Sensorics Exam

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5th March 2013

Name:	Task:	T1	T2	T3	T4	T5	T6	Sum
Mat.-No.:	Scores:	32	24	16	6	28	14	120
Grade:	Accomplished:							

Task 1: Comprehension Questions

Mark the correct answers clearly.
Every question has one or two correct answers!
For every correctly marked answer you will get one point. If there is one correct answer marked and one incorrect answer marked, you will get no point for that subtask.

a) Which statements are true about bridge circuits?

- ☐ A bridge circuit can only measure direct-current voltage.
- ☐ To fulfill the balance conditions, only the absolute values of electric resistance and impedance are important, not their ratios.
- ☐ If the circuit is balanced, the bridge voltage U_d is equal to zero.

b) Unsupervised Learning...

- ☐ ...does not require the desired output values.
- ☐ ...requires the desired output values.
- ☐ ...is often used for data pre-processing.

c) How can speed be measured?

- ☐ Via the measurement of the rotational speed and a subsequent division by the radius.
- ☐ Via the measurement of the acceleration and a subsequent differentiation.
- ☐ With the help of the Doppler-effect.

d) A non-causal filter...

- ☐ ...requires only current and previous input values.
- ☐ ...requires future output values.
- ☐ ...requires future input values.

e) An increase in the number of measurements has the following effect on the frequency resolution:
- [ ] It becomes finer.
- [ ] It remains unaffected.
- [ ] It becomes coarser.

f) How large should the internal resistance of a voltage meter be?
- [ ] It depends on the electrical circuit, what internal resistance is ideal.
- [ ] Very large, in the ideal case infinite large.
- [ ] Very small, in the ideal case zero.

g) Principal Component Analysis (PCA)...
- [ ] ...can be calculated with a filter.
- [ ] ...can be calculated by solving an eigenvalue problem.
- [ ] ...can be calculated with a singular value decomposition.

h) In order to make a meaningful statement about the contained frequencies in a non-stationary signal, ...
- [ ] ...the signal can be examined with a wavelet transform.
- [ ] ...the signal can be examined with a short-time DFT.
- [ ] ...the signal can not be examined with a short-time DFT.

i) Which statements are true for the measurement of temperatures?
- [ ] Thermocouples are suitable for point-wise measurements.
- [ ] Thermocouples are more accurate than resistance thermometer.
- [ ] Thermocouples have a smaller time constant than resistance thermometer.

j) A median filter...
- [ ] ...is always causal.
- [ ] ...has a step output as a response to a step input.
- [ ] ...can be used to remove outliers.

k) The so-called leakage effect can be reduced by ...
- [ ] ...subtraction of a window in the time domain.
- [ ] ...multiplication with a window in the time domain.
- [ ] ...convolution with a window in the time domain.

l) Principal Component Analysis (PCA)...
- [ ] ...is a tool for dimensionality reduction.
- [ ] ...is a linear axis transformation.
- [ ] ...is a non-linear axis transformation.

m) To process a measurement signal in the computer, the signal...

- [ ] ...has to be transformed into the frequency domain.
- [ ] ...has to be discretized.
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n) Parametric frequency analysis ...

- [ ] ...is more robust with respect to measurement noise as a non-parametric method.
- [ ] ...leads to a continuous amplitude spectrum.
- [ ] ...leads to a discrete amplitude spectrum.

o) Clustering...

- [ ] ...is a method for data pre-processing.
- [ ] ...is a method for data post-processing.
- [ ] ...belongs to the class of supervised learning algorithms.

p) When measuring low frequencies...

- [ ] ...the gate time is usually defined by an artificially generated frequency. While the gate is open, the frequency that should be measured is used to produce pulses that are counted to calculate the signal's frequency.
- [ ] ...it is recommended to count the number of cycles.
- [ ] ...the gate time is usually defined by the signal, that should be measured. While the gate is open an artificially generated frequency produces pulses that are counted and utilized to calculate the low frequency.

q) A temporal sequence of N measurements that is transformed with the DFT results in a number of ...

- [ ] ... $N/2$ discrete frequencies.
- [ ] ... N discrete frequencies.
- [ ] ... 2^N discrete frequencies.

r) What can be achieved with the discrete Fourier transform (DFT)?

- [ ] The transformation of a discrete signal from the time domain to the continuous range of frequencies.
- [ ] The transformation of a signal from the discrete frequency domain into the discrete time domain.
- [ ] The transformation of a discrete signal from the time domain to the discrete frequency domain.

s) Which of the following statements about measurement errors are correct?

- [ ] Reason and kind of the error action are known for systematic errors.
- [ ] Systematic errors can be reduced by calculating the mean of multiple measurements.
- [ ] Reason and kind of the error action are known for random errors.

t) The discrete Fourier transform is periodic ...

- [ ] ... only in frequency.
- [ ] ... in time and frequency.
- [ ] ... only in time.

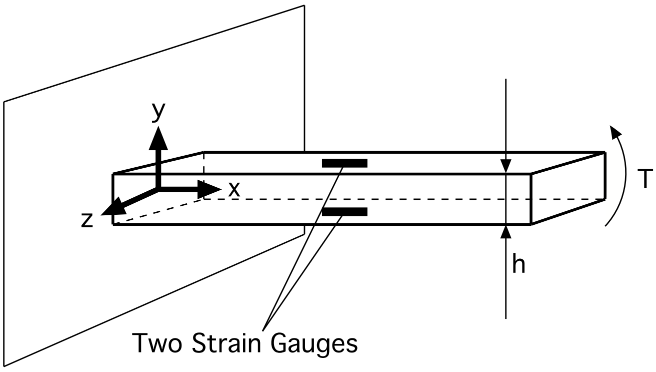
u) Which statements are true for strain gauges?

- [ ] Strain gauges utilize the resistance change caused by a change in length and a change of the cross section area of a metallic conductor for the measurement.
- [ ] Strain gauges utilize the change of its capacity for the measurement.
- [ ] Environmental influences can easily be compensated through a clever arrangement of the strain gauges.

v) Stationary signals ...

- [ ] ...can be considered as non-stationary, when viewed over a short period.
- [ ] ...are time-variant.
- [ ] ...are time-invariant.

Task 2: Measurement of Torque



In the picture above you can see a beam, that is loaded with a torque T on one end. The other end is fixed to a wall and two strain gauges are applied to the beam to measure the torque. The following equations are given:

$$|\rho| = \frac{EI}{|T|} \quad ; \quad \Delta R = R_0 K \epsilon \quad ; \quad \epsilon = \frac{y}{|\rho|}$$

ρ: Curvature of the deflected beam; E: Young's modulus; I: Second moment of area; T: Torque (constant over the beam length); ΔR: The change in resistance of one strain gauge; R_0: The resistance of the strain gauge without any applied torque; K: Sensitivity of the strain gauges; ϵ: Deformation of the strain gauge, h: Beam height; y: Coordinate of the beam height, that starts exactly at the half of the beam height

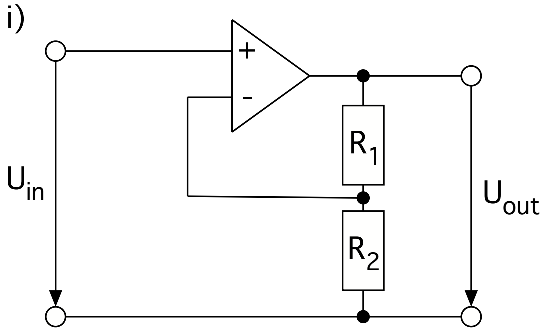
a) Evaluate the equation that describes the change of the resistances ΔR depending on the applied torque T. What relationship does the change of the resistance of the upper and the lower strain gauge have?

b) Sketch a bridge circuit that can be used to transform the change in resistance ΔR into a voltage U_d and derive the corresponding equation (Just use ΔR for the change of resistance - NOT the equation from a!).

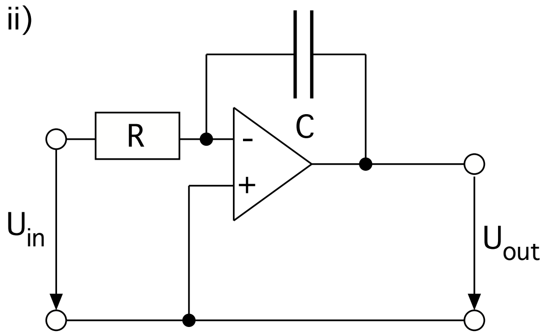
c) In what way can the sensitivity of the torque measurement be increased through a change in the bridge circuit?

d) The measured voltage should be amplified with the help of an operational amplifier (OpAmp) circuit. Which of the following two circuits amplifies the input voltage U_{in} (assume ideal OpAmps)?

i)



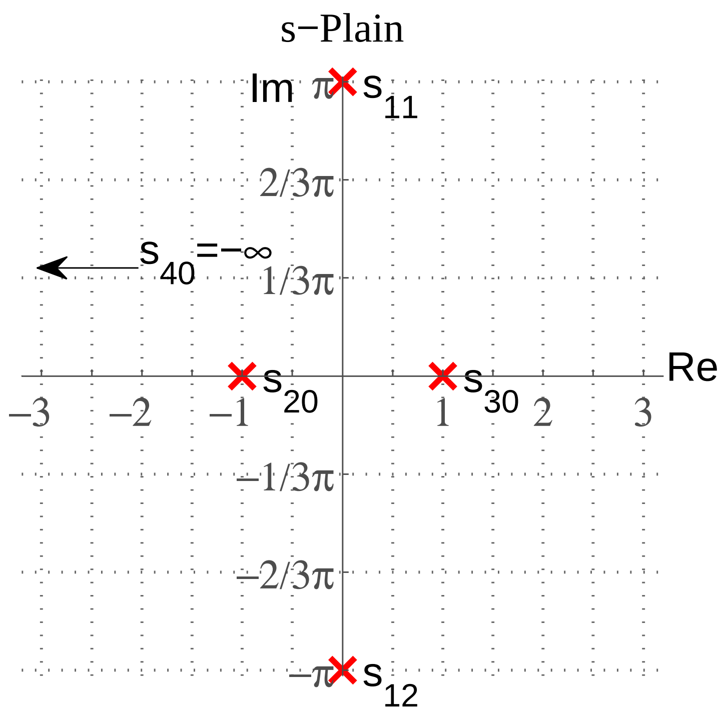
ii)

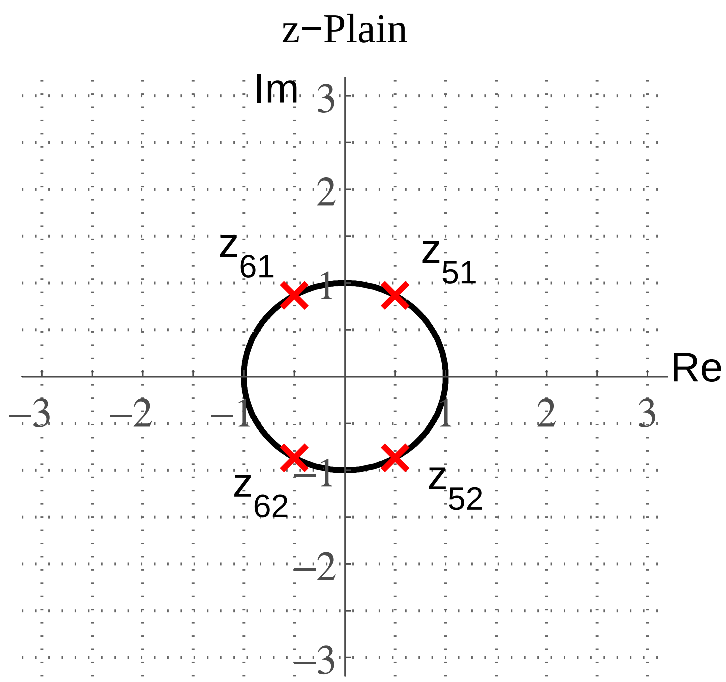


e) What relationship of R_1 and R_2 has to be fullfilled to achieve an amplification of the input voltage by a factor of four?

Task 3: Relation Between s-Plane and z-Plane

In the following figure are six poles, two in the z-domain and four in the s-domain. Perform the transformation of all points in the z-domain to the s-domain and from the s-domain to the z-domain and sketch them in the corresponding plain. Denote related points by the same indices, i.e. the point s_{ij} in the s-domain corresponds to the point z_{ij} in the z-domain. To perform the transformation assume a sampling time $T_0 = 1$ second. (Hint: Use the exact equation for the transformation!).





Task 4: Aliasing

The following periodic signal is given: $y(t) = \sin(2\pi \cdot 10\text{Hz} \cdot t)$.

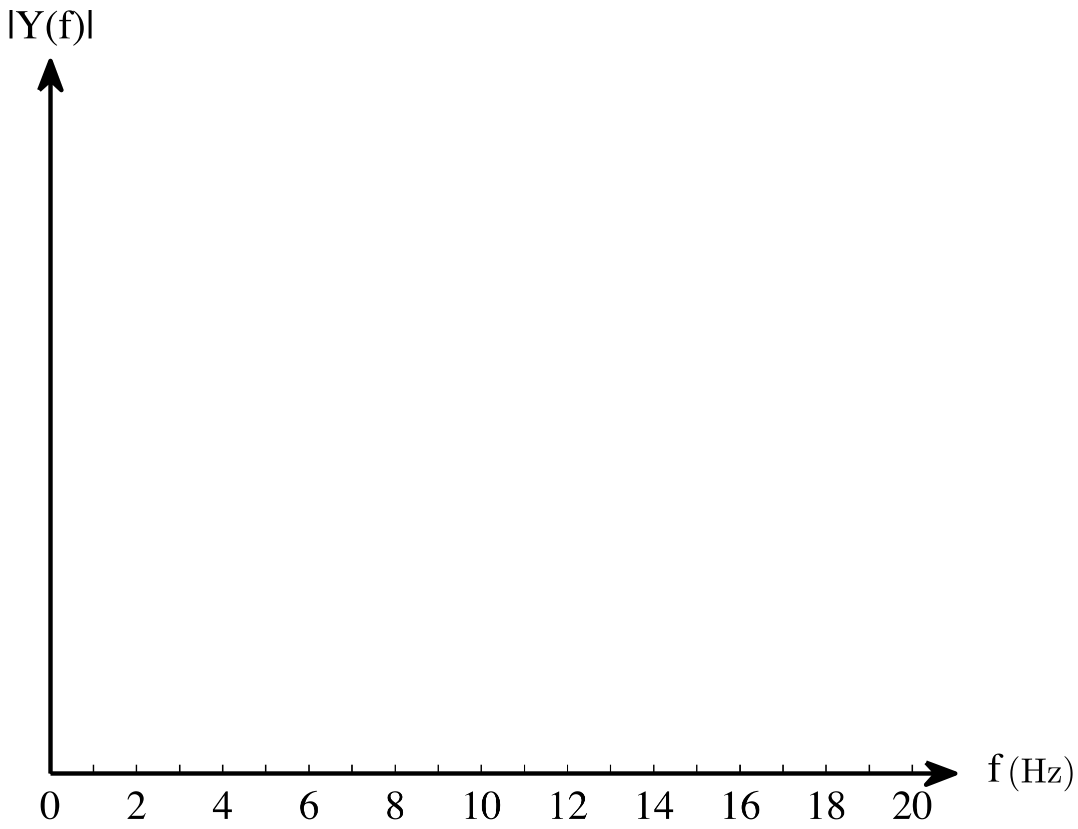
a) Sketch the amplitude spectrum for the given signal in the diagram below.

b) The signal is measured with the sampling frequency $f_0 = 8\text{Hz}$. Explain, why aliasing will occur in this case.

c) Highlight the Nyquist frequency (Shannon's sampling theorem) in the diagram below.

d) Sketch the shadow spectra that result from the sampling theorem.

e) What signal frequency can wrongly be guessed from the sampled signal?

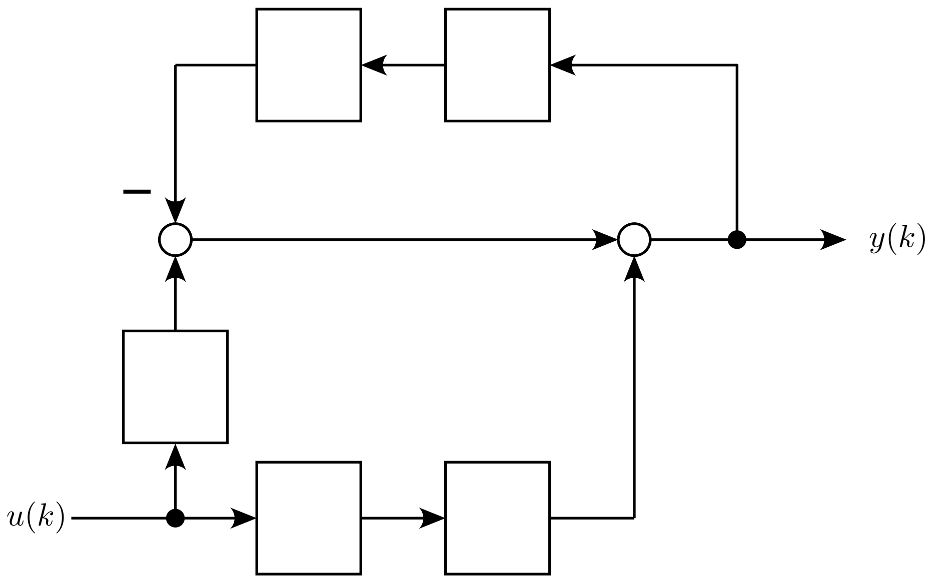


Task 5: Time-Discrete Systems

A first order time discrete system has in general the difference equation:

$$y(k) = b_0 u(k) + b_1 u(k-1) - a_1 y(k-1) \; .$$

a) Fill out the given block diagram so that it matches the given difference equation.



b) What is the crucial prerequisite for the system to have a direct throughput from the input to the output? Explain your answer.

c) What is the crucial prerequisite for the system to have a FIR-characteristics? Explain your Answer.

d) Calculate the impulse response $g(k)$ (initial condition: $y(k < 0) = 0$).

e) Determine the corresponding transferfunction in the z-domain ($G(z) = \ldots$)?

f) Calculate the z-transformed step response $H(z)$.

g) What is the value of the step response in the time domain $h(k)$ for $k \to \infty$? What is the initial value of the step response for $k = 0$?

h) Now assume that the coefficients have the following values: $b_0 = 0,3$, $b_1 = 0$ and $a_1 = -0{,}9$. In this case, what is the universal equation for the step response $h(k)$?

i) Assume that the coefficient a_1 has a positive algebraic sign, i.e. $a_1 = +0.95$. What effect could be observed on the impulse response $g(k)$? Which influence has the changed algebraic sign of the pole with respect to the stability?

Task 6: Filters

In the figure below, you can see the time signal $u(k)$ that has to be filtered. For k values lower than -2 and greater than 10 the signal $u(k)$ is equal to zero. Two possible filters are given. Firstly, the signal $u(k)$ is filtered with the following median filter:

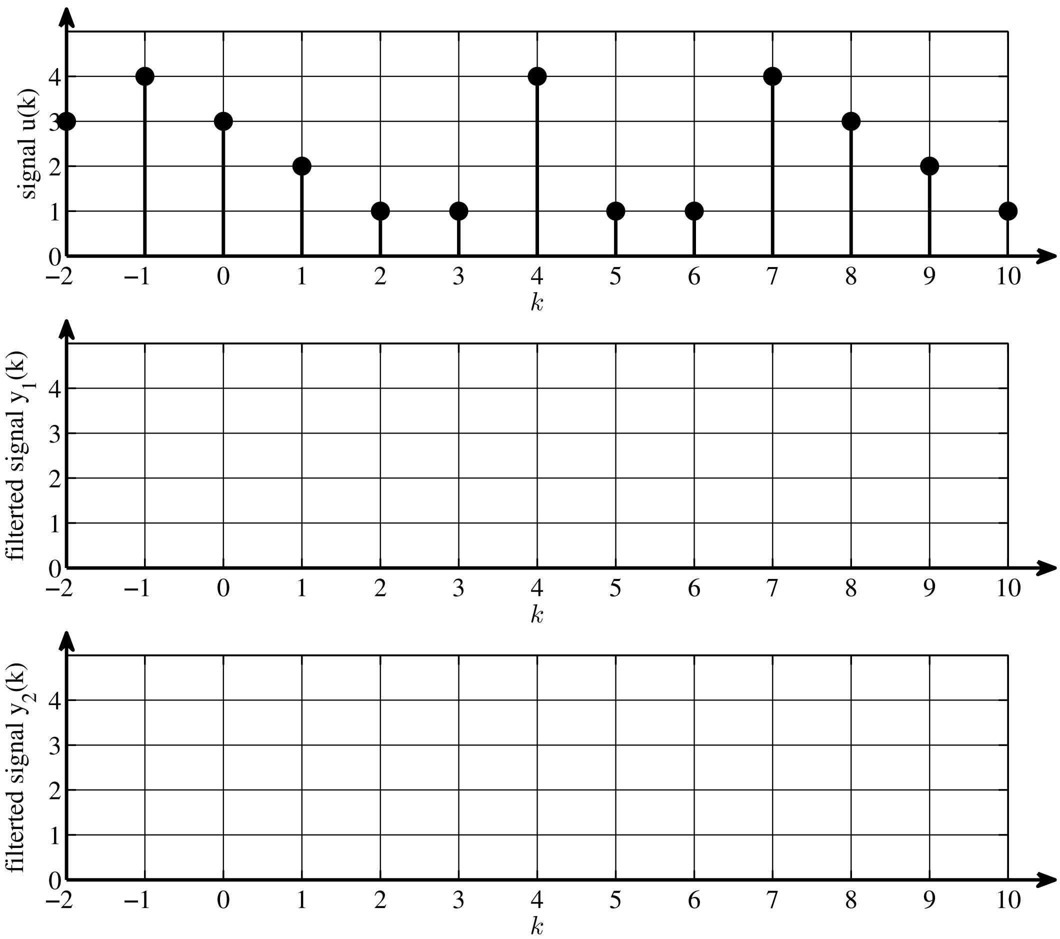
$$y_1(k) \quad = \quad \text{median}\{u(k), u(k-1), u(k-2)\} \ .$$

Secondly, the signal $u(k)$ has to be filtered with a linear average FIR filter of second order.

a) What is the difference equation of the causal linear average FIR filter of second order?

b) Calculate and sketch both, the signal $y_1(k)$ which is filtered with the median filter and the signal $y_2(k)$ which is filtered with the linear average FIR filter. Use the given diagrams for that purpose. Additionally, write down the resulting output sequences in the following form:
$y(k) = \{y(-2), y(-1), y(0), \ldots, y(10)\}$.

c) Explain, if both the median filter and the FIR filter are high-pass or low-pass filters, respectively.



Lösungen:

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$\sum$ 32

Task 2: Measurement of Torque

In the picture above you can see a beam, that is loaded with a torque T on one end. The other end is fixed to a wall and two strain gauges are applied to the beam to measure the torque. The following equations are given:

$$|\rho| = \frac{EI}{|T|} \quad ; \quad \Delta R = R_0 K \epsilon \quad ; \quad \epsilon = \frac{y}{|\rho|}$$

ρ: Curvature of the deflected beam; E: Young's modulus; I: Second moment of area; T: Torque (constant over the beam length); ΔR: The change in resistance of one strain gauge; R_0: The resistance of the strain gauge without any applied torque; K: Sensitivity of the strain gauges; ϵ: Deformation of the strain gauge, h: Beam height; y: Coordinate of the beam height, that starts exactly at the half of the beam height

a)

$$\Delta R = R_0 K \epsilon \tag{1}$$

$$|\rho| = \frac{EI}{|T|} \tag{2}$$

$$\epsilon = \frac{y}{|\rho|} \tag{3}$$

(2) in (3):

$$\epsilon = \frac{y|T|}{EI} \tag{4}$$

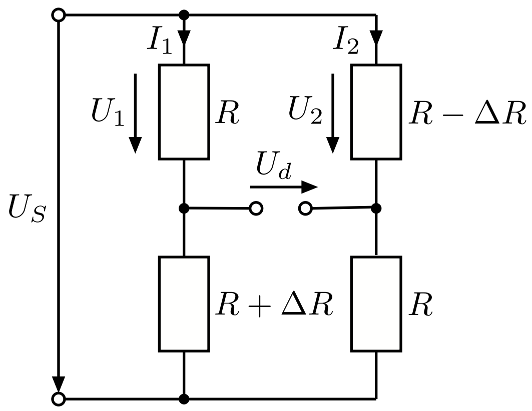
(4) in (1):

$$\Delta R = \frac{R_0 K y |T|}{EI} \tag{5}$$

$\boxed{5}$

Upper strain gauge ($y = +\frac{h}{2}$): $\Delta R_U = \frac{R_0 K h |T|}{2EI} \hat{=} \Delta R$
Lpper strain gauge ($y = -\frac{h}{2}$): $\Delta R_L = -\frac{R_0 K h |T|}{2EI} \hat{=} -\Delta R$

Relationship: $\Delta R_U = -\Delta R_L$ $\boxed{2}$



b) $\boxed{6}$

$$U_1 - U_2 + U_d = 0 \tag{6}$$
$$U_1 = RI_1 \tag{7}$$
$$U_2 = (R - \Delta R)I_2 \tag{8}$$
$$I_1 = \frac{U_S}{2R + \Delta R} \tag{9}$$
$$I_2 = \frac{U_S}{2R - \Delta R} \tag{10}$$

(9) in (7) and (10) in (8) :

$$U_1 = \frac{RU_S}{2R + \Delta R} \tag{11}$$
$$U_2 = \frac{(R - \Delta R)U_S}{2R - \Delta R} \tag{12}$$

(11) and (12) in (6):
(...)
$U_d = -\frac{\Delta R^2}{4R^2 - \Delta R^2} U_S$ $\boxed{4}$

c) If two additional strain gauges are applied to the beam, they can be used to realize a full bridge circuit, which leads to an increase of the sensitivity by nearly a factor of two. $\boxed{2}$

d) The first one (i) amplifies the voltage.

$$U_{in} = \frac{R_2}{R_1 + R_2} U_{out}$$
$$\Leftrightarrow U_{out} = (\frac{R_1}{R_2} + 1)U_{in}$$

$\boxed{4}$

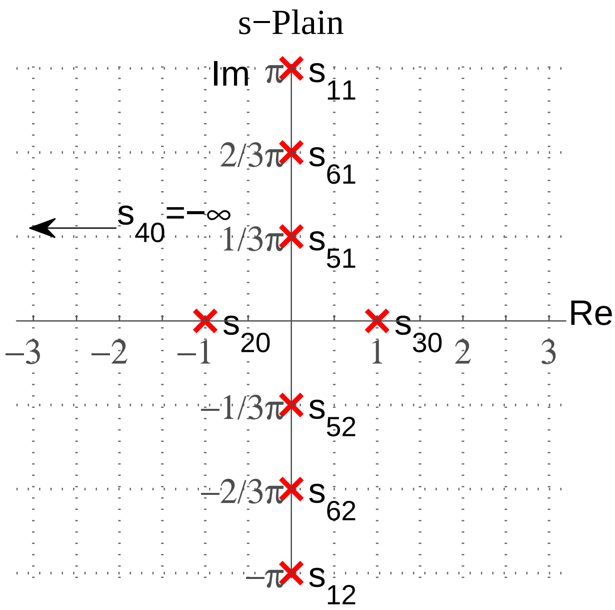
The second one (ii) is an integrator circuit.

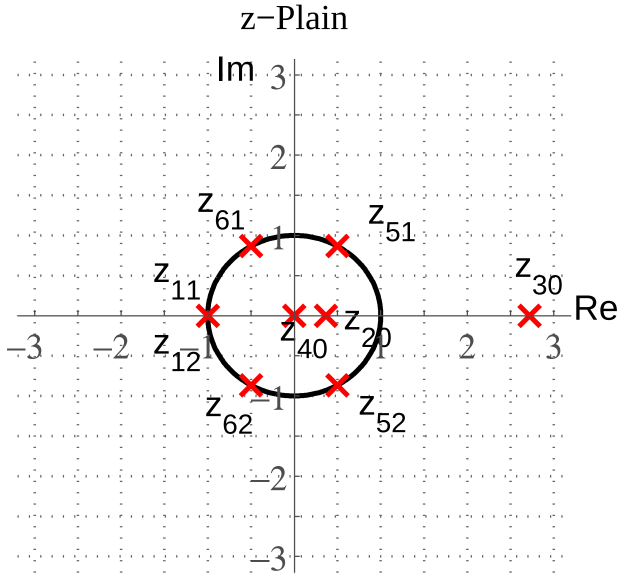
e) $R_1 = 3 \cdot R_2$ (see equation above). $\boxed{1}$

$\boxed{\sum 24}$

Task 3: Relation Between s-Plane and z-Plane

With $T_0 = 1$: $z = e^s$; $s = ln(z)$
$z_{11} = e^{0+i\pi} = e^0 \cdot e^{i\pi} = e^0[cos(\pi) + isin(\pi)] = 1 \cdot -1 = -1$ 1
$z_{12} - 1$ 1
$z_{20} = e^{-1} \approx 0.37$ 1
$z_{30} = e^1 \approx 2.72$ 1
$z_{40} = e^{-\infty} = 0$ 2
$s_{51} = ln(1 \cdot e^{i60°}) = ln(1 \cdot e^{i\frac{1}{3}\pi}) = 0 + i\frac{1}{3}\pi$ 2
$s_{52} = 0 - i\frac{1}{3}\pi$ 2
$s_{61} = 0 + i\frac{2}{3}\pi$ 2
$s_{62} = 0 - i\frac{2}{3}\pi$ 2





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$\sum 16$

Task 4: **Abtasttheorem**

The following periodic signal is given: $y(t) = \sin(2\pi \cdot 10\text{Hz} \cdot t)$.

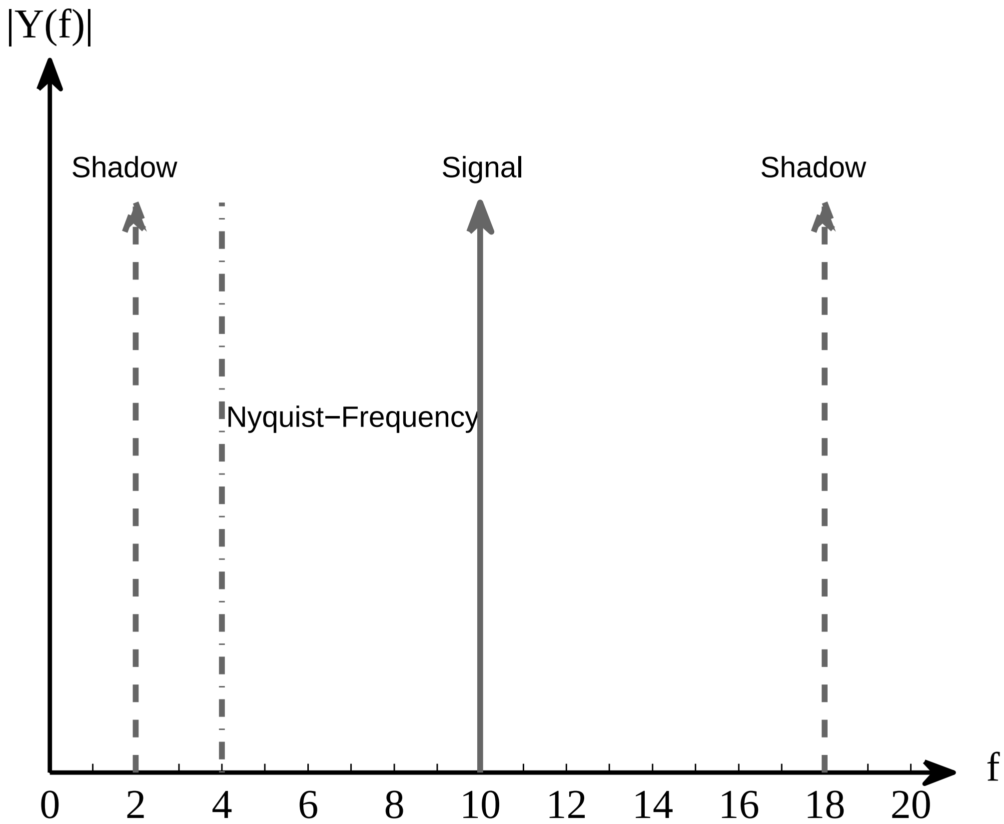
a) See figure below. $\boxed{1}$

b) Aliasing occurs, if the highest frequency contained in the signal is greater than or equal the half of the sampling frequency. Here the highest frequency, that is contained in the signal is 10Hz, which is greater than the half of the sampling frequency $\frac{f_0}{2} =$ 4Hz. $\boxed{1}$

c) Shannon-frequency $= \frac{f_0}{2} = 4$Hz (see figure below). $\boxed{1}$

d) See figure below. $\boxed{1}$

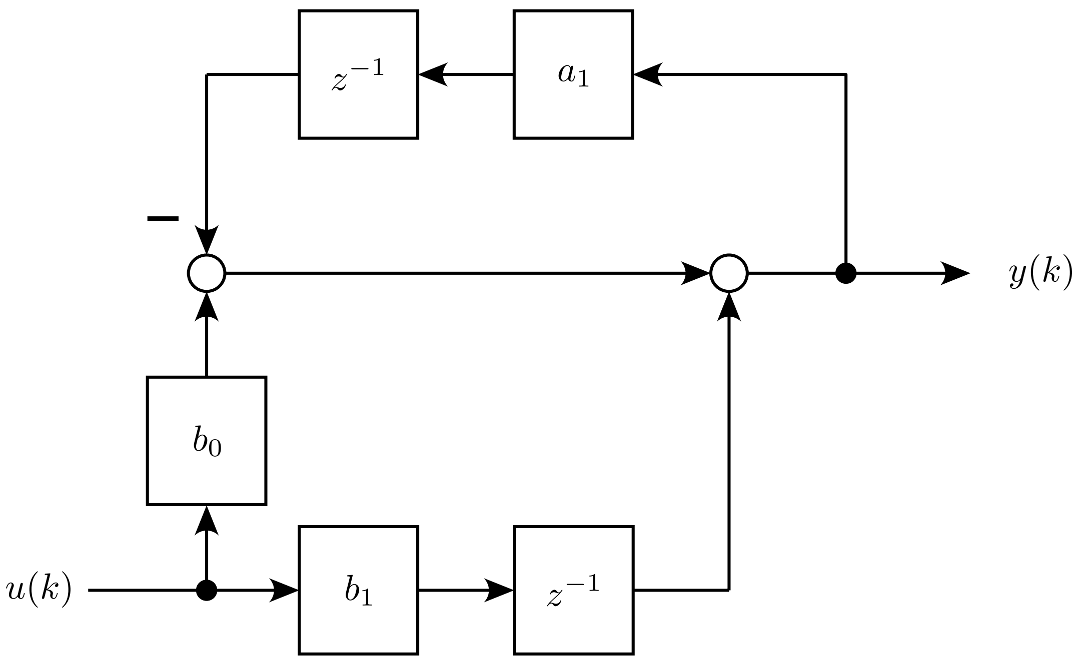
e) Here the sampled signal would have a frequency of 2Hz. $\boxed{2}$



$\boxed{\sum 6}$

Task 5: Time-Discrete Systems

a) The correct block diagram is:



$\boxed{5}$

b) To have a direct throughput from the input to the output the coefficient b_0 must not be zero. Therefore the output reacts immediately to changes of the input. $\boxed{1}$

c) To have a finite impulse response, the output of the system must be independent from further outputs. This means that the coefficient a_1 need to be zero. Therefore the output $y(k)$ must not be calculated recursively. $\boxed{1}$

d) The impulse response $g(k)$ is $y(k)$ for $u(k) = \delta_K(k)$. Calculating the first steps leads to the impulse respond at the k-th time step.

$$
\begin{aligned}
k = 0 &: g(0) = b_0 \cdot 1 + b_1 \cdot 0 + (-a_1) \cdot 0 = b_0 \\
k = 1 &: g(1) = b_0 \cdot 0 + b_1 \cdot 1 + (-a_1) \cdot b_0 = b_1 - a_1 \cdot b_0 \\
k = 2 &: g(2) = b_0 \cdot 0 + b_1 \cdot 0 + (-a_1) \cdot (b_1 - a_1 \cdot b_0) = (-a_1) \cdot b_1 + (-a_1)^2 \cdot b_0 \\
k = 3 &: g(3) = 0 + 0 + (-a_1) \cdot ((-a_1) \cdot b_1 + (-a_1)^2 \cdot b_0) = (-a_1)^2 \cdot b_1 + (-a_1)^3 \cdot b_0
\end{aligned}
$$

In general case k :

$$
g(k) = (-a_1)^{k-1} \cdot b_1 + (-a_1)^k \cdot b_0 = (-a_1)^{k-1} \cdot (b_1 - a_1 \cdot b_0)\ .
$$

$\boxed{6}$

e) The transferfunction $G(z)$ for the given difference equation is:

$$y(k) = b_0 u(k) + b_1 u(k-1) - a_1 y(k-1)$$

$$\circ\!\!-\!\!\bullet$$

$$Y(z) = b_0 \cdot U(z) + b_1 \cdot U(z) \cdot z^{-1} - a_1 \cdot Y(z) \cdot z^{-1}$$
$$Y(z) \cdot (1 + a_1) = U(z) \cdot \left(b_0 + b_1 \cdot z^{-1}\right)$$
$$G(z) = \frac{Y(z)}{U(z)} = \frac{b_0 + b_1 \cdot z^{-1}}{1 + a_1 \cdot z^{-1}}$$ $\boxed{3}$

f) The step respond in the z-domain is calculated by multiplication of $G(z)$ with the z-transformation of the unit step:

$$H(z) = G(z)\frac{1}{1 - z^{-1}} = \frac{b_0 + b_1\, z^{-1}}{(1 + a_1\, z^{-1})(1 - z^{-1})}$$ $\boxed{2}$

g) Final value:

$$h(k \to \infty) = \lim_{z \to 1}(z-1)H(z) = \lim_{z \to 1} G(z) = \frac{b_0 + b_1}{1 + a_1}\,.$$

Initial value:

$$h(k \to 0) = \lim_{z \to \infty} H(z) = b_0\,.$$ $\boxed{2}$

(Therefore direct throughput, if $b_0 \neq 0$!)

h) With the given coefficients the step respond ($u_k = \sigma(k)$) for $k = 0$ is:

$$k = 0 : h(0) = b_0 = 0{,}3\,.$$ $\boxed{1}$

For $k > 0$ the step respond is (in general):

$$\begin{aligned}
k = 1 : h(1) &= b_0 \cdot 1 + b_1 \cdot 1 + (-a_1) \cdot b_0 = b_1 + b_0 \cdot (1 - a_1) \\
k = 2 : h(2) &= b_0 + b_1 + (-a_1) \cdot h(1) = b_0 + b_1 + (-a_1) \cdot (b_1 + b_0 \cdot (1 - a_1)) \\
&= b_0 \cdot \left(1 + (-a_1) + (-a_1^2)\right) + b_1 \cdot (1 - a_1) \\
k = 3 : h(3) &= b_0 + b_1 + (-a_1) \cdot h(2) \\
&= b_0 + b_1 + (-a_1) \cdot \left(b_0 \cdot \left(1 + (-a_1) + (-a_1^2)\right) + b_1 \cdot (1 - a_1)\right) \\
&= b_0 \cdot \left(1 + (-a_1) + (-a_1)^2 + (-a_1)^3\right) + b_1 \cdot \left(1 + (-a_1) + (-a_1)^2\right) \\
h(k) &= b_0 \cdot \sum_{i=0}^{k}(-a_1)^i + b_1 \cdot \sum_{j=0}^{k-1}(-a_1)^j \\
&= b_0 \cdot \frac{1 - (-a_1)^{k+1}}{1 - (-a_1)} + b_1 \cdot \frac{1 - (-a_1)^k}{1 - (-a_1)}
\end{aligned}$$

Using the given values $b_0 = 0{,}3, b_1 = 0$ and $a_1 = -0{,}9$ the conclusion is:

$$h(k) = 0{,}3 \cdot \sum_{i=1}^{k}(0{,}9)^i = 0{,}3 \cdot \frac{1 - 0{,}9^{k+1}}{1 - 0{,}9} = 3 \cdot (1 - 0{,}9^{k+1})$$ $\boxed{6}$

i) If a_1 were equal to $+0{,}9$, the impulse respond would be alternating. The stability properties do not change. $\boxed{1}$

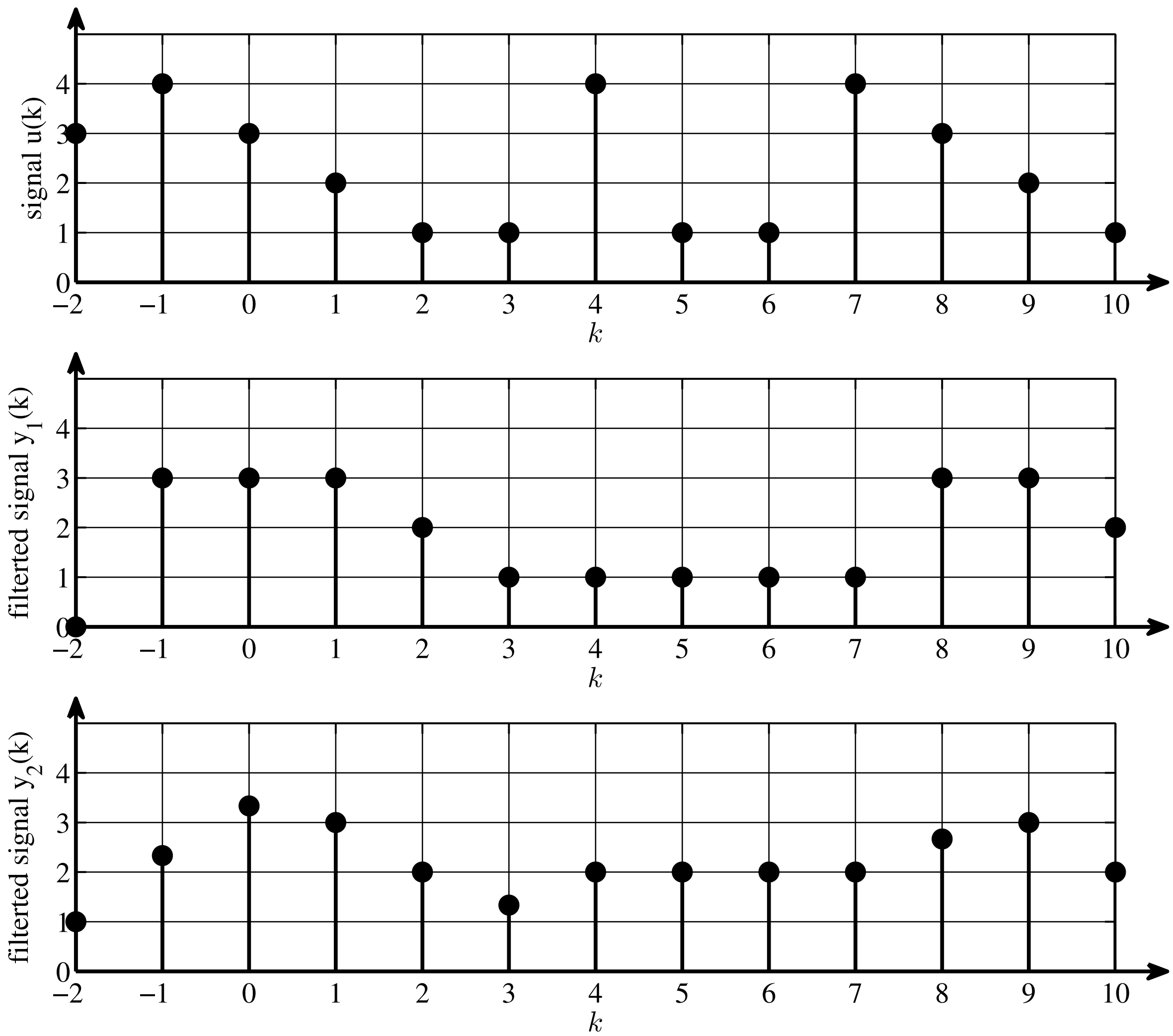
$\boxed{\sum 28}$

Task 6: Filter

a) $y_2(k) = \frac{1}{3}\left(u(k) + u(k-1) + u(k-2)\right)$. 2

b) See figure below. 6
$y_1(k) = \{0; 3; 3; 3; 2; 1; 1; 1; 1; 1; 3; 3; 2\}$ 2
$y_2(k) = \{1; \frac{7}{3}; \frac{10}{3}; 3; 2; \frac{4}{3}; 2; 2; 2; 2; \frac{8}{3}; 3; 2\}$ 2

c) Both, the median filter and the FIR filter are low-pass filters. 2



$\sum 14$