Sensorics Exam

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Universität Siegen

3rd of March 2020

Name:	
Mat.-No.:	
Grade:	

Task	T1	T2	T3	T4	T5	T6	T7	Sum
Score:	18	15	17	13	23	15	19	120
Accomplished:								

Exam duration: 2 hours

Permitted Tools: Pocket Calculator and 4 pages of formula collection

Task 1: Comprehension Questions (18 Points))

Mark the correct answers clearly.
Every question has one, two or three correct answers!
For every correctly marked answer you will get one point. For every wrong answer a point will be subtracted, but a question will never be rated with negative points.

a) Which statements are true regarding Clustering?

- ☐ Clustering is usually a supervised learning method.
- ☐ Clustering minimizes the total (weighted) distances of all data points to their associated cluster centers.
- ☐ The number of clusters has not to be known in advance, but is determined through the clustering algorithm.
- ☐ K-means clustering weights distances equally in all directions.

b) A system $G(z)$ in the z-domain is always …

- ☐ …stable if all poles have a negative imaginary part.
- ☐ …unstable if at least one pole is outside the unit circle.
- ☐ …stable if it is calculated from a stable G(s) by using bilinear transformation.
- ☐ …unstable if poles are complex conjugate.

c) A Confidence Intervall …

- ☐ … is the time interval after which a measuring device must be recalibrated.
- ☐ … does not depend on the probability distribution of the value to be estimated.
- ☐ … can only be calculated for the probabilities 95% or 99%.
- ☐ … is the span around the mean value of an estimate in which the estimate lies for a given probability of interest.

d) Assess following statements regarding errors:

- ☐ The quantization error is half as large if a 16 bit A/D converter is used instead of an 8 bit converter.
- ☐ The absolute error of a measurement is the difference between the measured value and the true value, divided by the true value.
- ☐ Stochastic errors can be reduced by averaging over several measurements.
- ☐ Random errors are typically reduced with $1/\sqrt{N}$, where N is the number of measurements.

e) What is the meaning of the terms *dead zero* and *life zero*?

- ☐ Live zero is a measurement technique which allows detection of a broken wire.
- ☐ Live zero means a measurement device is broken and has to be replaced.
- ☐ If a measurement of a signal $\neq 0$ has a value $= 0$, it is called a dead zero.
- ☐ If a measurement of a signal $= 0$ has a value $= 0$, it is called a dead zero.

.

f) Which statements are true with regard to a Flash Converter (parallel principle)?

- ☐ This converter is a D/A converter.
- ☐ This converter is very fast A/D converter, usually with a low resolution.
- ☐ This converter compares the measured voltage directly to a large number of reference values.
- ☐ This converter uses feedback und works similar to an integrative controller.

g) Explain apparent, active and reactive power.

- ☐ Apparent power is the power an electrical device (e.g. a motor) actually delivers.
- ☐ Active and reactive power depend on the phase shift φ between voltage and current.
- ☐ Active power is the power lost due to phase shifting and should usually be avoided.
- ☐ Reactive power is calculated from apparent power P_S as follows: $P_B = P_S \cdot \cos(\varphi)$.

h) Operational amplifiers ...

- ☐ ... are passive components (need no external energy source).
- ☐ ... are amplifiers with a very low gain.
- ☐ ... have a very high **output** resistance.
- ☐ ... have a very high **input** resistance.

i) The Singular Value Decomposition (SVD) ...

- ☐ ... computes the following matrix decomposition $\underline{U} = \underline{W} \cdot \underline{S} \cdot \underline{V}^T$, where $\underline{S}$ and $\underline{V}$ are always square matrices and $\underline{S}$ contains the singular values.
- ☐ ... can only decompose a square $(n \times n)$ matrix.
- ☐ ... is used e.g. for Principal Component Analysis (PCA).
- ☐ ... of $\underline{U}$ calculates singular values which are identical to the eigenvalues of $\underline{U}$.

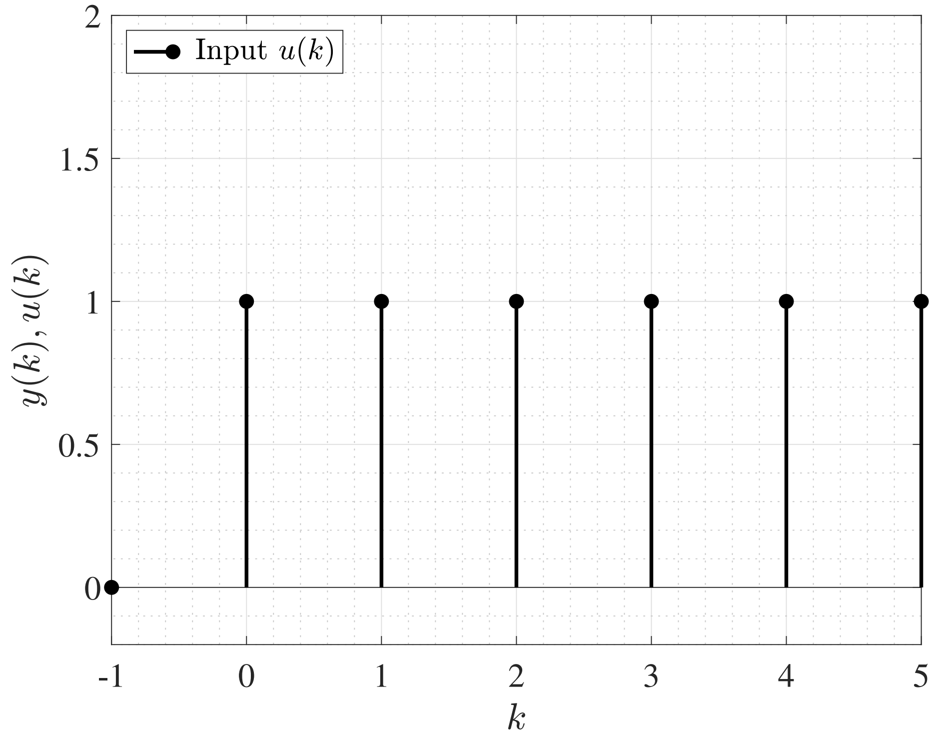
j) What is a principal component analysis (PCA)?

- ☐ PCA is a unsupervised learning method.
- ☐ PCA is often used for data preprocessing.
- ☐ PCA is used for feature **selection**, meaning some of the inputs are completely discarded from further use.
- ☐ PCA is used for feature **extraction**, meaning all original inputs may still be necessary to calculate a lower number of features for the next processing step.

Task 2: Time-Discrete Systems (15 Points)

A system with output $y(k) = b_0 \cdot u(k) - a_1 \cdot y(k-2)$ is given with $a_1 = -1$ and $b_0 = 0.3$.

a) Calculate the output of the system with the given input signal for $k = 0, 1, ..., 5$. Sketch the corresponding signal in the given figure. Use $y(k) = 0$ for all $k < 0$ as the initial condition.



b) What is the global behavior of the system ($P, PI, PD, PT_1, I, D, \ldots$)?

c) Sketch the block diagram of the given system.

d) Determine the poles of the given system.

e) Calculate the final value of the *impulse* response.

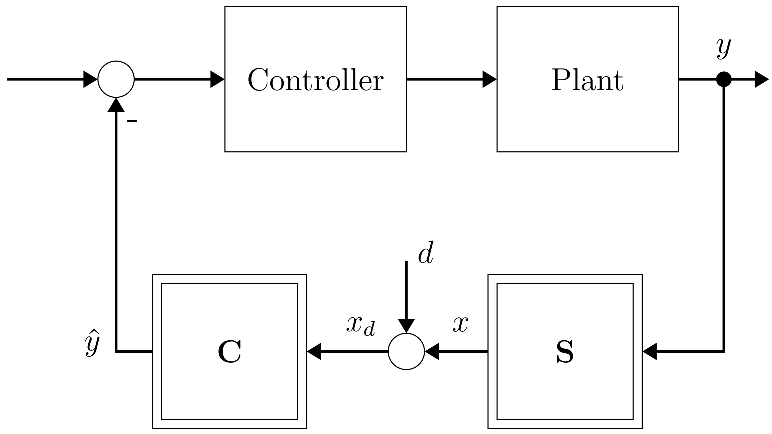
f) Which of the following statements is correct?

- ☐ The system is stable.
- ☐ The system is unstable.
- ☐ The system is marginally stable.

g) How can you modify the given system to meet the two non-checked statements in task f)?

Task 3: Compensator (17 Points)

In order to control a plant, the output of the system has to be measured. For many sensors (e.g. Thermocouples) the measurement of the output results in a voltage value. From this voltage value, the to be measured value can be calculated.
In our example we have a sensor in the feedback loop which measures the output y of our plant. The output of the sensor S is x. This value can suffer from additional disturbance d. x_d is then fed into the compensator C. The Compenstor C does not compensate the added noise d it just reconstructs y by $\hat{y}$ from x_d with the inverse characteristic from S. So with $d = 0$ the statement $\hat{y} = y$ is valid.



The equation of the sensor is

$$f_S(y) = \sqrt{e^y - 1} = x$$

and it is only operating in the positive range $y > 0$.

a) Find the equation for the Compensator $f_C(x_d) = \hat{y}$.
Hint: Leave the disturbance d out of the equation.

b) Find the sensitivity of the Compensator $\frac{d\hat{y}}{dx_d}$.

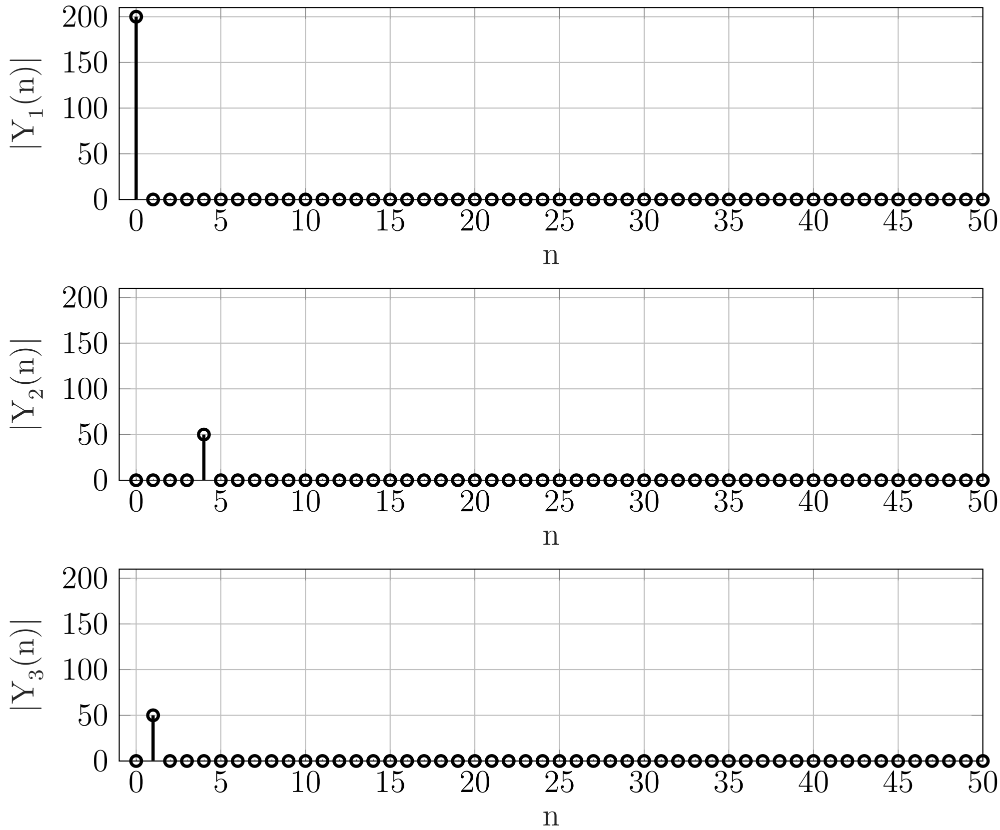
c) At which value of $x_d = x_{set}$ the sensitivity is the highest? Provide your answer with a calculation or explanation.

d) We now have systematic disturbance of $d = 0.1$. What are the values for $\hat{y}$, x_d and y if $x = x_{set}$?

e) What is the relative error e_{rel} and what is the absolute error e_{abs} with the disturbance $d = 1$ if $x = x_{set}$ in your measurement of y?

Task 4: **DFT (13 Points)**

In the following you can see the DFT results (left half of the spectrum) for different signals.
Hint: $|Y(n)| = \frac{N}{2} \cdot A$ *if* A *is the Amplitude of an oscillation.*
At $\omega = 0$ *holds:* $|Y(n)| = N \cdot A$.

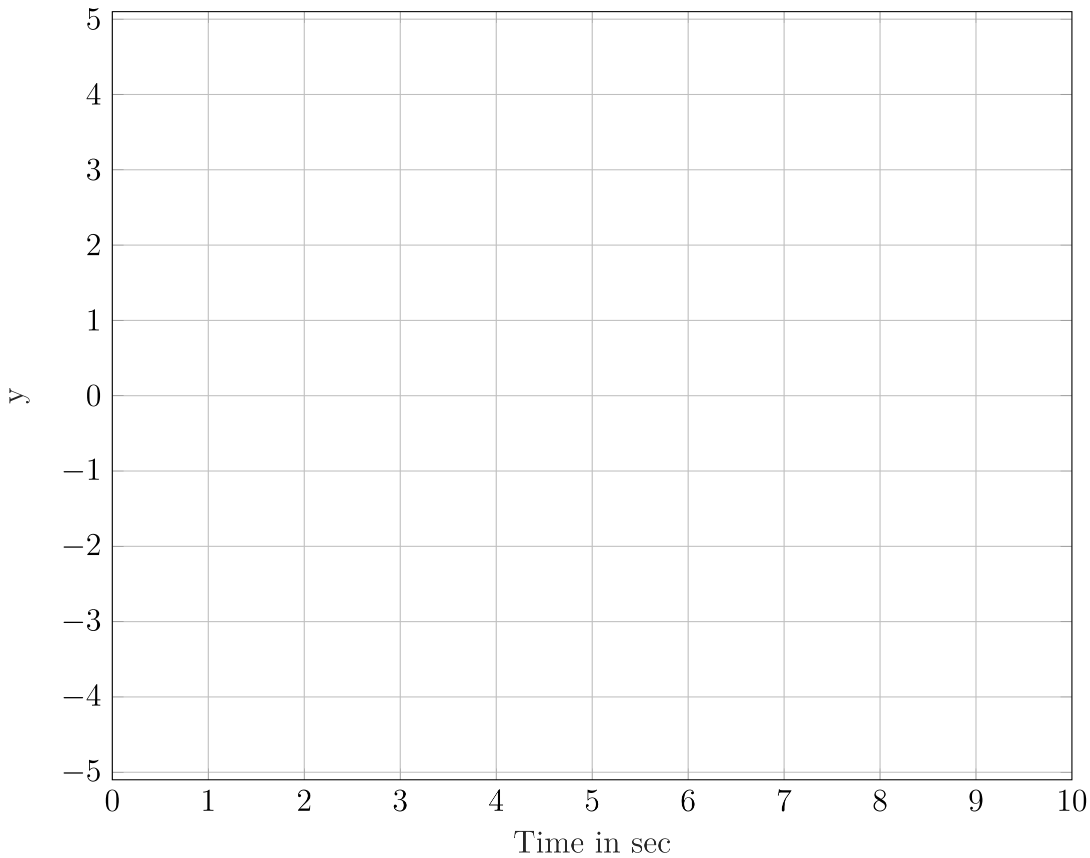


The signal were recorded with a sampling time of 0.1 sec in the time frame from 0 to 9.9 sec (first sample at t=0 sec).

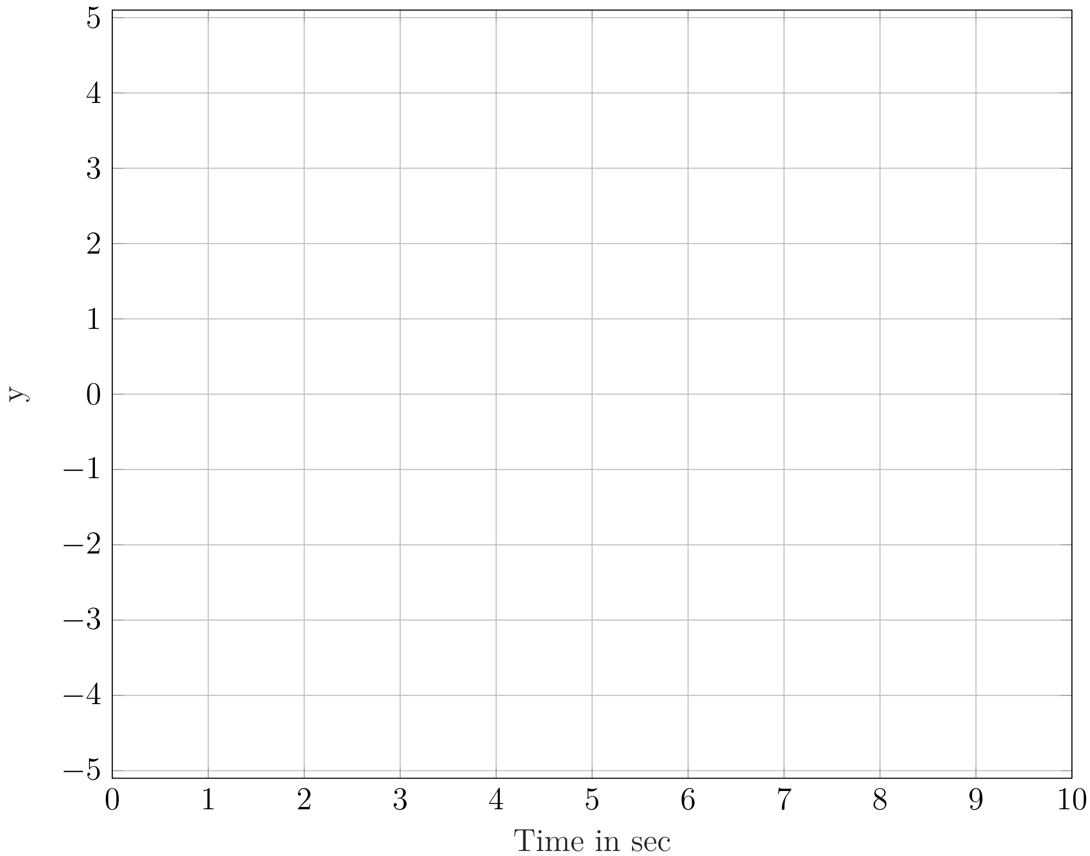
a) What is the sampling frequency f_0?

b) Determine N when N is the number of samples per signal.

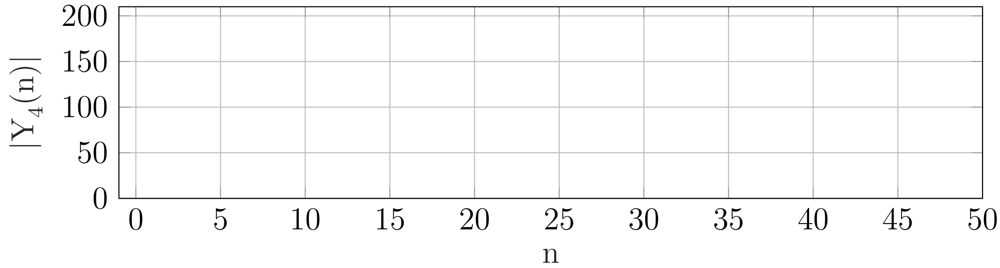
c) Draw the signals $y_1(t)$, $y_2(t)$ and $y_3(t)$ according to the shown DFT plots in the empty diagram. Make sure that one can determine which line represents which signal.



d) Draw $y_4(t) = y_1(t) + y_2(t) + y_3(t)$ into the following empty diagram.



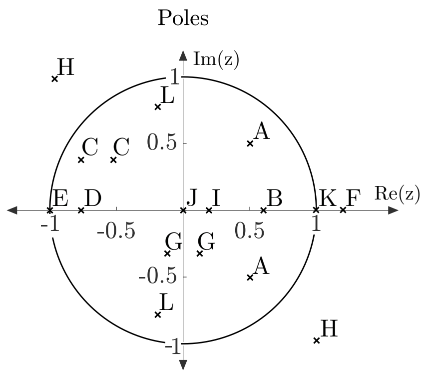
e) Plot the output of the DFT $|Y_4(n)|$ of your previously drawn signal $y_4(t)$ into the following empty diagram.



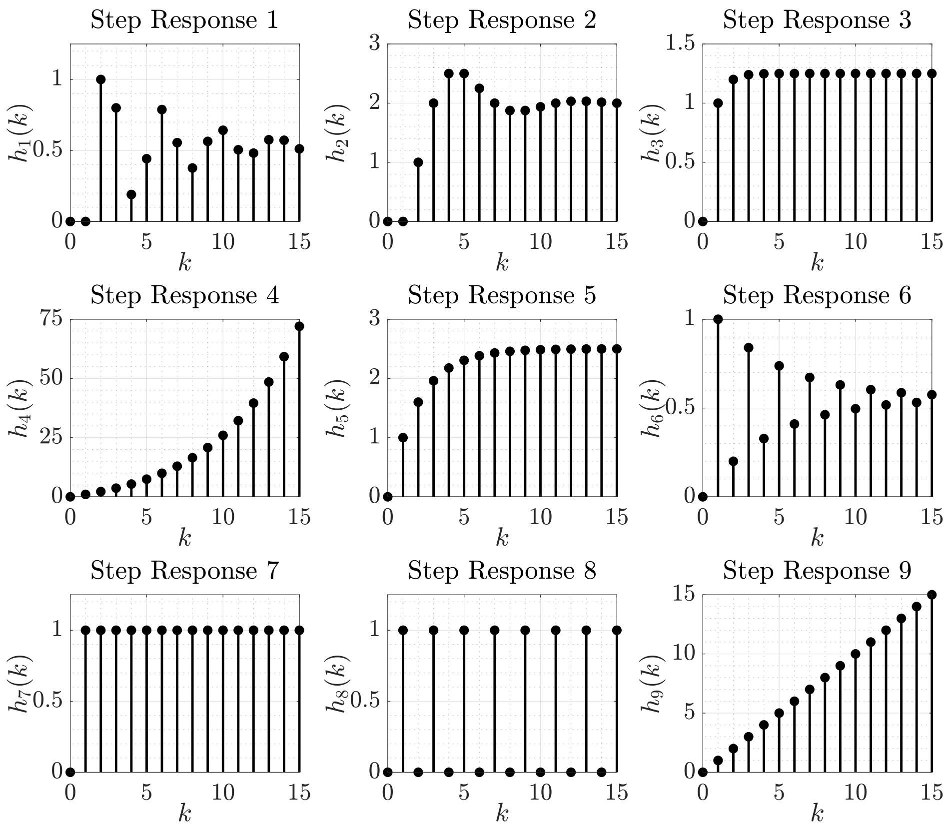
Task 5: Step Responses (23 Punkte)

Given are the pole locations for 12 different systems (A-L) in the pole zero map and 9 different step responses (1-9).

a) Match the pole locations to the step responses. Note down your answers in the given table. (Note: Three pole locations do not belong to any step response. All transfer functions have the structure $G(z) = \frac{1}{A(z)}$ with $A(z) = \prod_{i=1}^{n}(z - p_i)$.)

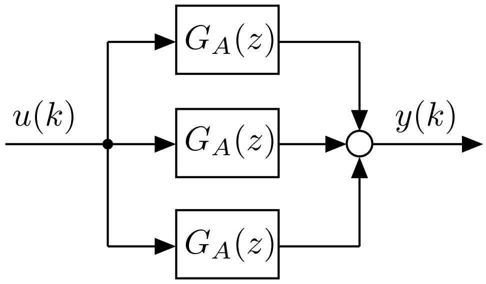


Step Response	Poles
1	
2	
3	
4	
5	
6	
7	
8	
9	



b) Assume that the poles of system A are at $p_1 = 0.5 + 0.5i$ and $p_2 = 0.5 - 0.5i$. Calculate the systems transfer function $G_A(z)$ and apply the final value theorem to the systems response with input signal $u(k) = 3 \cdot \sigma(k)$.

c) System $G_A(z)$ is now expanded to system $G_{AAA}(z)$ (see block diagram below), which is $G_A(z)$ three times parallel-connected. What is the gain of $G_{AAA}(z)$?



Task 6: Quantization (15 Points)

Two different AD-converters are available for a quantization task. Both are rounding the signal values *down* to the next quantization step. The parameters of the converters are as follows:

	10-bit AD-converter		**12-bit AD-converter**	
sampling frequency:	$f_0 = 100$	Hz	$f_0 = 100$	Hz
resolution:	10	bit	12	bit
measurement range:	$U_{\text{range}} = 0 \dots 1024$	mV	$U_{\text{range}} = 0 \dots 1024$	mV

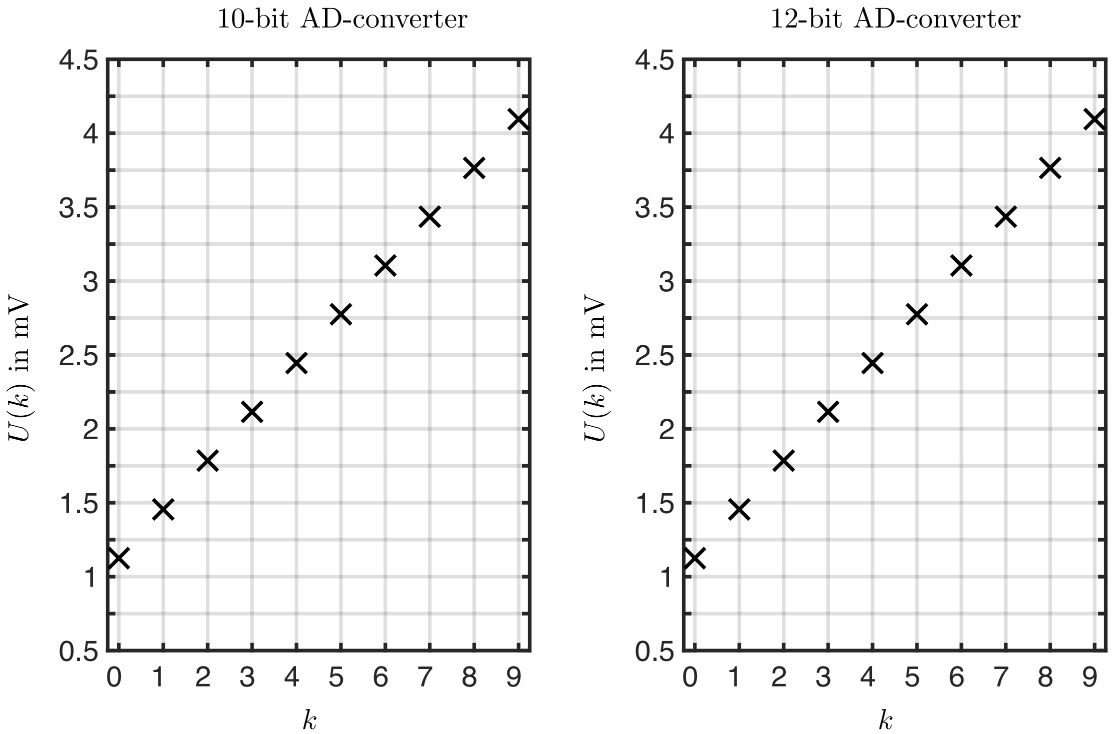
a) Calculate the step size ΔU and the maximal quantization error $e_{\text{Q max}}$ of both AD-converters.

b) How do you call the error signal ($U(k) - U_\text{Q}(k)$, $k = 1, \dots, N$) resulting from the quantization error?

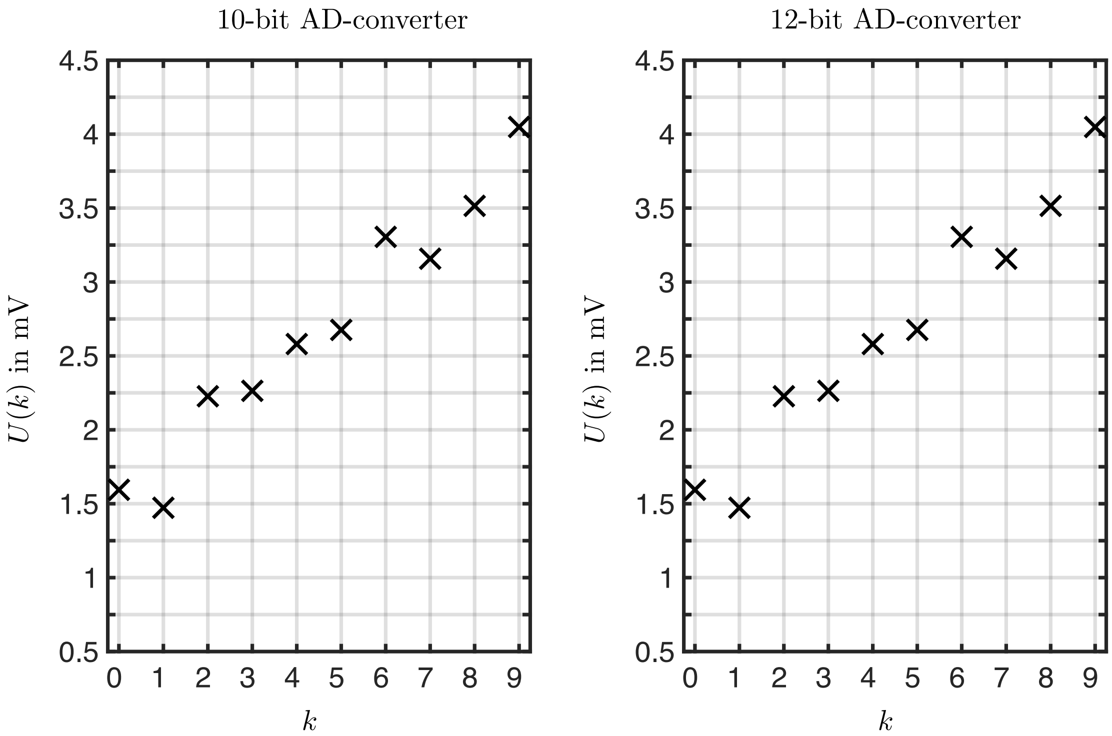
c) What type of error occurs from rounding the values down? Can this error be corrected and if so, how?

d) Assuming uniformly distributed signal, which has to be quantized. What is the probability distribution of the quantization error?

e) The values of an *ideal* ramp $U(k)$ shall be quantized using both AD-converters. Sketch the resulting signal $U_\text{Q}(k)$ in the given diagram.



f) The values of a *noisy* ramp $U(k)$ shall be quantized using both AD-converters. Sketch the resulting signal $U_\text{Q}(k)$ in the given diagram.



g) The quantized signal of the 12-bit AD-converter is closer to the real values. Therefore, the 12-bit AD-converter reconstructs the noise even better. State one way to get rid of the high frequency noise.

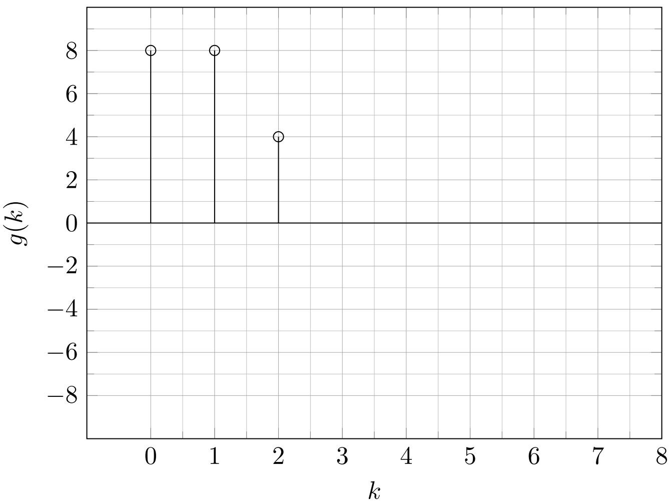
Task 7: Linear Filter (19 Points)

a) The linear transfer function of a filter

$$G_1(z) = \frac{b_0 z + b_1}{z + a_1} \tag{1}$$

is given. Calculate the corresponding difference equation.

b) From the part of the impulse response shown in the figure the coefficients b_0, b_1 and a_1 should be calculated.



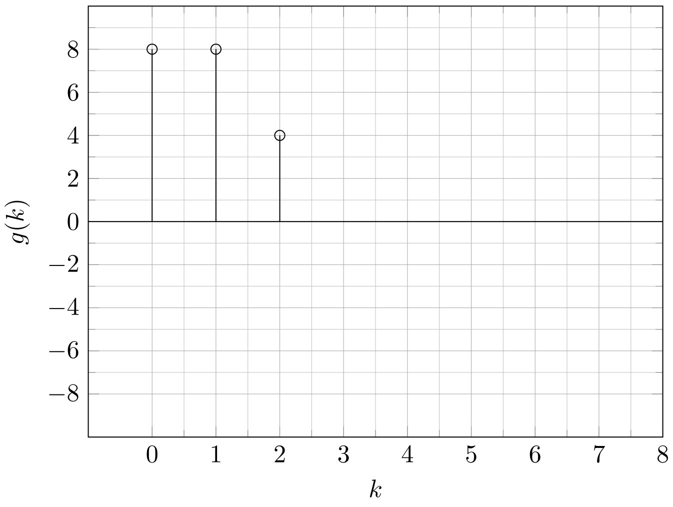
c) Draw the next 4 values (time steps $k = 3, \ldots, 6$) of the impulse response of $G_1(z)$ into the diagram.

d) Now, let the transfer function

$$G_2(z) = \frac{b_0 z^2}{z^2 + a_1 z + a_2} \tag{2}$$

be given. Calculate the corresponding difference equation.

e) Calculate from the shown part of the impulse response the coefficients b_0, a_1 and a_2 of $G_2(z)$.

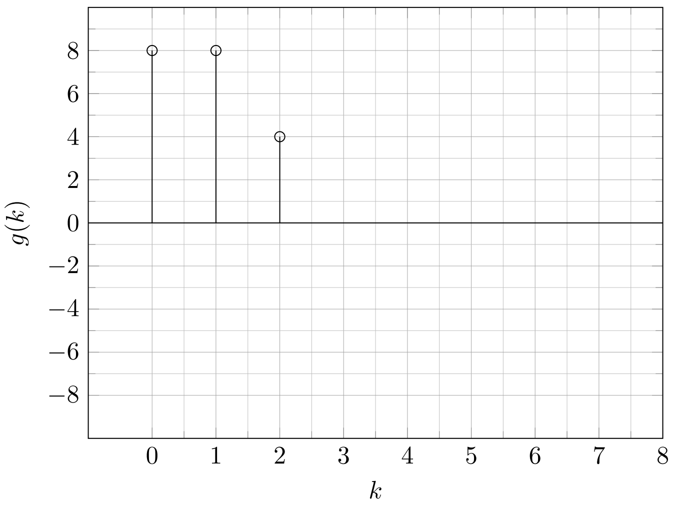


f) Draw the next 6 values of the impulse response (time steps $k = 3\ldots, 8$) of the impulse response of $G_2(z)$ into the diagram.

g) Finally, the transfer function

$$G_3(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2} \tag{3}$$

is given. Derive the coefficients b_0, b_1 and b_2 from the values provided in the diagram and plot the next 4 values (time step $k = 3, \ldots, 6$).



Solution:

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d) Assess following statements regarding errors:

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e) What is the meaning of the terms *dead zero* and *life zero*?

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☐ Live zero means a measurement device is broken and has to be replaced.

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f) Which statements are true with regard to a Flash Converter (parallel principle)?

☐ This converter is a D/A converter.

■ This converter is very fast A/D converter, usually with a low resolution.

■ This converter compares the measured voltage directly to a large number of reference values.

☐ This converter uses feedback und works similar to an integrative controller.

g) Explain apparent, active and reactive power.

☐ Apparent power is the power an electrical device (e.g. a motor) actually delivers.

■ Active and reactive power depend on the phase shift φ between voltage and current.

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h) Operational amplifiers …

☐ … are passive components (need no external energy source).

☐ … are amplifiers with a very low gain.

☐ … have a very high **output** resistance.

■ … have a very high **input** resistance.

i) The Singular Value Decomposition (SVD) …

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j) What is a principal component analysis (PCA)?

■ PCA is a unsupervised learning method.

■ PCA is often used for data preprocessing.

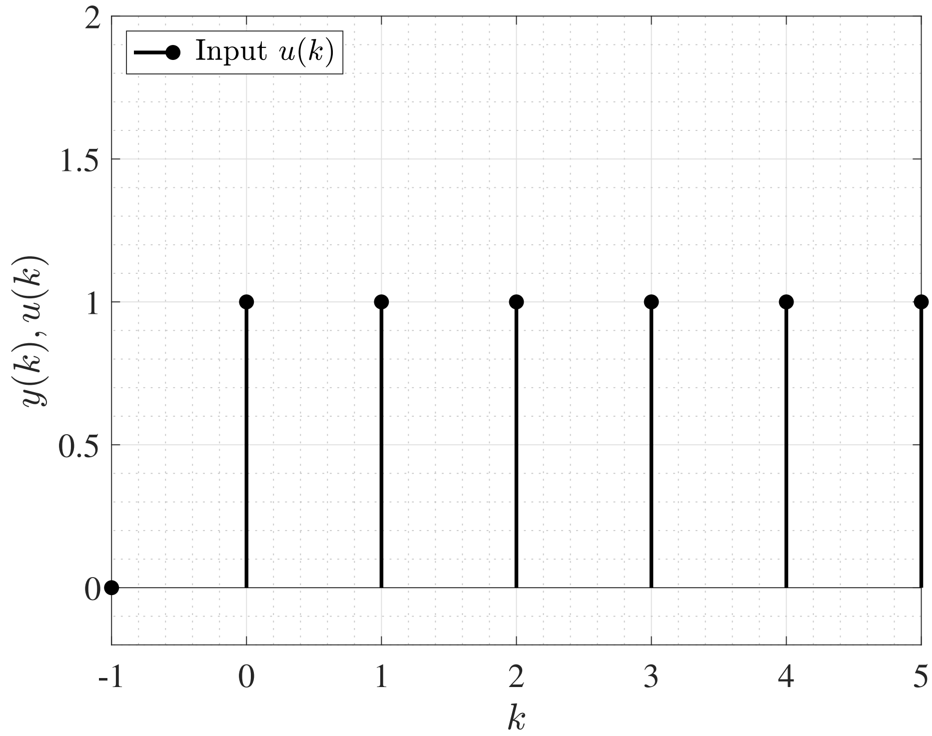
☐ PCA is used for feature **selection**, meaning some of the inputs are completely discarded from further use.

■ PCA is used for feature **extraction**, meaning all original inputs may still be necessary to calculate a lower number of features for the next processing step.

Task 2: Time-Discrete Systems (15 Points)

A system with output $y(k) = b_0 \cdot u(k) - a_1 \cdot y(k-2)$ is given with $a_1 = -1$ and $b_0 = 0.3$.

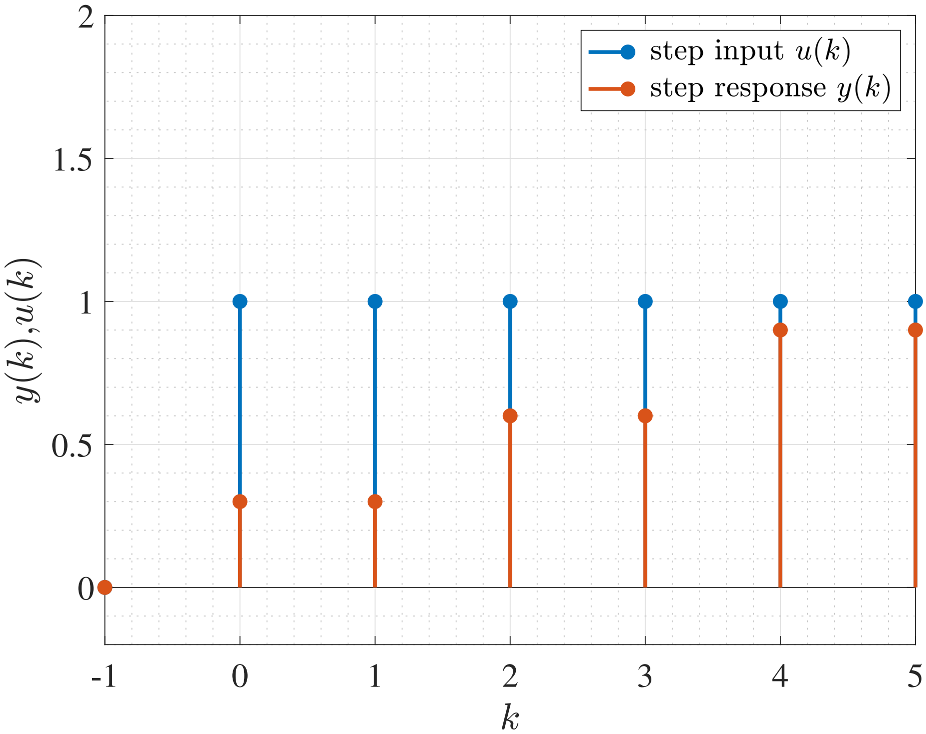
a) Calculate the output of the system with the given input signal for $k = 0, 1, ..., 5$. Sketch the corresponding signal in the given figure. Use $y(k) = 0$ for all $k < 0$ as the initial condition. **Answer**: The given input signal is



$$u(k) = \begin{cases} 1, & \text{for } k \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

Calculation of the values of $y(k)$ for $k = 0, 1, ..., 5$: $\boxed{3}$

$$y(k=0) = b_0 \cdot u(0) - a_1 \cdot y(0-2) = 0.3 \cdot 1 - (-1) \cdot 0 = 0.3$$
$$y(k=1) = 0.3 \cdot 1 - (-1) \cdot 0 = 0.3$$
$$y(k=2) = 0.3 \cdot 1 - (-1) \cdot 0.3 = 0.6$$
$$y(k=3) = 0.3 \cdot 1 - (-1) \cdot 0.3 = 0.6$$
$$y(k=4) = 0.3 \cdot 1 - (-1) \cdot 0.6 = 0.9$$
$$y(k=5) = 0.3 \cdot 1 - (-1) \cdot 0.6 = 0.9$$



1

b) What is the global behavior of the system ($P, PI, PD, PT_1, I, D, \ldots$)?

Answer: The system is an integrator (global I-behavior). 1

c) Sketch the block diagram of the given system.

Answer:

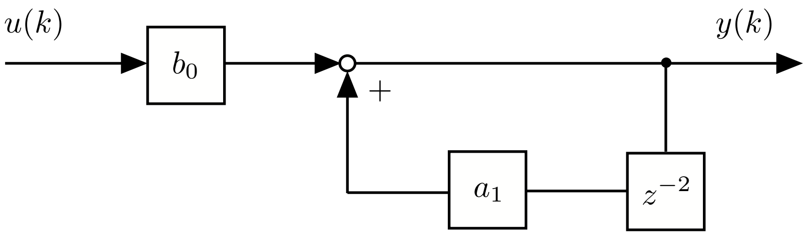


Figure 1: Block diagram of the system.

2

d) Determine the poles of the given system. **Answer**:

$$y(k) + a_1 \cdot y(k-1) = b_0 \cdot u(k)$$
$$Y(z) + a_1 \cdot Y(z) \cdot z^{-2} = b_0 \cdot U(z)$$
$$G(z) = \frac{b_0}{1 + a_1 \cdot z^{-2}}$$
$$p_1 = 1$$
$$p_2 = -1$$

3

e) Calculate the final value of the *impulse* response. **Answer**:

$$Y(z) = G(z) \cdot U(z)$$
$$y(k \to \infty) = \lim_{z \to 1} G(z) \cdot U(z) = (z-1) \cdot \frac{b_0}{1 + a_1 \cdot z^{-2}} \cdot \frac{z}{z} = \frac{b_0 \cdot z}{1} = b_0 = 0.3$$

2

f) Which of the following statements is correct?

- ☐ The system is stable.
- ☐ The system is unstable.
- ■ The system is marginally stable. 1

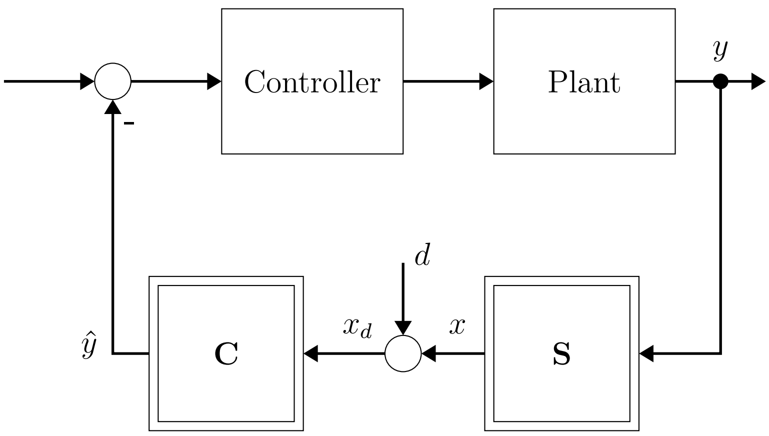
g) How can you modify the given system to meet the two non-checked statements in task f)? **Answer**: For an unstable system, $|a_1| > 1$. For a stable system, $|a_1| < 1$.

2

$\sum 15$

Task 3: Compensator (17 Points)

In order to control a plant, the output of the system has to be measured. For many sensors (e.g. Thermocouples) the measurement of the output results in a voltage value. From this voltage value, the to be measured value can be calculated.
In our example we have a sensor in the feedback loop which measures the output y of our plant. The output of the sensor S is x. This value can suffer from additional disturbance d. x_d is then fed into the compensator C. The Compenstor C does not compensate the added noise d it just reconstructs y by $\hat{y}$ from x_d with the inverse characteristic from S. So with $d = 0$ the statement $\hat{y} = y$ is valid.



The equation of the sensor is

$$f_S(y) = \sqrt{e^y - 1} = x$$

and it is only operating in the positive range $y > 0$.

a) Find the equation for the Compensator $f_C(x_d) = \hat{y}$.
Hint:Leave the disturbance d out of the equation.
Answer: $f_C(x_d) = \hat{y} = \ln(x^2 + 1)$ 3

b) Find the sensitivity of the Compensator $\frac{d\hat{y}}{dx_d}$.
Answer: $\frac{d\hat{y}}{dx_d} = \frac{2x_d}{x_d^2 + 1}$ 2

c) At which value of $x_d = x_{set}$ the sensitivity is the highest? Provide your answer with a calculation or explanation.
Answer:
One way to find the maximum: Take the derivative and set it to 0.

$$\frac{d^2\hat{y}}{dx^2} = \frac{-2x^2 + 2}{x^4 + 2x^2 + 1} = 0$$
$$x^2 = 1$$
$$x_{d1} = 1; x_{d2} = -1$$

The maximum must be $x_{d1} = 1 = x_{set}$ since the operating range is for positive values of y which also means, that all values of x must be positive. 6

d) We now have systematic disturbance of $d = 0.1$. What are the values for $\hat{y}$, x_d and y if $x = x_{set}$?
Answer:

$$x_d = 1.1$$
$$\hat{y} = 0.79$$
$$y = 0.69$$

3

e) What is the relative error e_{rel} and what is the absolute error e_{abs} with the disturbance $d = 1$ if $x = x_{set}$ in your measurement of y?
Answer:

$$y = 0.69$$
$$x_d = 1 + 1 = 2$$
$$\hat{y} = \ln\left((1+1)^2 + 1\right) = 1.61$$
$$e_{abs} = \hat{y} - y = 0.92$$
$$e_{rel} = \frac{e_{abs}}{y} = 1.33$$

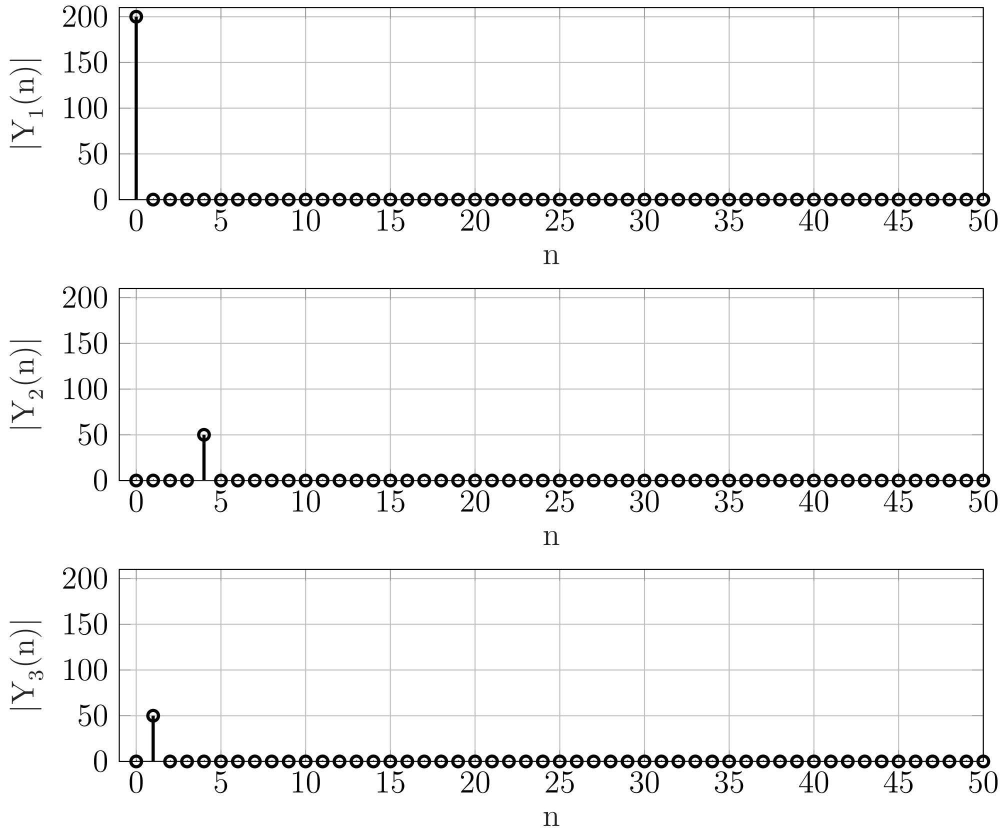
3

$\sum 17$

Task 4: DFT (13 Points)

In the following you can see the DFT results (left half of the spectrum) for different signals.

Hint: $|Y(n)| = \frac{N}{2} \cdot A$ *if A is the Amplitude of an oscillation.*
At $\omega = 0$ *holds:* $|Y(n)| = N \cdot A$.

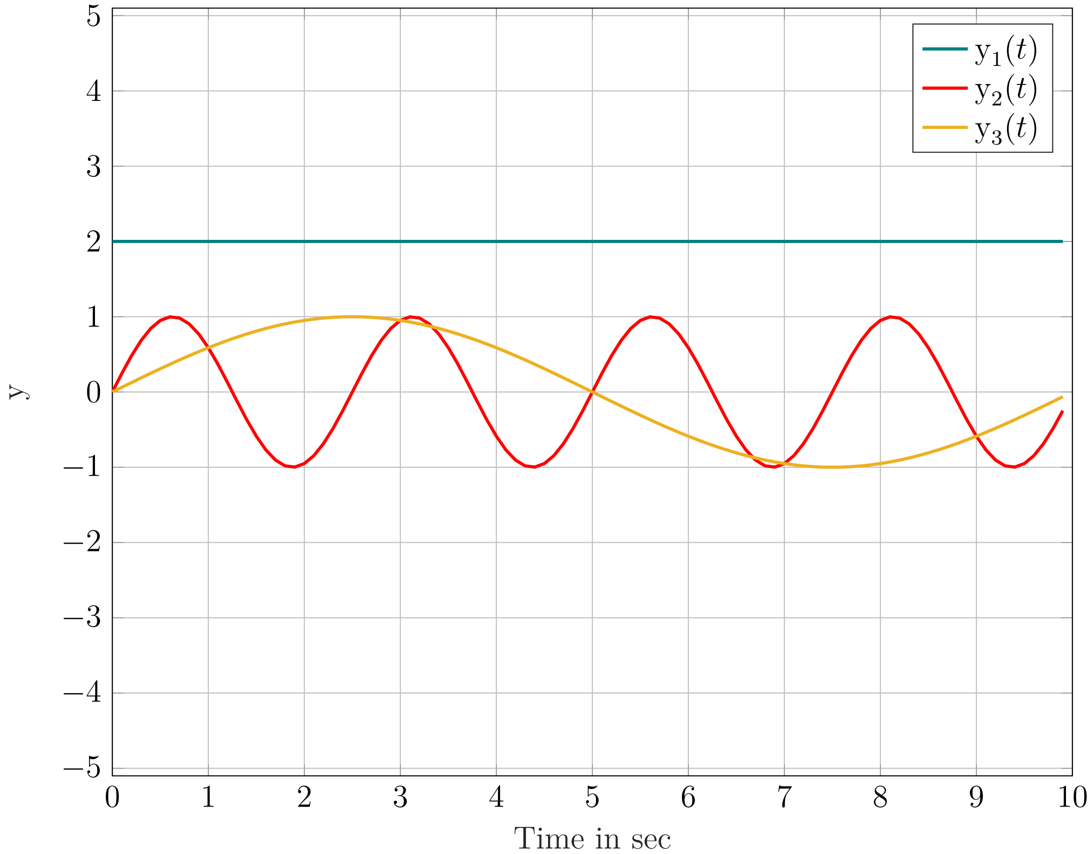


The signal were recorded with a sampling time of 0.1 sec in the time frame from 0 to 9.9 sec (first sample at t=0 sec).

a) What is the sampling frequency f_0?
Answer: $f_0 = \frac{1}{0.1 \text{ sec}} = 10$ Hz 1

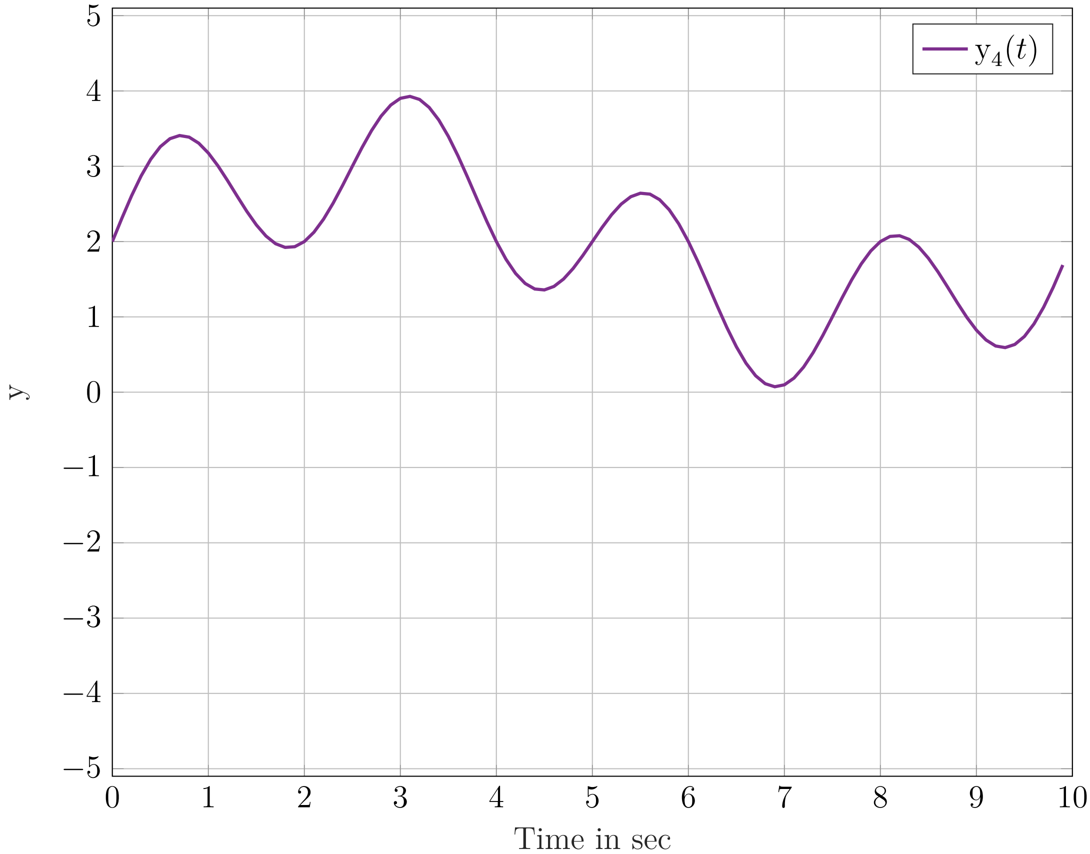
b) Determine N when N is the number of samples per signal.
Answer: $N = 100 = \frac{9.9}{0.1} + 1$ 1

c) Draw the signals $y_1(t)$, $y_2(t)$ and $y_3(t)$ according to the shown DFT plots in the empty diagram. Make sure that one can determine which line represents which signal.
Answer:
$y_1(t)$ is a constant with value 2.
$y_2(t)$ is a sine wave with the frequency $\frac{1}{100} \cdot 10$ Hz and an amplitude of 1.
$y_3(t)$ is a sine wave with the frequency $\frac{4}{100} \cdot 10$ Hz and an amplitude of 1.



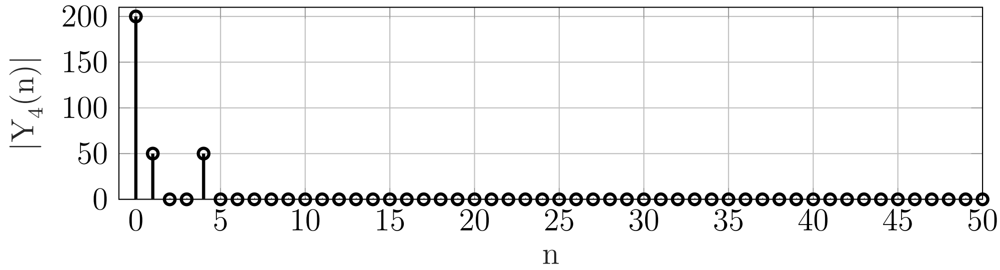
6

d) Draw $y_4(t) = y_1(t) + y_2(t) + y_3(t)$ into the following empty diagram.



2

e) Plot the output of the DFT $|Y_4(n)|$ of your previously drawn signal $y_4(t)$ into the following empty diagram.
Answer:
The spectrum $|Y_4(N)| = |Y_1(N)| + |Y_2(N)| + |Y_3(N)|$ is a sum of the first 3 spectra, since the DFT is a linear transformation.



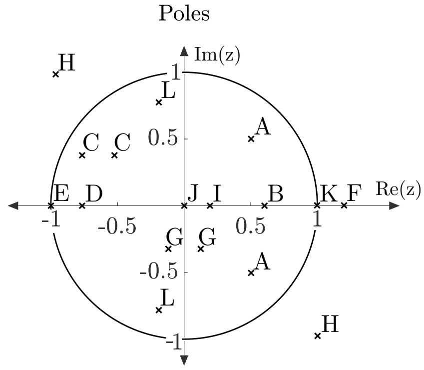
3

$\sum$ 13

Task 5: Step Responses (23 Punkte)

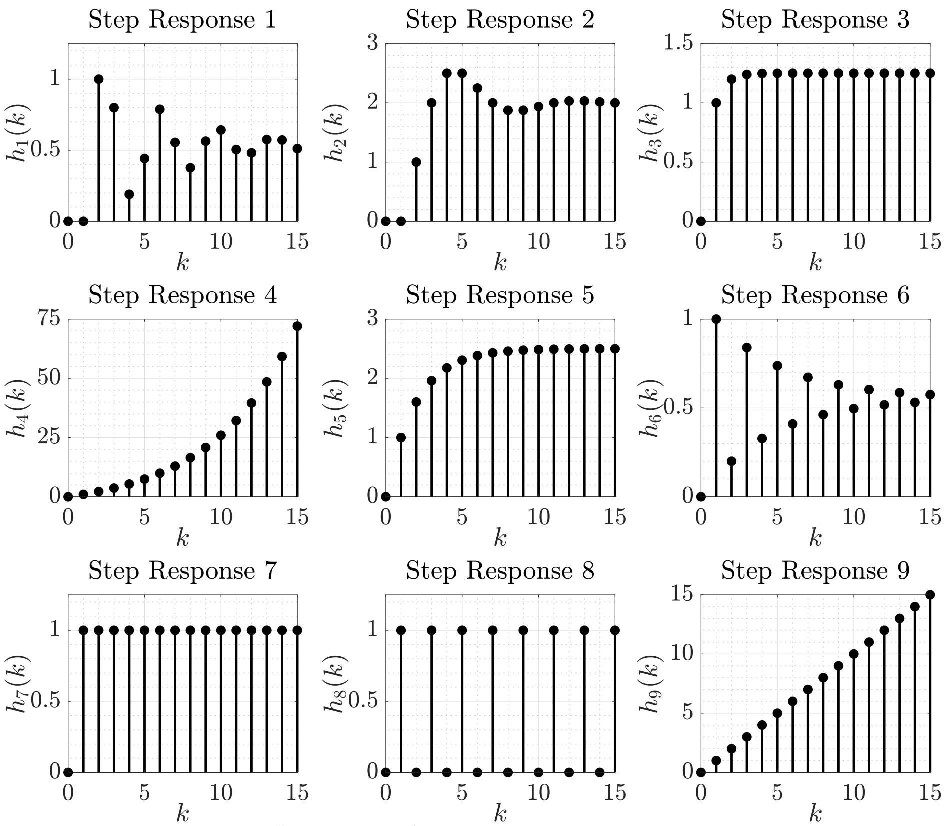
Given are the pole locations for 12 different systems (A-L) in the pole zero map and 9 different step responses (1-9).

a) Match the pole locations to the step responses. Note down your answers in the given table. (Note: Three pole locations do not belong to any step response. All transfer functions have the structure $G(z) = \frac{1}{A(z)}$ with $A(z) = \prod_{i=1}^{n}(z - p_i)$.)



Step Response	Poles
1	L
2	A
3	I
4	F
5	B
6	D
7	J
8	E
9	K

2 2 2 2 2 2 2 2 2



b) Assume that the poles of system A are at $p_1 = 0.5 + 0.5i$ and $p_2 = 0.5 - 0.5i$. Calculate the systems transfer function $G_A(z)$ and apply the final value theorem to the systems response with input signal $u(k) = 3 \cdot \sigma(k)$.

$$G_A(z) = \frac{1}{(z - (0.5 + 0.5i))(z - (0.5 - 0.5i))} = \frac{1}{z^2 - z + 0.5} = \frac{z^{-2}}{1 - z^{-1} + 0.5z^{-2}}$$

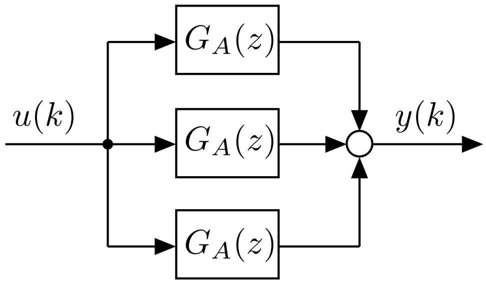
$$U(z) = \mathcal{Z}\left\{u(k)\right\} = 3 \cdot \frac{z}{z-1}$$

$$\lim_{z\to 1}(z-1)Y(z) = \lim_{z\to 1}(z-1)G_A(z)U(z)$$

$$= \lim_{z\to 1}\cancel{(z-1)}\cdot\frac{1}{z^2-z+0.5}\frac{3z}{\cancel{z-1}} = 6$$

3

c) System $G_A(z)$ is now expanded to system $G_{AAA}(z)$ (see block diagram below), which is $G_A(z)$ three times parallel-connected. What is the gain of $G_{AAA}(z)$?



$$G_{AAA}(z) = 3\cdot G_A(z) = \frac{3}{z^2-z+0.5}$$

$$\lim_{z\to 1}(z-1)Y(z) = \lim_{z\to 1}(z-1)G_{AAA}(z)U(z) = \lim_{z\to 1}\cancel{(z-1)}\cdot\frac{3}{z^2-z+0.5}\frac{z}{\cancel{z-1}} = 6$$

2

$\sum 23$

Task 6: Quantization (15 Points)

Two different AD-converters are available for a quantization task. Both are rounding the signal values *down* to the next quantization step. The parameters of the converters are as follows:

	10-bit AD-converter		12-bit AD-converter	
sampling frequency:	$f_0 = 100$	Hz	$f_0 = 100$	Hz
resolution:	10	bit	12	bit
measurement range:	$U_{\text{range}} = 0 \dots 1024$	mV	$U_{\text{range}} = 0 \dots 1024$	mV

a) Calculate the step size ΔU and the maximal quantization error $e_{\text{Q max}}$ of both AD-converters.
The step size ΔU is calculated by partitioning the measurement range in 2^n equidistant parts:

$$\text{In general:} \quad \Delta U = \frac{U_{\max} - U_{\min}}{2^n}$$

$$\Delta U_{\text{10-bit}} = \frac{1024\,\text{mV} - 0\,\text{mV}}{2^{10}}$$

$$\Delta U_{\text{10-bit}} = \frac{1024\,\text{mV}}{1024}$$

$$\Delta U_{\text{10-bit}} = 1\,\text{mV}$$

$$\Delta U_{\text{12-bit}} = \frac{1024\,\text{mV} - 0\,\text{mV}}{2^{12}}$$

$$\Delta U_{\text{12-bit}} = \frac{1024\,\text{mV}}{4096}$$

$$\Delta U_{\text{12-bit}} = \frac{1}{4}\,\text{mV}\,.$$

The maximal quantization error is identical to the step size, when rounding the real values down:

$$e_{\text{Q max}} = \frac{U_{\max} - U_{\min}}{2^n}$$ 2

$$e_{\text{Q max 10-bit}} = 1\,\text{mV}$$

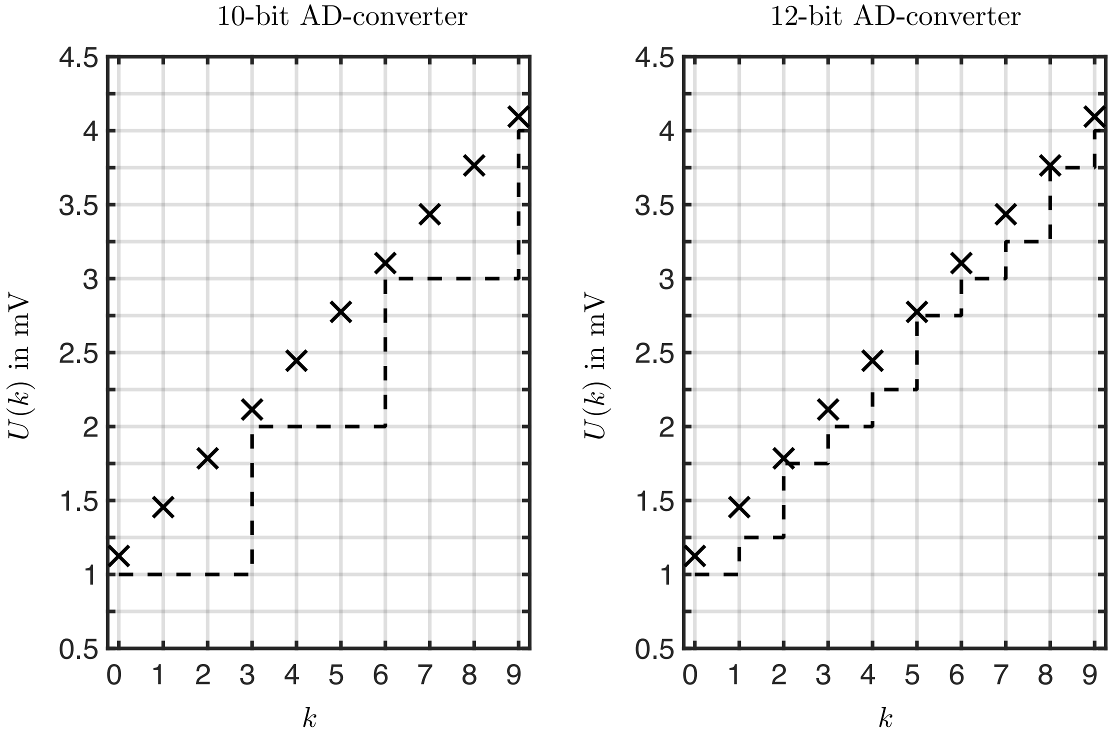
$$e_{\text{Q max 12-bit}} = \frac{1}{4}\,\text{mV}$$ 2

b) How do you call the error signal ($U(k) - U_{\text{Q}}(k)$, $k = 1, \dots, N$) resulting from the quantization error?
Quantization noise. 1

c) What type of error occurs from rounding the values down? Can this error be corrected and if so, how?
This is a systematic error. 1
The signal values are corrected by adding the half step size $\Delta U/2$. 1

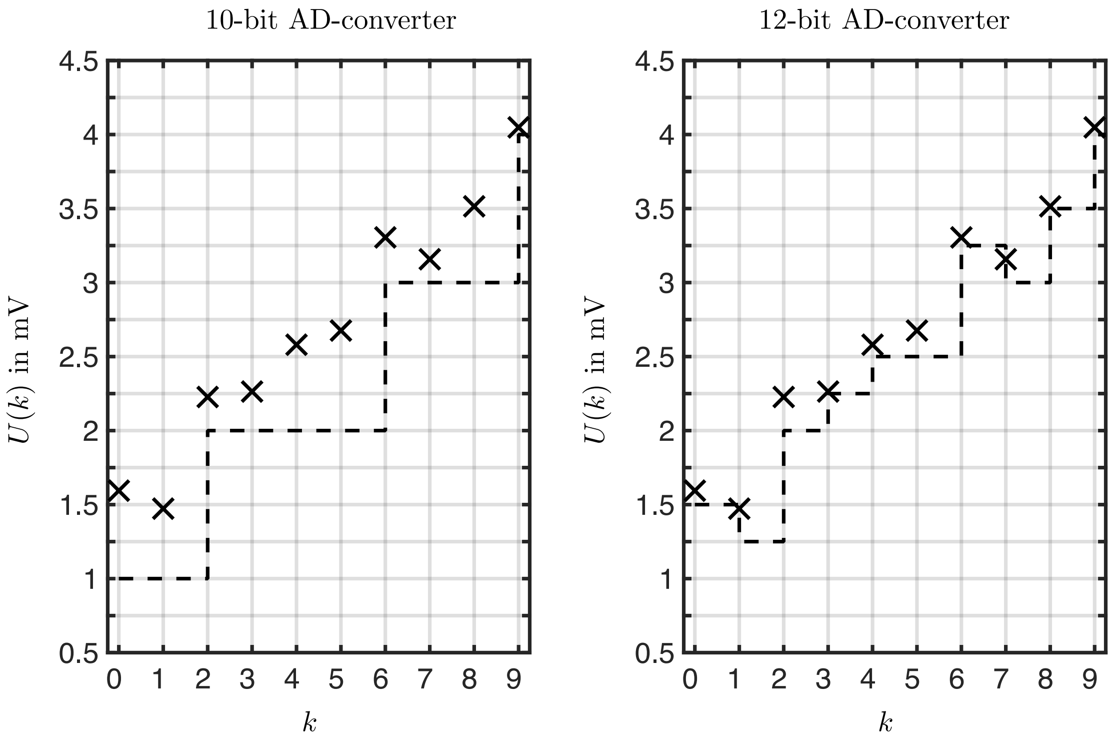
d) Assuming uniformly distributed signal, which has to be quantized. What is the probability distribution of the quantization error?
The quantization error is uniformly distributed as well. 1

e) The values of an *ideal* ramp $U(k)$ shall be quantized using both AD-converters. Sketch the resulting signal $U_{\mathrm{Q}}(k)$ in the given diagram.



3

f) The values of a *noisy* ramp $U(k)$ shall be quantized using both AD-converters. Sketch the resulting signal $U_{\mathrm{Q}}(k)$ in the given diagram.



3

g) The quantized signal of the 12-bit AD-converter is closer to the real values. Therefore, the 12-bit AD-converter reconstructs the noise even better. State one way to get rid of the high frequency noise.
The high frequent parts of the noise can be filtered out by using an low-pass filter. 1

$\sum$ 15

Task 7: Linear Filter (19 Points)

a) The linear transfer function of a filter

$$G_1(z) = \frac{b_0 z + b_1}{z + a_1} \tag{4}$$

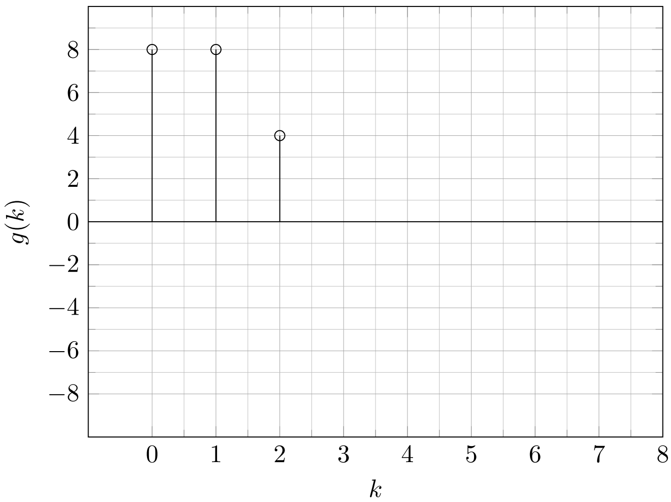
is given. Calculate the corresponding difference equation.

The corresponding difference equation is

$$y(k) = b_0 u(k) + b_1 u(k-1) - a_1 y(k-1). \tag{5}$$

1

b) From the part of the impulse response shown in the figure the coefficients b_0, b_1 and a_1 should be calculated.

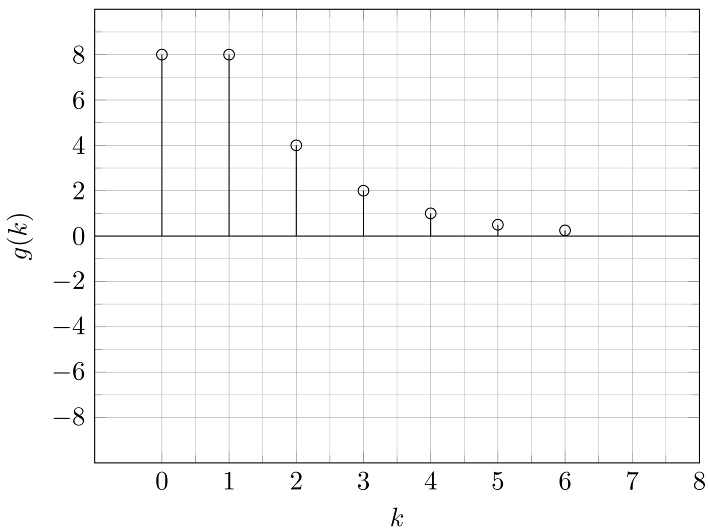


It holds that

$$g(0) = 8 = b_0 \cdot 1 + b_1 \cdot 0 - a_1 \cdot 0 \rightarrow b_0 = 8 \tag{6}$$
$$g(2) = 4 = b_0 \cdot 0 + b_1 \cdot 0 - a_1 \cdot 8 \rightarrow a_1 = -0.5 \tag{7}$$
$$g(1) = 8 = b_0 \cdot 0 + b_1 \cdot 1 - 0.5 \cdot 8 \rightarrow a_2 = 4. \tag{8}$$

4

c) Draw the next 4 values (time steps $k = 3, \ldots, 6$) of the impulse response of $G_1(z)$ into the diagram.



2

d) Now, let the transfer function

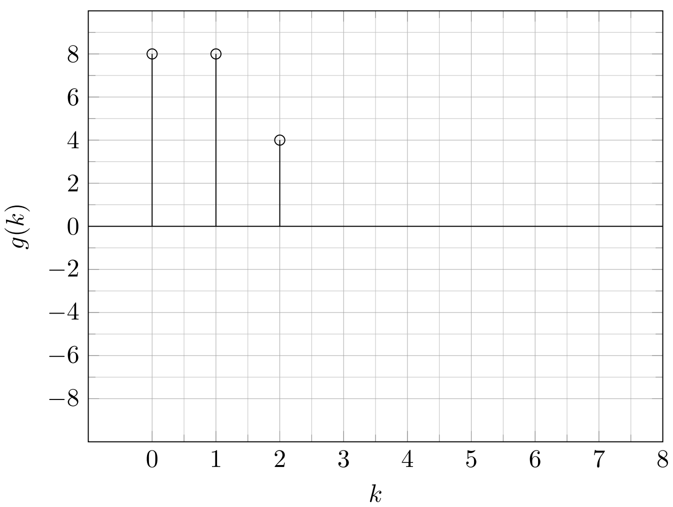
$$G_2(z) = \frac{b_0 z^2}{z^2 + a_1 z + a_2} \tag{9}$$

be given. Calculate the corresponding difference equation. The corresponding difference equation is

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) + b_0 u(k). \tag{10}$$

1

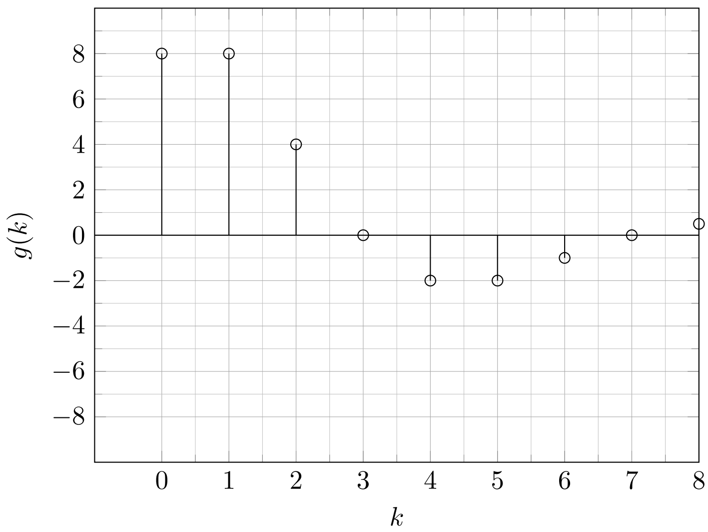
e) Calculate from the shown part of the impulse response the coefficients b_0, a_1 and a_2 of $G_2(z)$.



$$g(0) = 8 = b_0 \cdot 1 - a_1 \cdot 0 - a_2 \cdot 0 \rightarrow b_0 = 8 \tag{11}$$
$$g(1) = 8 = b_0 \cdot 0 - a_1 \cdot 8 - a_2 \cdot 0 \rightarrow a_1 = -1 \tag{12}$$
$$g(2) = 8 = b_0 \cdot 0 + 8 - a_2 \cdot 8 = 4 \rightarrow a_2 = 0.5 \tag{13}$$

4

f) Draw the next 6 values of the impulse response (time steps $k = 3\ldots,8$) of the impulse response of $G_2(z)$ into the diagram.

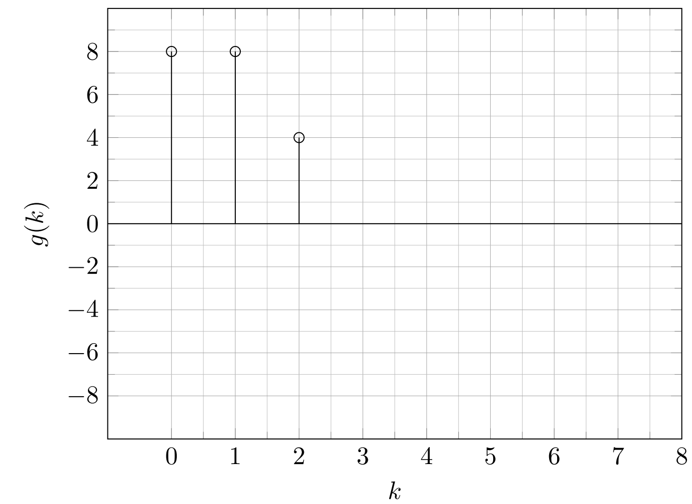


2

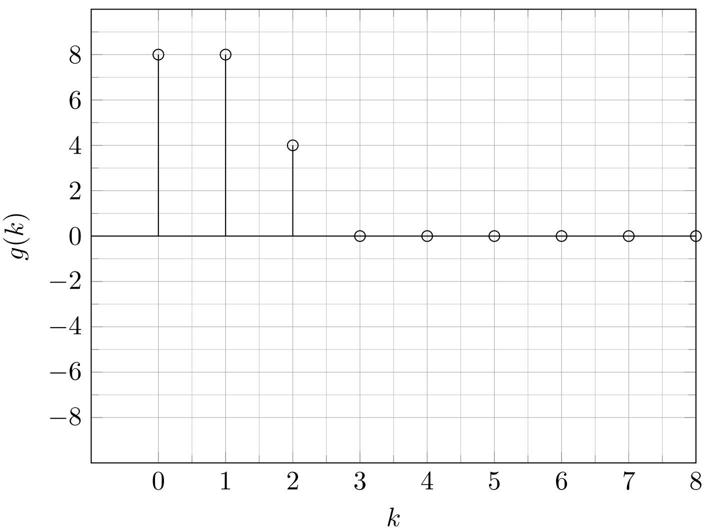
g) Finally, the transfer function

$$G_3(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2} \tag{14}$$

is given. Derive the coefficients b_0, b_1 and b_2 from the values provided in the diagram and plot the next 4 values (time step $k = 3, \ldots, 6$).



Since the system is an FIR system, the coefficients correspond to values of the impulse response. Thus it holds that: $b_0 = 8$, $b_1 = 8$ and $b_2 = 4$. Starting at $k = 3$ the impulse response is 0.



5

$\sum$ 19