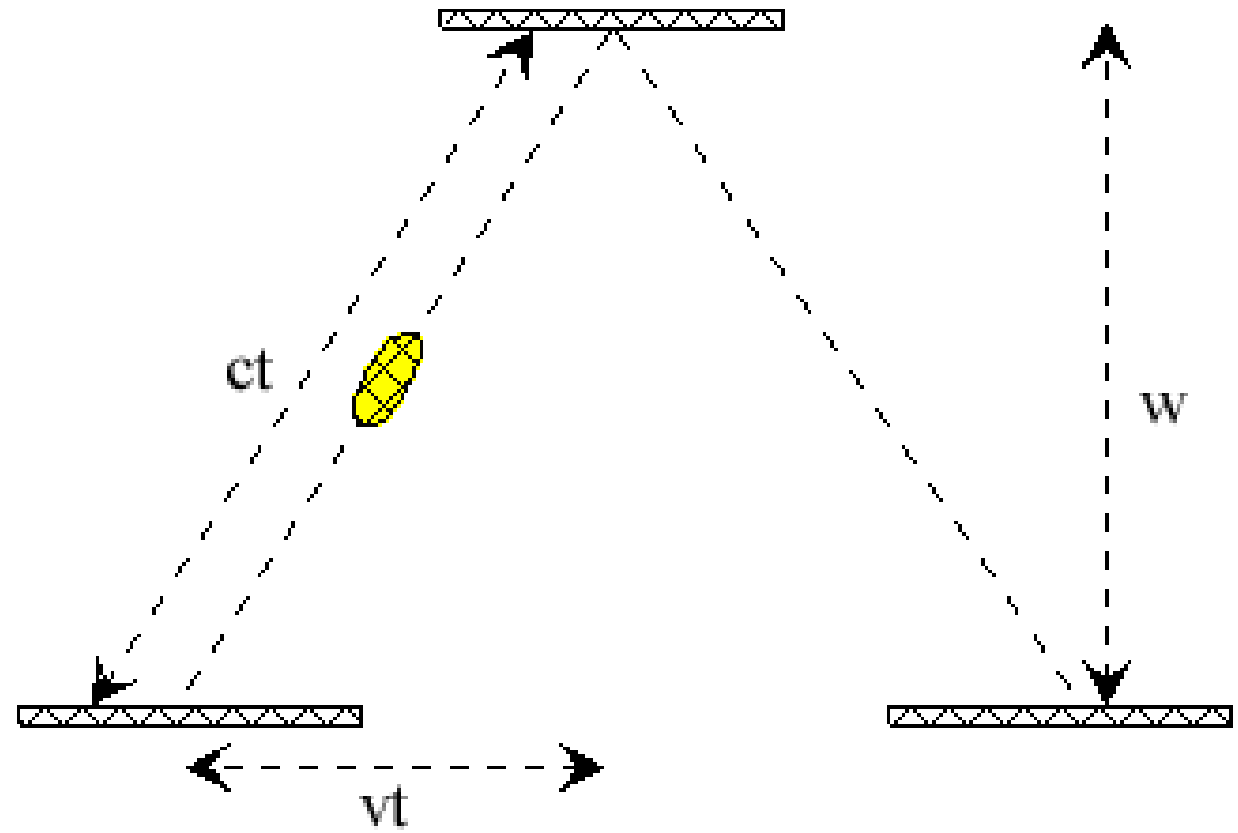


Warping time physically

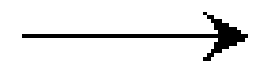
mirror



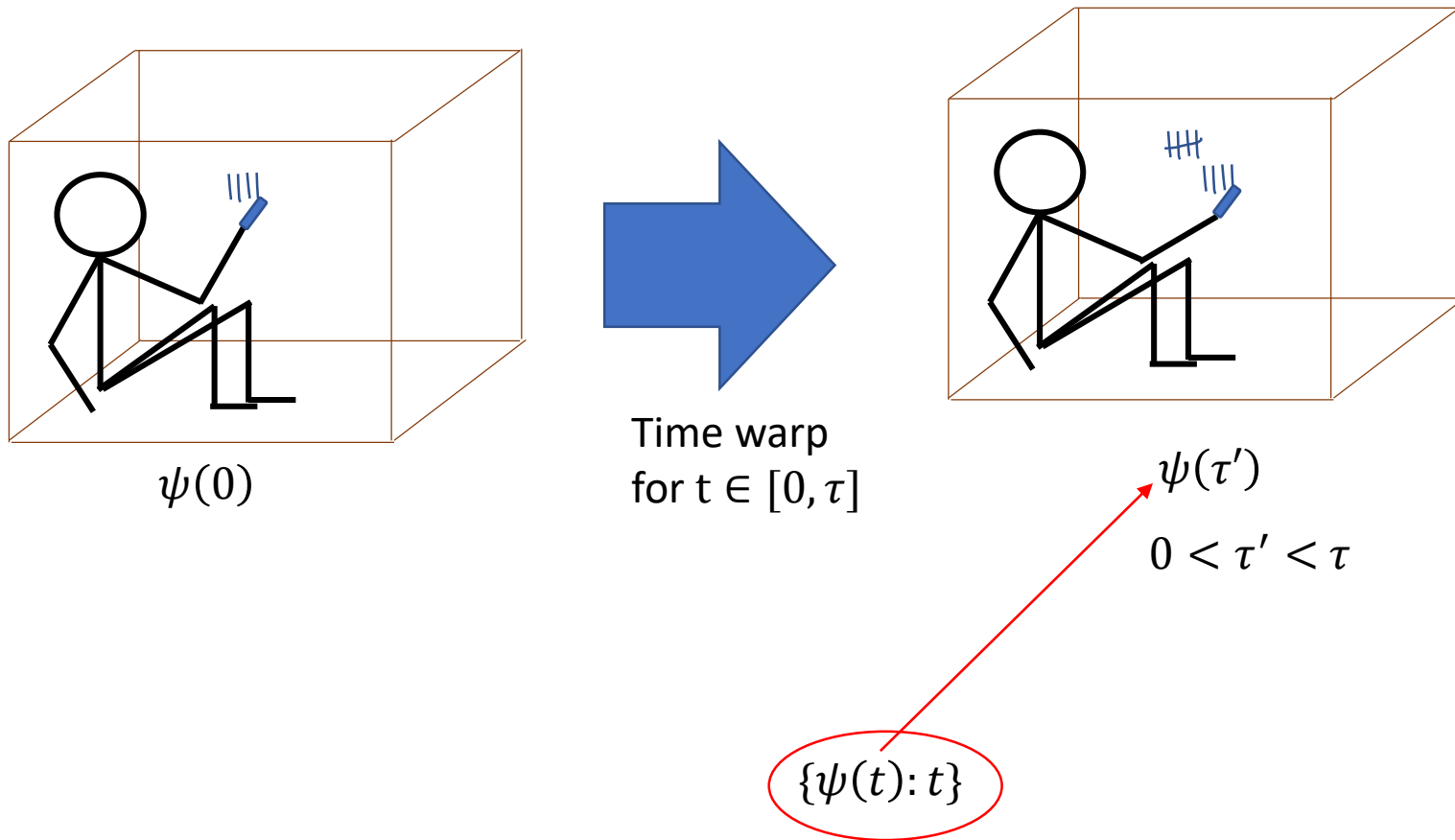
clock at rest



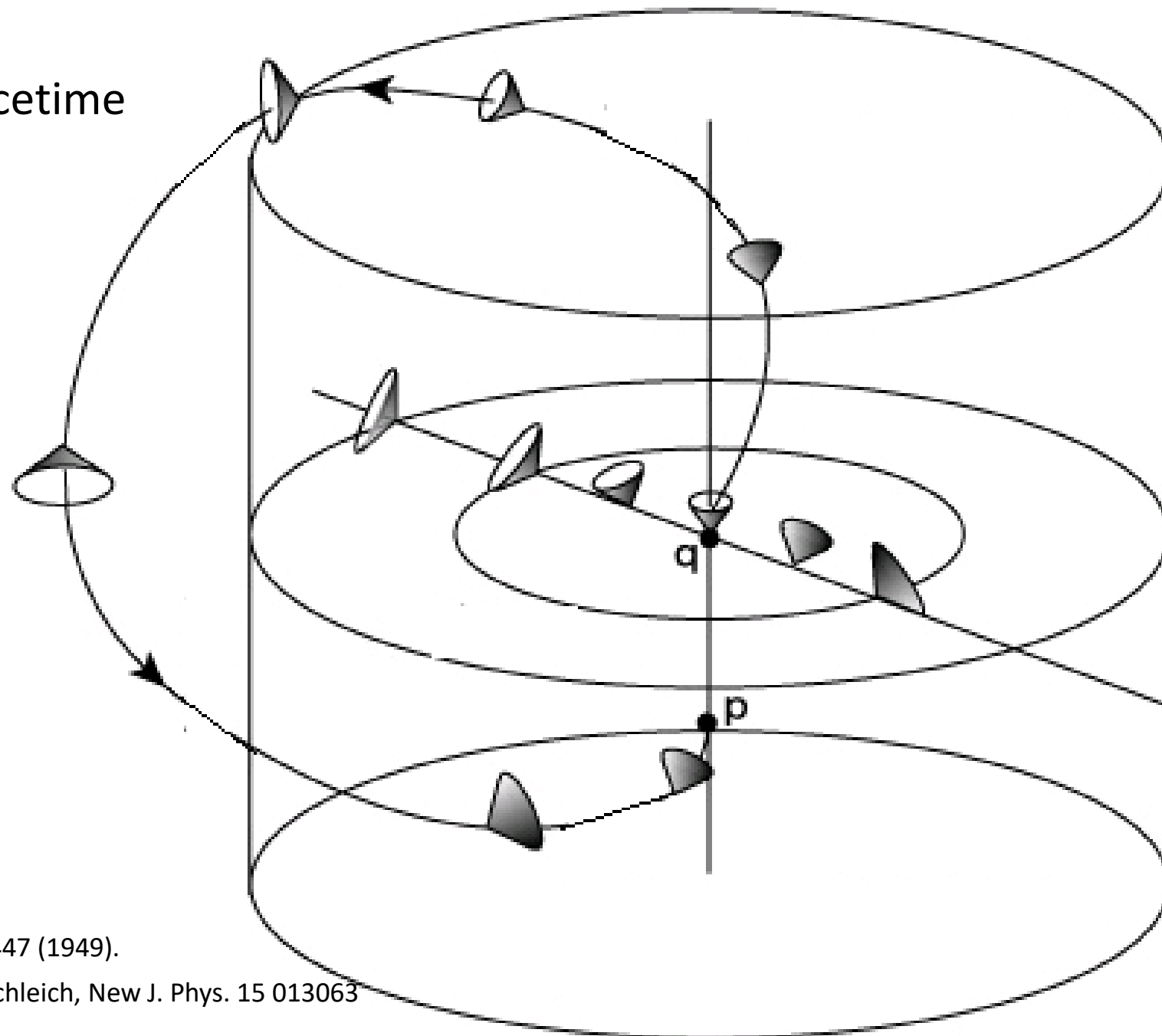
clock moving at v



Time warp in special relativity



Gödel spacetime



K. Gödel, *Rev. Mod. Phys.* **21** 447 (1949).

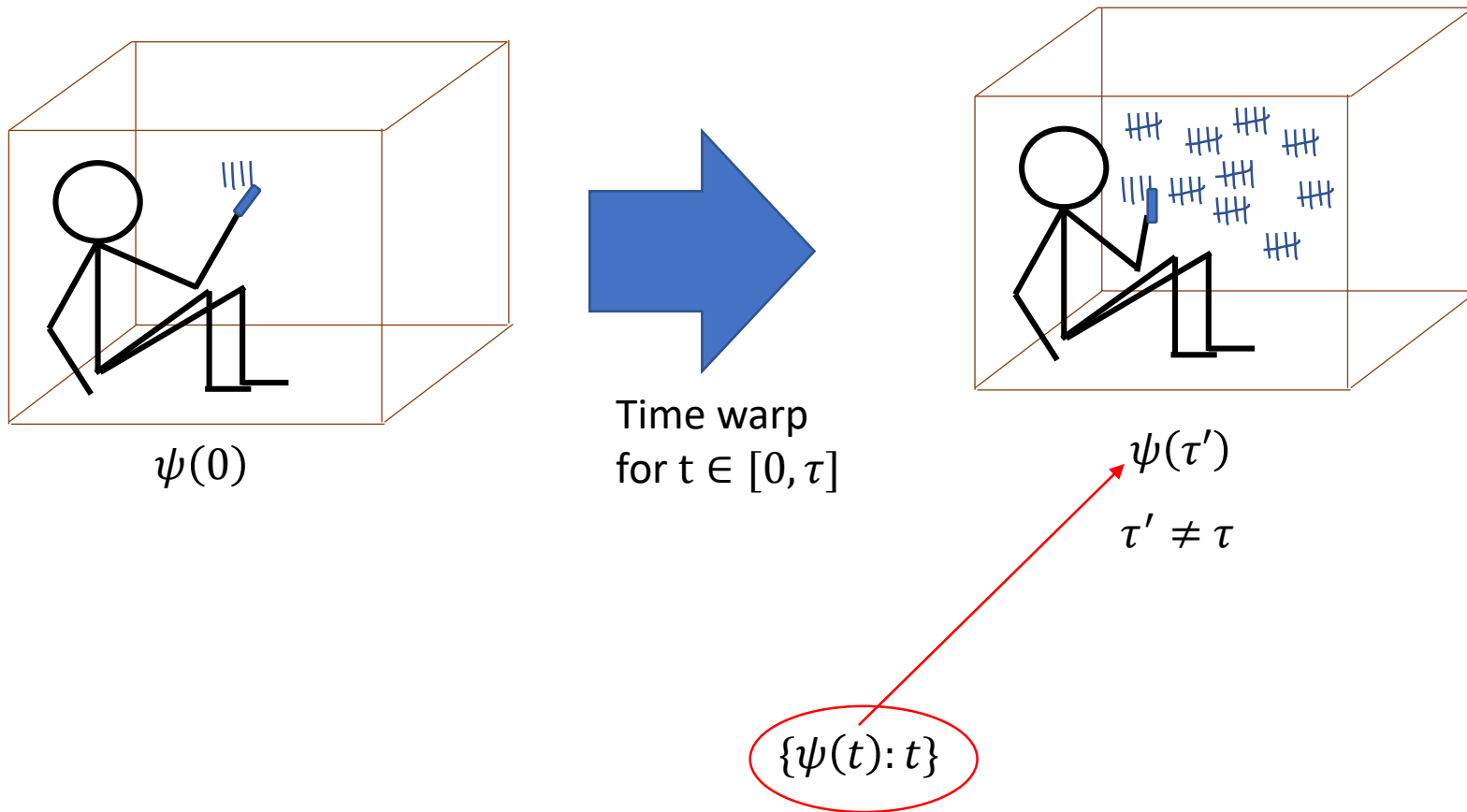
M. Buser, E. Kajari and W. P. Schleich, *New J. Phys.* **15** 013063 (2013).

Time travel
with
wormholes

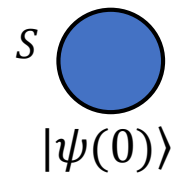


K. Thorne, *Black Holes and Time Warps: Einstein's
Outrageous Legacy*, Commonwealth Fund Book Program
(1994).

Time warp with time machines



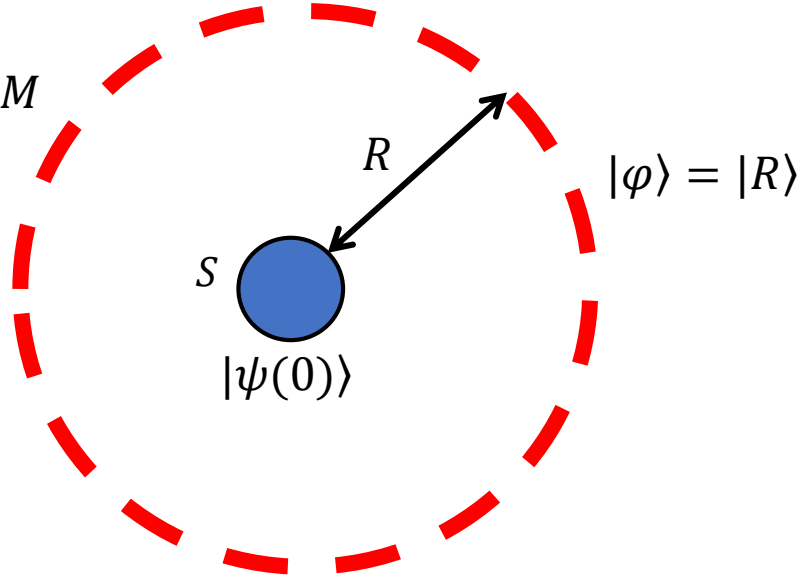
The time translator



Evolution after time T

$$e^{-iH_0T} |\psi(0)\rangle$$

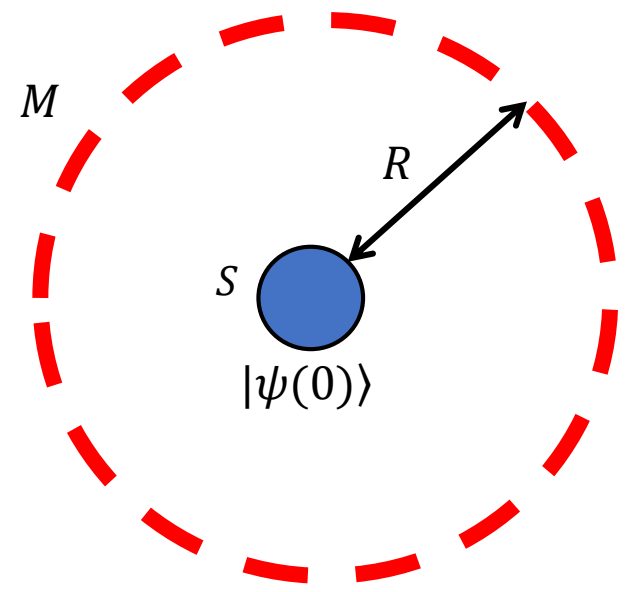
The time translator



Evolution after time T

$$e^{-iH_0\gamma_R T} |\psi(0)\rangle |R\rangle$$

The time translator

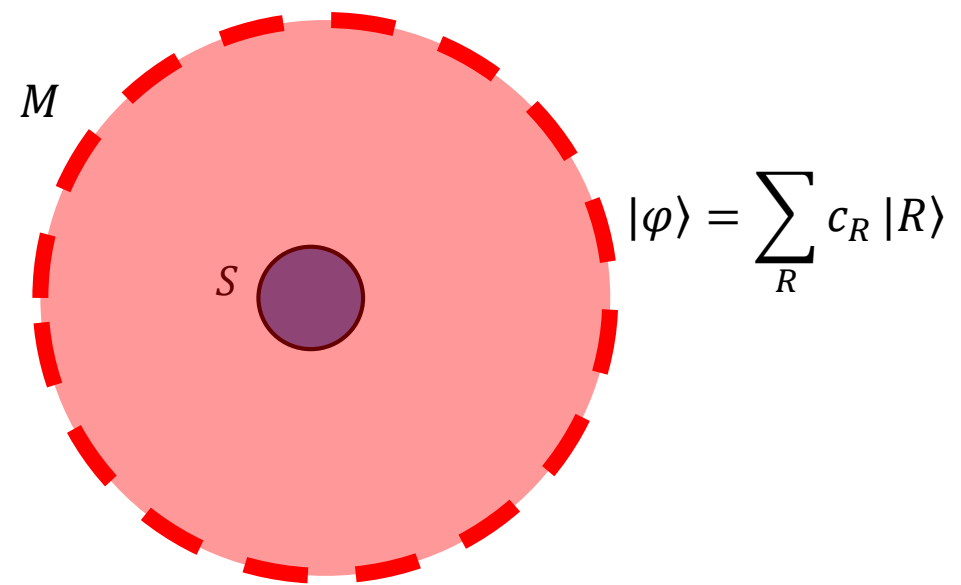


Evolution after time T

$$e^{-iH_0 \gamma_R T} |\psi(0)\rangle |R\rangle$$

$$\gamma_R = \sqrt{1 - \frac{2GM}{Rc^2}}$$

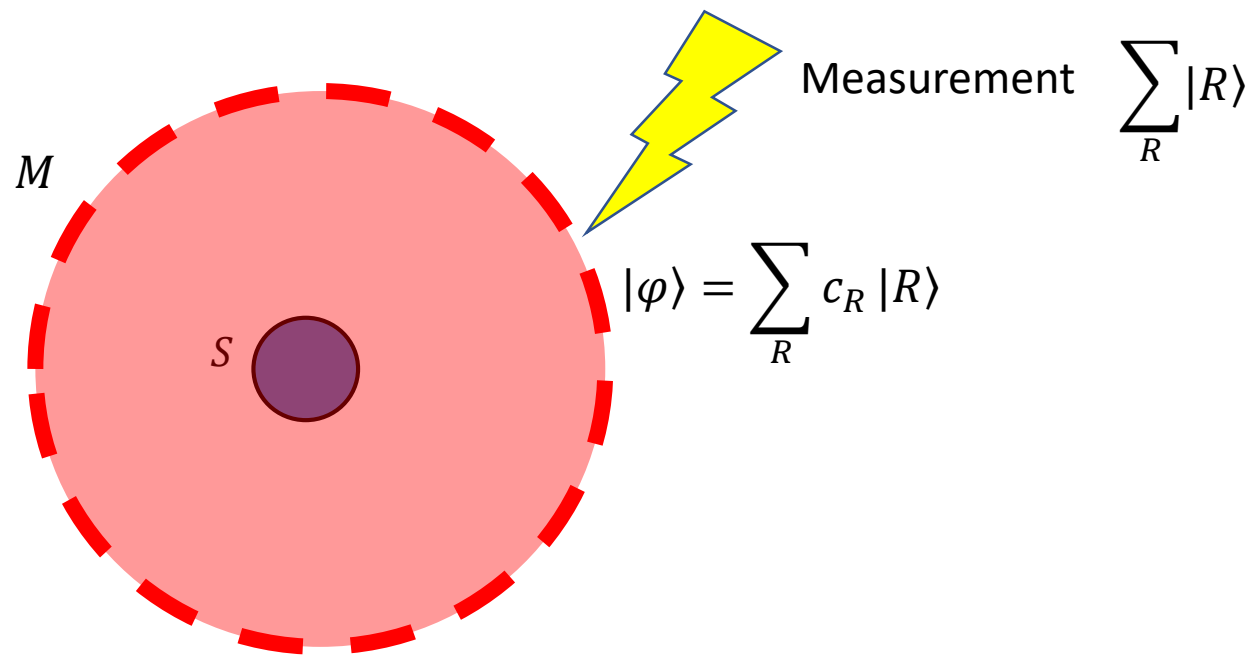
The time translator



Evolution after time T

$$\sum_R c_R e^{-iH_0\gamma RT} |\psi(0)\rangle |R\rangle$$

The time translator

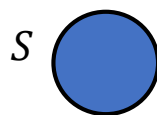


Evolution after time T

$$\sum_R c_R e^{-iH_0\gamma_R T} |\psi(0)\rangle |R\rangle$$

The time translator

M

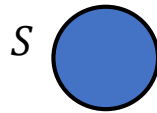


Evolution after time T

$$\sum_R c_R e^{-iH_0 \gamma_R T} |\psi(0)\rangle$$

The time translator

M

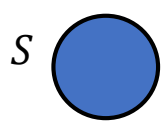


Evolution after time $T \ll 1$

$$\sum_R c_R e^{-iH_0 \gamma_R T} |\psi(0)\rangle \approx e^{-iH_0 \alpha T} |\psi(0)\rangle$$

The time translator

M

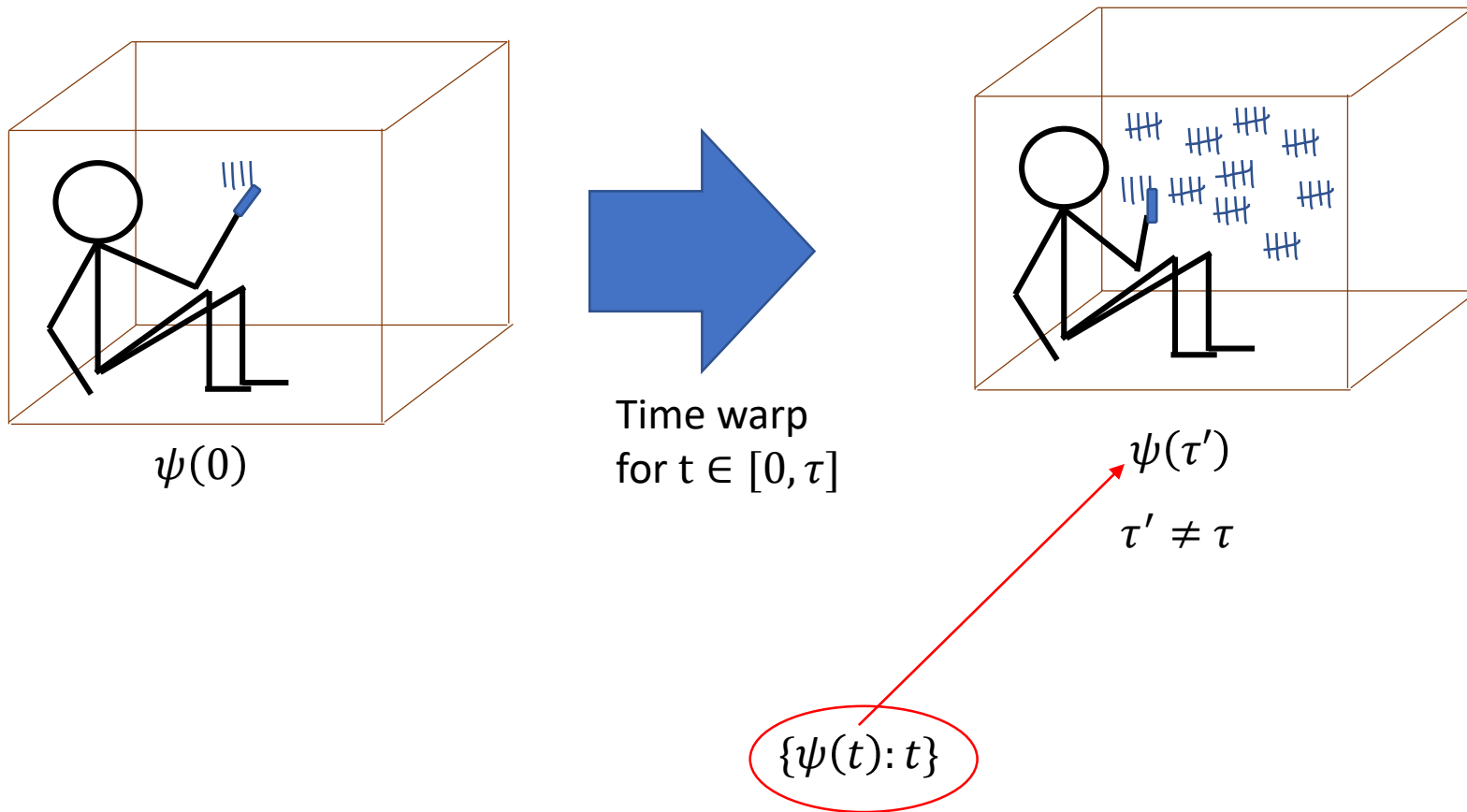


Interesting cases $\left\{ \begin{array}{l} \alpha \gg 1 \\ \alpha \leq 0 \end{array} \right.$

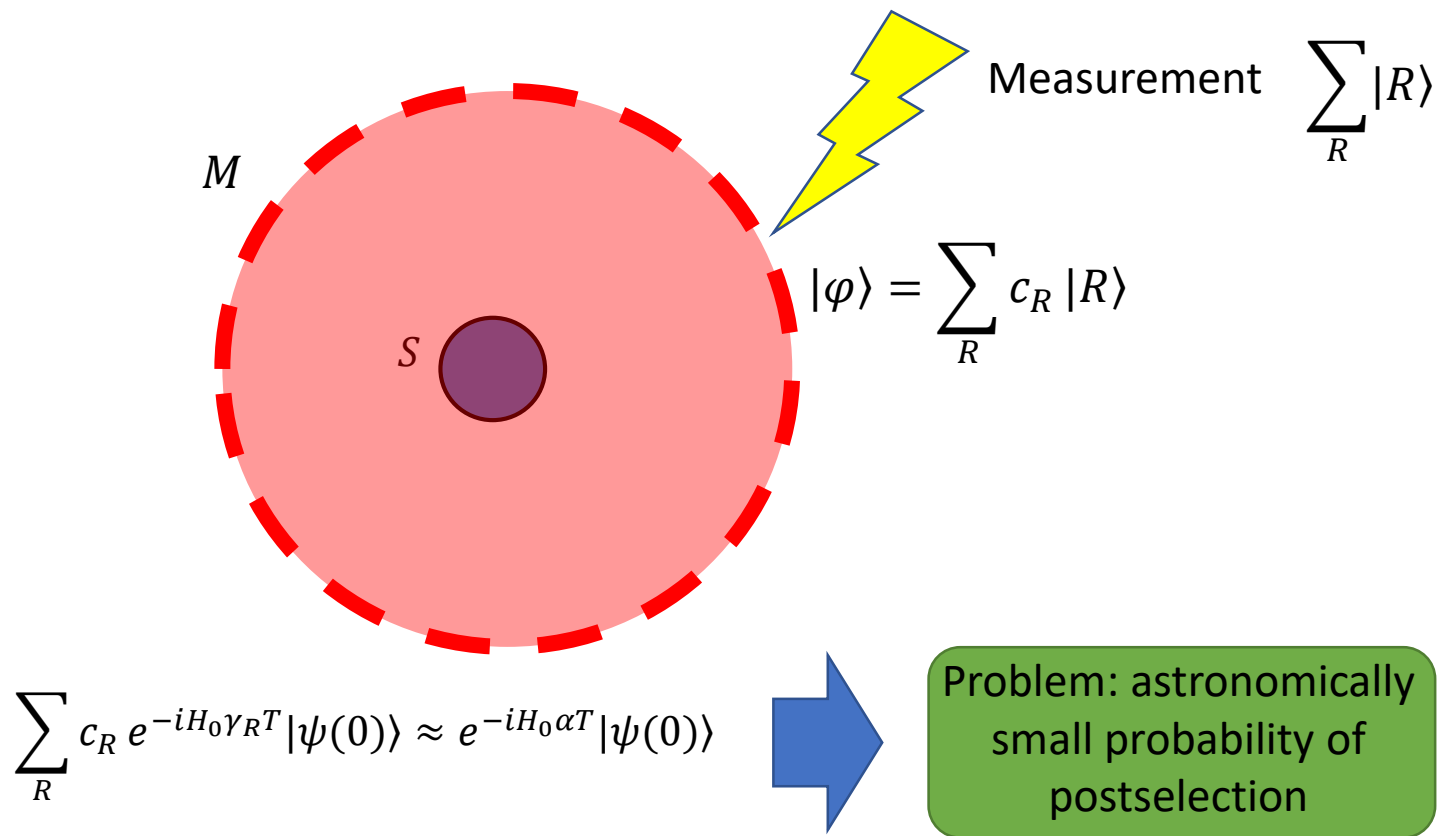
Evolution after time $T \ll 1$

$$\sum_R c_R e^{-iH_0 \gamma_R T} |\psi(0)\rangle \approx e^{-iH_0 \alpha T} |\psi(0)\rangle$$

Time warp with the time translator



The time translator

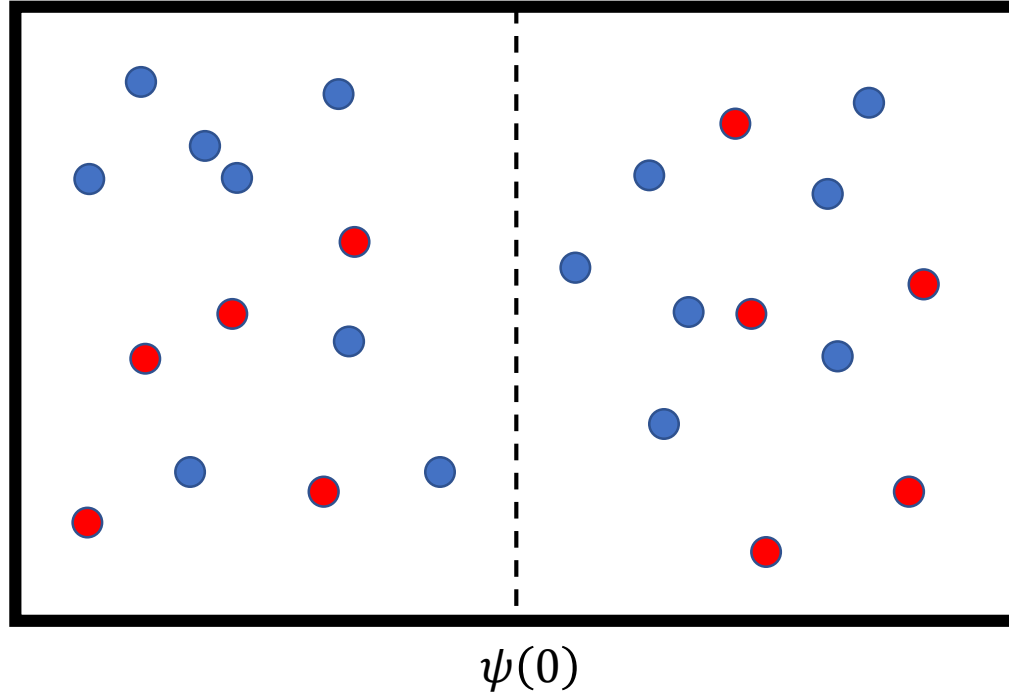




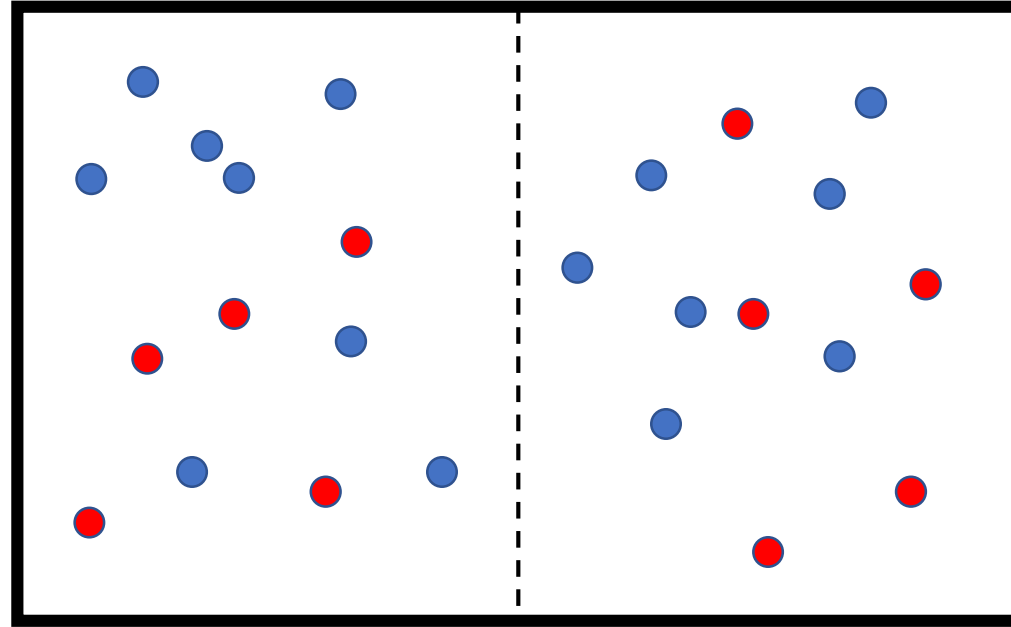
“The time translator has the same chances of succeeding as I have of delocalizing and relocalizing somewhere else”

L. Vaidman

Time warp, the lame way



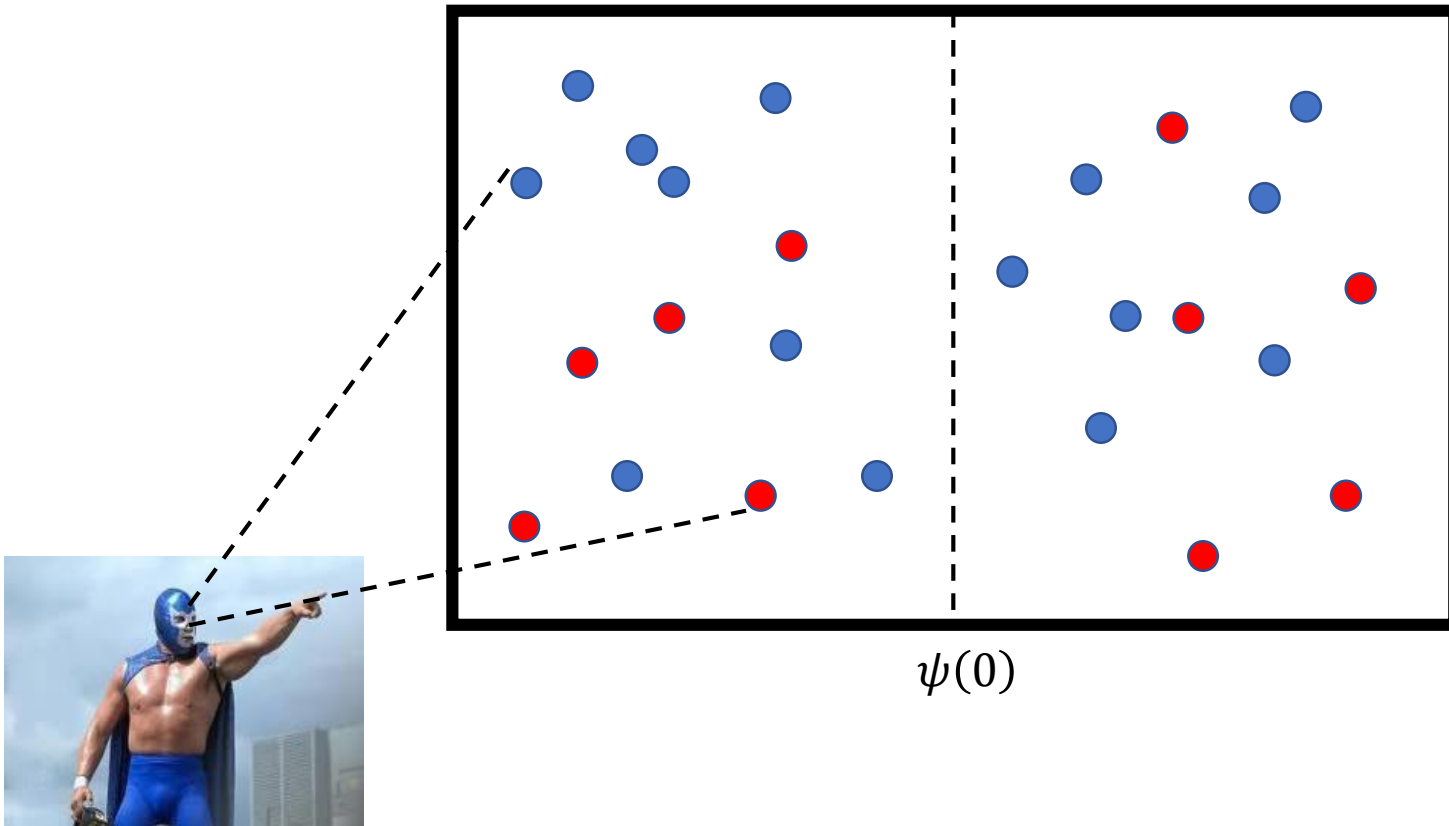
Time warp, the lame way



$\psi(0)$

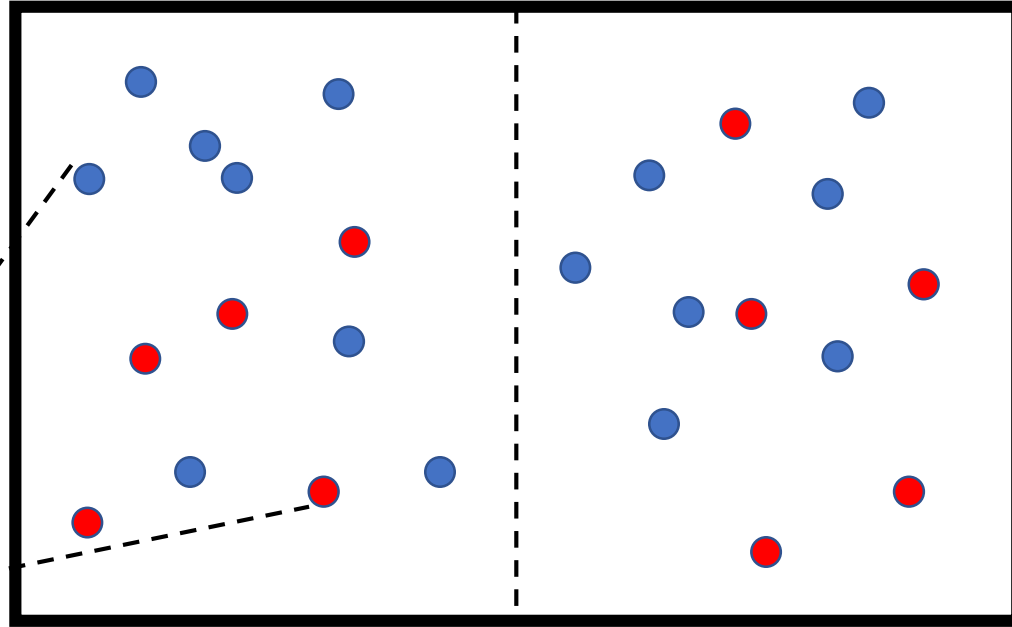


Time warp, the lame way



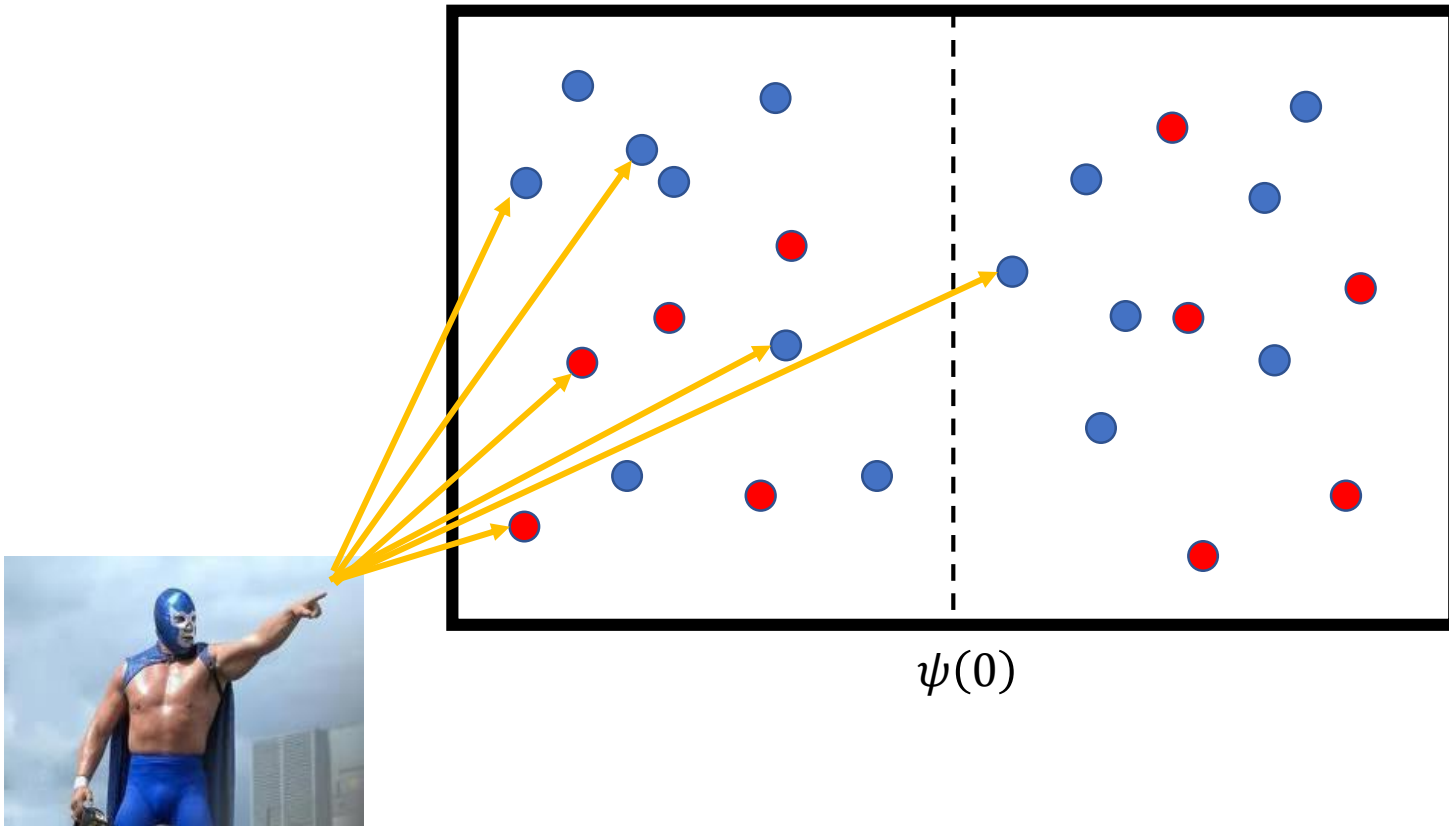
Time warp, the lame way

(0.3,0.2,0.9),
(0.4,0.7,0.1),
(0.5,0.6,0.3),

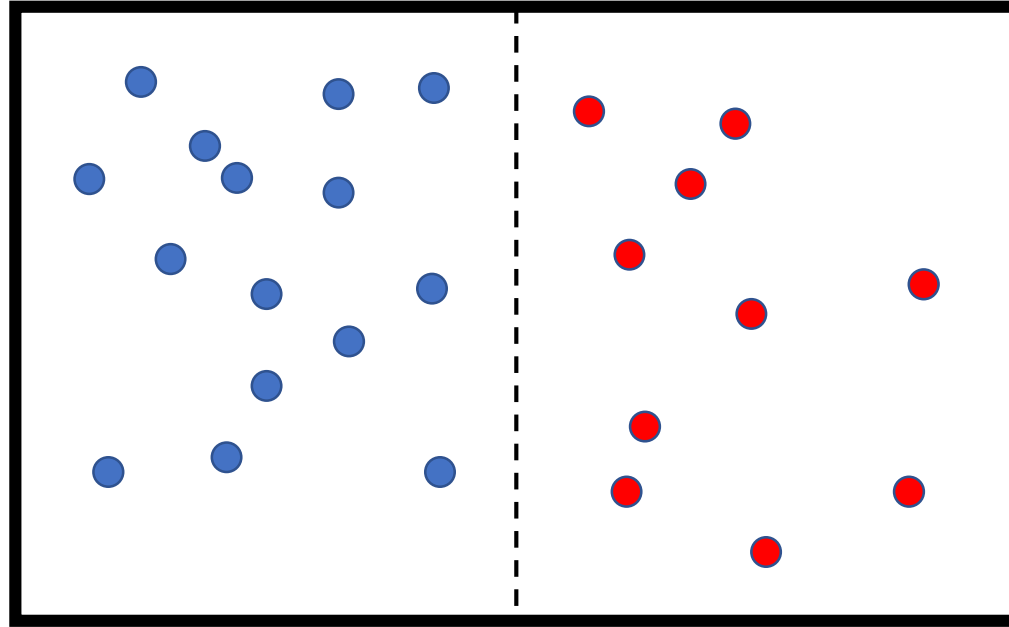


$\psi(0)$

Time warp, the lame way

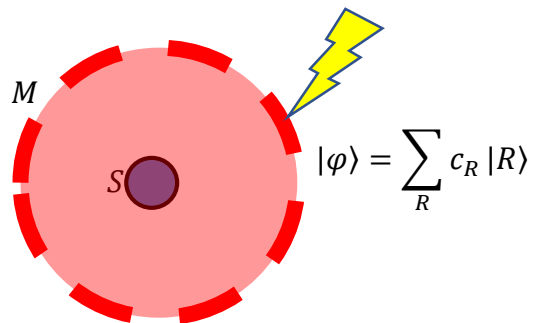
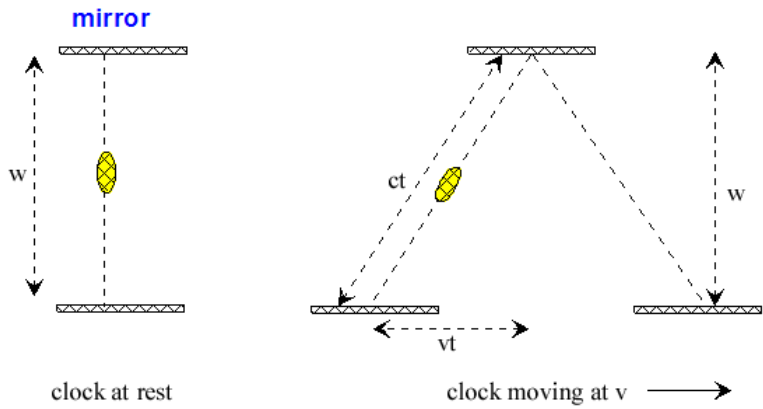


Time warp, the lame way

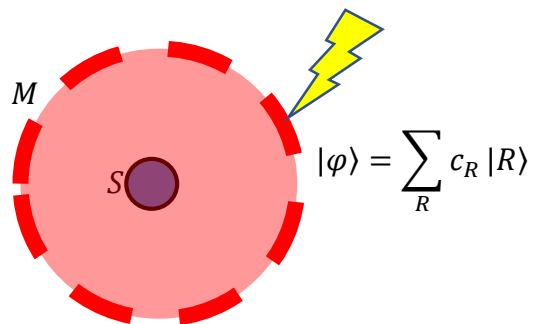
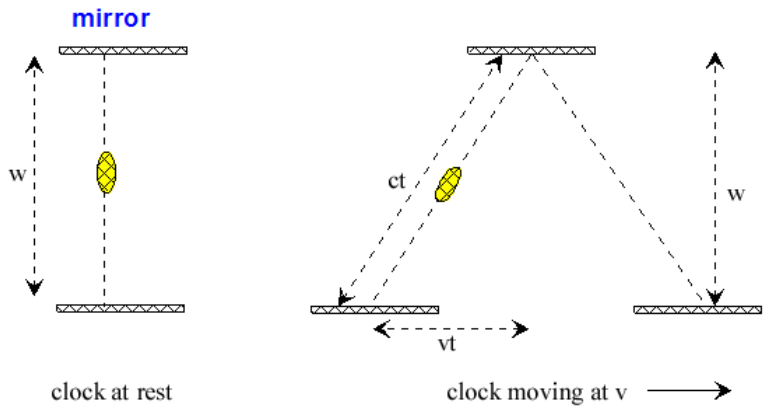


$\psi(-5732)$



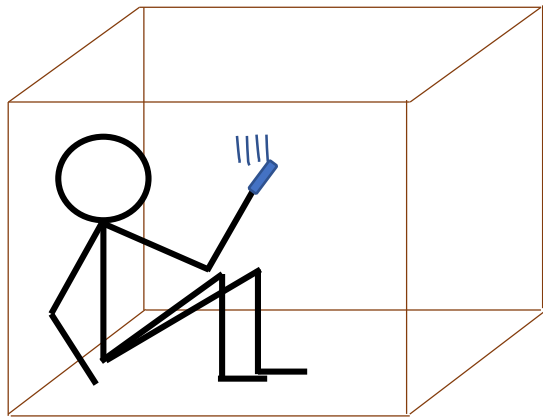


These proposals do not require control or knowledge of the physical system that we influence



All interesting proposals to achieve time warp rely on special or general relativity

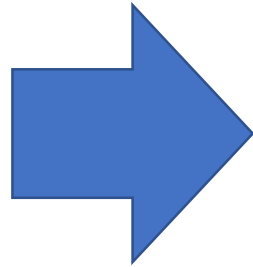
Main result



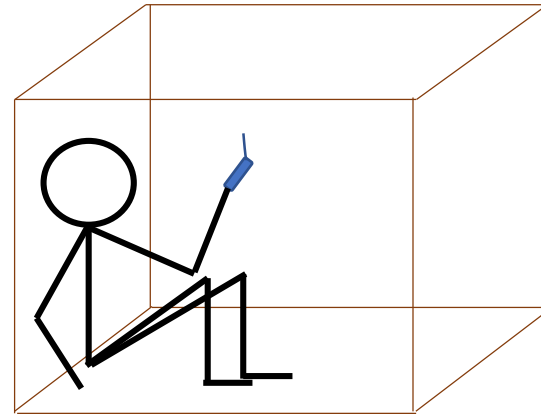
$$\psi(0)$$

Uncontrolled system

Non-relativistic
quantum physics



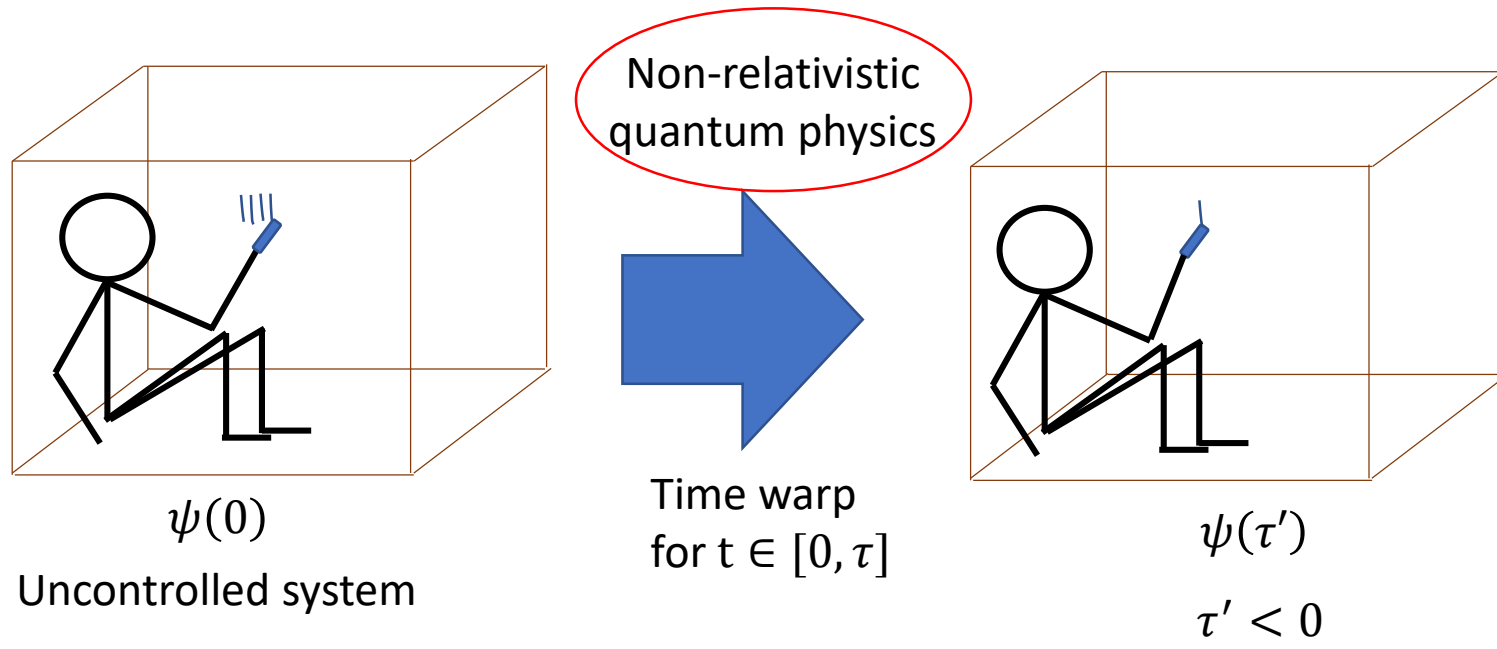
Time warp
for $t \in [0, \tau]$



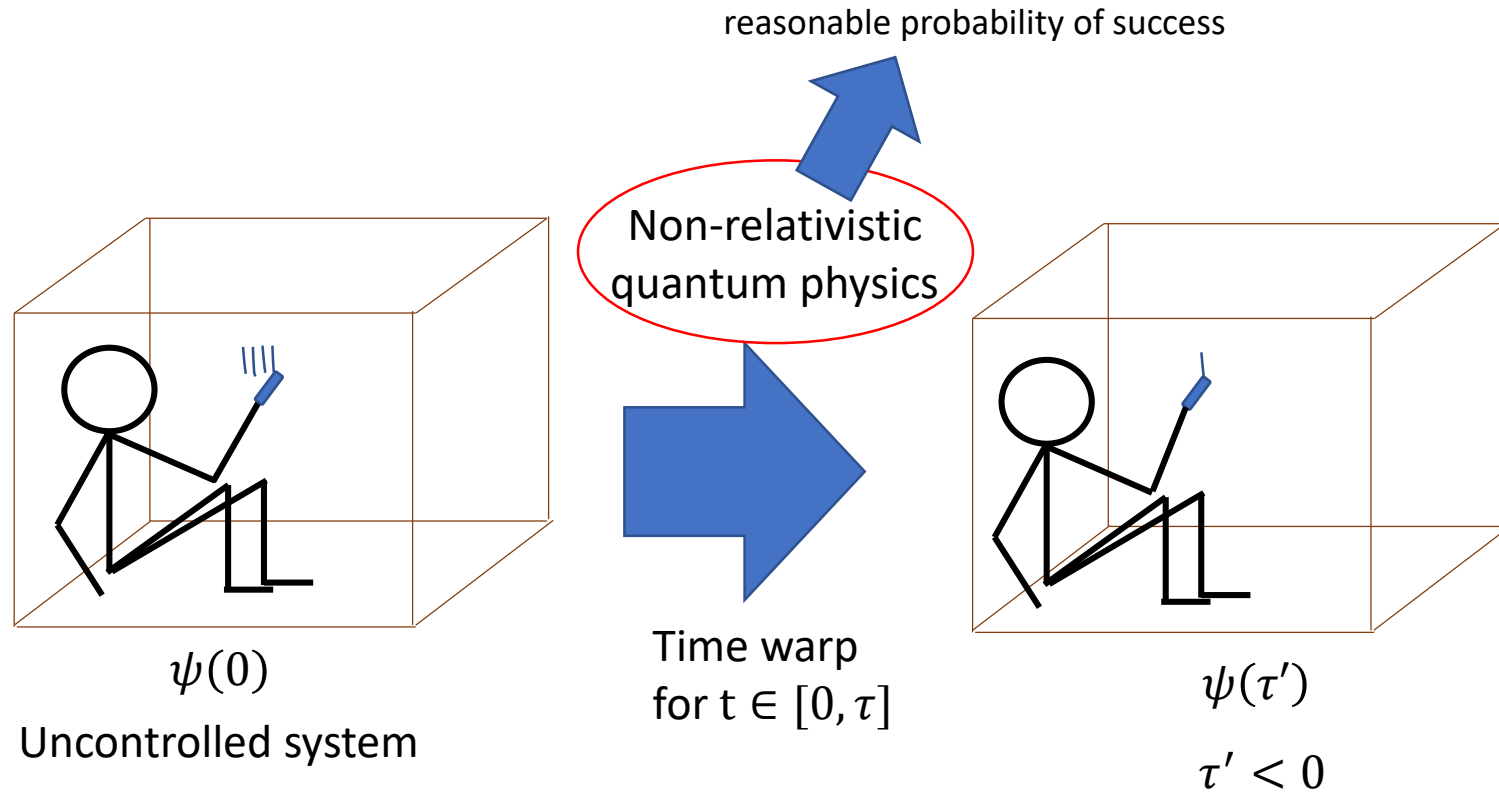
$$\psi(\tau')$$

$$\tau' < 0$$

Main result



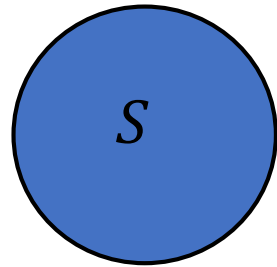
Main result



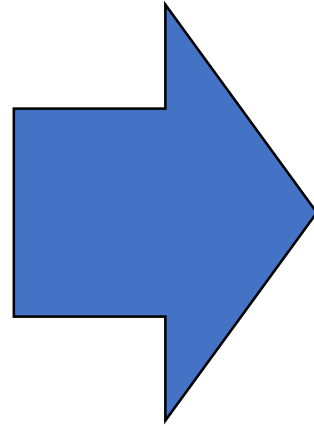
Scenario

Goal

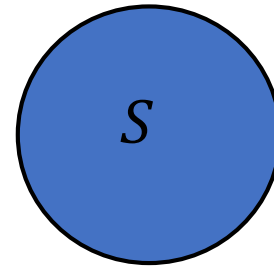
$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$



$$t = T > 0$$



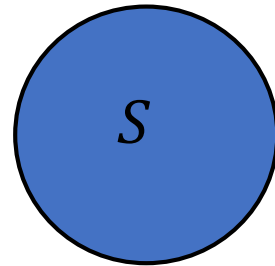
$$|\psi(0)\rangle$$



$$t = T + \Delta$$

Obvious solution

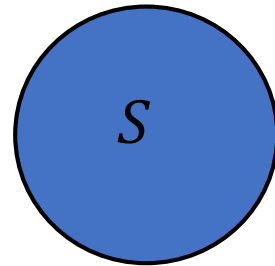
$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$



$$|\psi(0)\rangle = e^{+iH_0T} |\psi(T)\rangle$$

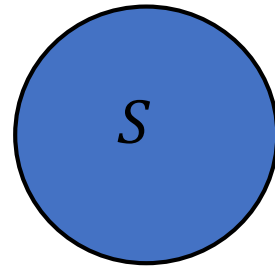
Obvious solution

$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$



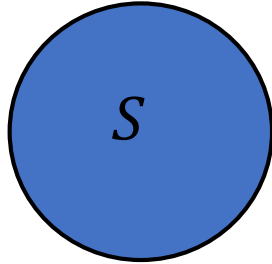
~~$$|\psi(0)\rangle = e^{+iH_0T} |\psi(T)\rangle$$~~

$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$

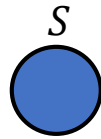
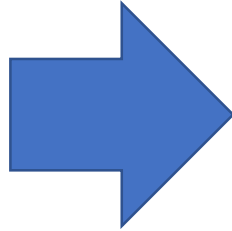


S, uncontrolled

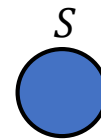
$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$



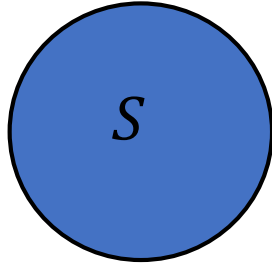
S , uncontrolled



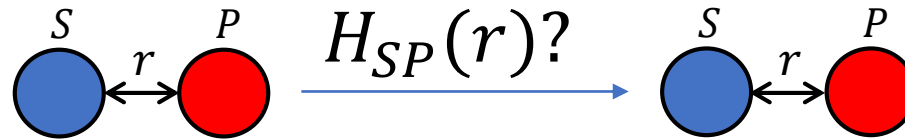
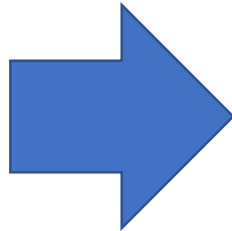
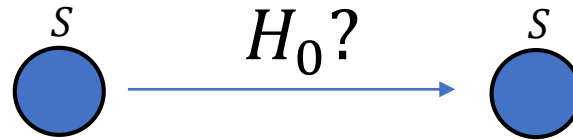
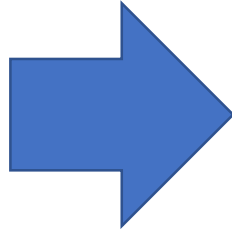
$H_0?$



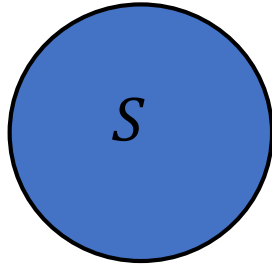
$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$



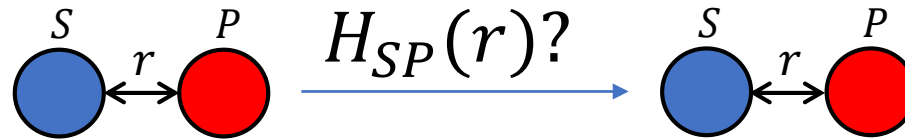
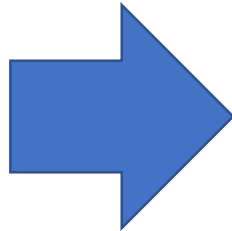
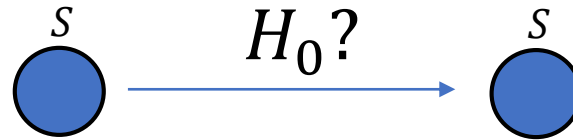
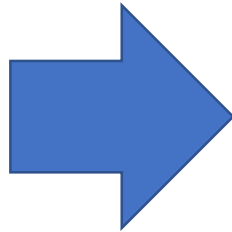
S, uncontrolled



$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$

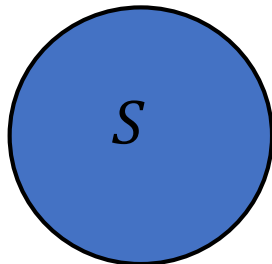


S, uncontrolled

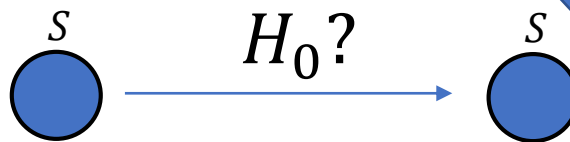
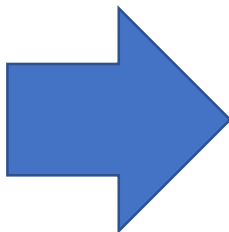


$$|\psi(0)\rangle = e^{+iH_0T} |\psi(T)\rangle?$$

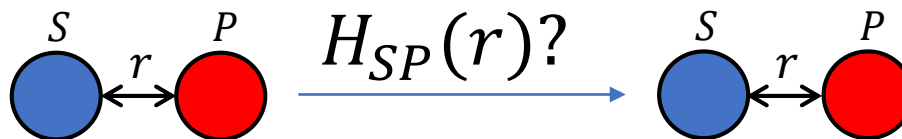
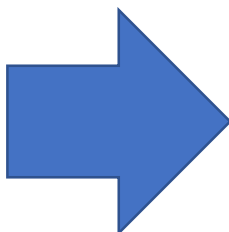
$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$



S, uncontrolled

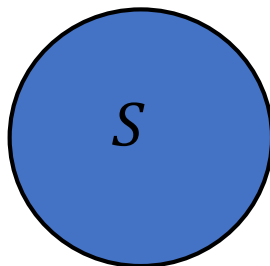


We don't know H_0 .

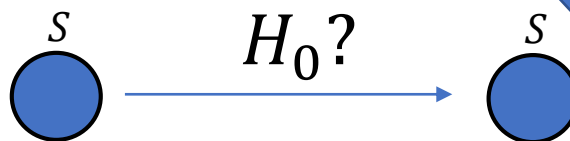
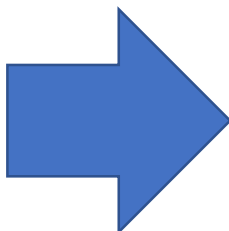


~~$$|\psi(0)\rangle = e^{+iH_0T} |\psi(T)\rangle?$$~~

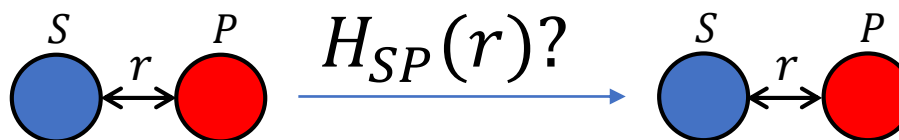
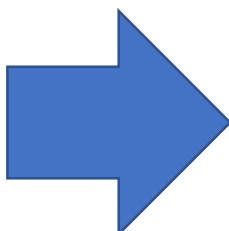
$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$



S, uncontrolled



We don't know H_0 .

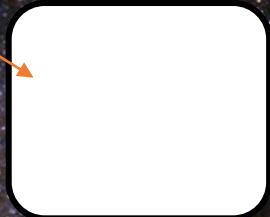


Even if we knew H_0 , we wouldn't know how to implement e^{-iH_0T} on S.

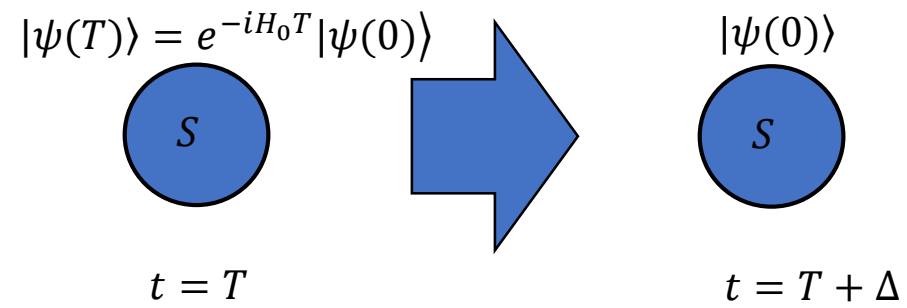
~~$$|\psi(0)\rangle = e^{+iH_0T} |\psi(T)\rangle?$$~~

S , uncontrolled

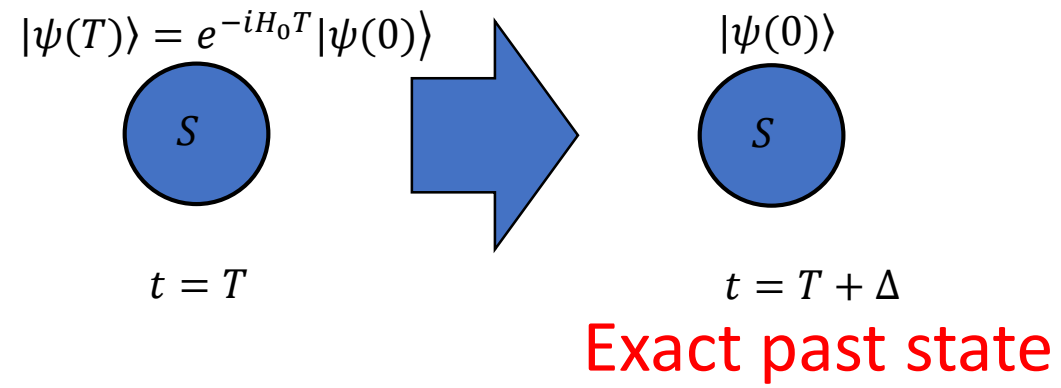
Controlled lab



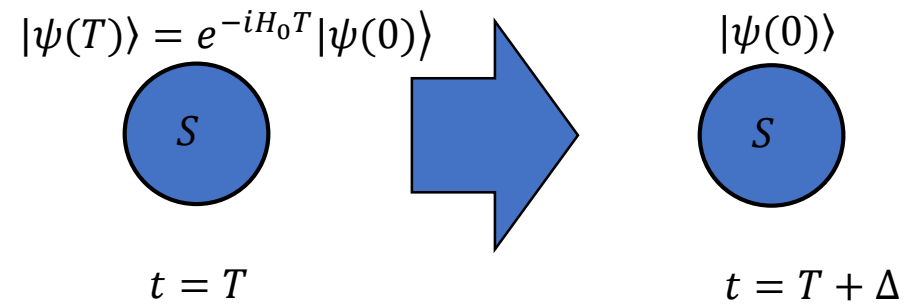
Resetting



Resetting



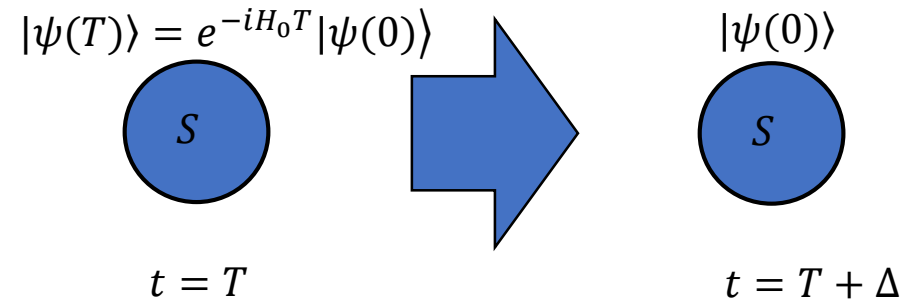
Resetting



We ignore how S evolves (unitarily) by itself and with other quantum systems

We know $d_S = \dim(H_S)$

Resetting



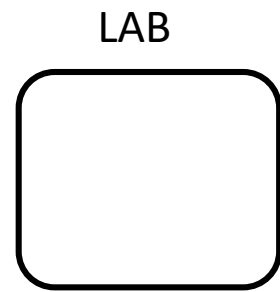
We ignore how S evolves (unitarily) by itself and with other quantum systems

We know $d_S = \dim(H_S)$

Impossible if we drop any
of the two assumptions

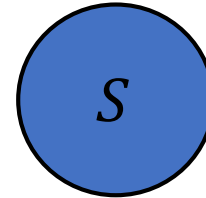
Sketch of a quantum resetting protocol

Initial conditions

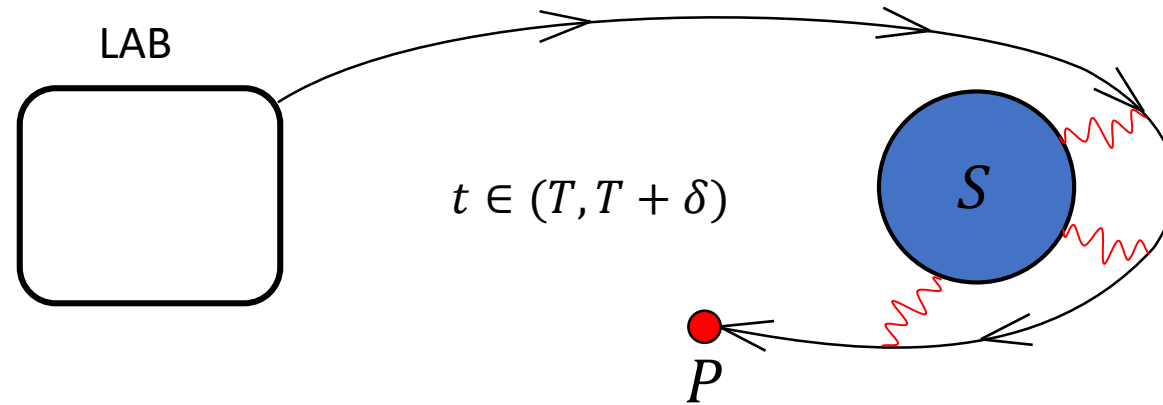


$t = T$

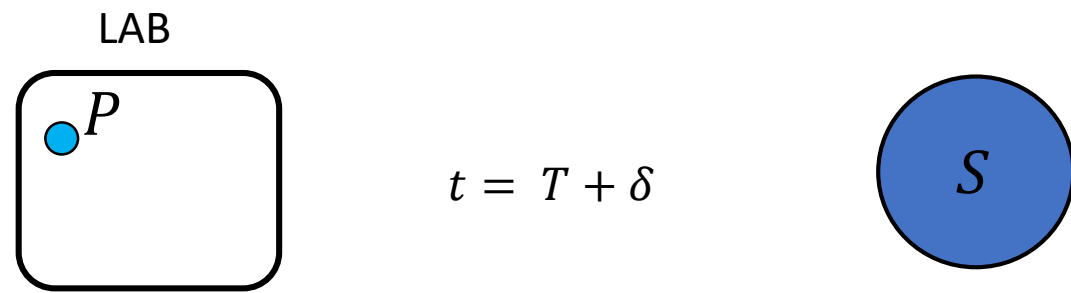
$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$



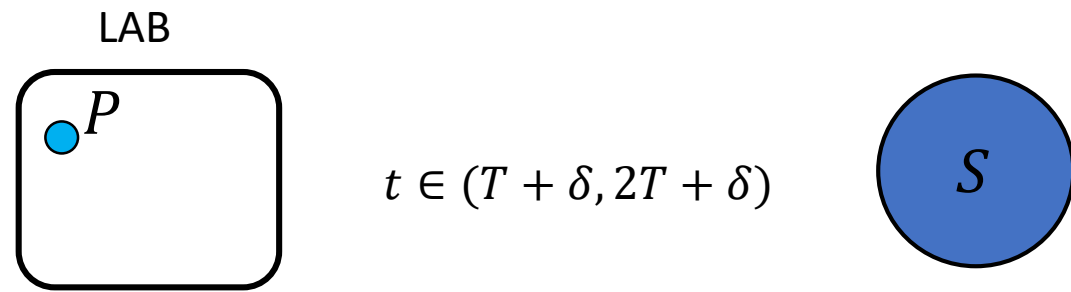
(a) Probe interaction



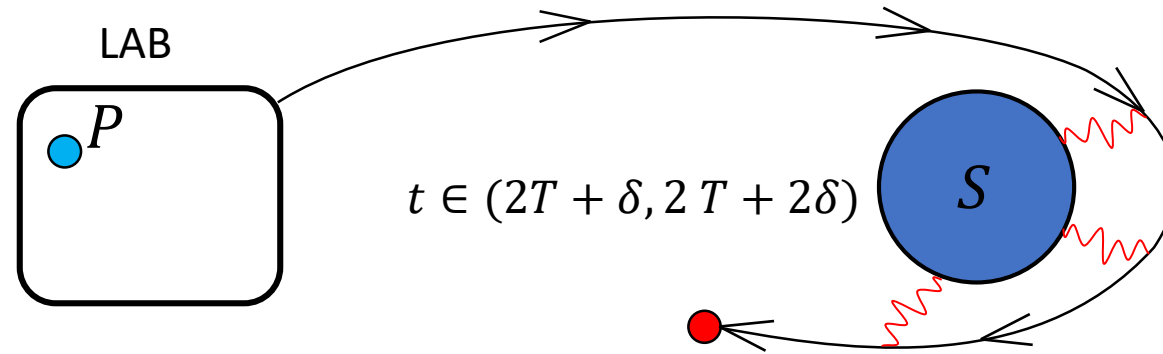
(a) Probe interaction



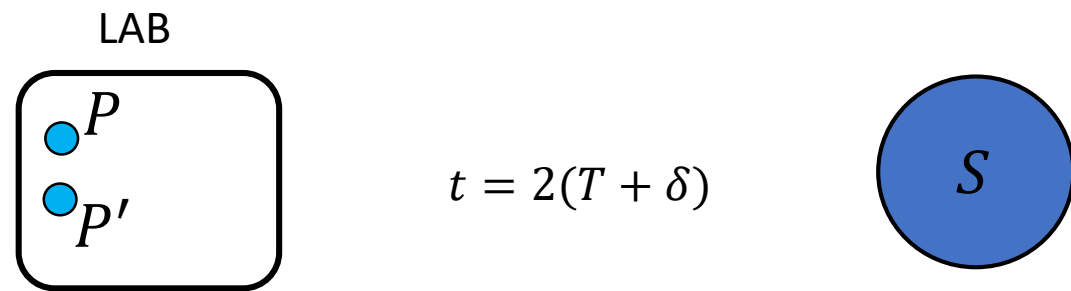
(b) Rest



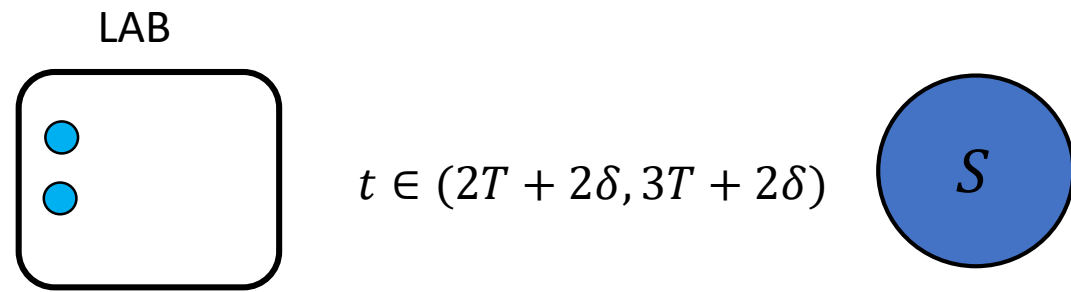
(a) Probe interaction



(a) Probe interaction

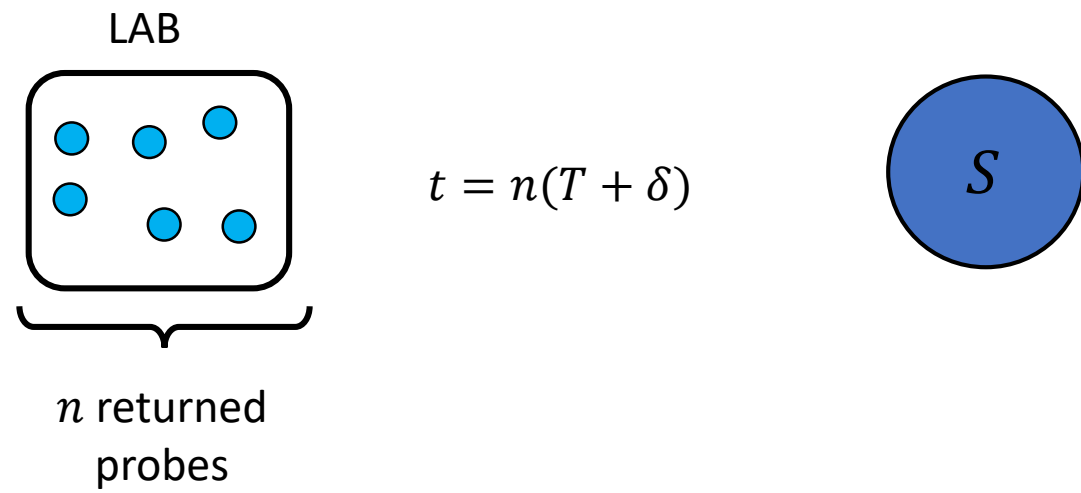


(b) Rest

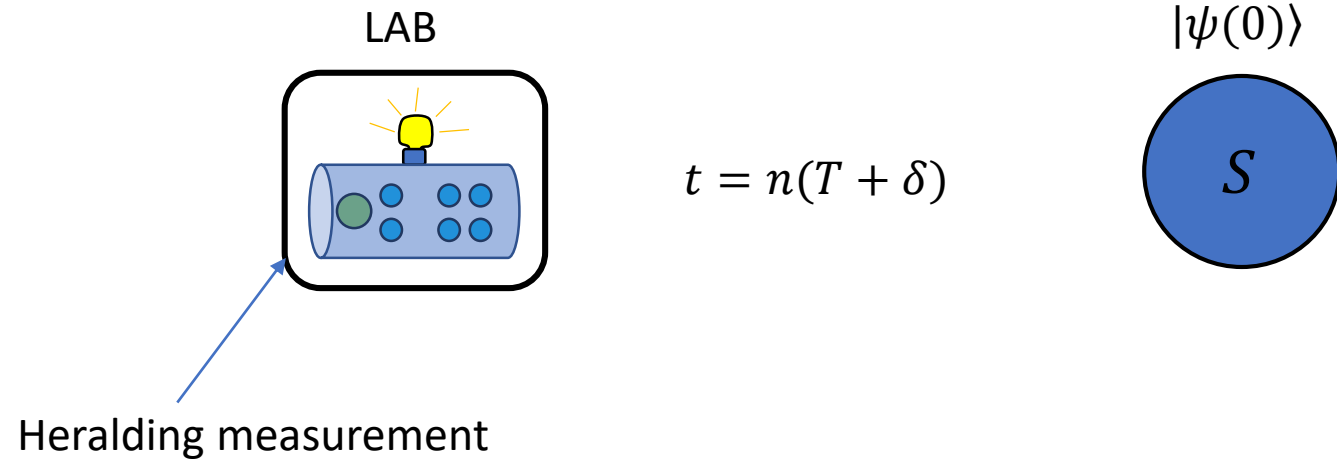




(a) Probe interaction

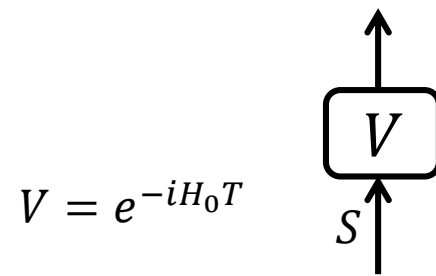
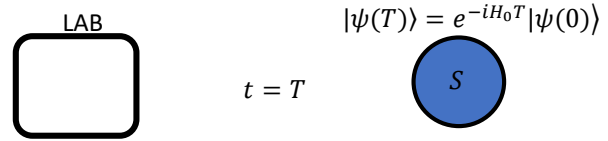


(c) Probe processing

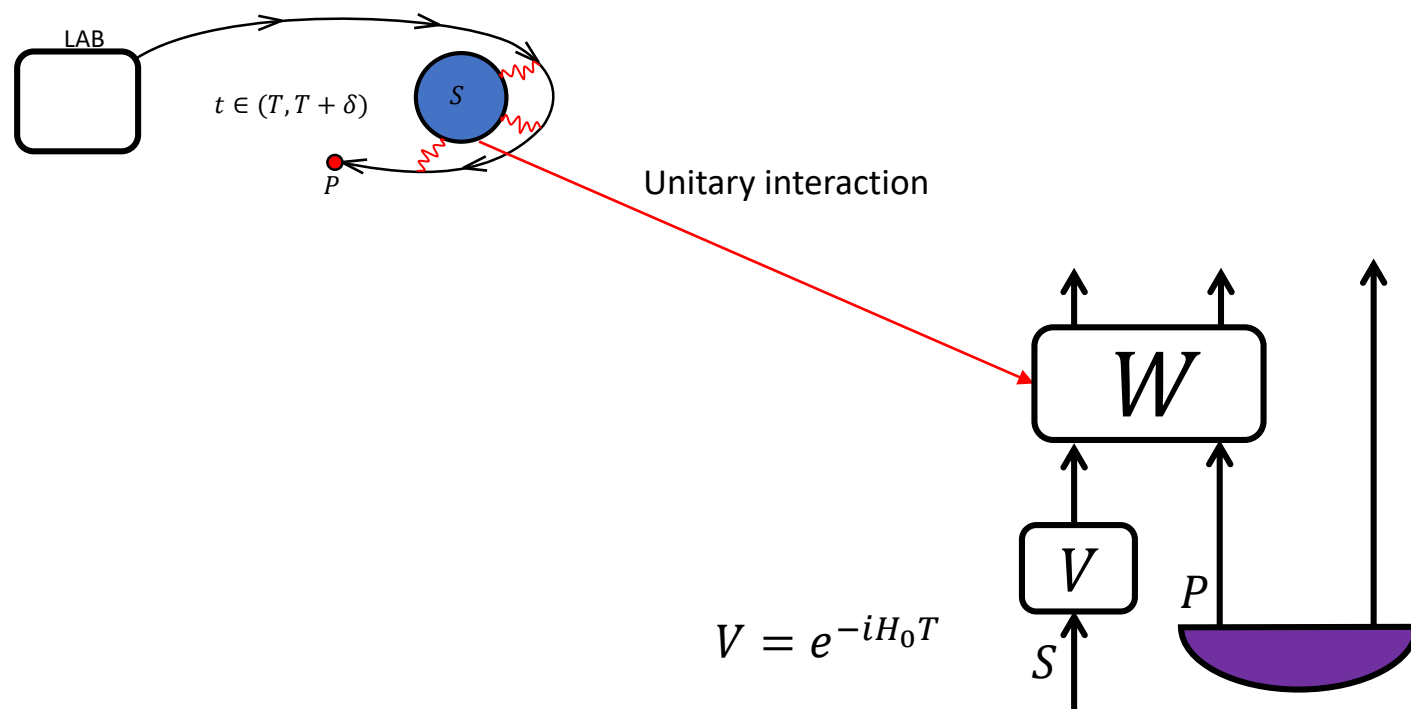


In the language of process diagrams

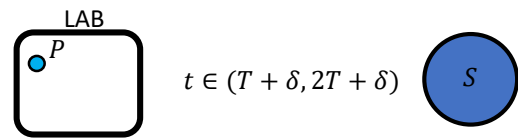
Initial conditions



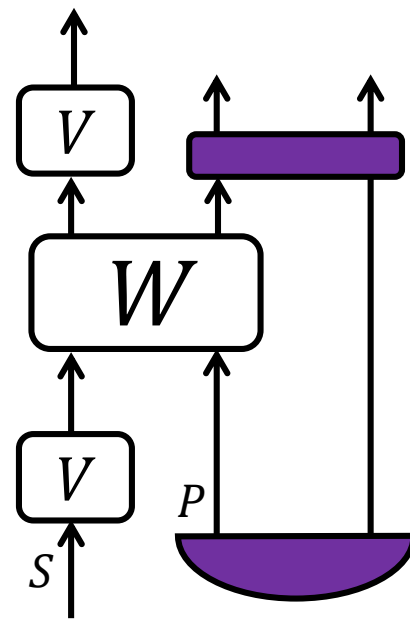
(a) Probe interaction



(b) Rest

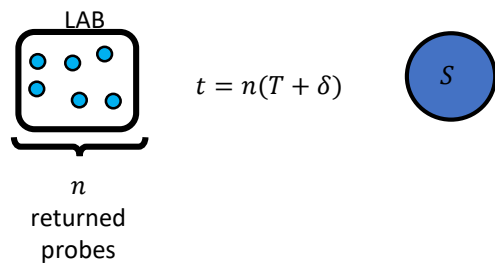


$$V = e^{-iH_0 T}$$

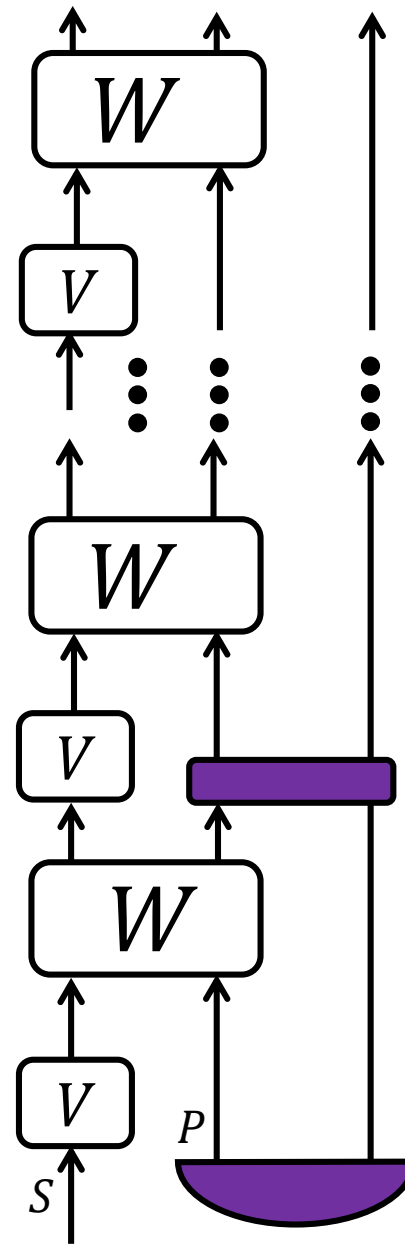




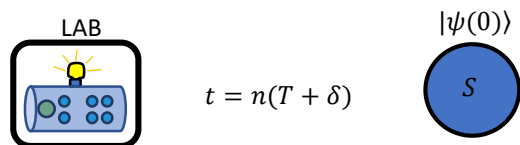
(a) Probe interaction



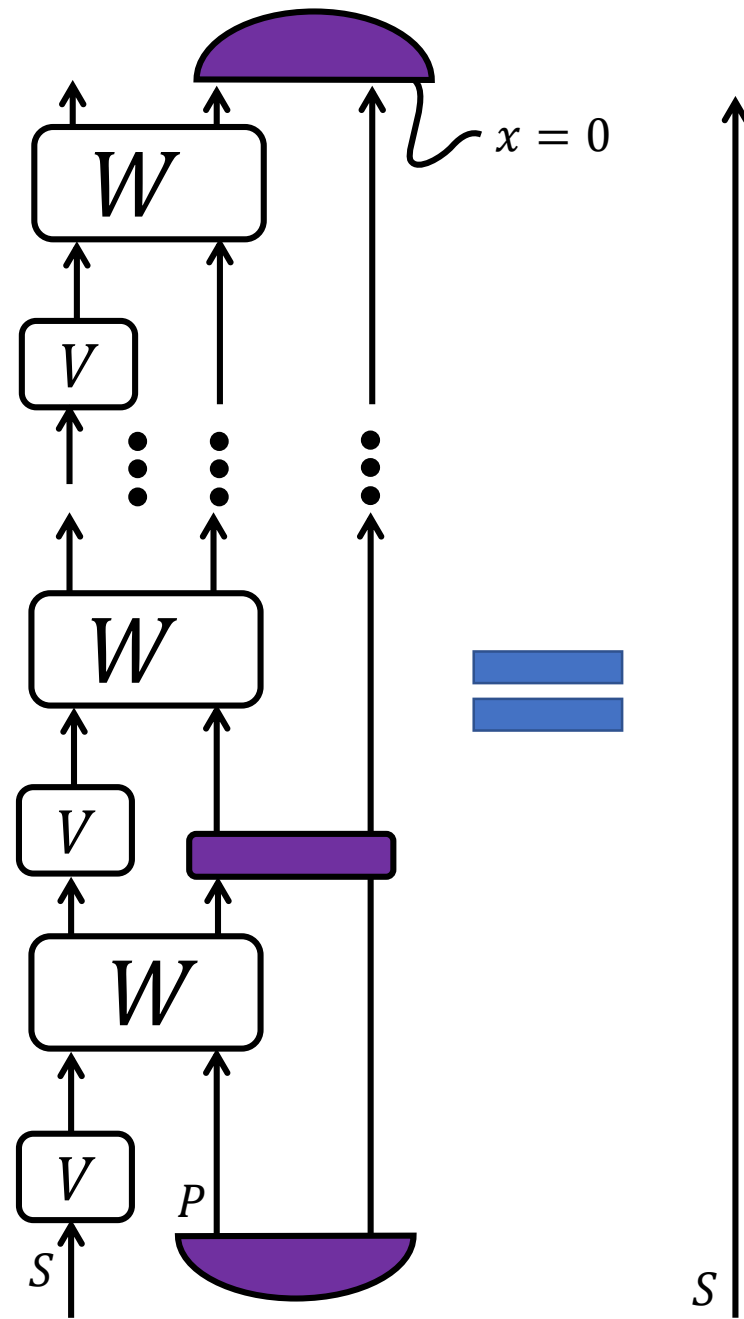
$$V = e^{-iH_0T}$$

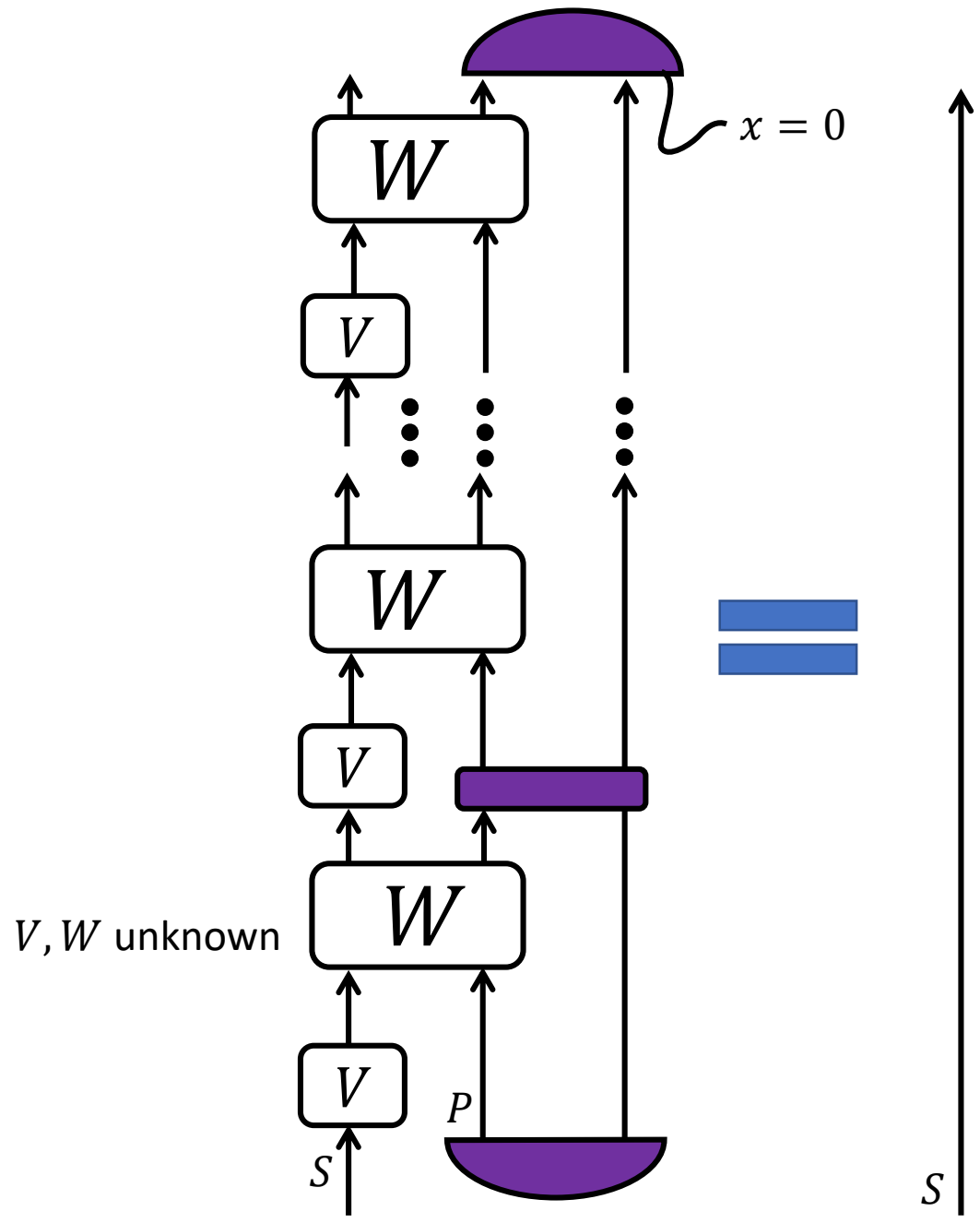


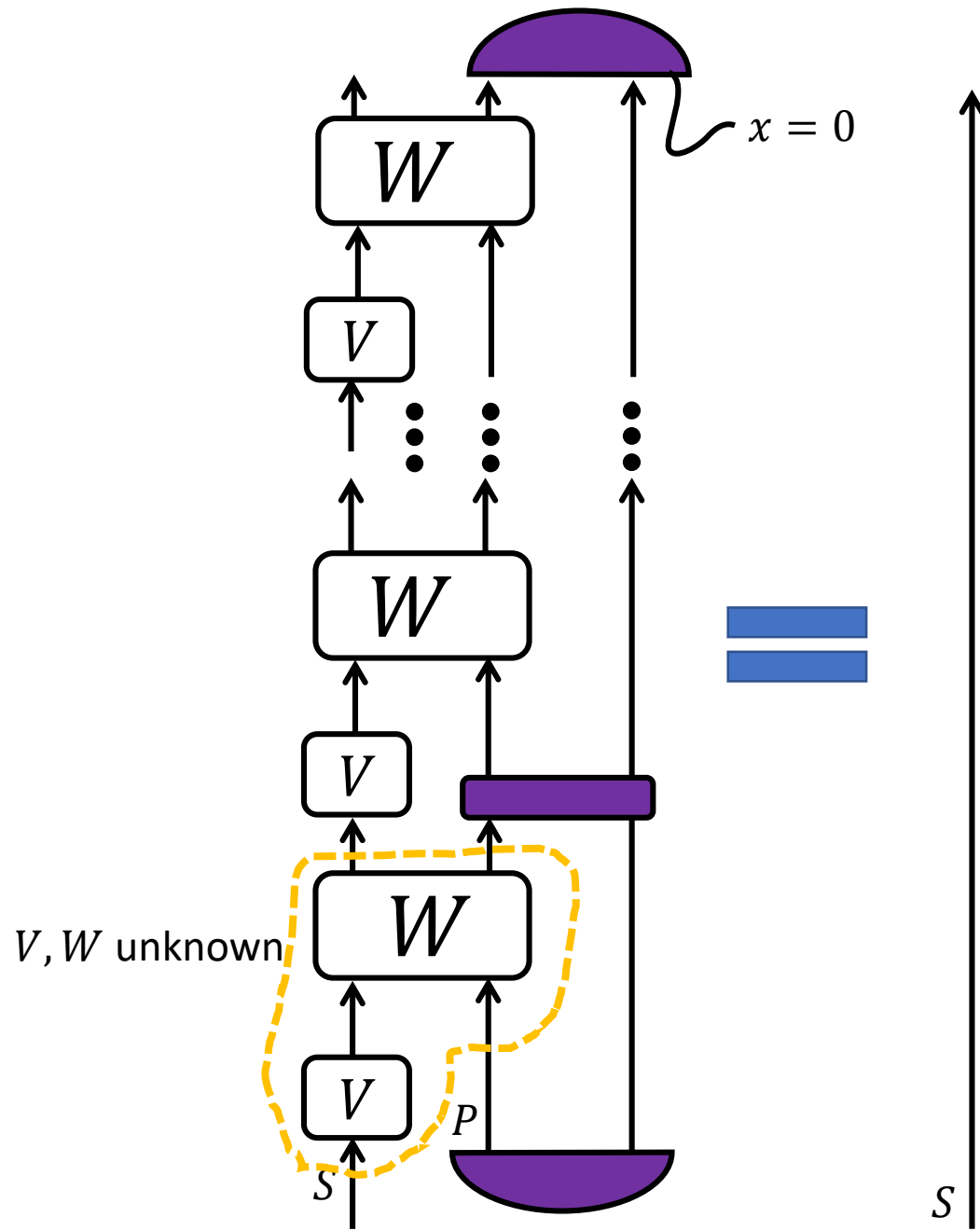
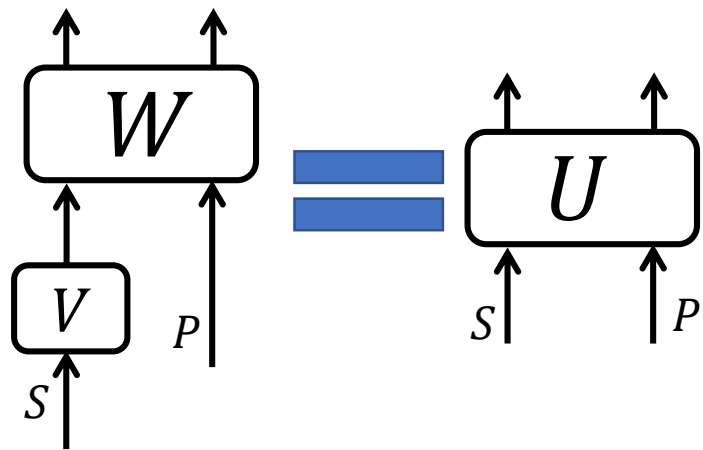
(c) Probe processing



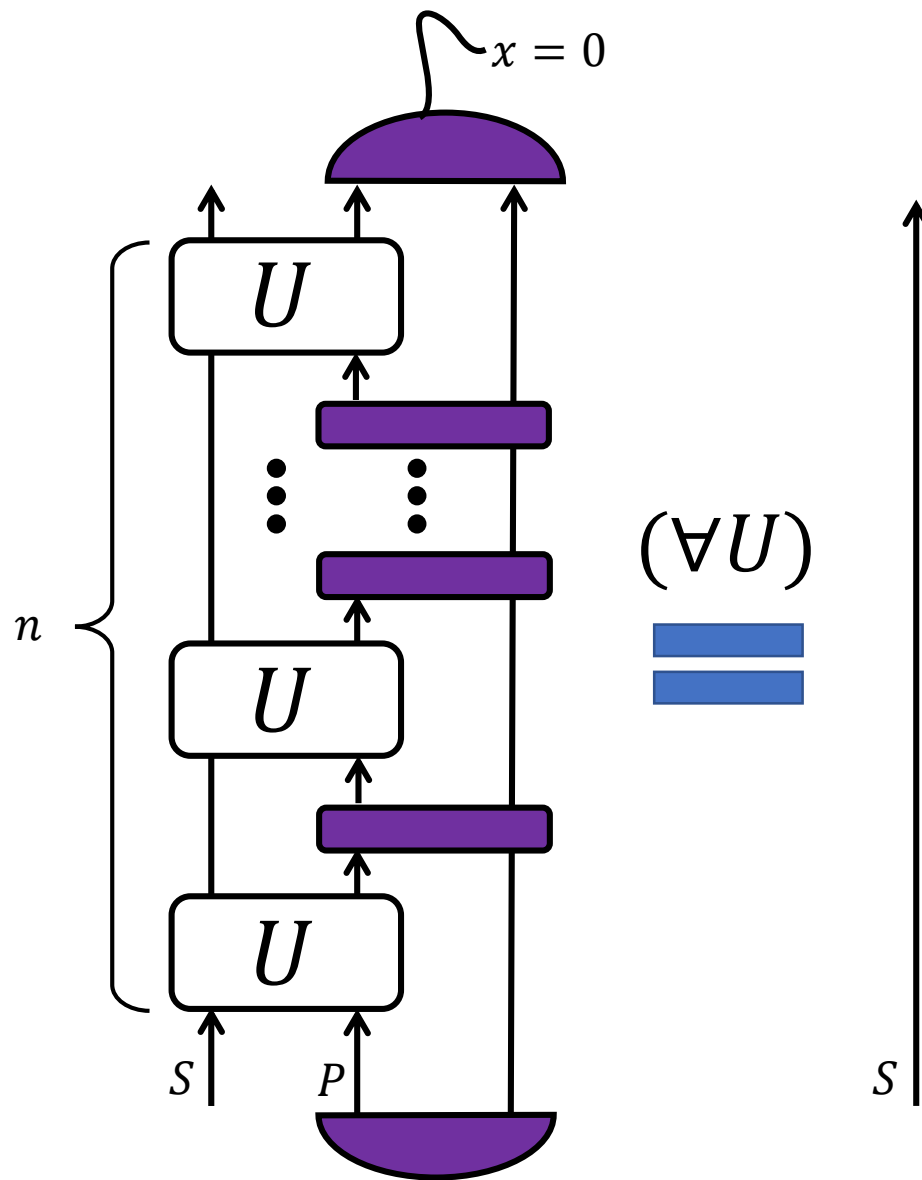
$$V = e^{-iH_0T}$$



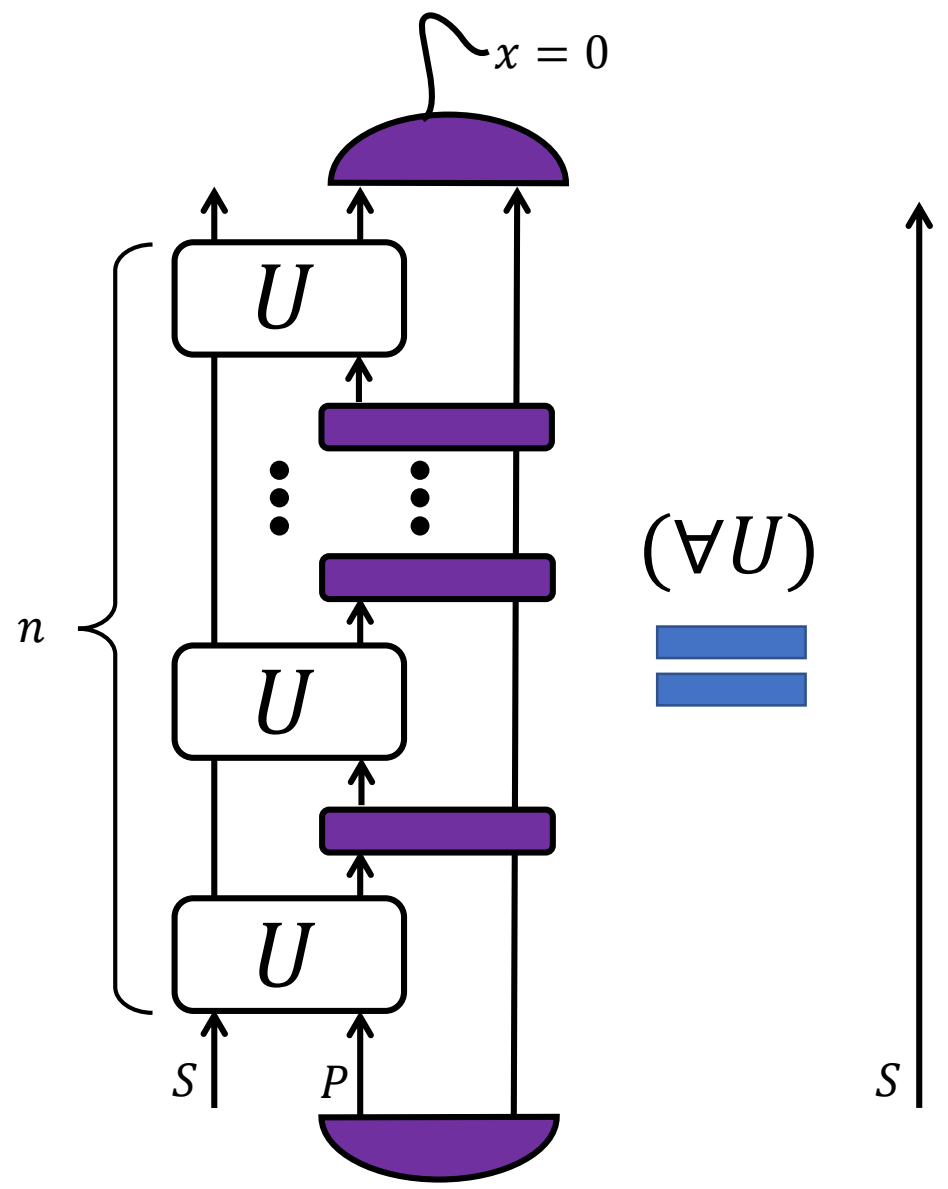




Quantum resetting
protocol



Quantum resetting protocol

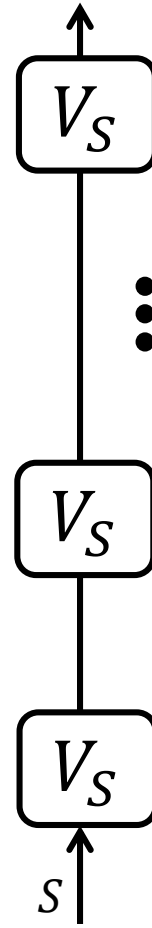
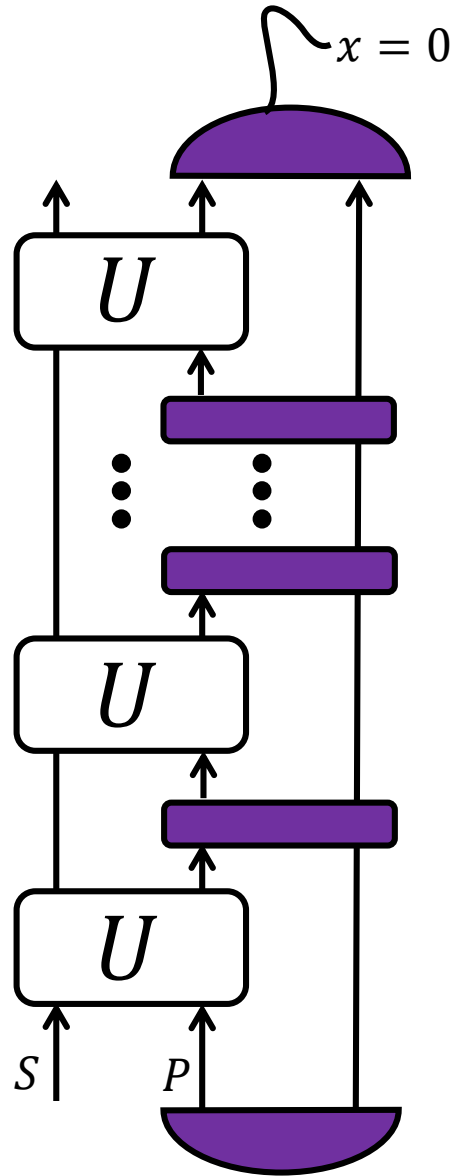


Desideratum: $P(x = 0|U) \neq 0$, for all U

$$U = V_S \otimes V_P$$



n

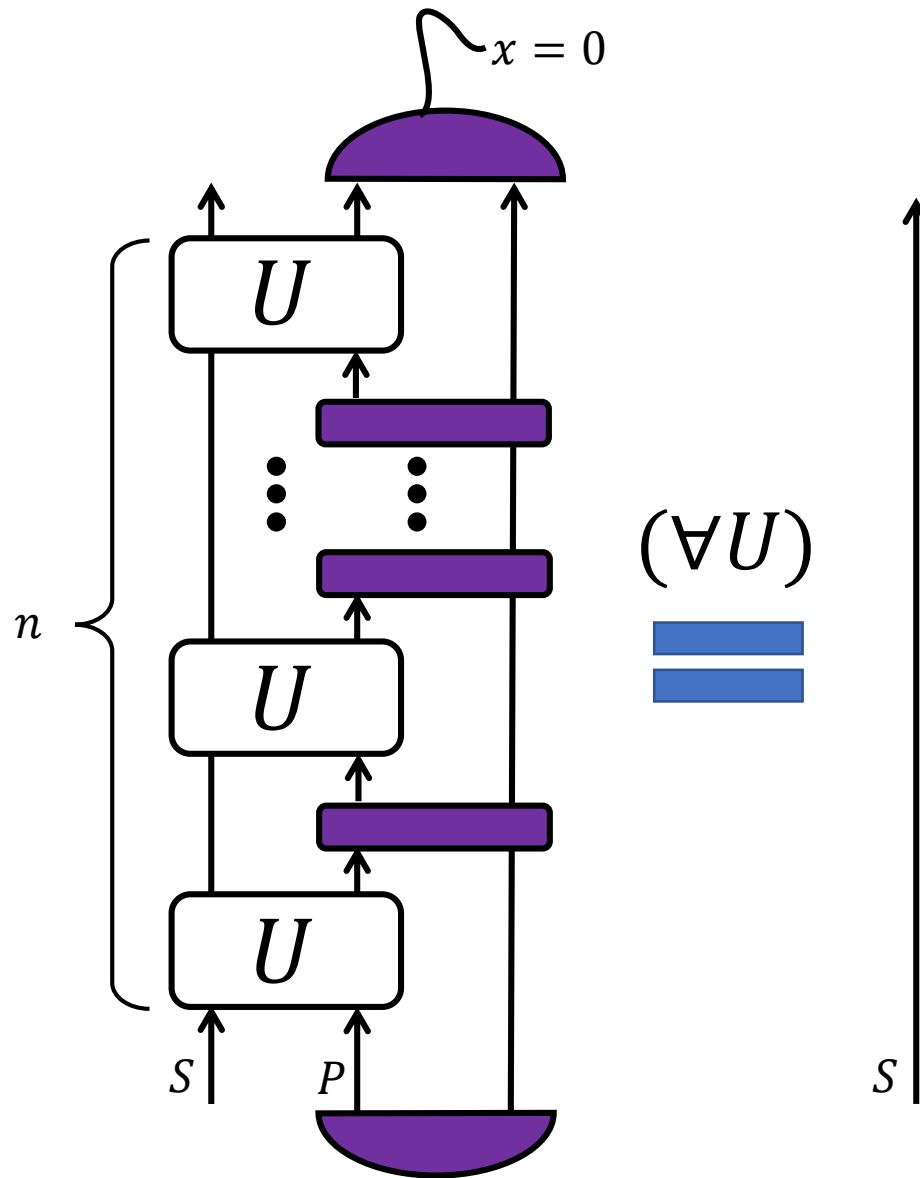


$$U = V_S \otimes V_P$$



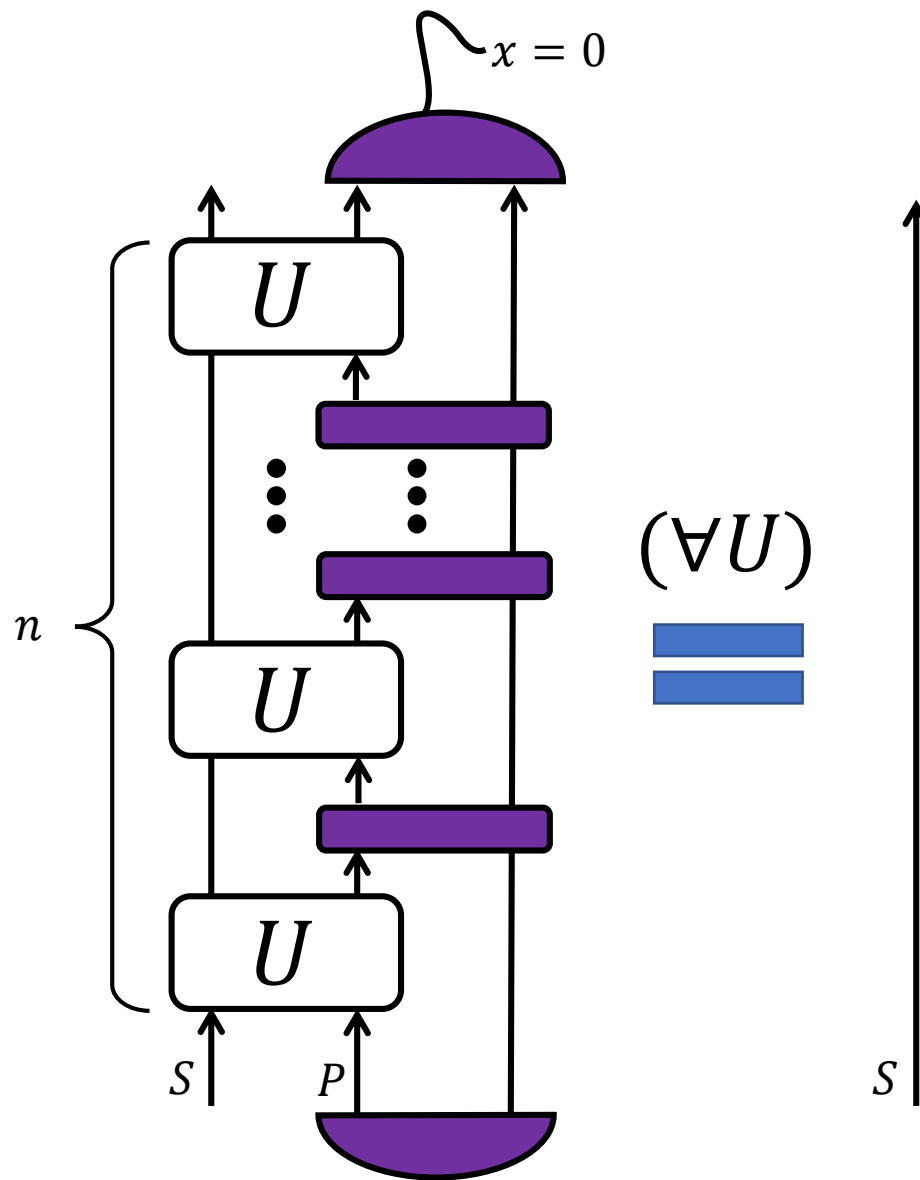
Resetting protocol will fail with probability 1

Quantum resetting protocol



Desideratum: ~~$P(x = 0|U) \neq 0$, for all U~~

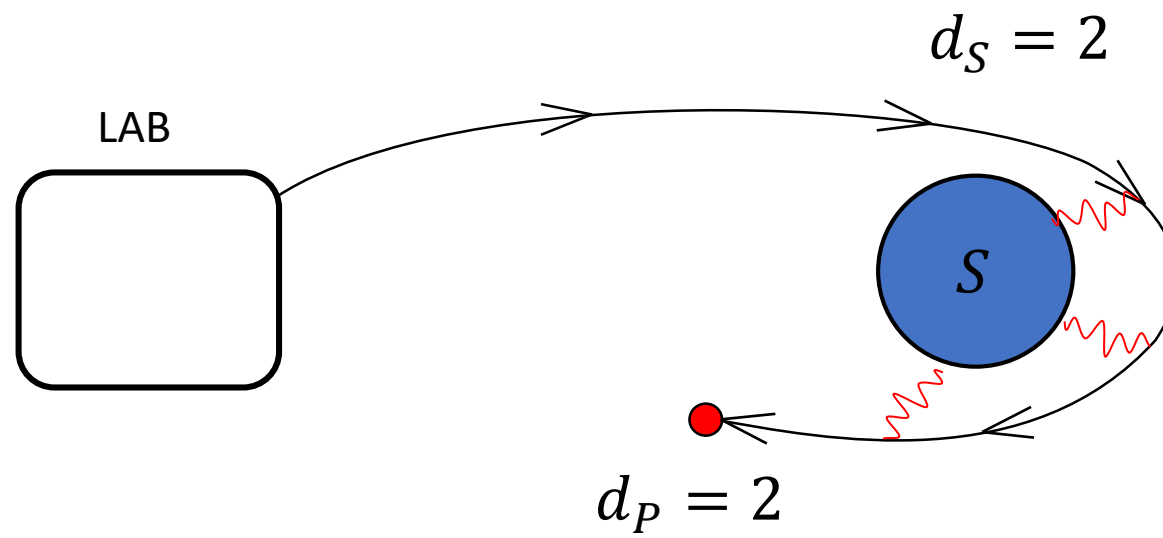
Quantum resetting protocol



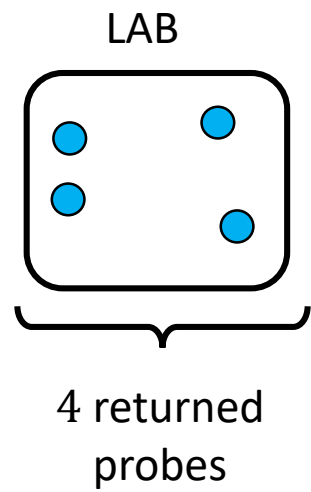
$P(x = 0|U) \neq 0$, except for a subset of unitaries of zero measure

Do quantum resetting protocols exist?

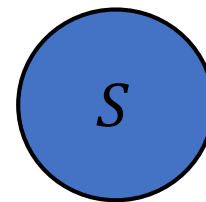
Scenario

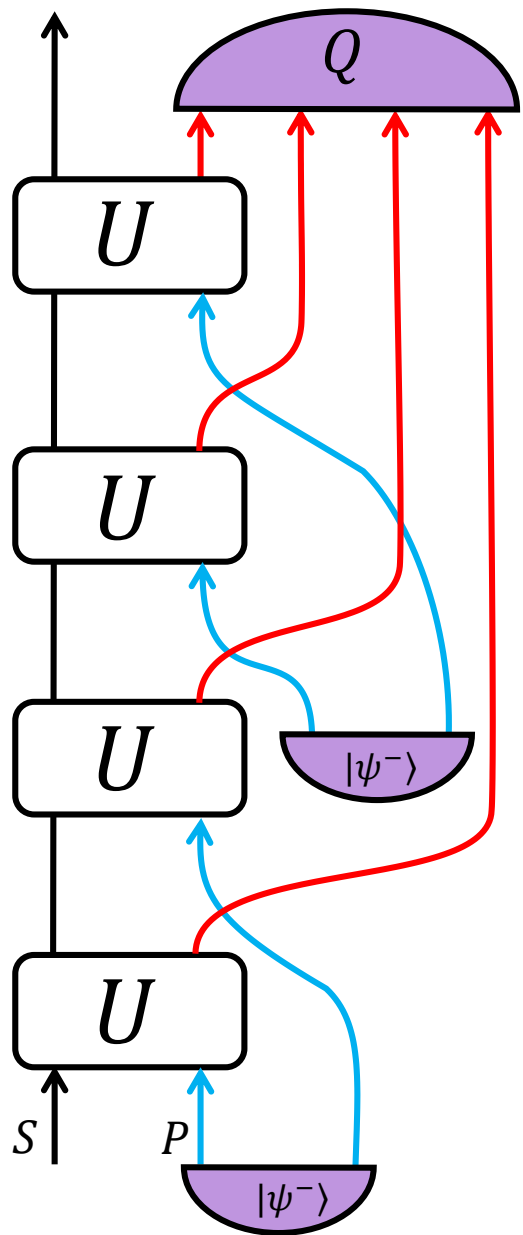


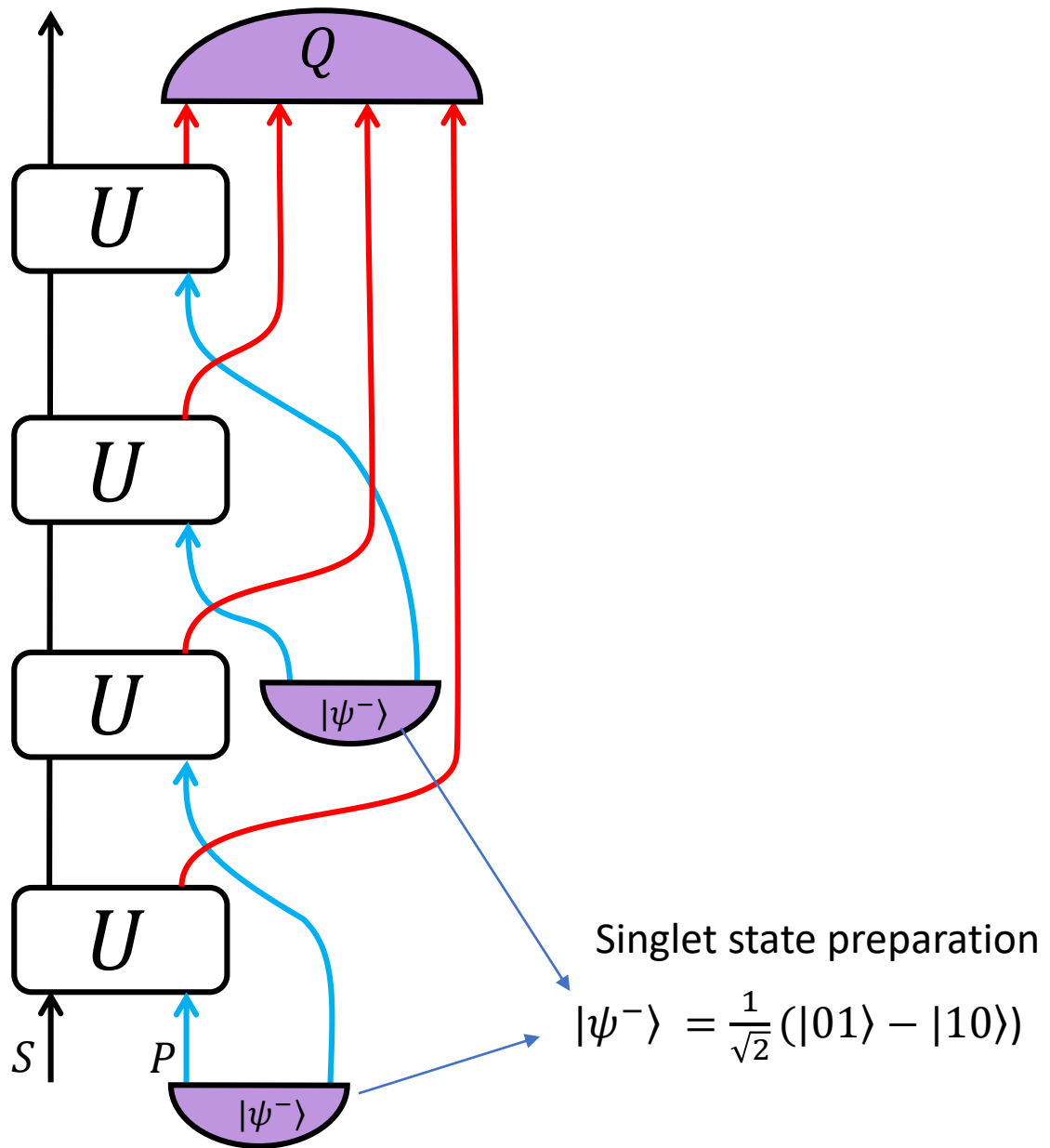
Scenario

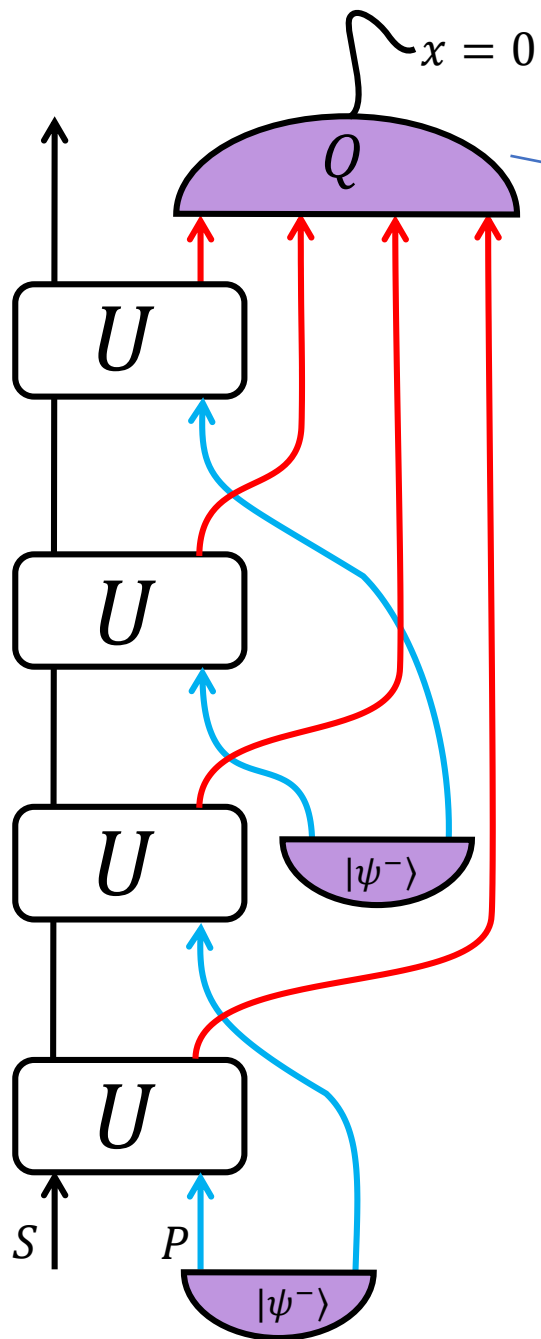


$$t = 4(T + \delta)$$









Projection onto the space spanned by

$$|m_1\rangle = |0, 0, 0, 0\rangle,$$

$$|m_2\rangle = \frac{1}{2}(|1, 0, 0, 0\rangle + |0, 1, 0, 0\rangle + |0, 0, 1, 0\rangle + |0, 0, 0, 1\rangle),$$

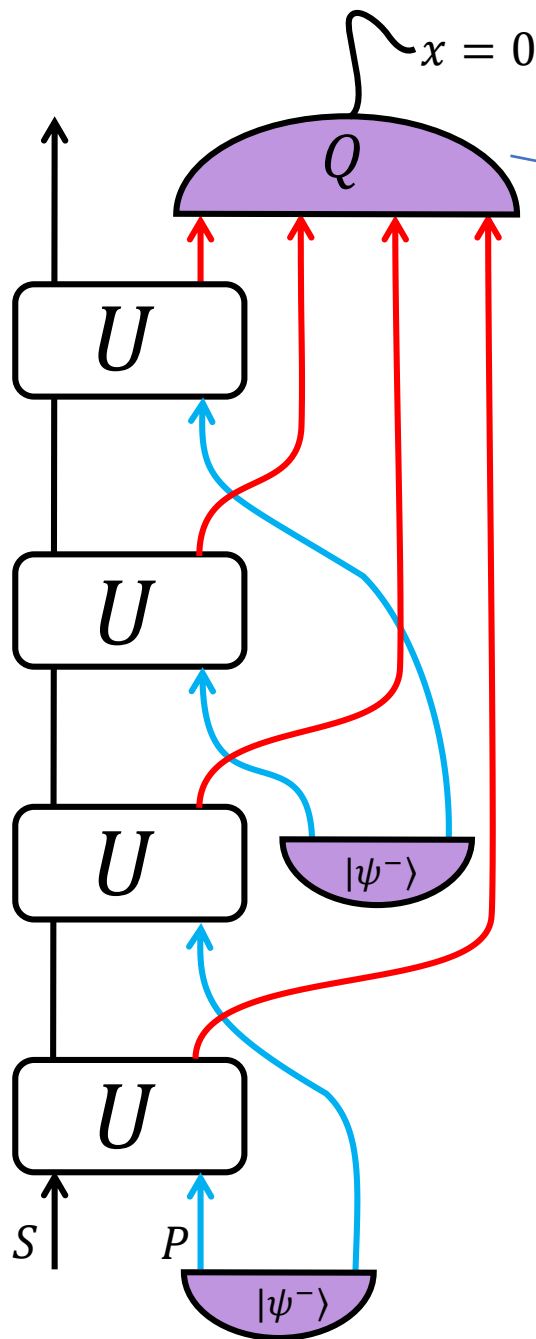
$$|m_3\rangle = \frac{1}{2}(|1, 0, 1, 0\rangle + |0, 1, 0, 1\rangle + |1, 0, 0, 1\rangle + |0, 1, 1, 0\rangle),$$

$$|m_4\rangle = \frac{1}{\sqrt{2}}(|0, 0, 1, 1\rangle + |1, 1, 0, 0\rangle),$$

$$|m_5\rangle = \frac{1}{2}(|1, 1, 1, 0\rangle + |0, 1, 1, 1\rangle + |1, 0, 1, 1\rangle + |1, 1, 0, 1\rangle),$$

$$|m_6\rangle = |1, 1, 1, 1\rangle$$

Why does this work?



Projection onto the space spanned by

$$|m_1\rangle = |0, 0, 0, 0\rangle,$$

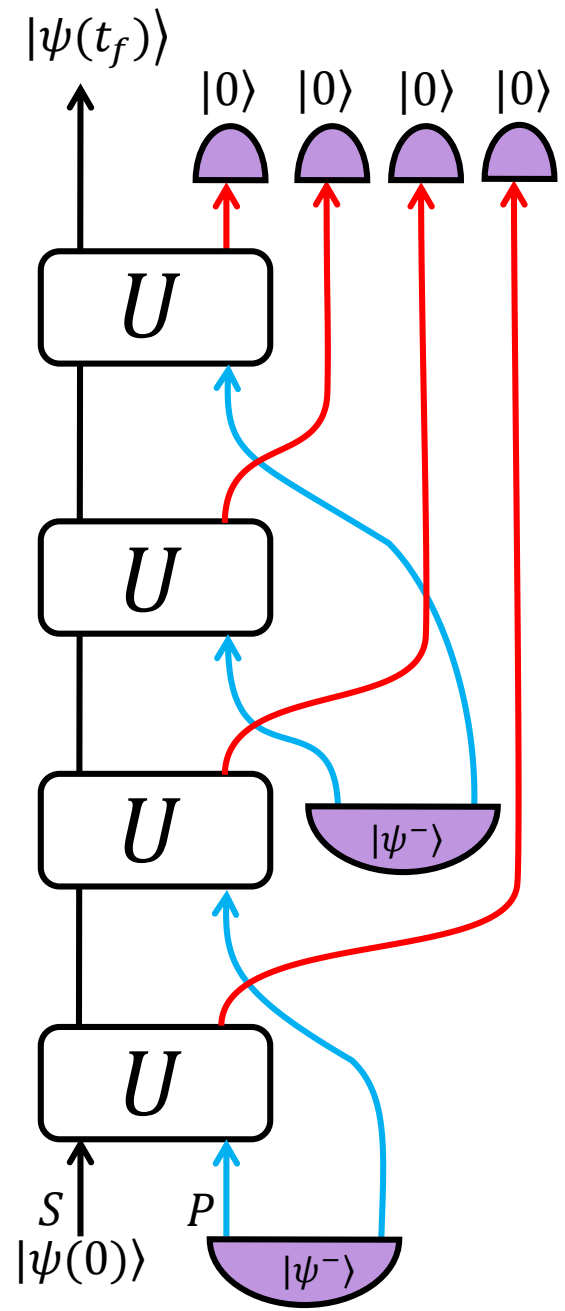
$$|m_2\rangle = \frac{1}{2}(|1, 0, 0, 0\rangle + |0, 1, 0, 0\rangle + |0, 0, 1, 0\rangle + |0, 0, 0, 1\rangle),$$

$$|m_3\rangle = \frac{1}{2}(|1, 0, 1, 0\rangle + |0, 1, 0, 1\rangle + |1, 0, 0, 1\rangle + |0, 1, 1, 0\rangle),$$

$$|m_4\rangle = \frac{1}{\sqrt{2}}(|0, 0, 1, 1\rangle + |1, 1, 0, 0\rangle),$$

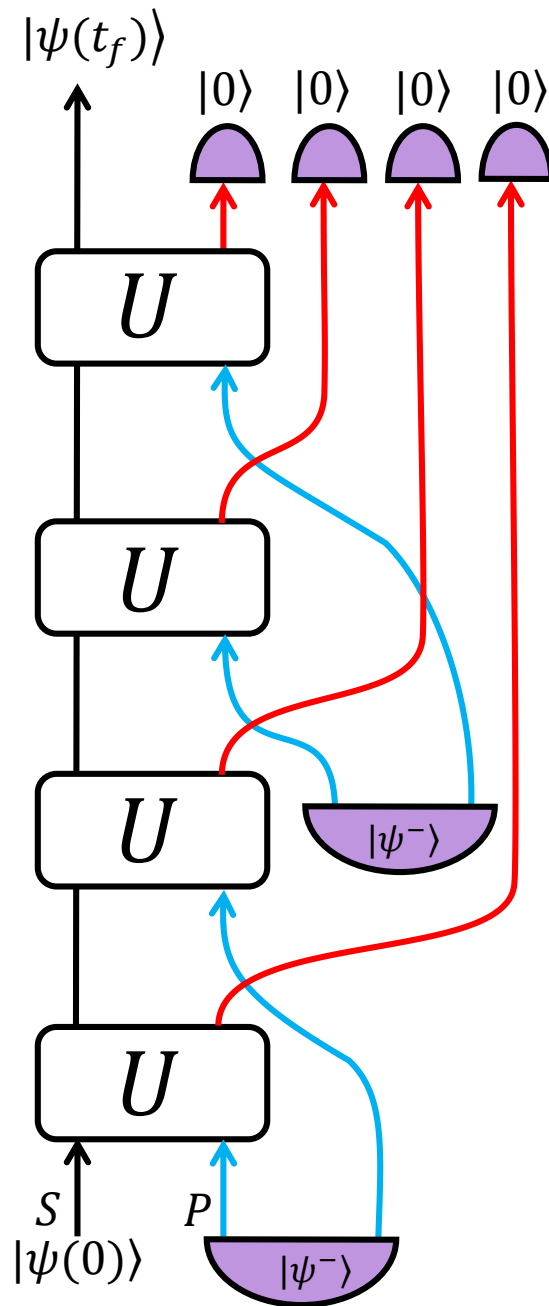
$$|m_5\rangle = \frac{1}{2}(|1, 1, 1, 0\rangle + |0, 1, 1, 1\rangle + |1, 0, 1, 1\rangle + |1, 1, 0, 1\rangle),$$

$$|m_6\rangle = |1, 1, 1, 1\rangle$$



$$|\psi(t_f)\rangle = \frac{1}{2} [U_{0,0}, U_{0,1}]^2 |\psi(0)\rangle$$

$U_{ij} = (\mathbb{I}_S \otimes \langle i|_P) U (\mathbb{I}_S \otimes |j\rangle_P)$
 2x2 complex matrices



$A, B, 2 \times 2$ matrices

$A, B, 2 \times 2$ matrices

$$[A, B] = \sum_{i=0,1,2,3} c_i \sigma_i$$

σ_i , Pauli matrices

$A, B, 2 \times 2$ matrices

$$[A, B] = \sum_{i=0,1,2,3} c_i \sigma_i$$

σ_i , Pauli matrices

$$\text{Tr}([A, B]) = 0 \rightarrow c_0 = 0$$

$A, B, 2 \times 2$ matrices

$$[A, B] = \sum_{i=1,2,3} c_i \sigma_i$$

σ_i , Pauli matrices

$A, B, 2 \times 2$ matrices

$$[A, B]^2 = \left(\sum_{i=1,2,3} c_i \sigma_i \right)^2 = \left(\sum_{i=1,2,3} c_i^2 \right) \mathbb{I}$$

σ_i , Pauli matrices

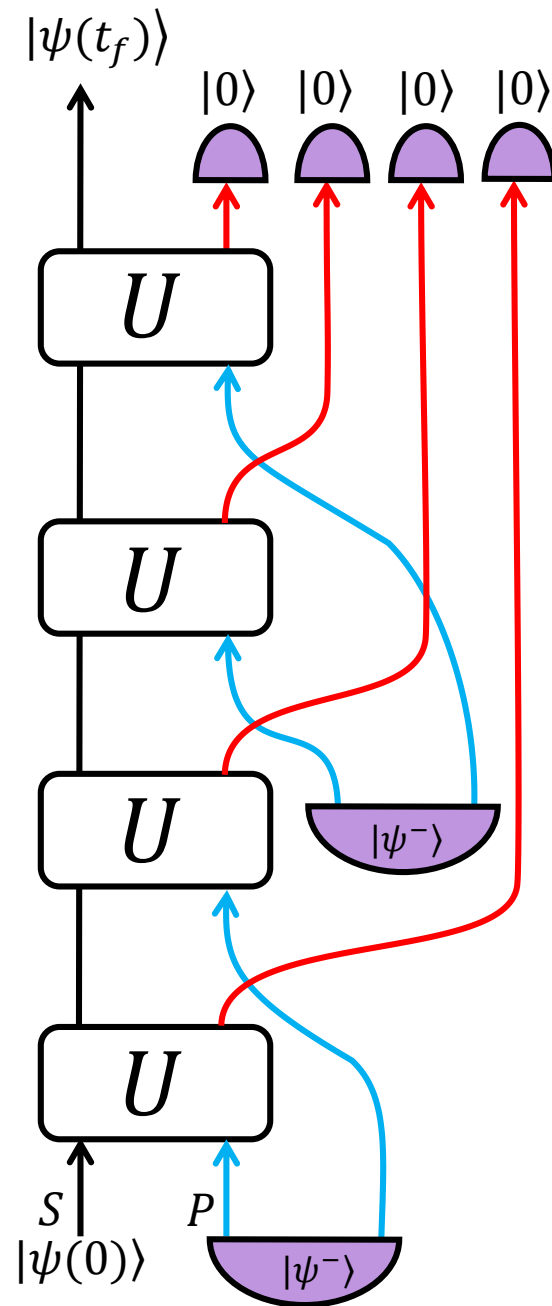
Central polynomial for dimension 2

$A, B, 2 \times 2$ matrices

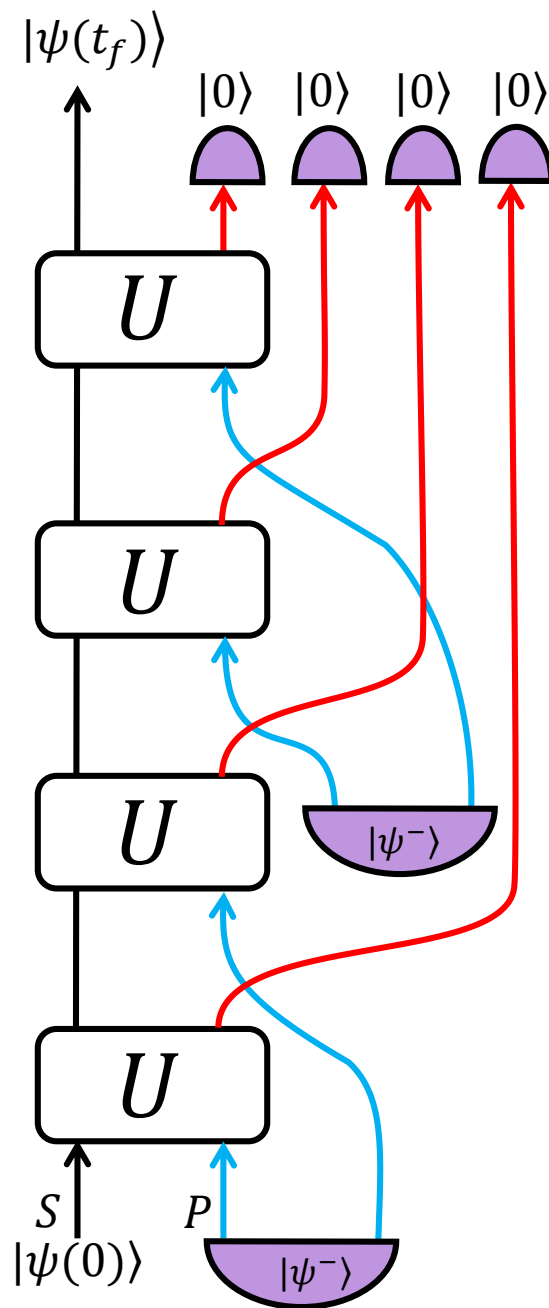
$$[A, B]^2 = \left(\sum_{i=1,2,3} c_i \sigma_i \right)^2 = \left(\sum_{i=1,2,3} c_i^2 \right) \mathbb{I}$$

σ_i , Pauli matrices

$$|\psi(t_f)\rangle = \frac{1}{2} [U_{0,0}, U_{0,1}]^2 |\psi(0)\rangle$$



$$|\psi(t_f)\rangle = \frac{1}{2} [U_{0,0}, U_{0,1}]^2 |\psi(0)\rangle \propto |\psi(0)\rangle$$



Similarly,

$$|\psi(t_f)\rangle \propto |\psi(0)\rangle$$

$$|m_1\rangle = |0, 0, 0, 0\rangle,$$

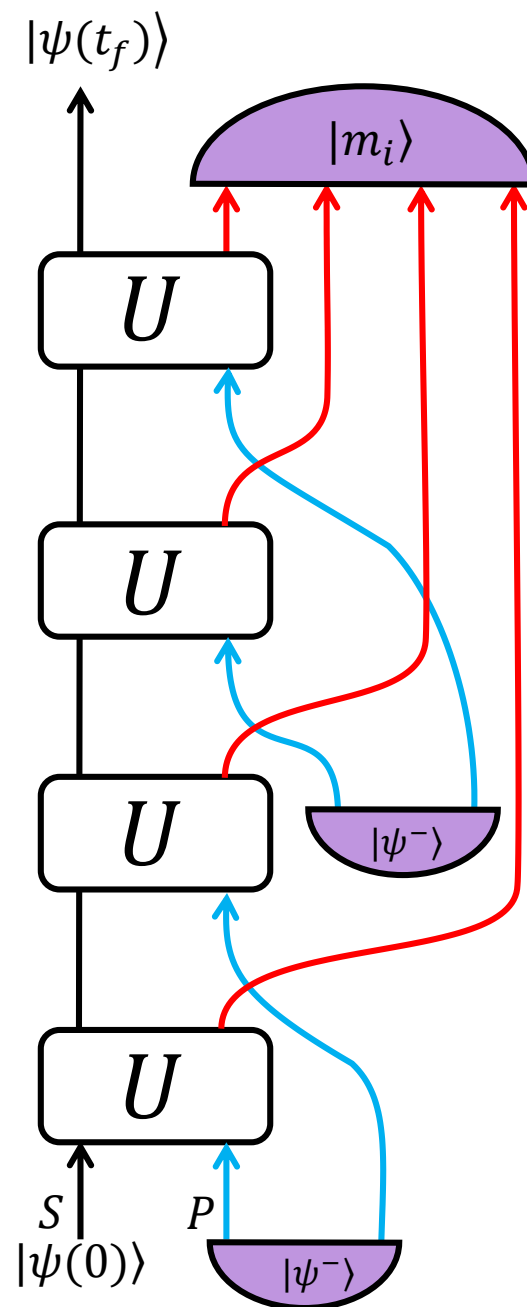
$$|m_2\rangle = \frac{1}{2}(|1, 0, 0, 0\rangle + |0, 1, 0, 0\rangle + |0, 0, 1, 0\rangle + |0, 0, 0, 1\rangle),$$

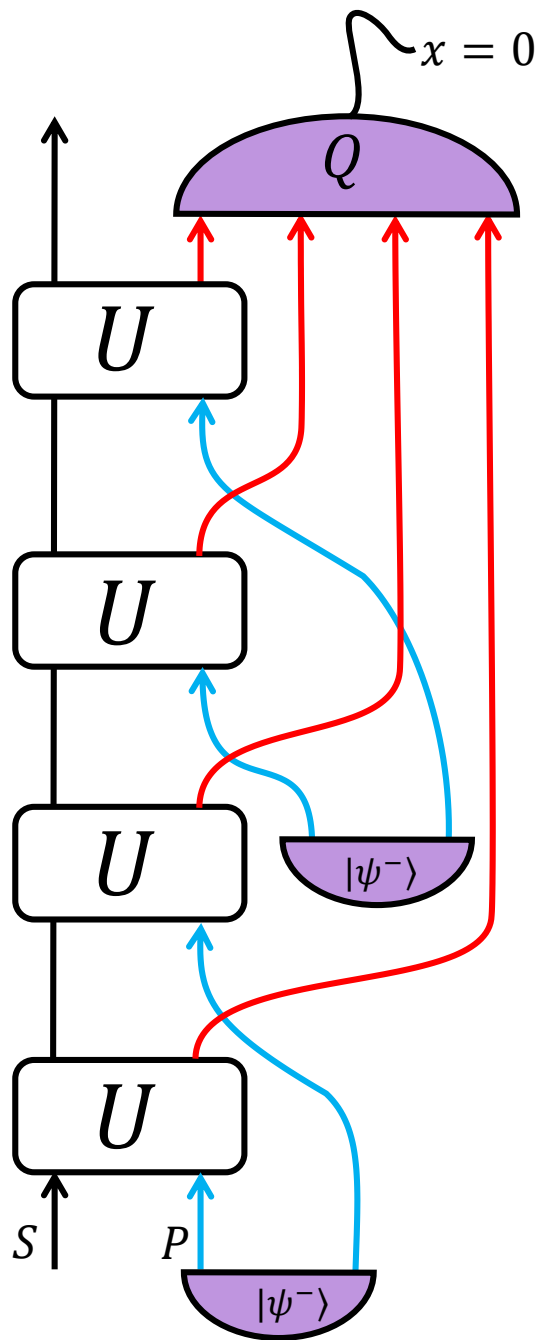
$$|m_3\rangle = \frac{1}{2}(|1, 0, 1, 0\rangle + |0, 1, 0, 1\rangle + |1, 0, 0, 1\rangle + |0, 1, 1, 0\rangle),$$

$$|m_4\rangle = \frac{1}{\sqrt{2}}(|0, 0, 1, 1\rangle + |1, 1, 0, 0\rangle),$$

$$|m_5\rangle = \frac{1}{2}(|1, 1, 1, 0\rangle + |0, 1, 1, 1\rangle + |1, 0, 1, 1\rangle + |1, 1, 0, 1\rangle),$$

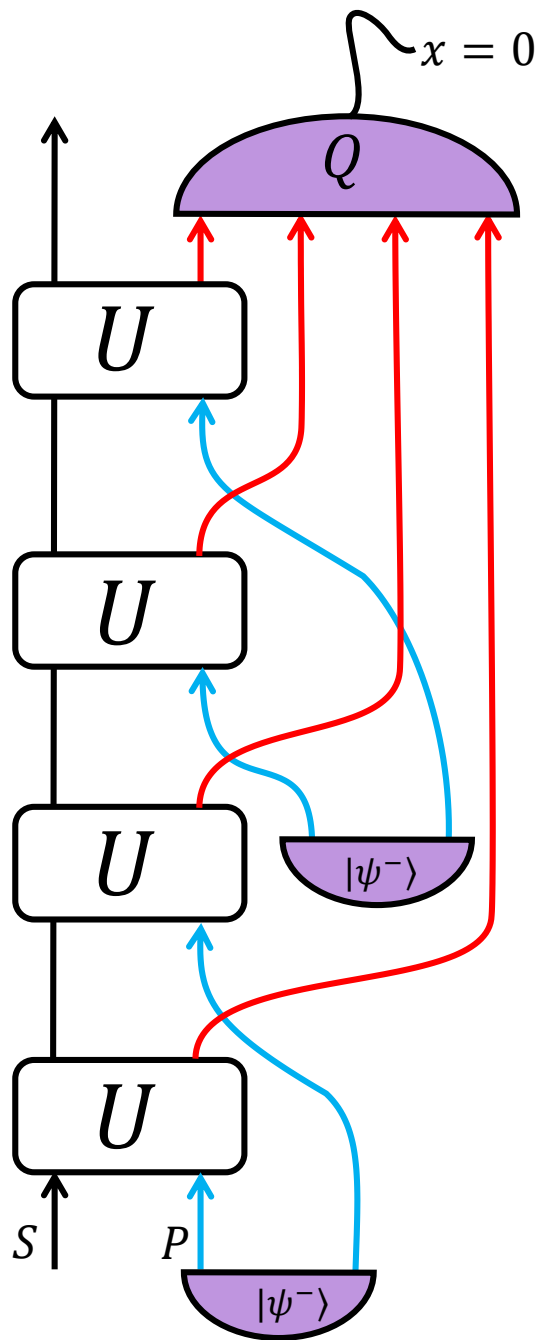
$$|m_6\rangle = |1, 1, 1, 1\rangle$$





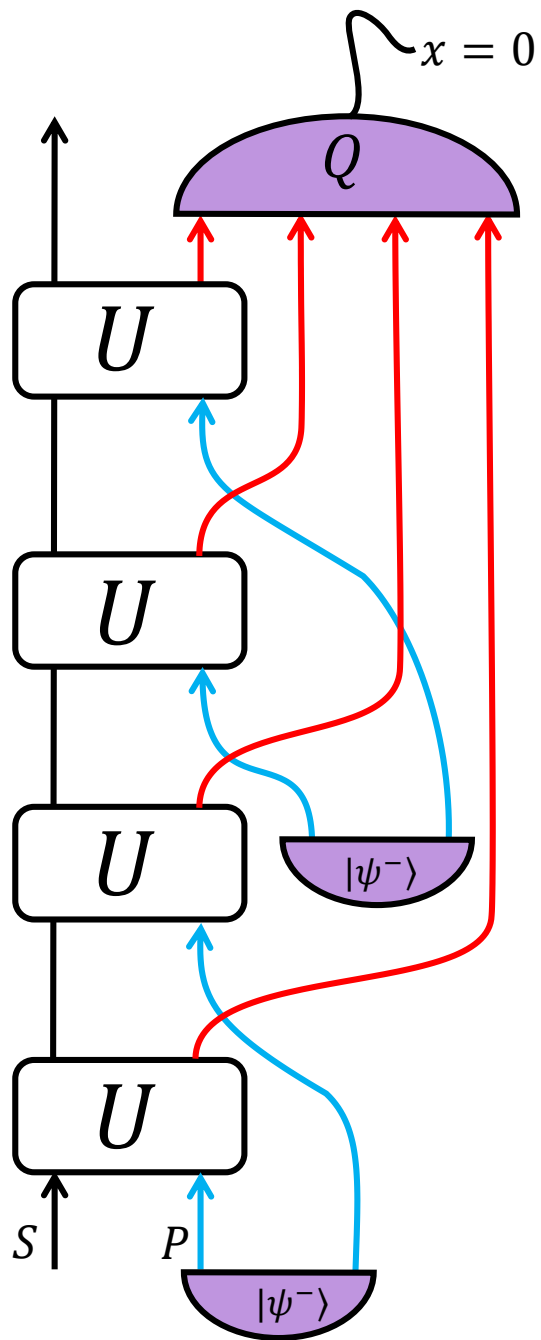
$$P(x = 0 | U) \in [0, 1]$$

E.g.: $U = V_S \otimes V_P$



$$P(x = 0 | U) \in [0, 1]$$

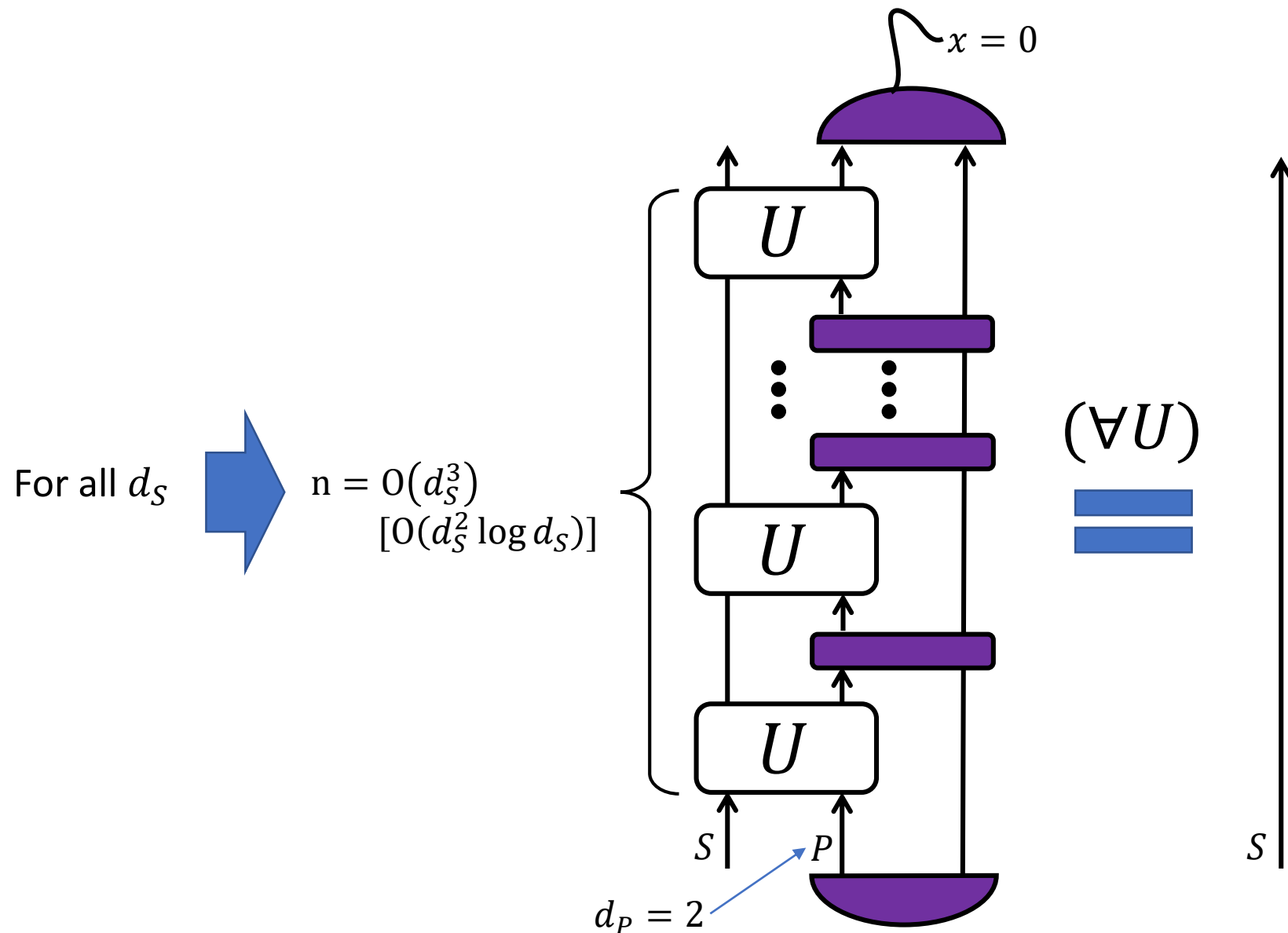
E.g.: $U = \frac{\sigma_x \otimes \sigma_z + i \sigma_y \otimes \sigma_x}{\sqrt{2}}$



Average probability of success for completely unknown U

$$\int dU P(x = 0|U) \approx 0.2170$$

Generalization



$P(x = 0|U) \neq 0$, except for a subset of unitaries of zero measure

Characterization of all quantum resetting protocols (practical for $n=4$)

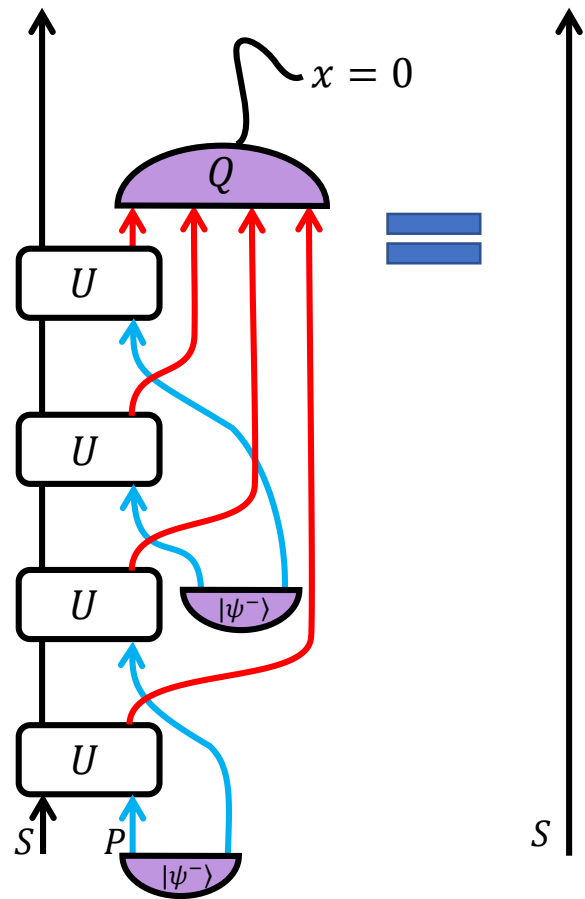
$$\begin{aligned}
 & \max \operatorname{tr}(M_0^T X(\rho)) \\
 \text{s.t. } & \operatorname{supp}(M_0^T) \in \mathcal{H}^c, M_0, M_1 \geq 0 \\
 & M_0 + M_1 = \mathbb{I}_{A_n^{\text{out}}} \otimes \Gamma^{(n)}, \\
 & \operatorname{tr}_{I_k}(\Gamma^{(k)}) = \mathbb{I}_{O_{k-1}} \otimes \Gamma^{(k-1)},
 \end{aligned}$$

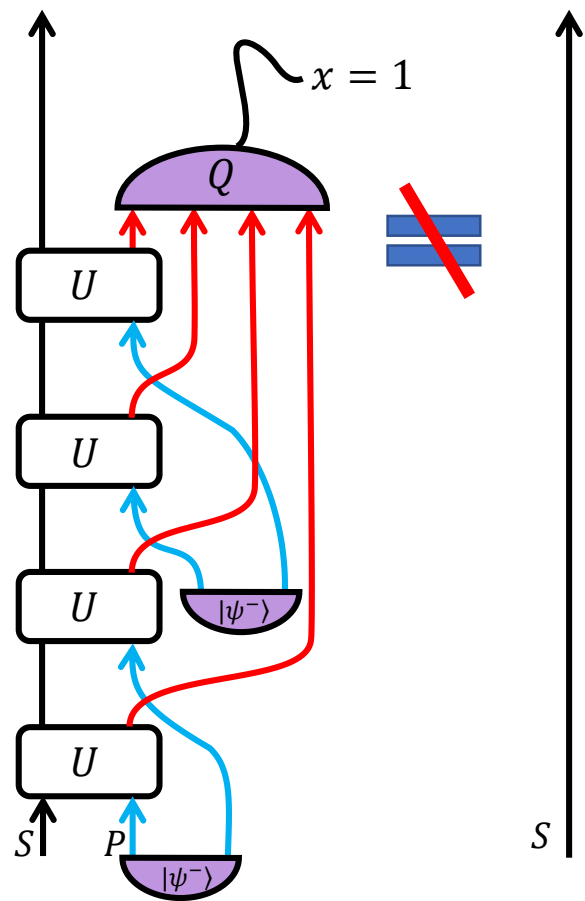
Heuristics to identify optimal strategies for high numbers of probes

$$\mathcal{W}_8, \tilde{\mathcal{W}}_8, \quad n = 8, d_S = 2$$

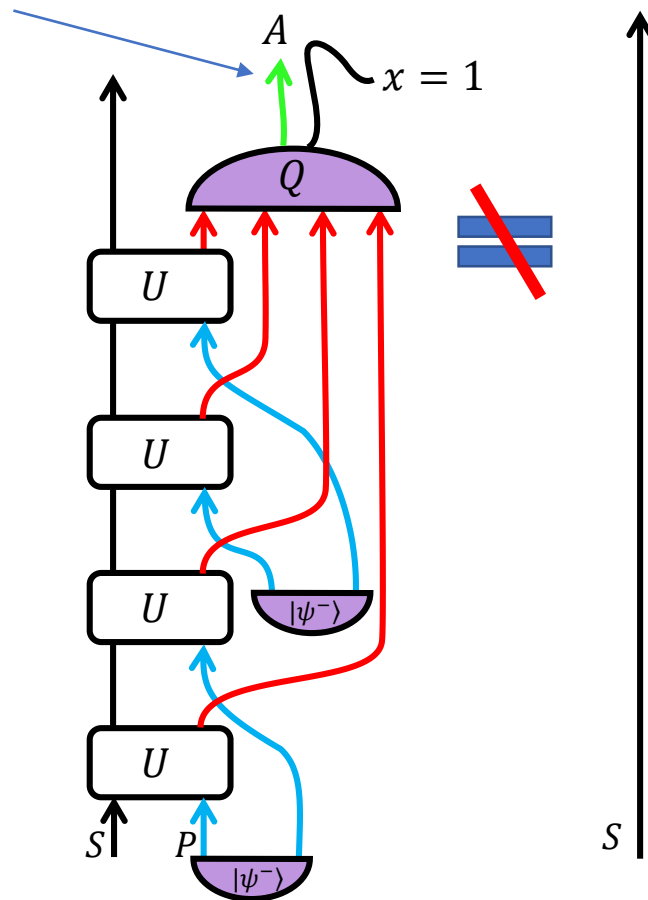
$$\mathcal{W}_9, \quad n = 9, d_S = 3$$

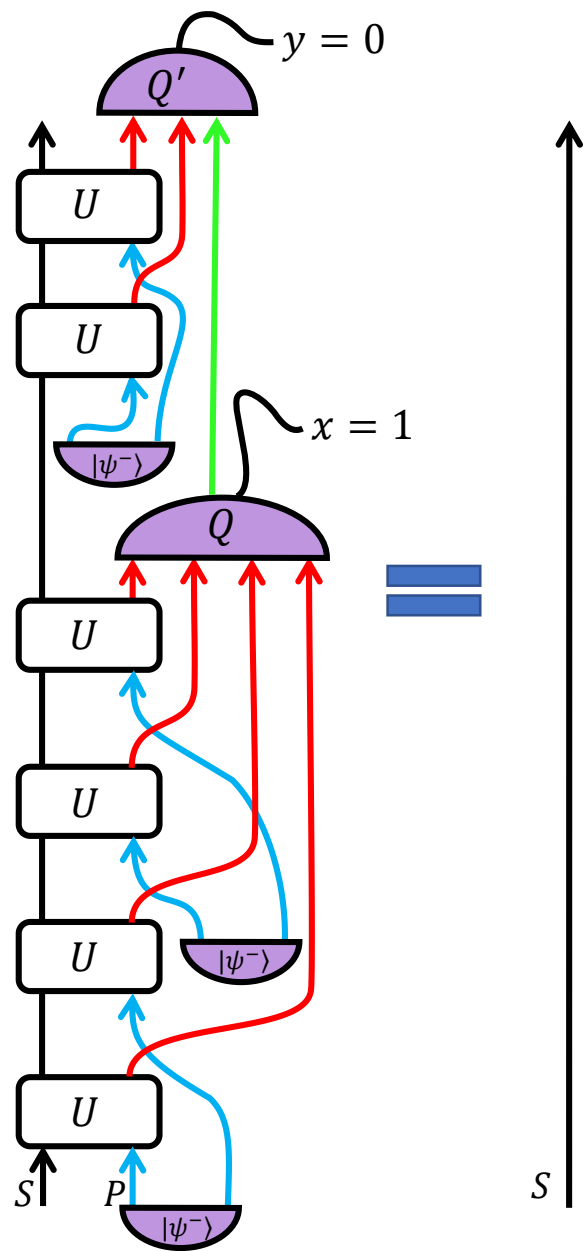
What if the protocol fails?





Suppose that the last measurement is a non-demolition one

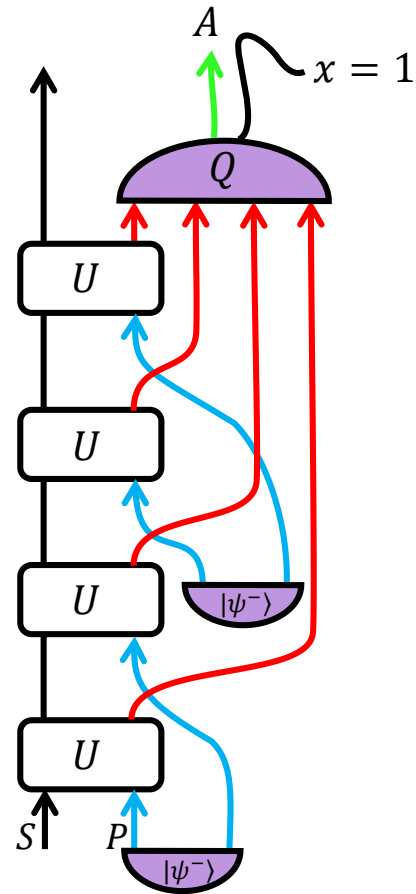




$$|\omega\rangle_{SA} = \sum_i f_i(U) |\psi\rangle_S |i\rangle_A$$

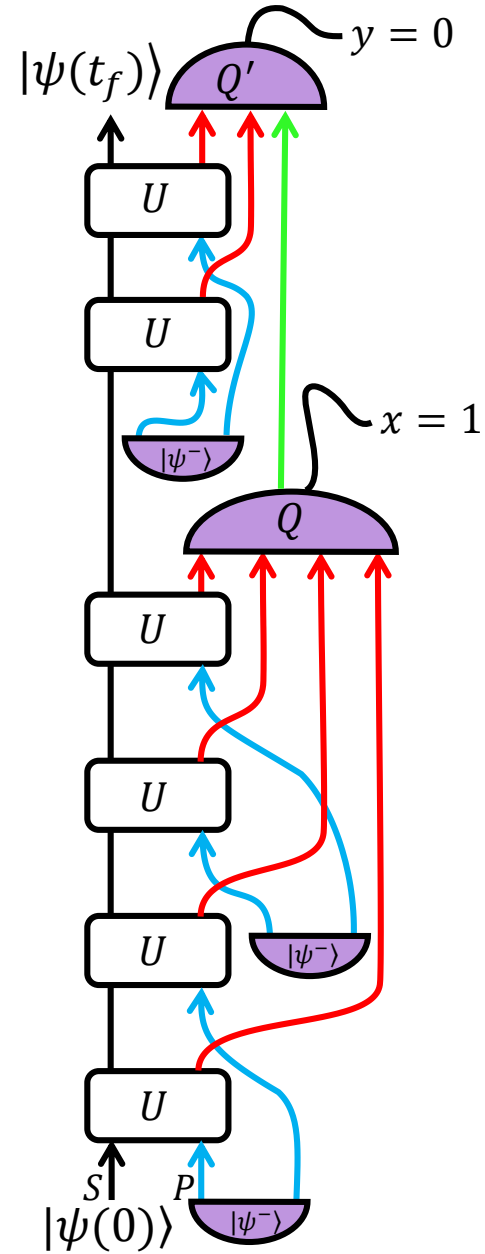
$f_i(U)$, non-central matrix polynomials
of degree 4 on the variables

$$U_{ij} = (\mathbb{I}_S \otimes \langle i|_P) U (\mathbb{I}_S \otimes |j\rangle_P)$$

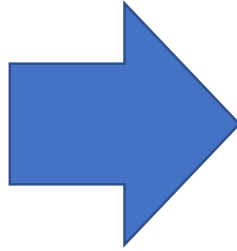


$$|\psi(t_f)\rangle_S = \sum_i g_i(U) f_i(U) |\psi(0)\rangle_S$$

$g_i(U)$, matrix polynomials of degree 2, such that $\sum_i g_i(U) f_i(U)$ is a central polynomial



Undoing six possible failures

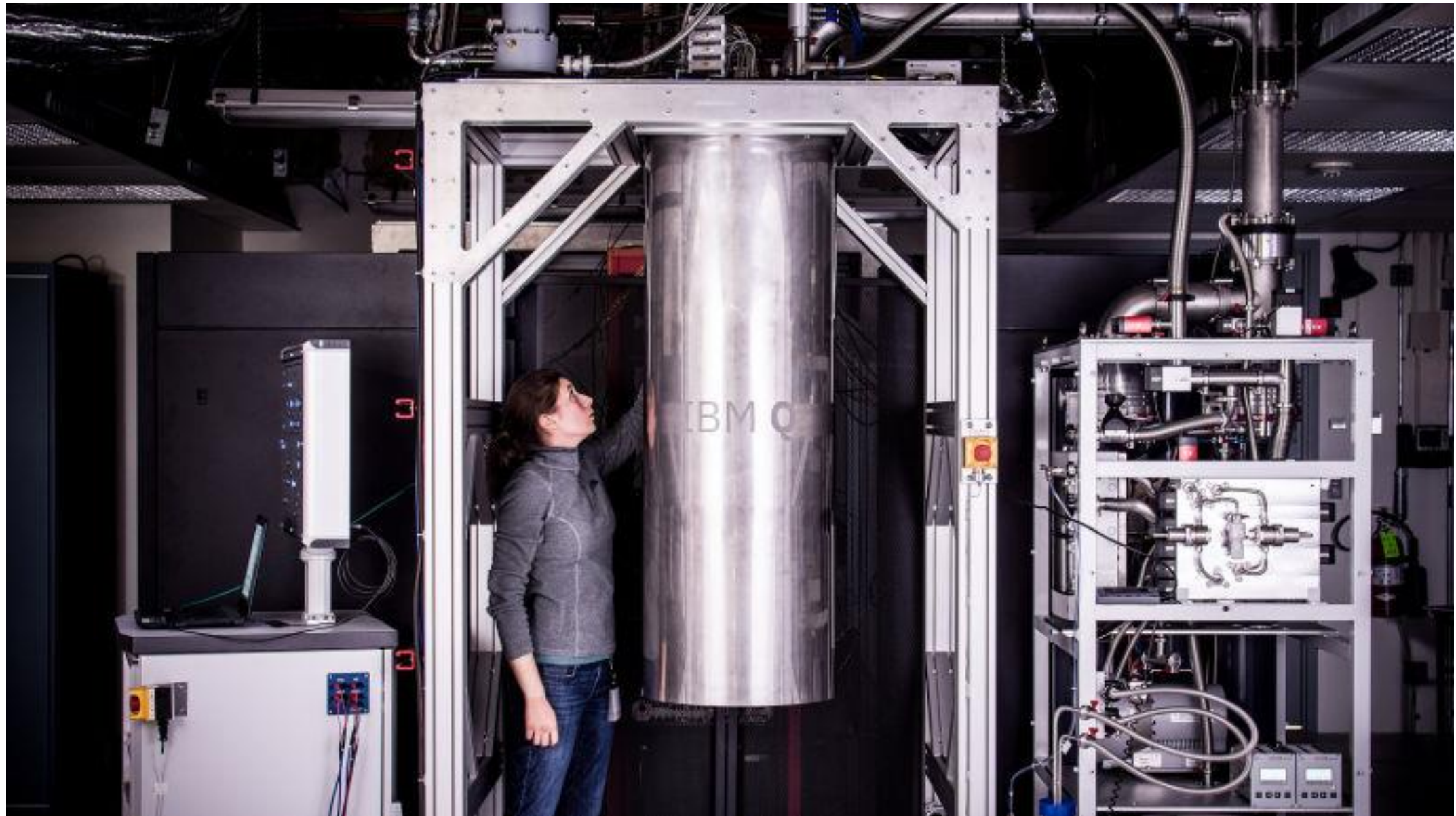


Average probability of success for completely unknown U

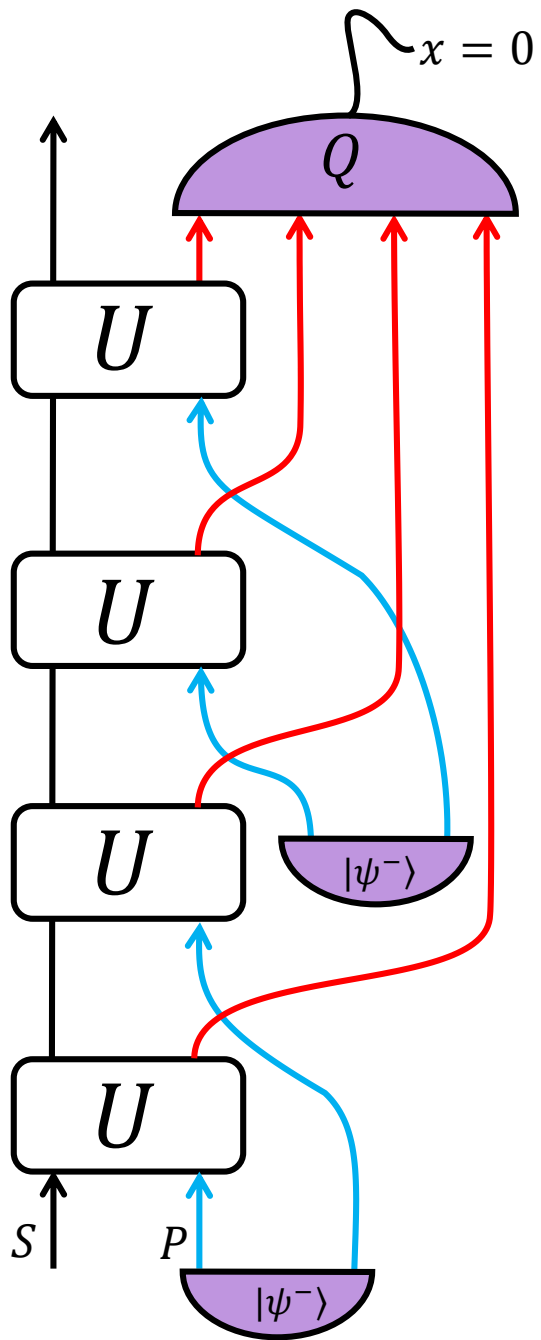
$$\int dU P(x = 0|U) \approx 0.6585$$

A man in a dark suit and light shirt is seated in a room filled with complex, vintage-style scientific or mechanical equipment. To his left is a large, circular, copper-colored device with a grid pattern and intricate mechanical components. In the foreground, there is a red cylindrical component with a patterned surface. To the right, there is a black spherical device with a glowing yellow ring and a decorative cross-like structure. The background features a brick wall and shelves with various glass bottles and containers. The overall atmosphere is that of a laboratory or workshop from a classic science fiction or pulp magazine era.

Experimental implementation?

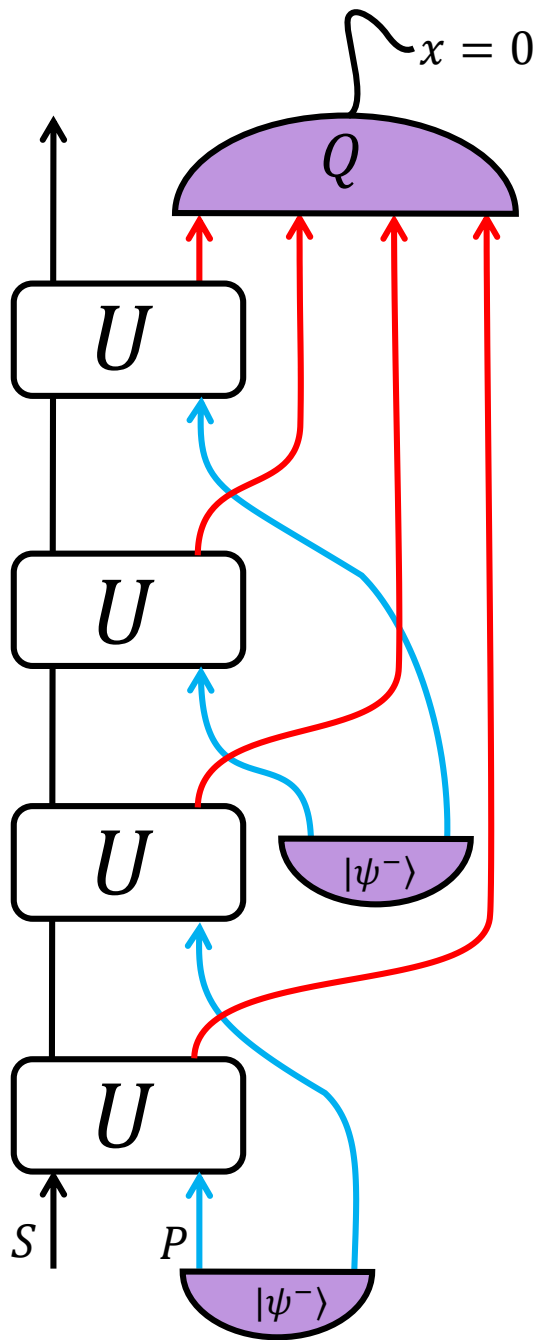


IBM Q Experience

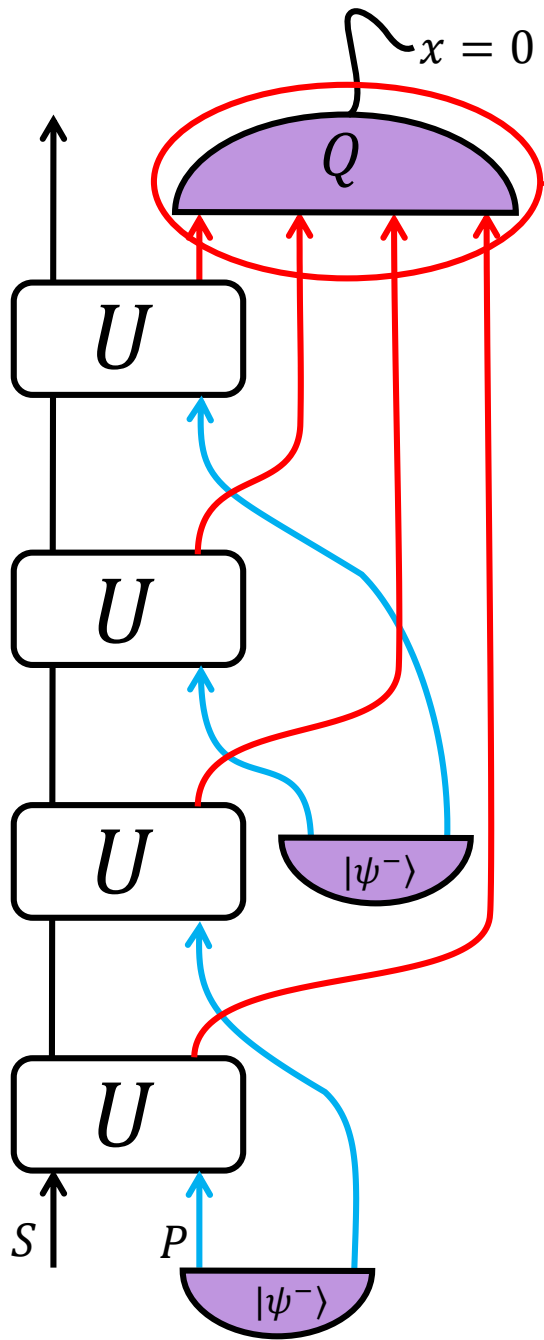


Experimental implementation: Prototype II

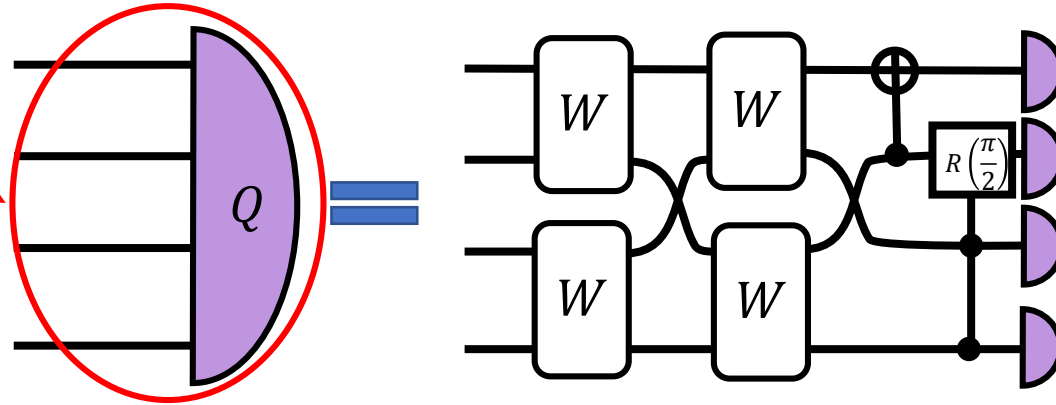
Experimental implementation: Prototype II



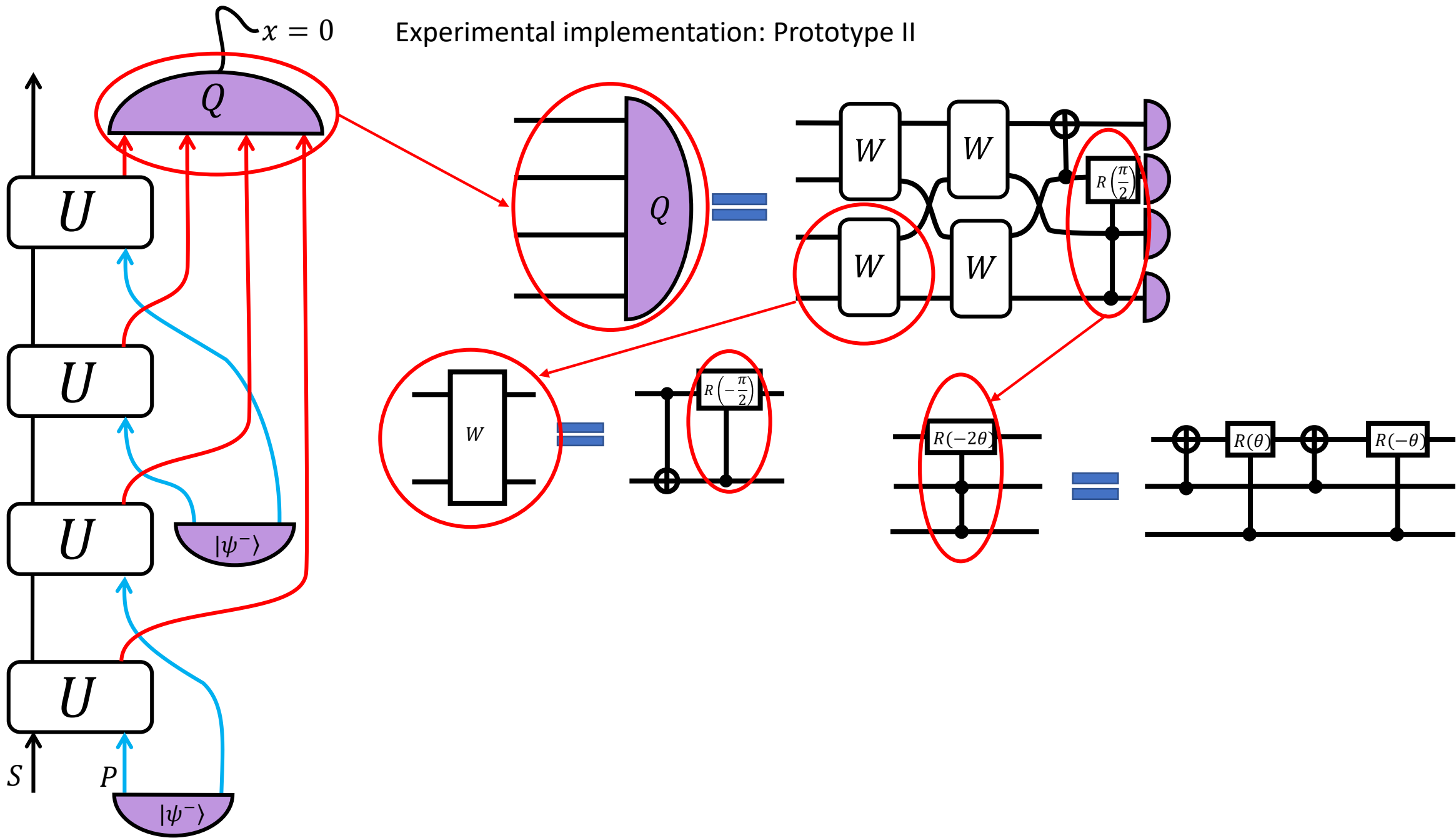
Implementation with single-qubit gates and CNOTS?



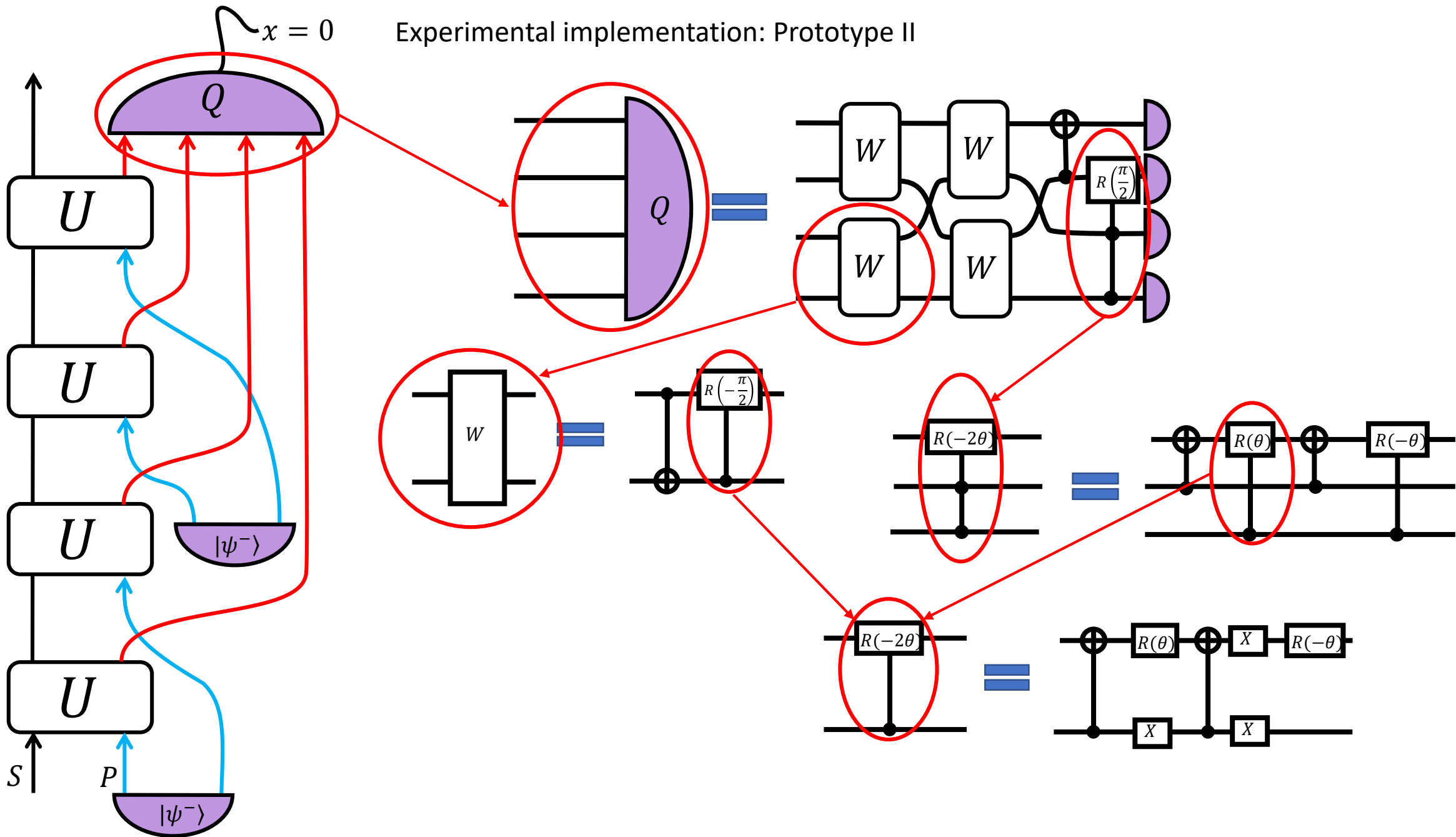
Experimental implementation: Prototype II



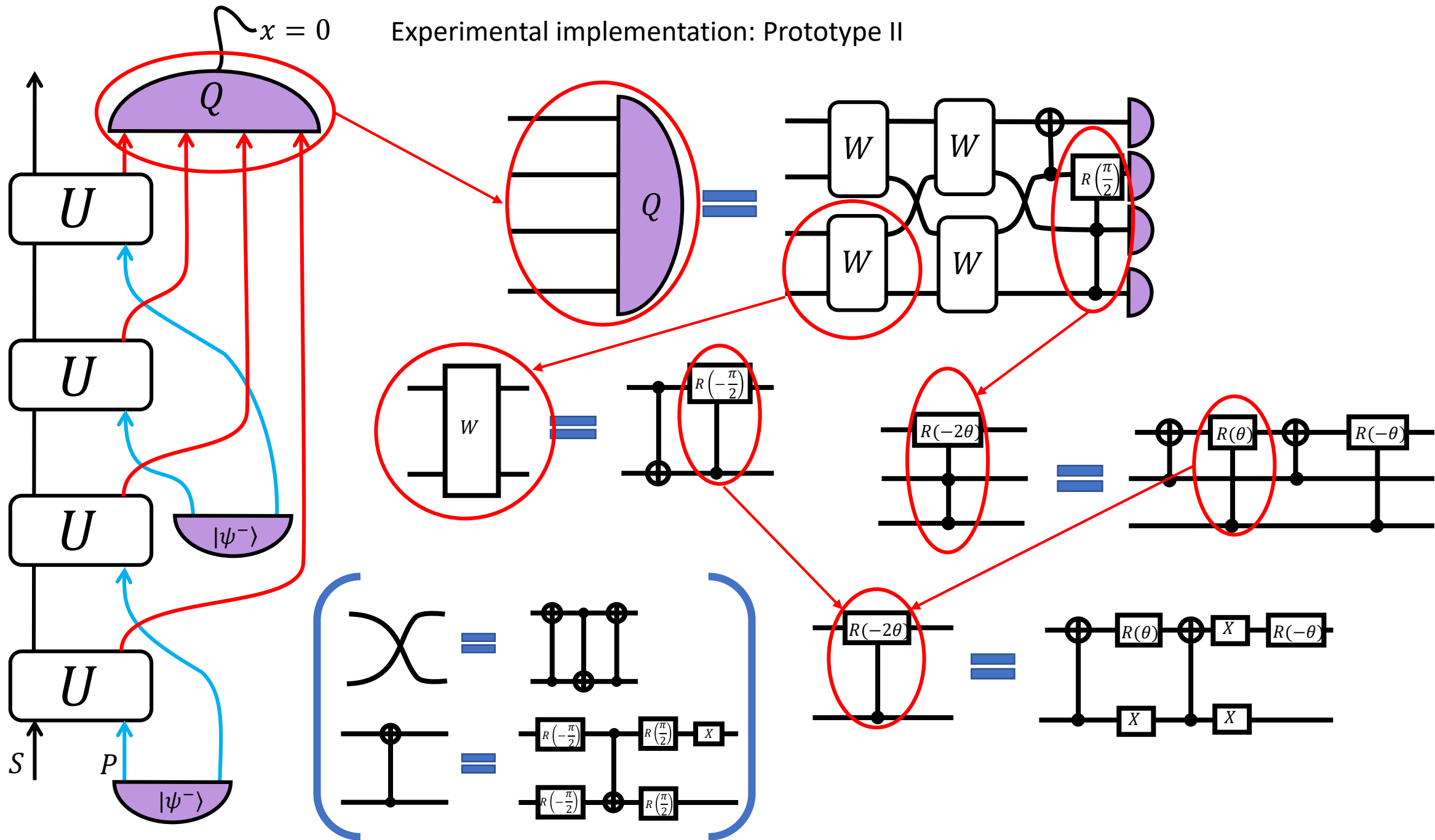
Experimental implementation: Prototype II

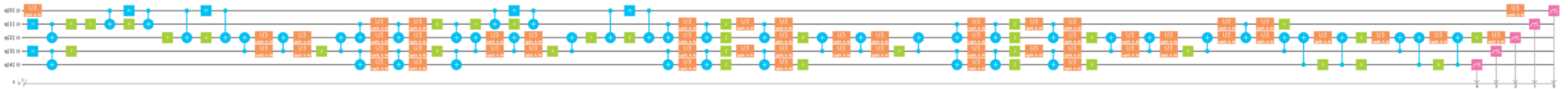
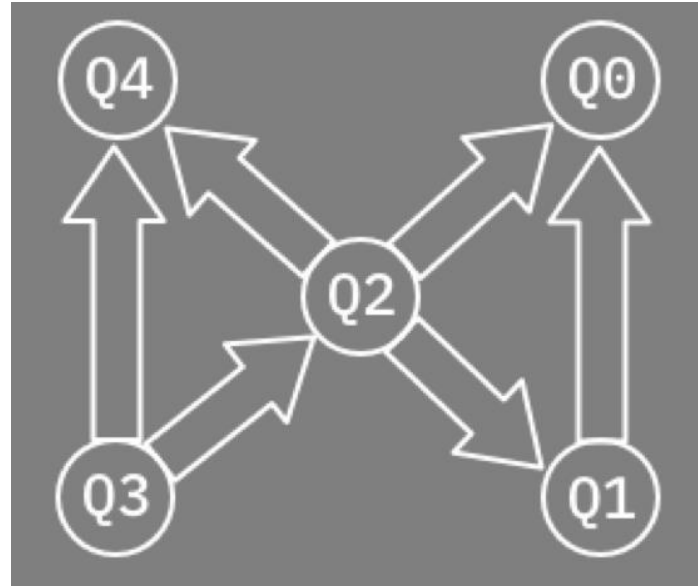


Experimental implementation: Prototype II



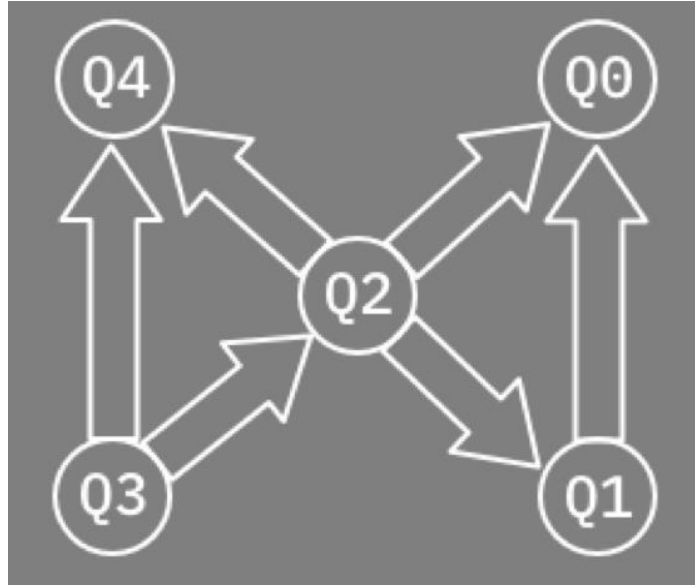
Experimental implementation: Prototype II



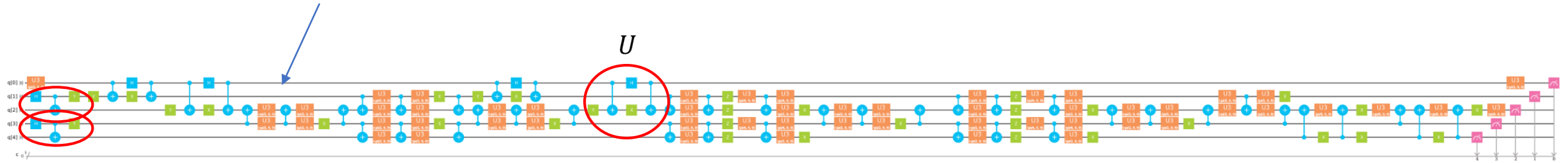


Largest circuit depth allowed!

IBM QX4: Raven



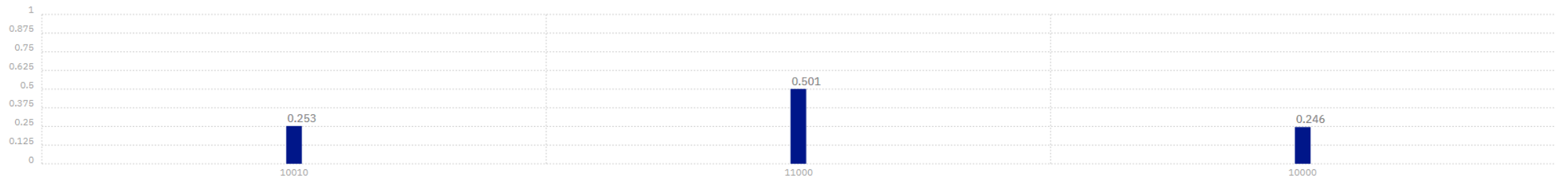
Target system



Preparation of $|\psi^+\rangle^{\otimes 2}$

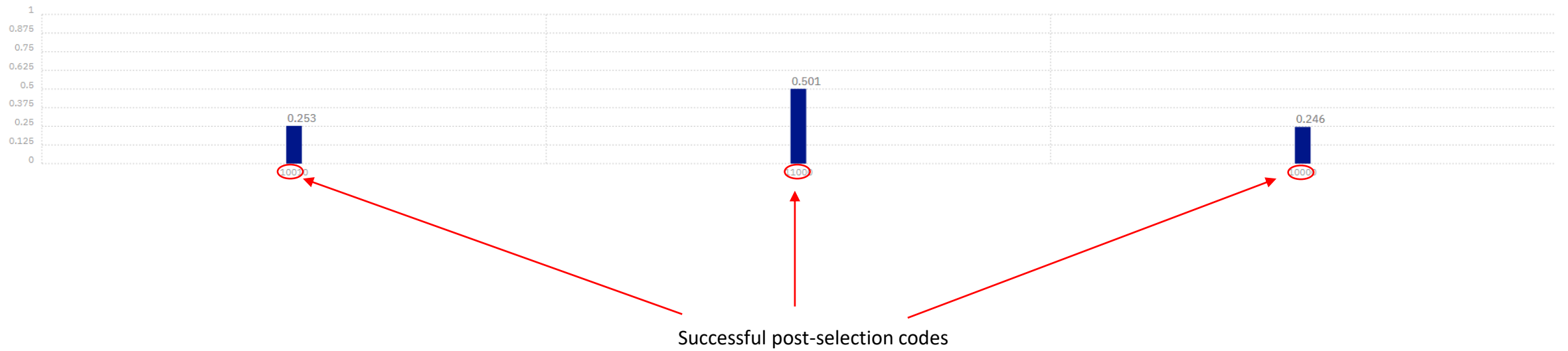
IBM QX4: Raven

Theoretical simulation



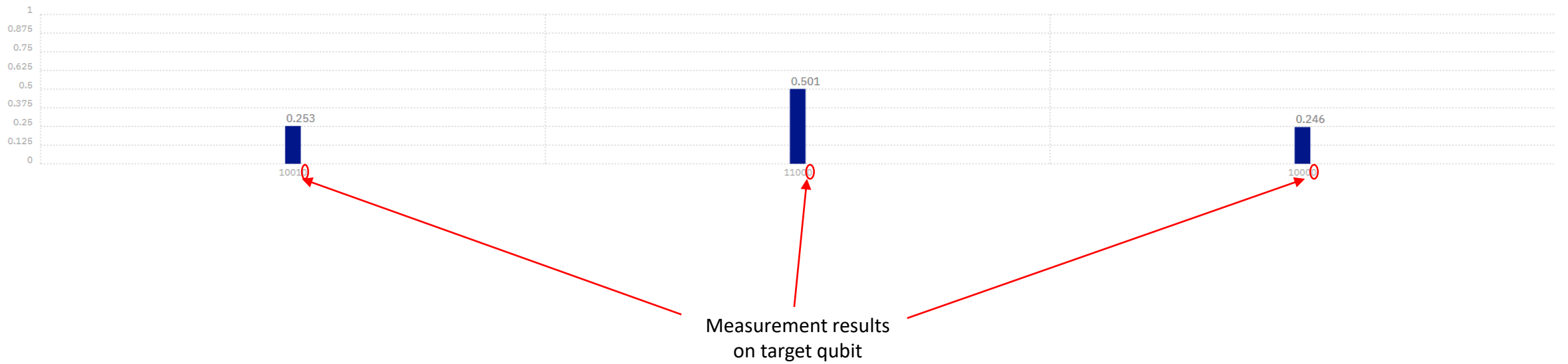
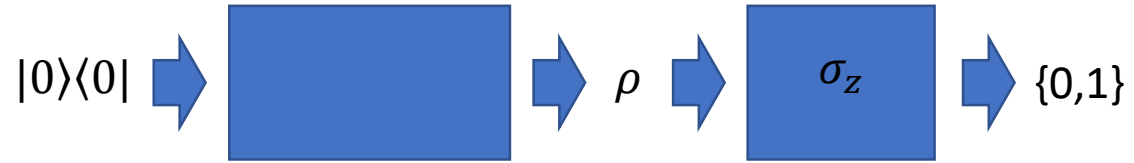
IBM QX4: Raven

Theoretical simulation



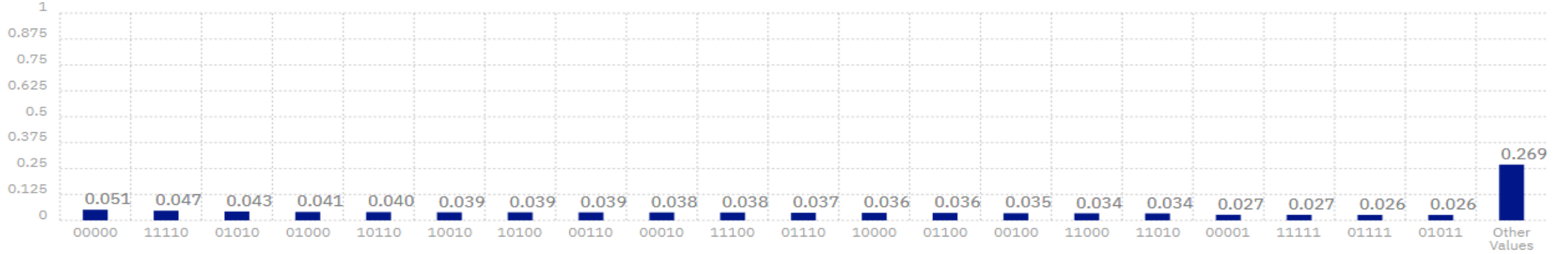
IBM QX4: Raven

Theoretical simulation

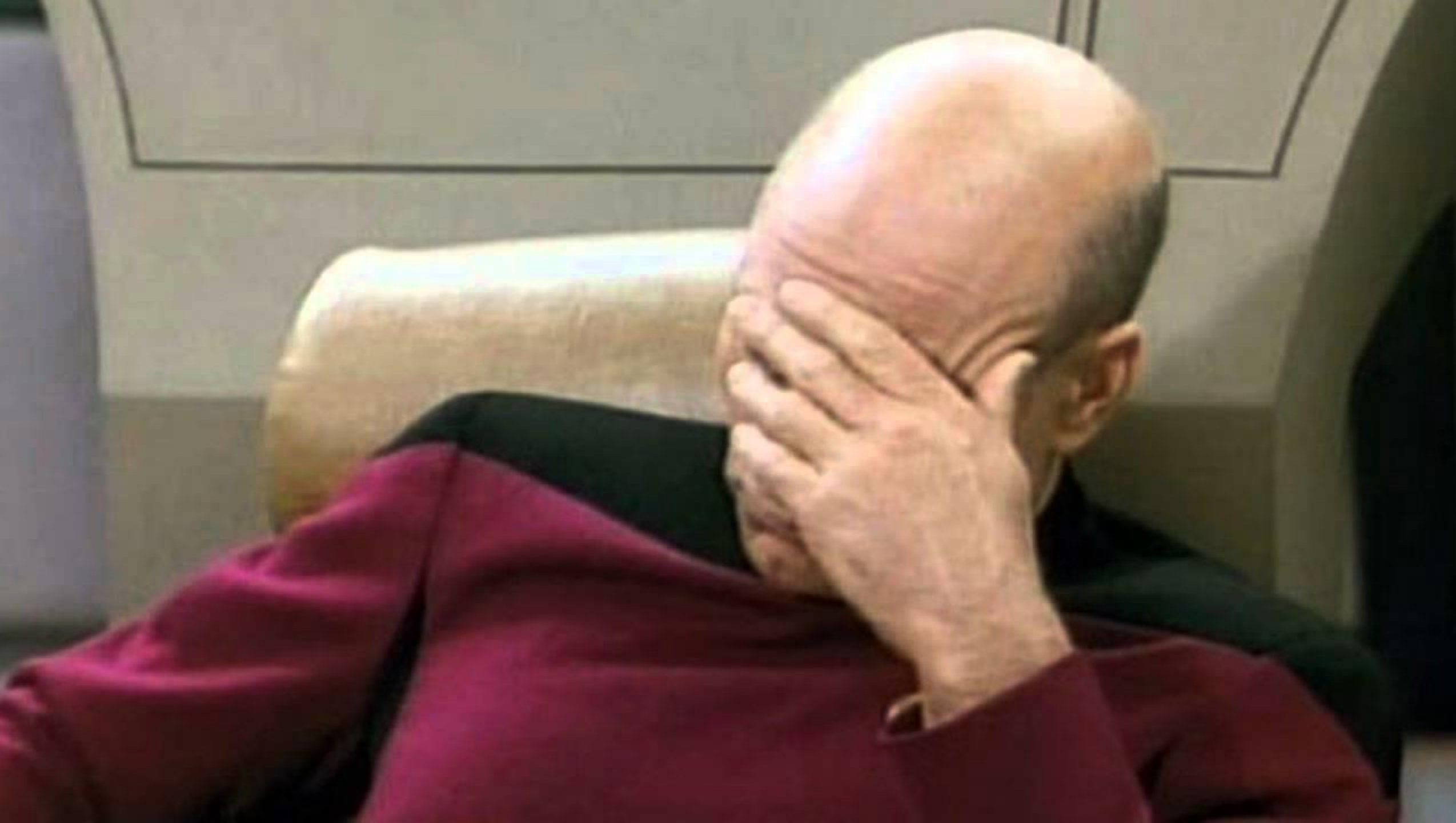


IBM QX4: Raven

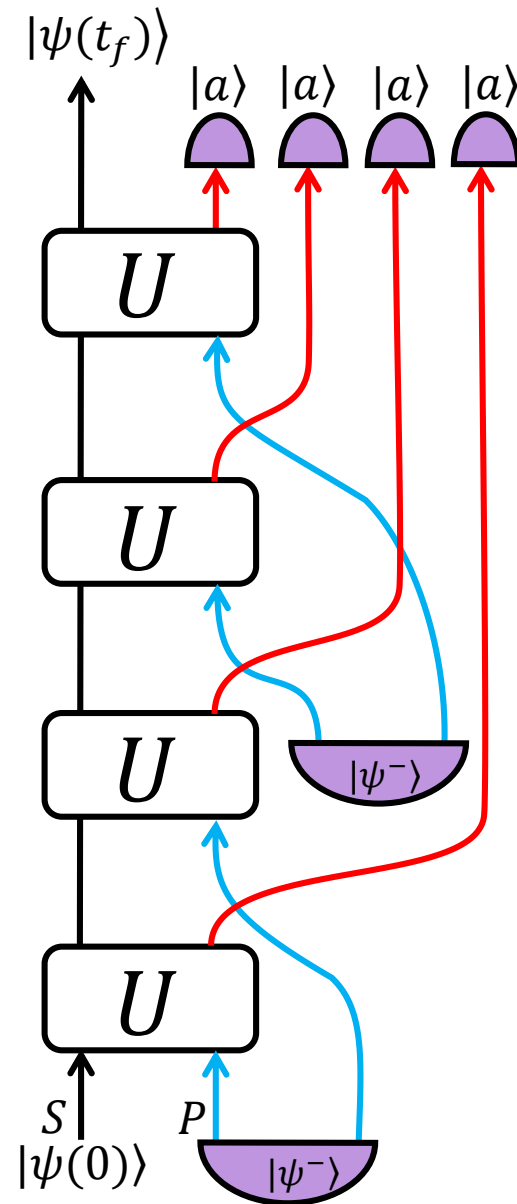
Crude reality

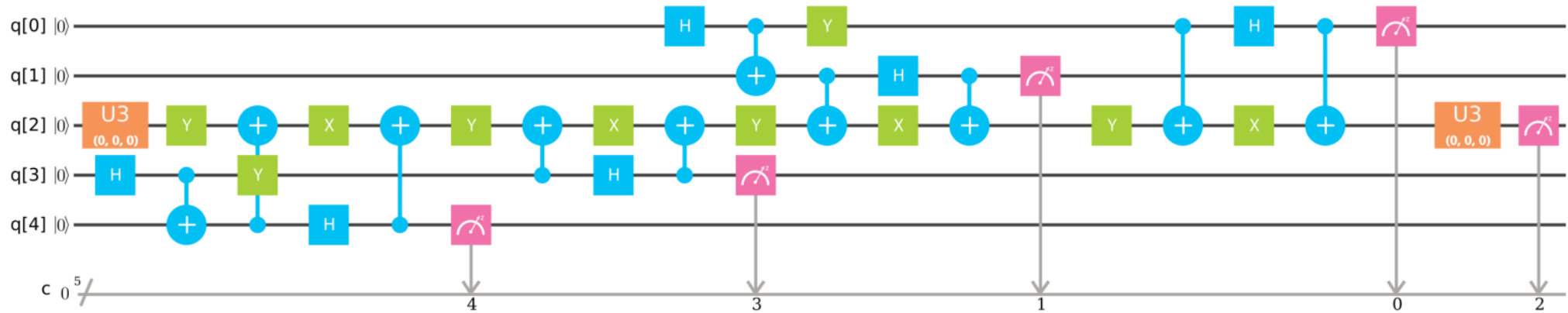
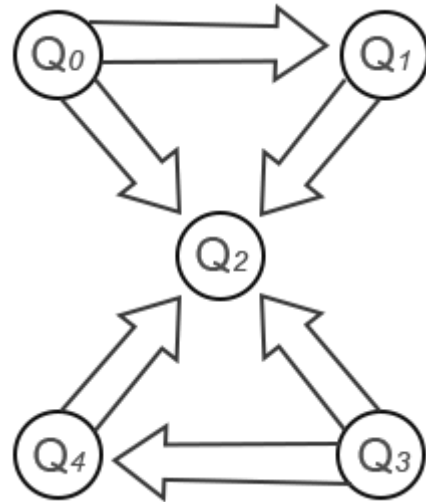


IBM QX4: Raven

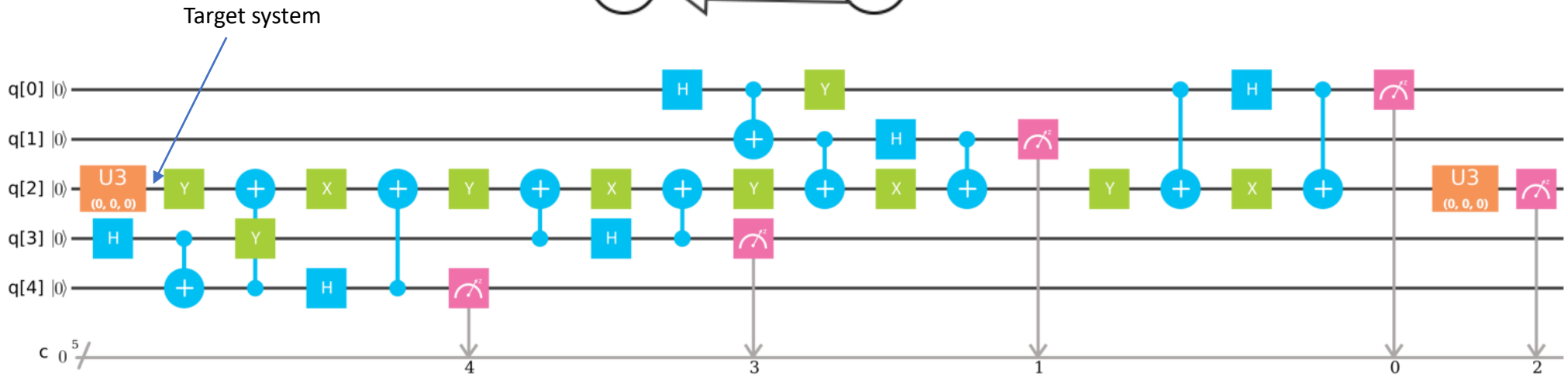
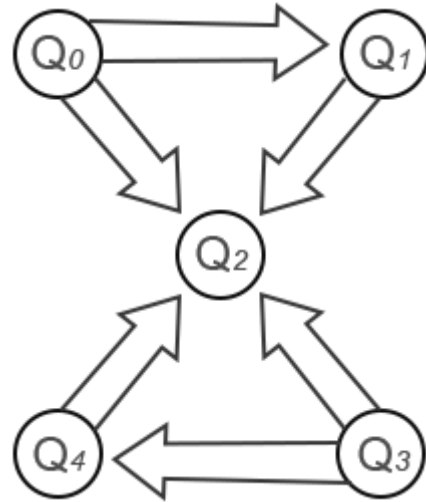


Experimental implementation: Prototype II



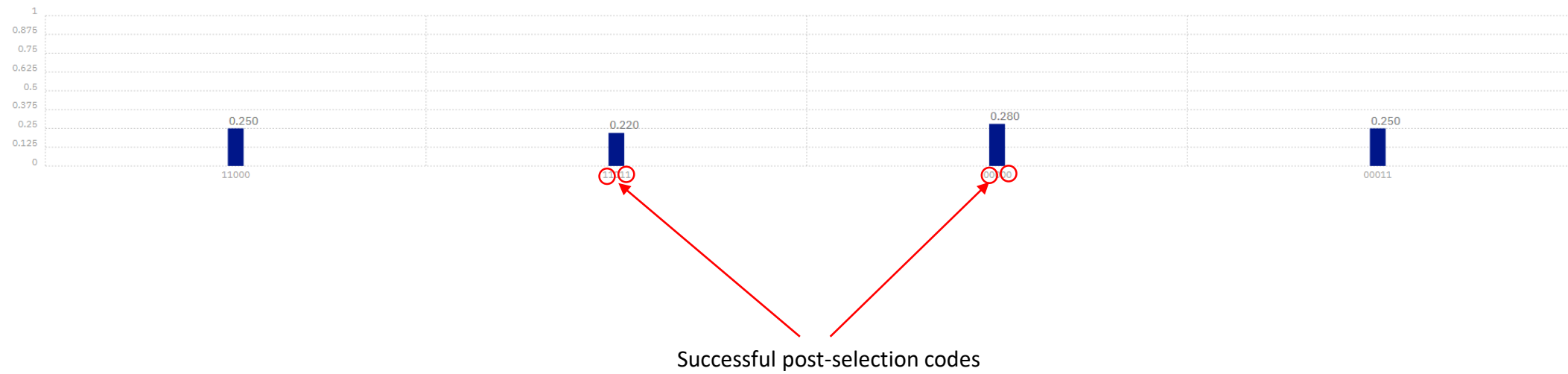


IBM QX2: Sparrow



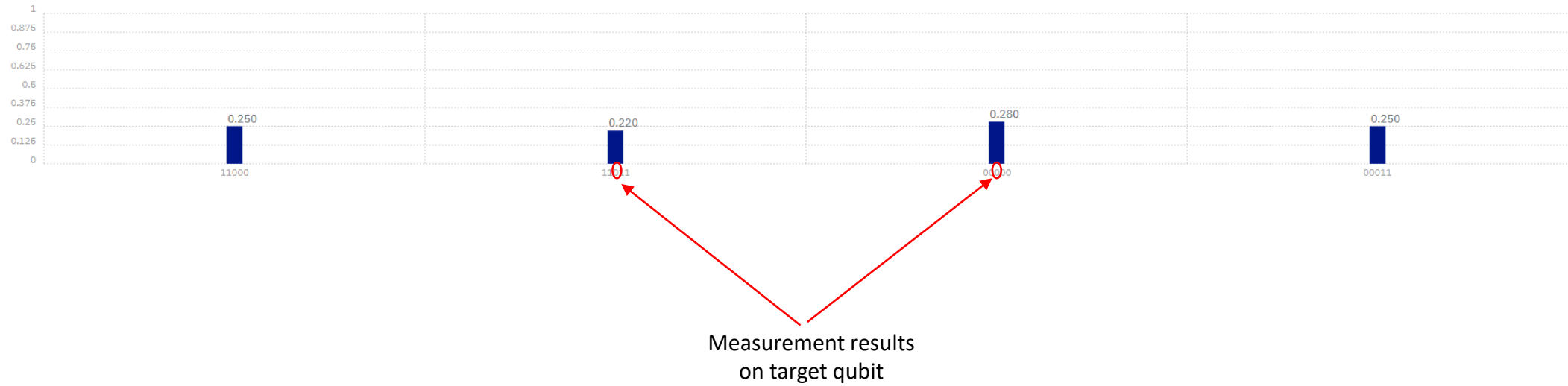
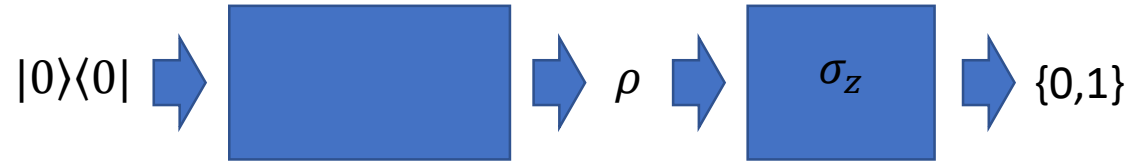
IBM QX2: Sparrow

Theoretical simulation



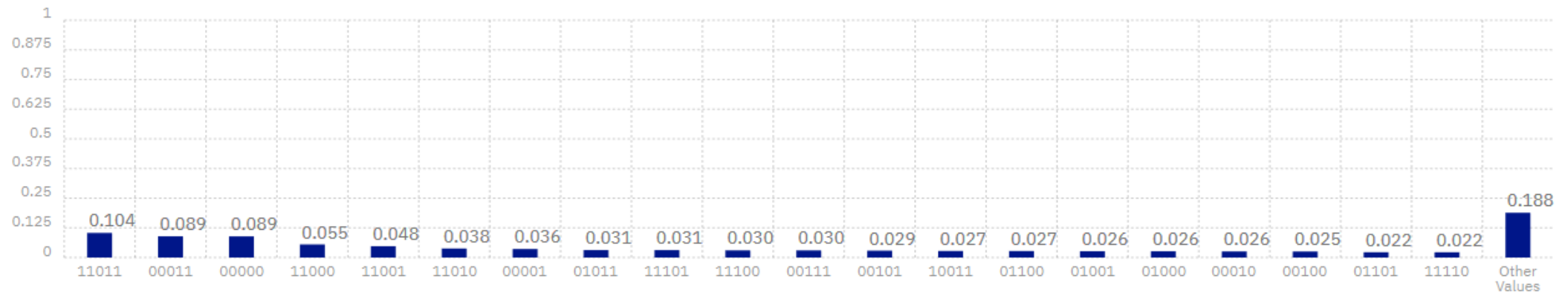
IBM QX2: Sparrow

Theoretical simulation

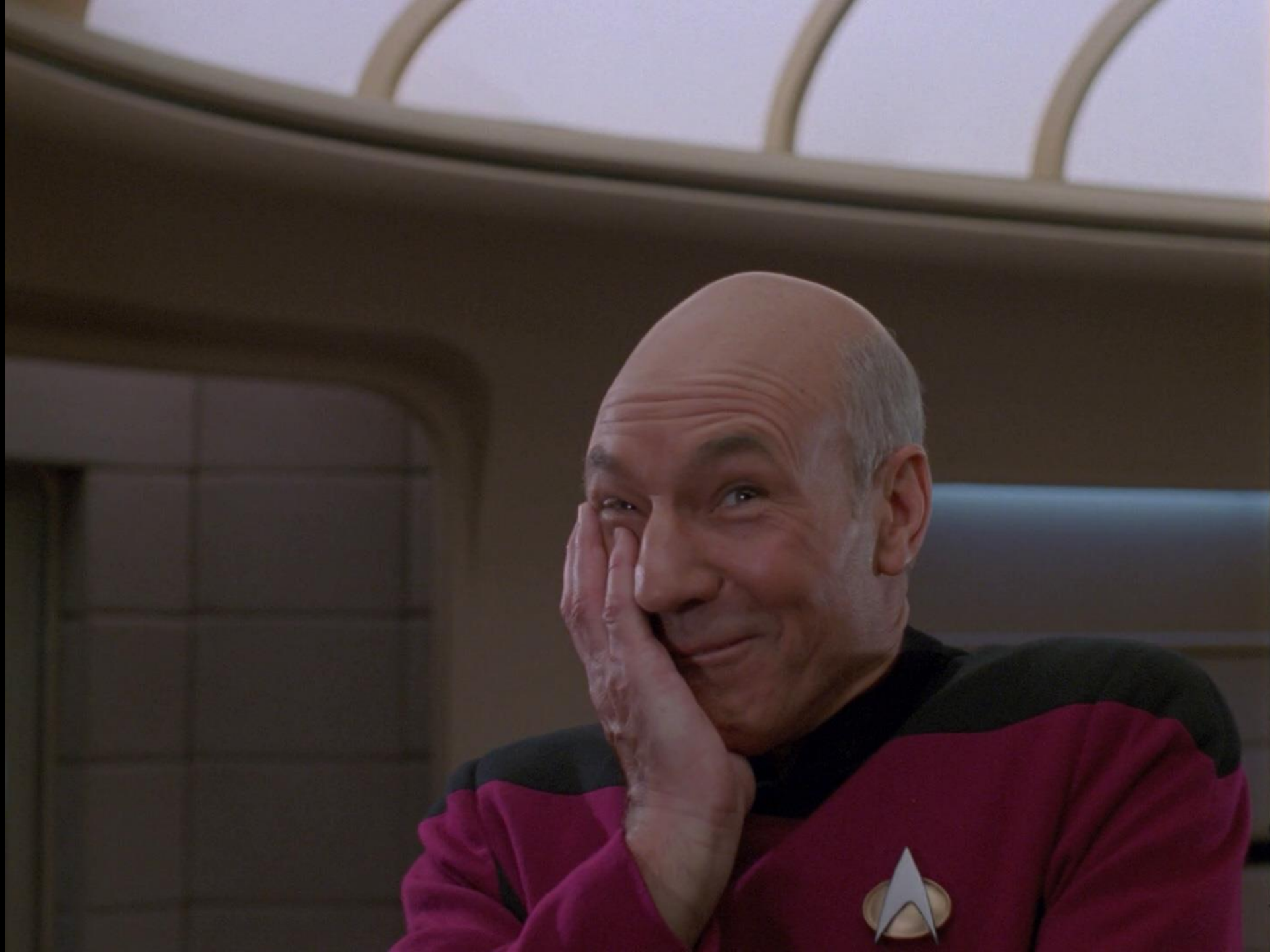


IBM QX2: Sparrow

Crude reality



IBM QX2: Sparrow

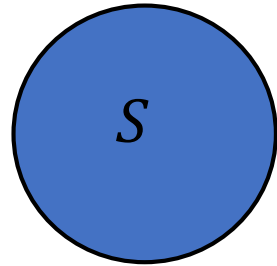


Conclusion

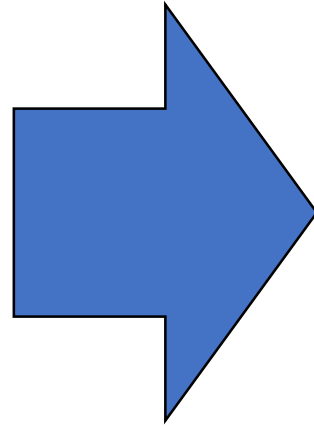
Do there exist protocols with average probability of success (with prior dU) arbitrarily close to 1?

Can we shorten resetting protocols?

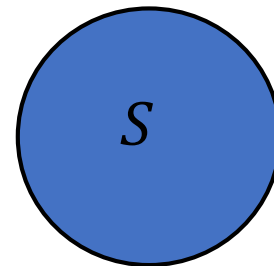
$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$



$t = T$



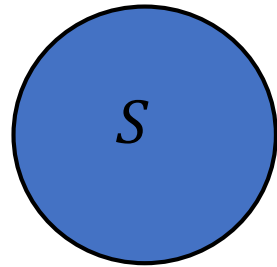
$$|\psi(0)\rangle$$



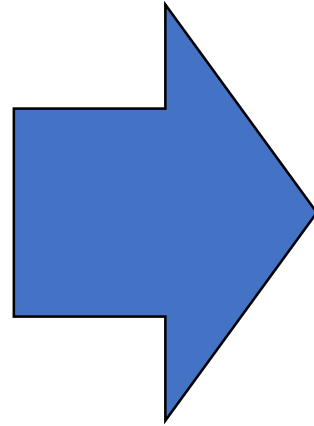
$t = T + \Delta$

Can we shorten resetting protocols?

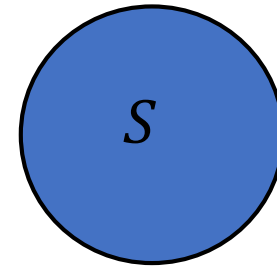
$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$



$t = T$



$$|\psi(0)\rangle$$

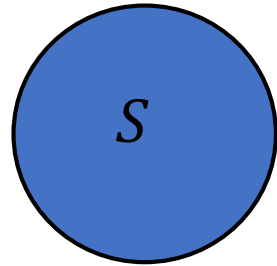


$t = T + \Delta$

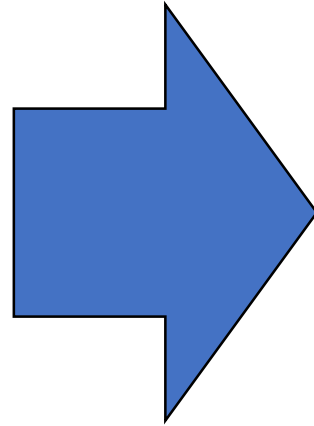
$$\Delta \geq 3T$$

Can we shorten resetting protocols?

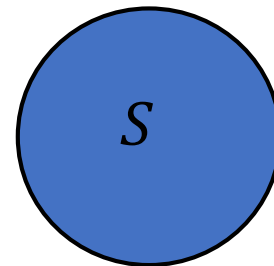
$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$



$t = T$



$$|\psi(0)\rangle$$

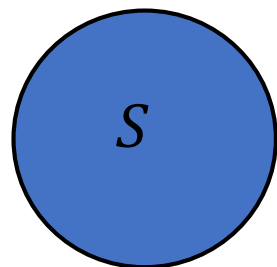


$t = T + \Delta$

$\Delta \ll T?$

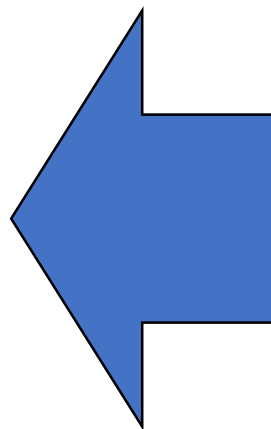
Can we fast-forward?

$$|\psi(T)\rangle = e^{-iH_0T} |\psi(0)\rangle$$

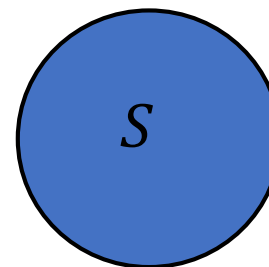


$t = \Delta$

$\Delta \ll T?$

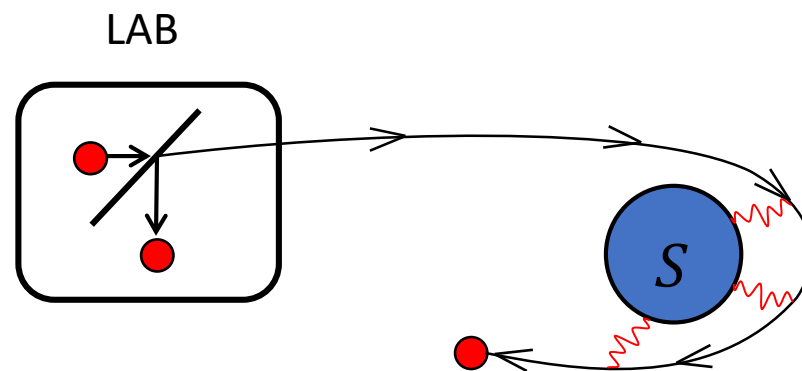


$|\psi(0)\rangle$



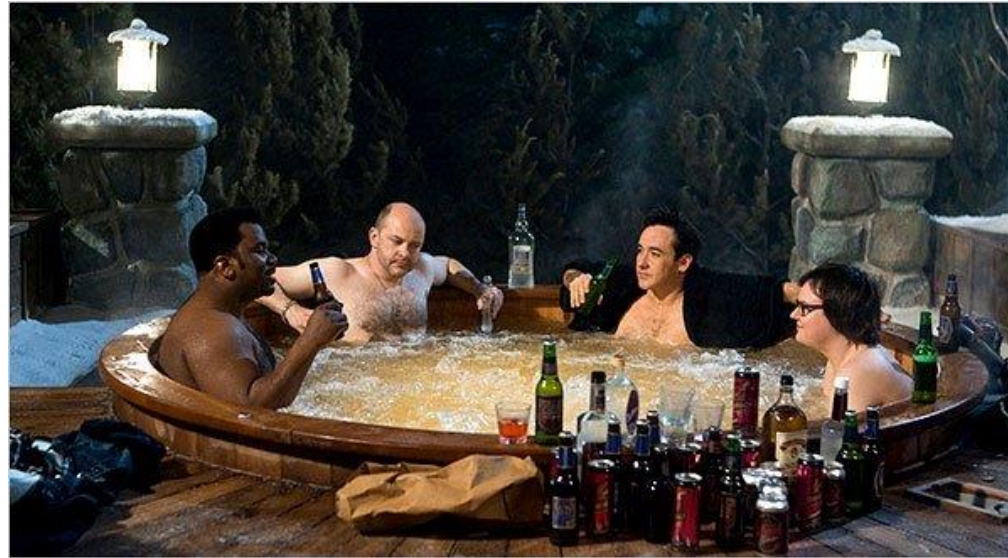
$t = 0$

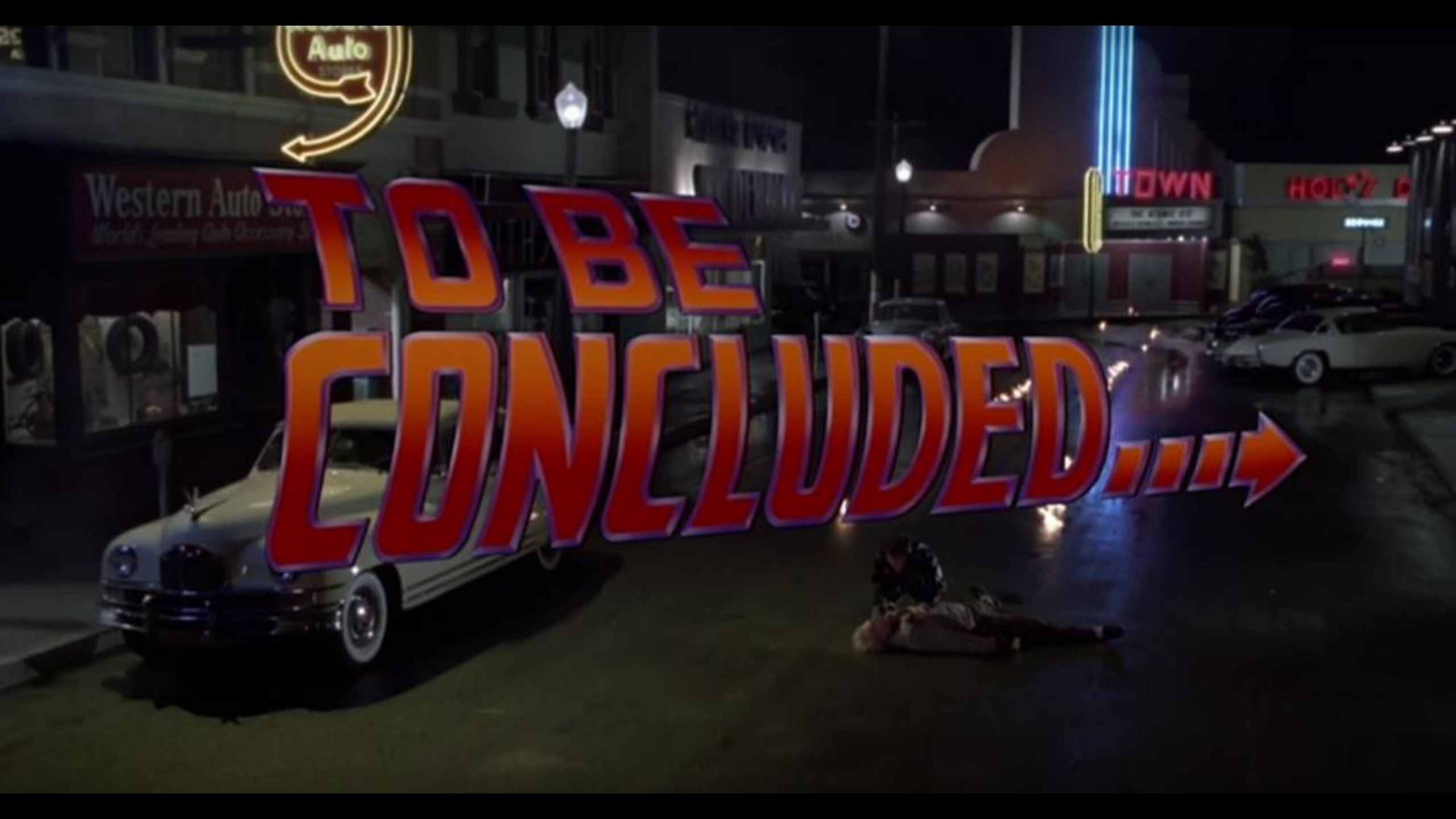
Can we shorten resetting protocols?/Can we fast-forward?



Hint: use which-path superpositions.

Simple experimental implementation?





Auto
Storage

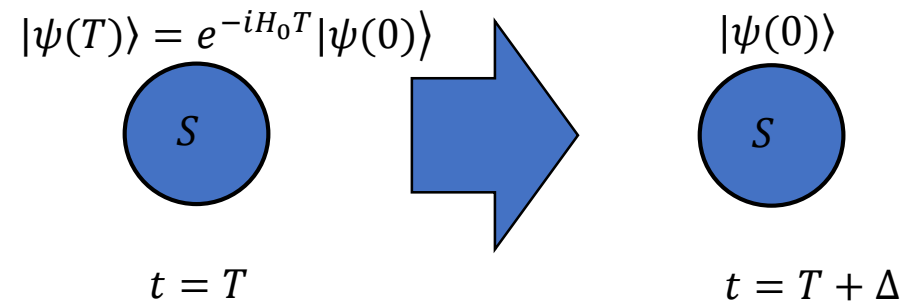
Western Auto
World's leading Auto Accessories

TO BE
CONCLUDED

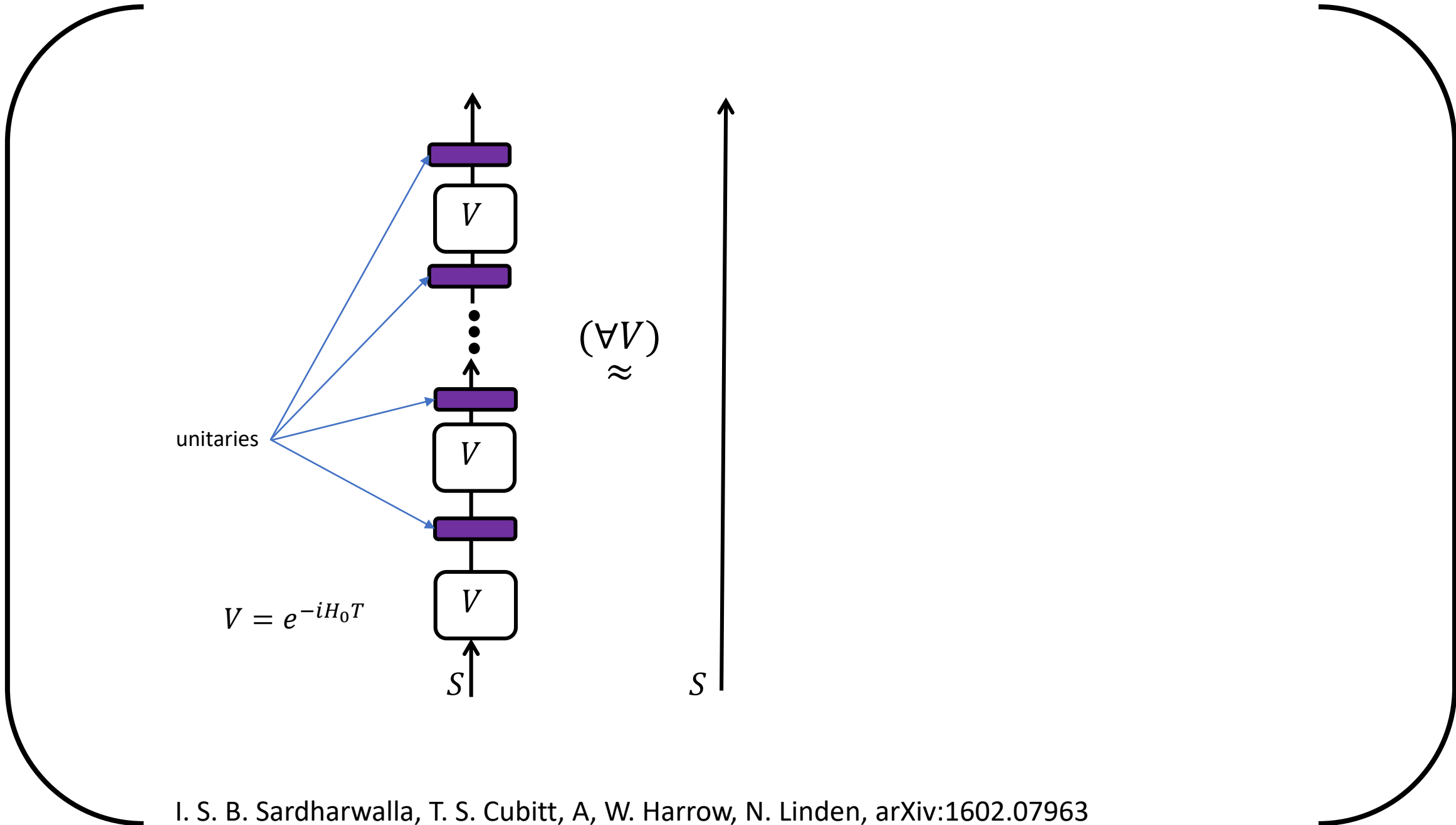
TOWN

HOTEL

Refocusing

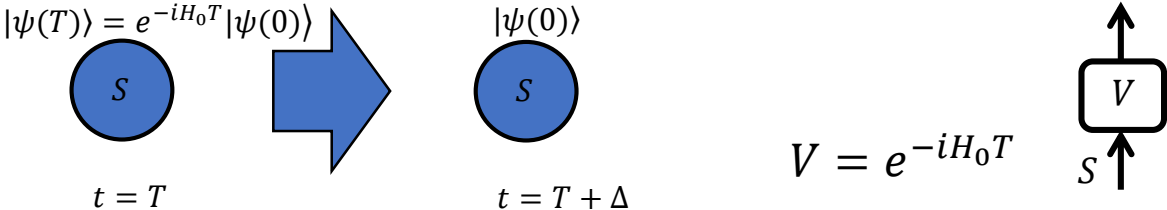


We ignore H_0 , but any operation on S is allowed

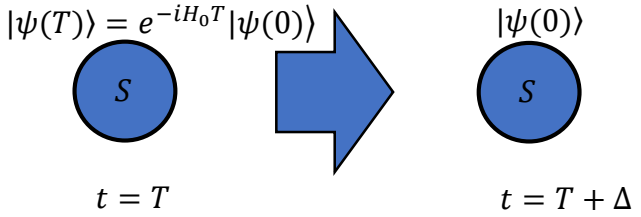


$$V = e^{-iH_0T}$$

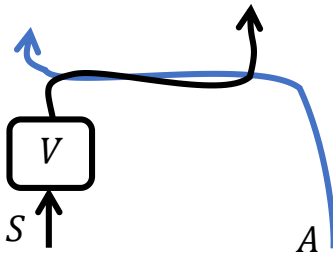
Refocusing (obvious solution)



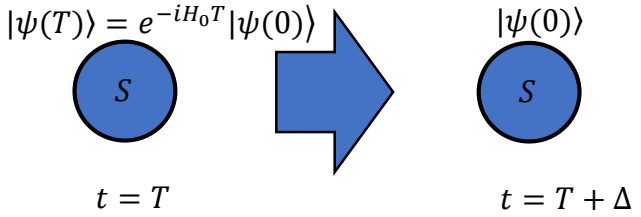
Refocusing (obvious solution)



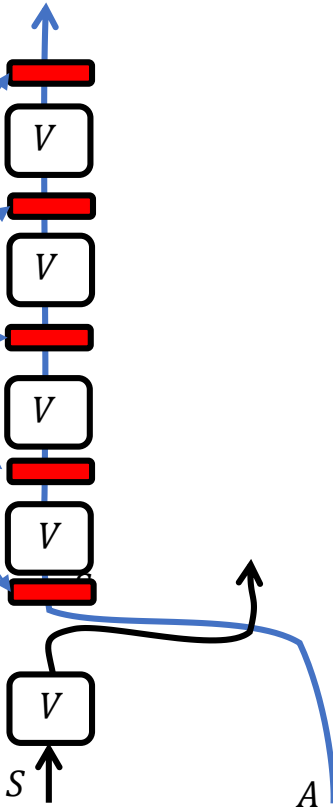
$$V = e^{-iH_0T}$$



Refocusing (obvious solution)



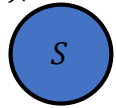
Channel tomography



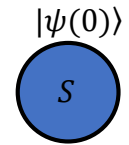
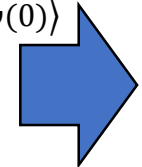
Refocusing (obvious solution)

Channel tomography

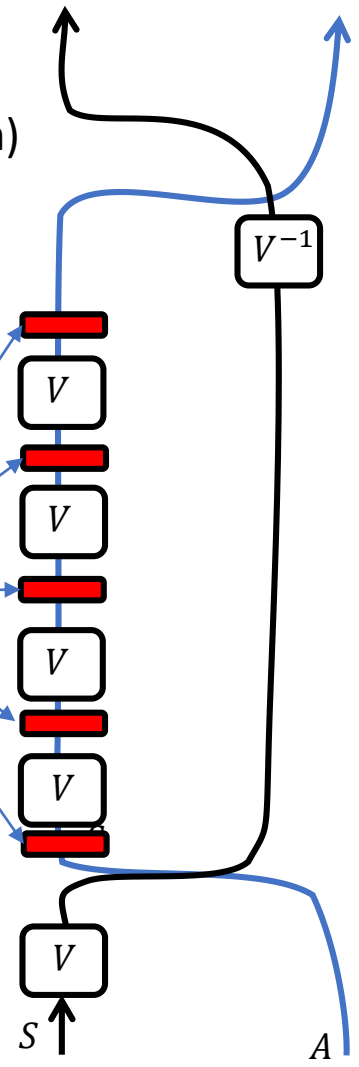
$$|\psi(T)\rangle = e^{-iH_0 T} |\psi(0)\rangle$$



$t = T$



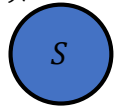
$t = T + \Delta$



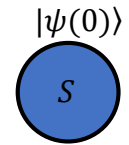
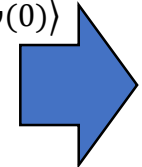
Refocusing (obvious solution)

Channel tomography

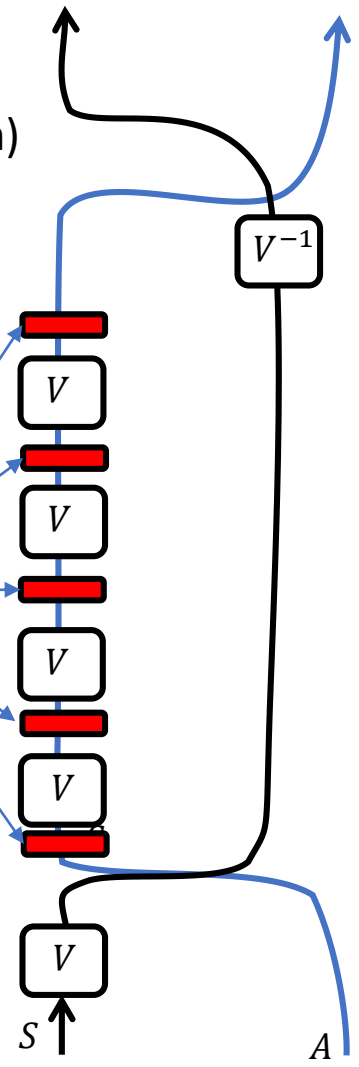
$$|\psi(T)\rangle = e^{-iH_0 T} |\psi(0)\rangle$$



$t = T$



$t = T + \Delta$



$$(VA) \approx$$

$|\psi(0)\rangle$

S
 $|\psi(0)\rangle$