Sensorics Exam

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Name:	
Mat.-No.:	
Grade:	

Task	T1	T2	T3	T4	T5	T6	T7	Sum
Score:	24	10	16	22	19	17	12	120
Accomplished:								

Exam duration: 2 hours

Permitted Tools: Pocket Calculator and 4 pages of formula collection

Task 1: **Short Tasks (24 Points)**

a) Explain briefly the three different measurement techniques *direct measurement, indirect measurement and incremental measurement* and give one example for each of them? (6 points)

b) The Tustin Formula (bilinear transformation) $G_{cont}(s) = G_{disc}\left(\frac{2(1-z^{-1})}{T_0(1+z^{-1})}\right)$ can be used to approximately transform a time continuous transfer function $G_{cont}(s)$ into a time discrete transfer function $G_{disc}(z)$ (with sample interval T_0). Deduce the time discrete approximation $y(k)$ for the output of an integrator $\frac{Y(s)}{U(s)} = \frac{1}{s}$ using the Tustin Formula. Explain briefly (in words or with a drawing) how that equation approximates the integral and what kind of input $u(k)$ is integrated without error by this equation. (6 points)

c) Explain briefly 3 different methods to measure speed (3 points).

d) Assume you have a movie camera which records with 60 frames per second. You want to film a rotating wheel with 5 spokes without any visual aliasing effects. What is the maximum allowed rotational speed of the wheel in this case (brief explanation)? (2 points)

e) Briefly explain the functional principle of a *Parallel* or *Flash A/D Converter*. State one advantage and one disadvantage of this type of converter. (3 points)

f) Your company is buying measurement devices in order to measure the volumetric flow rate of your secret product. You know two ways of measuring this flow rate:

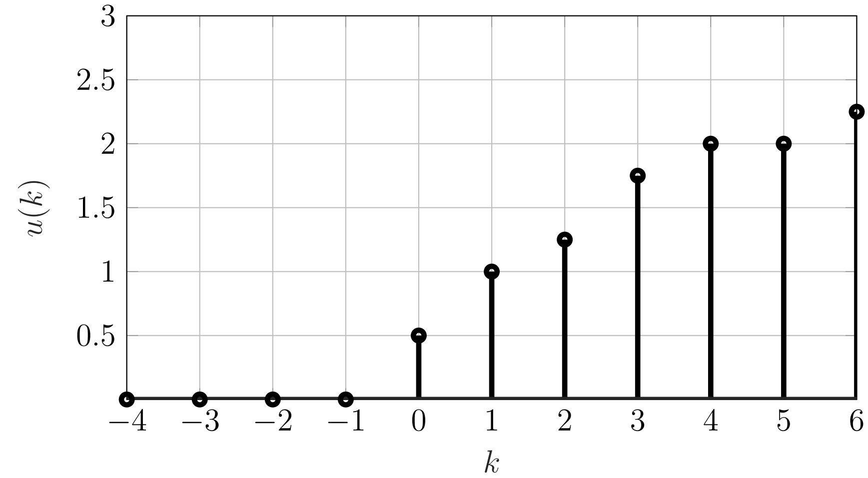
$$Q_1 = V/t$$
$$Q_2 = v \cdot A$$

Q_1 is calculated with the measurements of volume V and time t from two different devices.
Q_2 is calculated with the measurements of flow velocity v and cross-sectional area A from two different devices.
An insider gave you the hint, that all measurement devices from your supplier tend to overestimate their quantity. (4 points)

1) How are these types of measurement called?
2) Which way of measuring the volumetric flow do you prefer? Prove your answer by applying the Gaussian error propagation.

Task 2: Median Filter (10 Points)

The following figure shows the input signal $u(k)$.



A causal median FIR filter of 3^{rd} order should be used to filter the input signal $u(k)$.

a) Determine and draw the output signal $y_1(k)$ of the filter.

b) Determine the transfer function $G_1(z)$ of a linear filter which also calculates the output $y_1(k)$ (task part a)) from the given input $u(k)$. Explain how it is possible that two different filters yield the same output sequence.

In the following subtasks, a causal median FIR filter 5^{th} order will be investigated.

c) Calculate and draw the new output signal $y_2(k)$ of the filter.

d) Which transfer function $G_2(z)$ of a linear filter yields the output sequence $y_2(k)$ (task part c)) for the given input $u(k)$.

Task 3: Bode plot for discrete systems (16 Punkte)

The following discrete transfer function $G(z) = 1 + 0.5z^{-1} + z^{-2}$ is given. The transfer function is determined with a sample time $T_0 = 1\,\text{sec}$.

a) Determine the Shannon frequency ω_S in $\frac{\text{rad}}{\text{sec}}$ that belongs to the given sampling time.

b) Calculate the magnitude $|G(i\omega)|_{\text{dB}}$ explicitly for $\omega = 0.1\,\frac{\text{rad}}{\text{sec}}, \omega = 1\,\frac{\text{rad}}{\text{sec}}$, $\omega = 2\,\frac{\text{rad}}{\text{sec}}$ and $\omega = 3\,\frac{\text{rad}}{\text{sec}}$. Divide therefore the fuction in frequency dependent gain and phase. Is there any undefined point of $|G(i\omega)|_{\text{dB}}$ in the interval $\omega \in [0, \omega_S]\frac{\text{rad}}{\text{sec}}$?

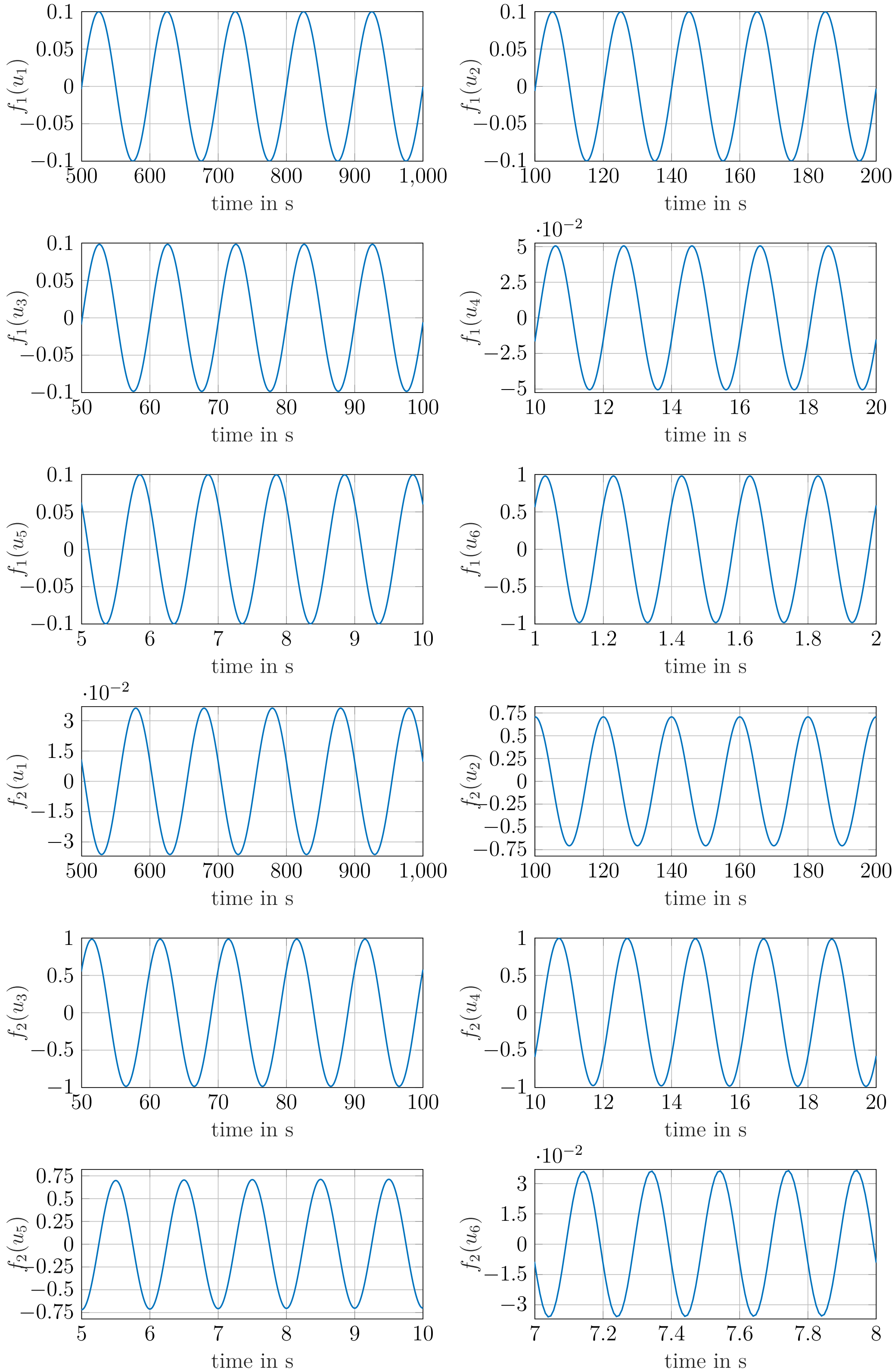
c) Calculate the phase $\varphi(\omega)$ explicitly in degrees for above given ω. Do not forget the sign of the frequency dependent gain.

d) Sketch a bode plot for the frequency intervall $\omega \in [0.1, 10]\,\frac{\text{rad}}{\text{sec}}$. Mark the previously calculated values clearly. What is the differnece between a bode plot of a discrete transfer function in comparision to a continous transfer function with regard to ω?

e) Now, the sampling time is only a third of the initial value. What is the effect on the Shannon frequency? What is the impact of this to the bode plot?

Task 4: Frequency Response of Filters (22 Points)

In the following, the outputs of two filters (f_1 and f_2) are given. The depicted graphs show the response of these filters to six different sinusoidal inputs. For tasks a) and b), it is recommended to make a common table.



a) Calculate the frequencies and angular frequencies of the input signals u_{1-6}. List them in Hz and rad/s.

b) Calculate the magnitude of the frequency responses of both filters. List them as a factor and in dB.

c) Sketch the Bode magnitude plot qualitatively for both filters. Make sure important properties are noticeable.

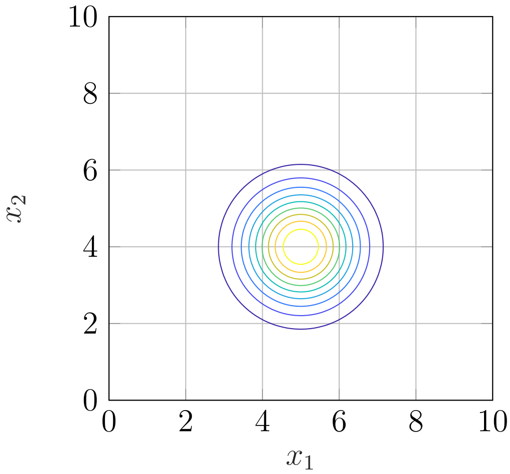
d) Classify both filters according following categories and briefly explain why.
highpass, lowpass, bandpass, stopband

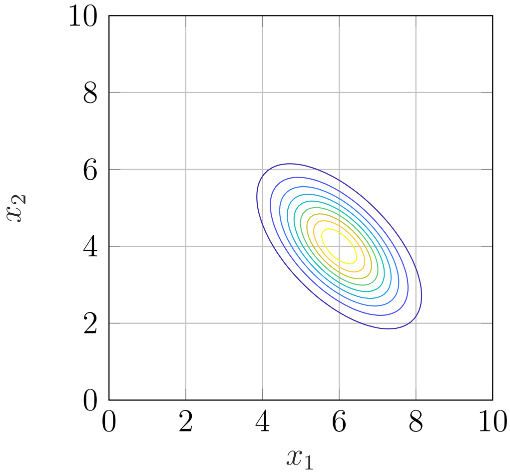
e) Classify both filters according following categories and briefly explain why.
Butterworth, Chebychev Type I, Chebychev Type II, Cauer

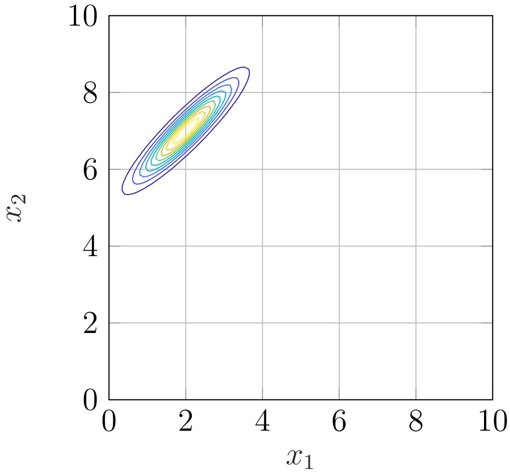
f) Now the input signal $u_7(t) = 2 + \sin(2\pi \cdot 0.3t) + 0.2\sin(2\pi \cdot 3t)$ is filtered with f_1 and f_2.
Sketch the filter outputs $f_1(u_7)$ and $f_2(u_7)$ (in steady state) in a graph. Make sure important properties can be seen.

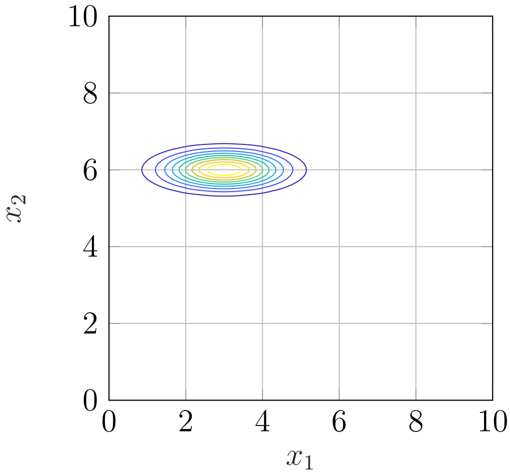
Task 5: Probabilities and Sampling (19 Points)

a) In the following, contour plots of two-dimensional normal distributions are displayed. Make up suitable samples for the distributions. Plot samples over the number of the random experiment. Also draw the means of x_1 and x_2 in the same graph as lines. Make sure your graphs are distinguishable and important properties can be seen.









b) A perfect six-sided dice is given. This dice contains the numbers 1, 2 and 3 on two faces each. Plot the discrete probability distribution $p_1(x)$ for the events '1', '2' and '3' in a graph. What is the name of this type of distribution?

c) If you roll 2 of these dice and add the shown numbers for each roll, what are the possible events? Plot the discrete probability distribution $p_2(x)$ for the possible events in a graph. Describe differences and similarities in both distributions and state their mean values.

Task 6: Filters (17 Points)

The difference equation of a time discrete system is given

$$a_0 y(k) = b_1 u(k-1) + b_2 u(k-2) - a_2 y(k-2)$$

with $a_0 = 0.5$, $a_2 = 0.2$, $b_1 = 0.5$ and $b_2 = 0.6$.

a) Determine the corresponding transfer function in the z-domain.

b) Draw the block diagram of the system.

c) Calculate the poles and zeros of the transfer function with the given coefficients $a_0, ..., b_2$.

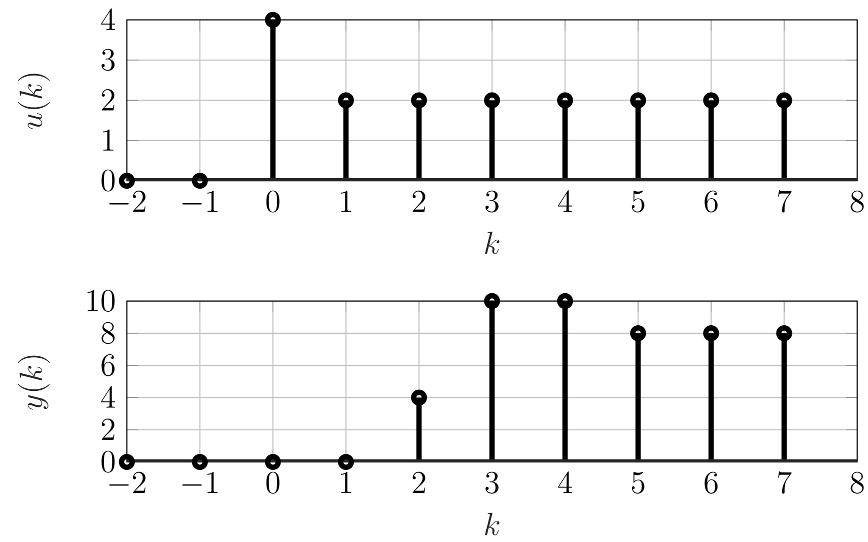
d) Calculate the z-transformed step response $H(z)$.

e) What is the value of the step response in the time domain $h(k)$ for $k \to \infty$ with the given coefficients $a_0, ..., b_2$?

f) Now, an FIR filter with zeros at $n_1 = -1$, $n_2 = -2$, a sampling time $T_0 = 1\,\text{s}$ and a dead time of $3\,\text{s}$ is given. Calculate the transfer function and draw the block diagram of the filter.

Task 7: System Identification FIR (12 Points)

The following figure shows the response $y(k)$ of a unknown system to the input signal $u(k)$.



The system can be described by an FIR filter of finite order. Goal of the task is to determine the coefficients of the FIR filter using the given response.

a) The given system is causal and has no direct feedthrough. Additionally it contains a dead time. What is the dead time of the system? Explain your answer.

b) The system can be represented in the form of an FIR filter. Determine the order of the FIR filter leading to the system response shown. Explain your answer.

c) Give the difference equation of an FIR filter with the order determined in part b). Calculate the coefficients of this filter based on the given system response.

d) Give the transfer function of the determined filter.
If you were unable to determine a filter, use the following difference equation to determine the transfer function:

$$y(k) = 3u(k) + 3u(k-1) + 5u(k-2) + 2u(k-3)$$

e) Draw the block diagram of the FIR filter determined.

Solution:

Task 1: Short Tasks (24 Points)

a) Explain briefly the three different measurement techniques *direct measurement, indirect measurement and incremental measurement* and give one example for each of them? (6 points)

Answer: *Direct measurement:* Compare with a gauge, e.g. length measurement with a ruler. *Indirect measurement:* Measure other quantities and calculate the required quantity, e.g. electrical power by measuring voltage and current and subsequent multiplication. *Incremental measurement:* Counting increments of a known value from a reference point, e.g. angle measurement with incremental rotary encoders. 6

b) The Tustin Formula (bilinear transformation) $G_{cont}(s) = G_{disc}\left(\frac{2(1-z^{-1})}{T_0(1+z^{-1})}\right)$ can be used to approximately transform a time continuous transfer function $G_{cont}(s)$ into a time discrete transfer function $G_{disc}(z)$ (with sample interval T_0). Deduce the time discrete approximation $y(k)$ for the output of an integrator $\frac{Y(s)}{U(s)} = \frac{1}{s}$ using the Tustin Formula. Explain briefly (in words or with a drawing) how that equation approximates the integral and what kind of input $u(k)$ is integrated without error by this equation. (6 points)

Answer:

$$\frac{Y(s)}{U(s)} = \frac{1}{s} \approx \frac{Y(z)}{U(z)} = \frac{T_0(1+z^{-1})}{2(1-z^{-1})} \Leftrightarrow 2Y(z)(1-z^{-1}) = T_0U(z)(1+z^{-1})$$

$$\Rightarrow 2(y(k)-y(k-1)) = T_0(u(k)+u(k-1)) \Leftrightarrow \boxed{y(k) = y(k-1) + \frac{T_0}{2}(u(k)+u(k-1))}$$

The equation (also called the Trapezoidal Rule of Integration) adds the average of the current $u(k)$ and previous input $u(k-1)$ multiplied by the sample interval to the previous output $y(k-1)$. Every (piecewise) linear (or constant) input will be integrated without error. 6

c) Explain briefly 3 different methods to measure speed (3 points).

Answer: Measure the rotational speed of a wheel and multiply with radius. Measure the acceleration and integrate. Use electromagnetic or sound waves and employ the Doppler Effect. 3

d) Assume you have a movie camera which records with 60 frames per second. You want to film a rotating wheel with 5 spokes without any visual aliasing effects. What is the maximum allowed rotational speed of the wheel in this case (brief explanation)? (2 points)

Answer: The Shannon Theorem states that sampling has to be twice as fast as the highest frequency in the signal. In this case this would allow a rotational speed of 30 Hz. But since the wheel has 5 spokes the wheel looks the same after a fifth of a complete rotation. Therefore the maximum allowed speed is 6 Hz. 2

e) Briefly explain the functional principle of a *Parallel* or *Flash A/D Converter*. State one advantage and one disadvantage of this type of converter. (3 points)

Answer: The measurement is simultaneously compared to all quantization levels. Very fast, only feasible for low resolutions (hardware complexity increases exponentially with resolution). 3

f) Your company is buying measurement devices in order to measure the volumetric flow rate of your secret product. You know two ways of measuring this flow rate:

$$Q_1 = V/t$$
$$Q_2 = v \cdot A$$

Q_1 is calculated with the measurements of volume V and time t from two different devices.
Q_2 is calculated with the measurements of flow velocity v and cross-sectional area A from two different devices.
An insider gave you the hint, that all measurement devices from your supplier tend to overestimate their quantity. (4 points)

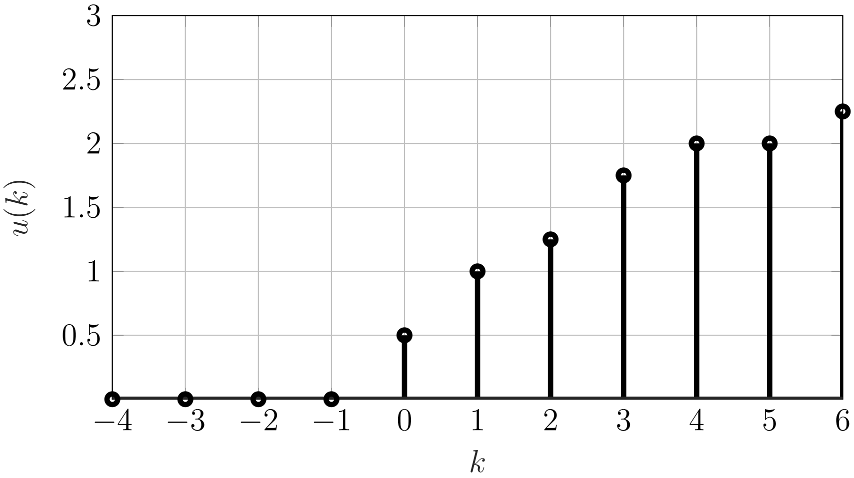
1) How are these types of measurement called? **Answer**:Indirect measurement 1
2) Which way of measuring the volumetric flow do you prefer? Prove your answer by applying the Gaussian error propagation.
Answer:

$$\Delta Q_1 = \frac{\partial Q_1}{\partial V}\Delta V + \frac{\partial Q_1}{\partial t}\Delta t$$
$$\Delta Q_1 = \frac{1}{t}\Delta V - \frac{V}{t^2}\Delta t$$
$$\Delta Q_2 = \frac{\partial Q_2}{\partial v}\Delta v + \frac{\partial Q_2}{\partial A}\Delta A$$
$$\Delta Q_2 = A\Delta v + v\Delta A$$

Q_1 is preferred, because ΔV and Δt contribute in opposite manner to ΔQ_1, the systematic overestimation is less problematic than in Q_2, where both Δv and ΔA contribute both positive to ΔQ_2 3

Task 2: Median Filter (10 Points)

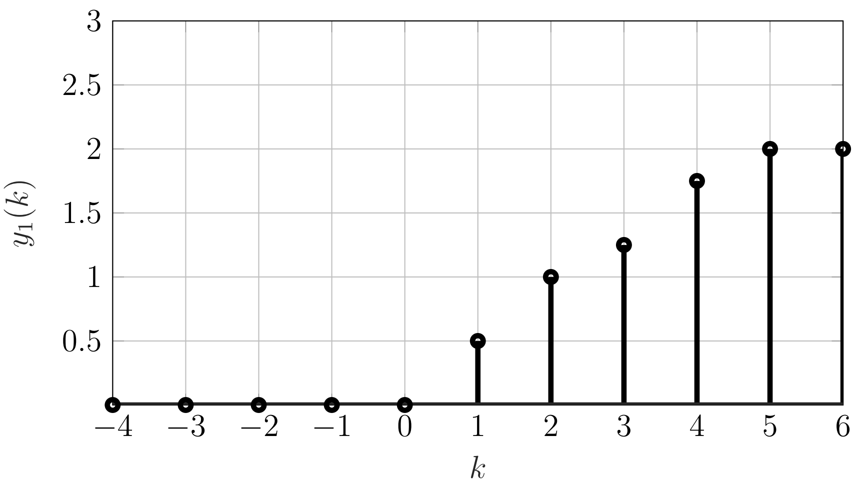
The following figure shows the input signal $u(k)$.



A causal median FIR filter of 3rd order should be used to filter the input signal $u(k)$.

a) Determine and draw the output signal $y_1(k)$ of the filter.
Answer: The output of a median filter 3rd order can be calculated by

$$y_1(k) = \text{median}(u(k), u(k-1), u(k-2)).$$



4

b) Determine the transfer function $G_1(z)$ of a linear filter which also calculates the output $y_1(k)$ (task part a)) from the given input $u(k)$. Explain how it is possible that two different filters yield the same output sequence.
Answer: Due to the monotonically increasing input sequence, the input value of $u(k-1)$ is determined as output $y_1(k)$ in the median filter. This results in the transfer function:

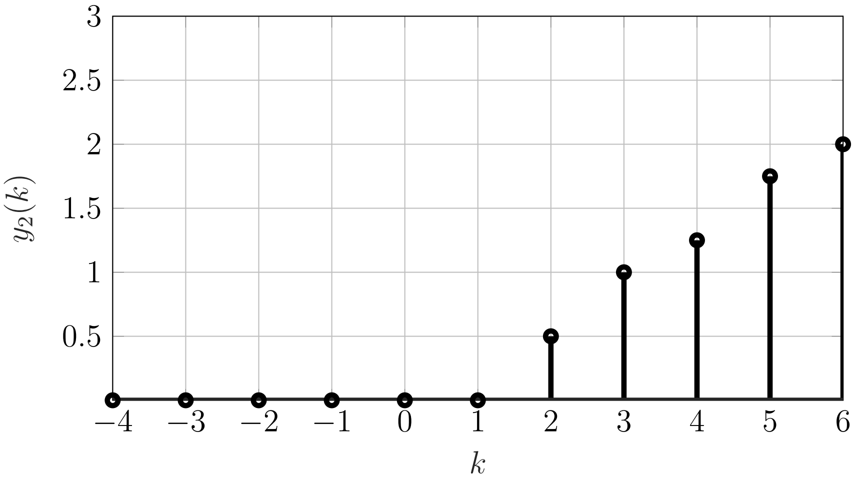
$$G_1(z) = z^{-1}.$$

Two different linear filters cannot have the same output response to the same input signal. This is only possible because the median filter is a nonlinear filter. 2

In the following subtasks, a causal median FIR filter 5^{th} order will be investigated.

c) Calculate and draw the new output signal $y_2(k)$ of the filter.
Answer: The output of a median filter 5^{th} order can be calculated by

$$y_2(k) = \text{median}(u(k), u(k-1), u(k-2), u(k-3), u(k-4)).$$



3

d) Which transfer function $G_2(z)$ of a linear filter yields the output sequence $y_2(k)$ (task part c)) for the given input $u(k)$.
Answer: Now, $y_2(k) = u(k-2)$ holds, thus the transfer function is given by

$$G_2(z) = z^{-2}.$$

1

$\sum$ 10

Task 3: Bode plot for discrete systems (16 Punkte)

The following discrete transfer function $G(z) = 1 + 0.5z^{-1} + z^{-2}$ is given. The transfer function is determined with a sample time $T_0 = 1\,\mathrm{sec}$.

a) Determine the Shannon frequency ω_S in $\frac{\mathrm{rad}}{\mathrm{sec}}$ that belongs to the given sampling time.

Answer:

$$\omega_S = \frac{\pi}{T_0}$$
$$\omega_S = \pi\,\frac{\mathrm{rad}}{\mathrm{sec}}$$

1

b) Calculate the magnitude $|G(i\omega)|_{\mathrm{dB}}$ explicitly for $\omega = 0.1\,\frac{\mathrm{rad}}{\mathrm{sec}}, \omega = 1\,\frac{\mathrm{rad}}{\mathrm{sec}}$, $\omega = 2\,\frac{\mathrm{rad}}{\mathrm{sec}}$ and $\omega = 3\,\frac{\mathrm{rad}}{\mathrm{sec}}$. Divide therefore the fuction in frequency dependent gain and phase. Is there any undefined point of $|G(i\omega)|_{\mathrm{dB}}$ in the interval $\omega \in [0, \omega_S]\frac{\mathrm{rad}}{\mathrm{sec}}$?

Answer:

- Rearranging the equation

$$\begin{aligned} G(z) &= 1 + 0.5z^{-1} + z^{-2} \\ &= z^{-1}\left(0.5 + z^1 + z^{-1}\right) \\ G(i\omega) &= \left(2\cos(\omega T_0) + 0.5\right) e^{-i\omega T_0} \text{ with } z = e^{i\omega T_0} \text{ and } z^1 + z^{-1} = 2\cos(\omega T_0) \end{aligned}$$

- Divide in frequency dependent gain and phase

$$G(i\omega) = \underbrace{\left(2\cos(\omega T_0) + 0.5\right)}_{\text{frequency dependent gain}} e^{\overbrace{-i\omega T_0}^{\text{phase}}}$$

  real magnitude

- Calculate magnitude for points

$$\begin{aligned} |G(i\omega)| &= |2\cos(\omega T_0) + 0.5| \\ |G(i\omega)|_{\mathrm{dB}} &= 20\log|2\cos(\omega T_0) + 0.5| \end{aligned}$$

$$\begin{aligned} \omega = 0.1\,\frac{\mathrm{rad}}{\mathrm{sec}} &: |G(i\omega)|_{\mathrm{dB}} = 7.92\,\mathrm{dB} \\ \omega = 1\,\frac{\mathrm{rad}}{\mathrm{sec}} &: \quad |G(i\omega)|_{\mathrm{dB}} = 3.98\,\mathrm{dB} \\ \omega = 2\,\frac{\mathrm{rad}}{\mathrm{sec}} &: \quad |G(i\omega)|_{\mathrm{dB}} = -9.57\,\mathrm{dB} \\ \omega = 3\,\frac{\mathrm{rad}}{\mathrm{sec}} &: \quad |G(i\omega)|_{\mathrm{dB}} = 3.41\,\mathrm{dB} \end{aligned}$$

- Frequency dependent gain equals zero?

$$
\begin{aligned}
0 &= 2\cos(\omega T_0) + 0.5 \\
-\frac{1}{4} &= \cos(\omega T_0) \\
\omega &= \frac{\arccos(-\frac{1}{4})}{T_0} \\
\omega &\approx 1.82
\end{aligned}
$$

For $\omega = \frac{\arccos(-\frac{1}{4})}{T_0}$, $\log(0)$ is not defined.
Frequency dependent gain for $\omega > \frac{\arccos(-\frac{1}{4})}{T_0}$ is negative. $\boxed{5}$

c) Calculate the phase $\varphi(\omega)$ explicitly in degrees for above given ω. Do not forget the sign of the frequency dependent gain.
Answer:

- With already calculated relation
Magnitude is real and has no effect on phase.

$$
\begin{aligned}
\angle G(i\omega) &= \angle e^{-i\omega T_0} \\
\varphi(\omega) &= T_0\omega
\end{aligned}
$$

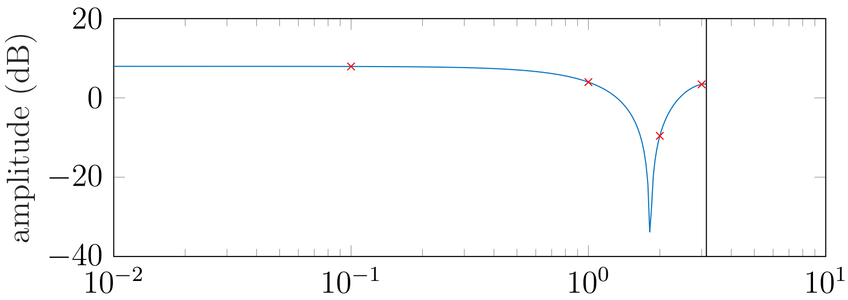
If frequency dependent gain is negative, add π to φ. Therefore:

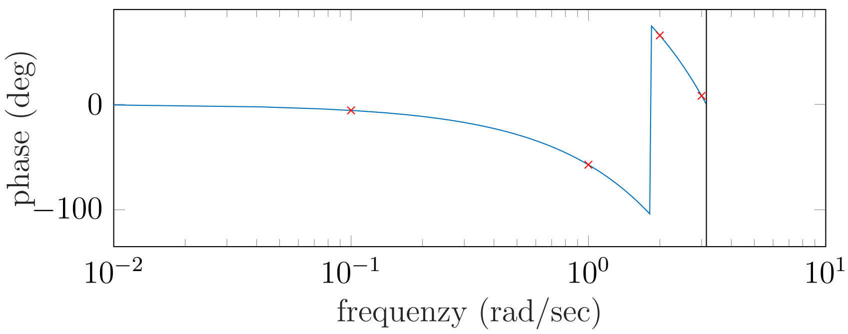
$$
\varphi(\omega) = \begin{cases} T_0\omega & \omega < \frac{\arccos(-\frac{1}{4})}{T_0} \\ T_0\omega + \pi & \omega > \frac{\arccos(-\frac{1}{4})}{T_0} \end{cases}
$$

$$
\begin{aligned}
\omega = 0.1\,\frac{\text{rad}}{\text{sec}} &: \varphi(\omega) = -5.73\,^\circ \\
\omega = 1\,\frac{\text{rad}}{\text{sec}} &: \quad \varphi(\omega) = -57.3\,^\circ \\
\omega = 2\,\frac{\text{rad}}{\text{sec}} &: \quad \varphi(\omega) = 65.41\,^\circ \\
\omega = 3\,\frac{\text{rad}}{\text{sec}} &: \quad \varphi(\omega) = 8.11\,^\circ
\end{aligned}
$$

$\boxed{4}$

d) Sketch a bode plot for the frequency intervall $\omega \in [0.1, 10]\ \frac{\text{rad}}{\text{sec}}$. Mark the previously calculated values clearly. What is the differnece between a bode plot of a discrete transfer function in comparision to a continous transfer function with regard to ω?
Answer:
To facilitate interpretation of a discrete transfer function, only the upper half of the unit circle up to the Shannon frequency is visualized.





5

e) Now, the sampling time is only a third of the initial value. What is the effect on the Shannon frequency? What is the impact of this to the bode plot?
Answer: Shannon frequency:

$$\omega_S = \frac{\pi}{T_{\text{new}}} \text{ with } T_{\text{new}} = \frac{T_0}{3}$$
$$\omega_S = 3\pi\,\frac{\text{rad}}{\text{sec}}$$

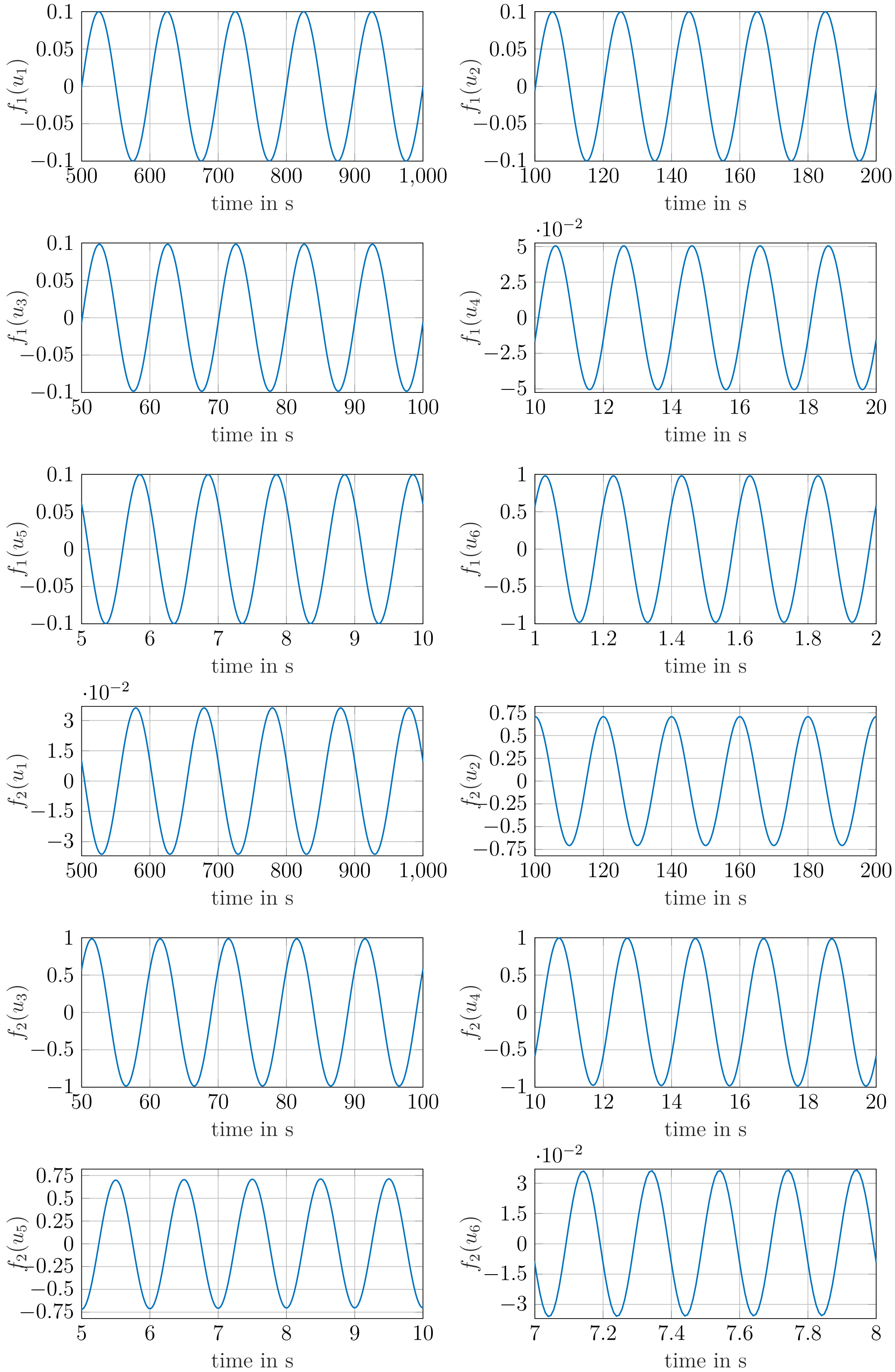
Shannon frequency is larger, thus the upper bound of ω increases, if only the upper half of the unit circle is considered.
The curves are scaled over the frequency axis.

3

$\sum 18$

Task 4: Frequency Response of Filters (22 Points)

In the following, the outputs of two filters (f_1 and f_2) are given. The depicted graphs show the response of these filters to six different sinusoidal inputs. For tasks a) and b), it is recommended to make a common table.



a) Calculate the frequencies and angular frequencies of the input signals u_{1-6}. List them in Hz and rad/s.
Answer: see table in b). 3

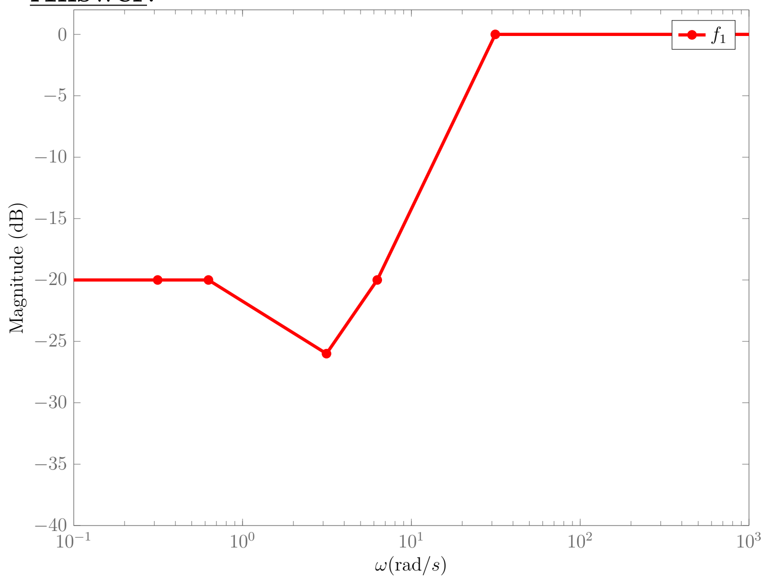
b) Calculate the magnitude of the frequency responses of both filters. List them as a factor and in dB.

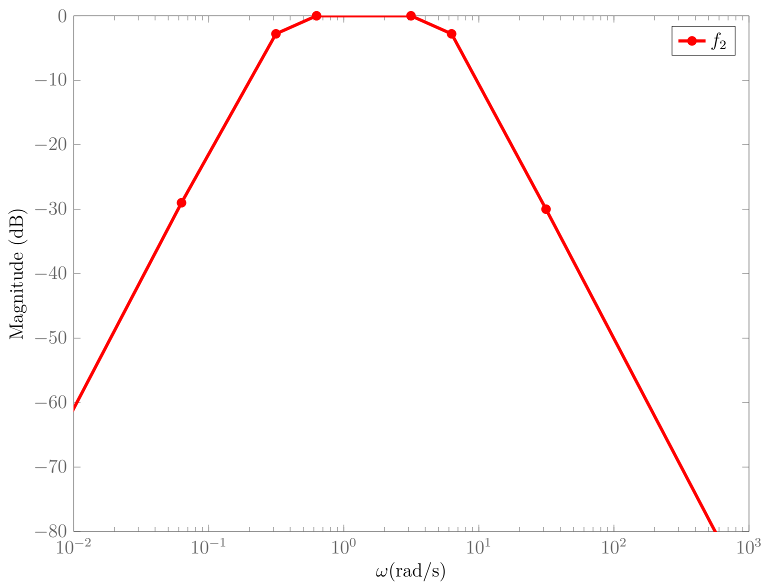
Answer:

	u_1	u_2	u_3	u_4	u_5	u_6
frequency in Hz	0.01	0.05	0.1	0.5	1	5
ang. frequency in rad/s	0.0628	0.3142	0.628	3.142	6.28	31.42
$\lvert G_{f_1}(i\omega)\rvert$	0.1	0.1	0.1	0.05	0.1	1
$\lvert G_{f_1}(i\omega)\rvert$ in dB	-20	-20	-20	-26	-20	0
$\lvert G_{f_2}(i\omega)\rvert$	0.032	0.7	1	1	0.7	0.035
$\lvert G_{f_2}(i\omega)\rvert$ in dB	-30	-3	0	0	-3	-29

6

c) Sketch the Bode magnitude plot qualitatively for both filters. Make sure important properties are noticeable.
Answer: 4



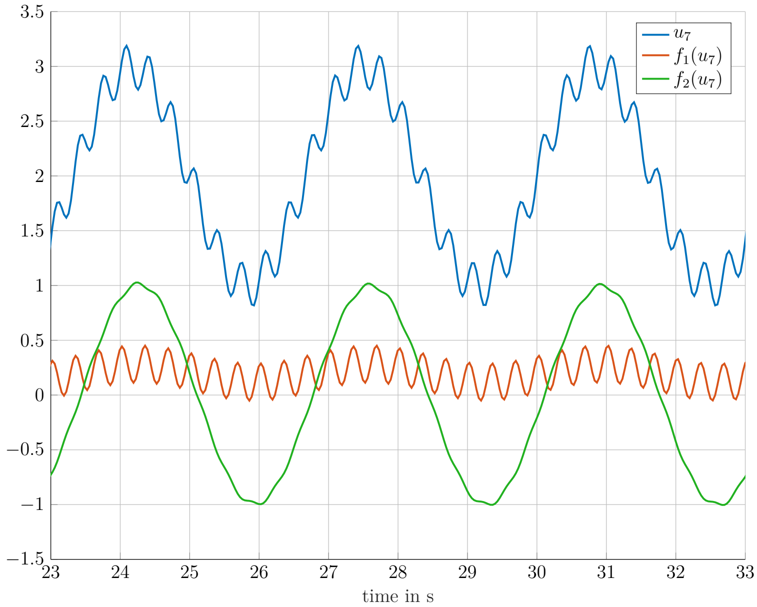


d) Classify both filters according following categories and briefly explain why.
highpass, lowpass, bandpass, stopband
Answer: 2

f_1 is a highpass filter because frequencies lower than 6 rad/s are suppressed by factor 10.
f_2 is a bandpass filter because frequencies below 0.3 rad/s and above 6 rad/s are suppressed.

e) Classify both filters according following categories and briefly explain why.
Butterworth, Chebychev Type I, Chebychev Type II, Cauer
Answer: 2
f_1 is a Chebychev Type II filter, because no ripples can be seen in the pass band but it has ripples in the stop band. It could also be a Cauer filter, since there might be not enough samples in the pass band.
f_2 is a Butterworth filter. Both stop bands show monotonic behavior.

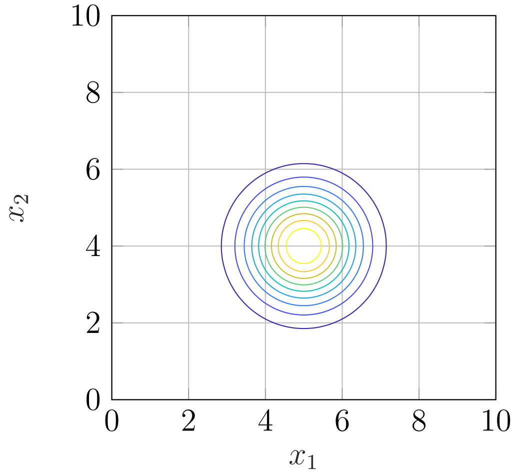
f) Now the input signal $u_7(t) = 2 + \sin(2\pi \cdot 0.3t) + 0.2\sin(2\pi \cdot 3t)$ is filtered with f_1 and f_2.
Sketch the filter outputs $f_1(u_7)$ and $f_2(u_7)$ (in steady state) in a graph. Make sure important properties can be seen.
Answer:



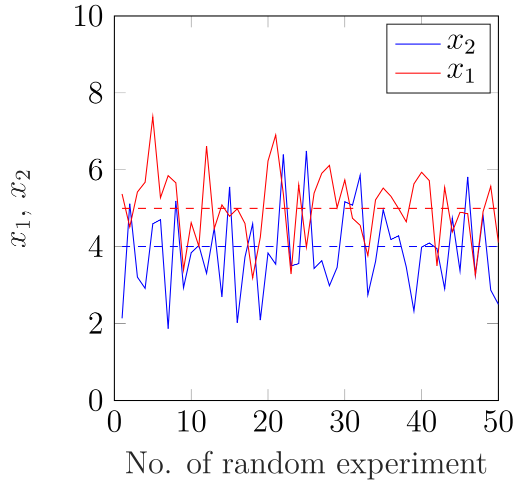
important properties:
f_1 reduces offset and slow sine component by factor 10 (bonus points). Quicker sine component remains in full amplitude.
f_2 removes the offset and the faster sine component almost completely. Slower sine component remains in full amplitude. 5

Task 5: Probabilities and Sampling (19 Points)

a) In the following, contour plots of two-dimensional normal distributions are displayed. Make up suitable samples for the distributions. Plot samples over the number of the random experiment. Also draw the means of x_1 and x_2 in the same graph as lines. Make sure your graphs are distinguishable and important properties can be seen.



Answer:



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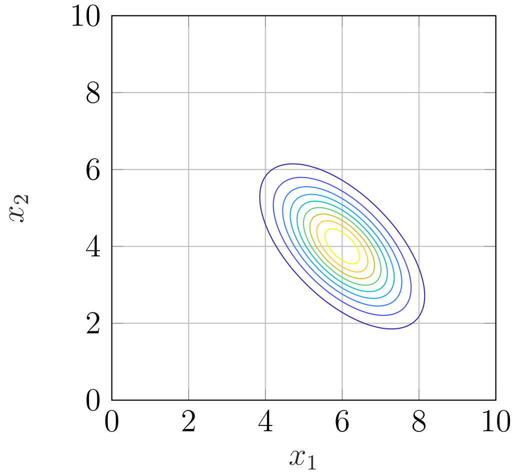
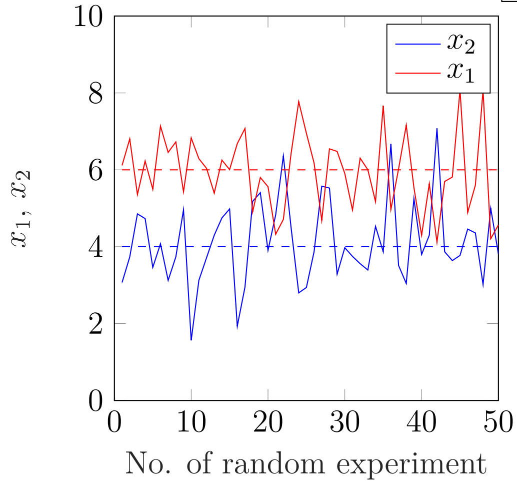


Answer:



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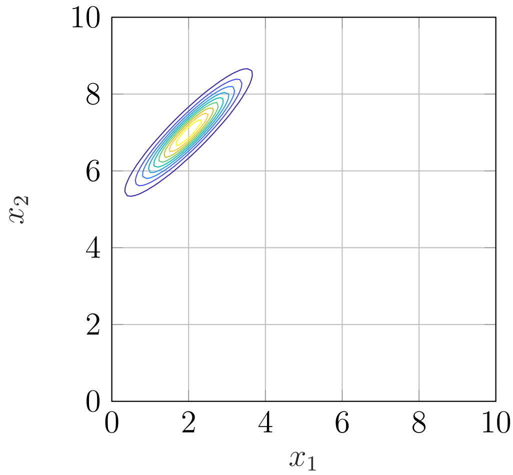
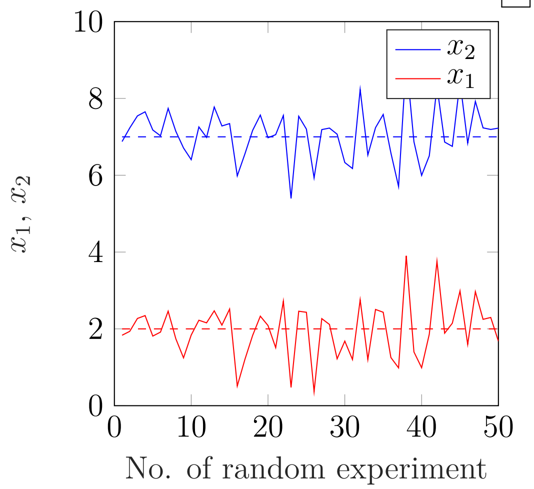


Answer:



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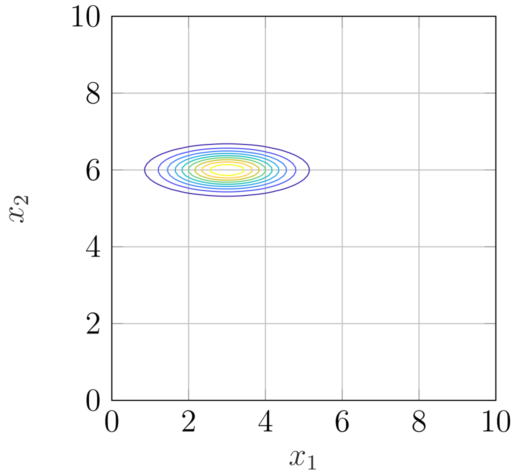
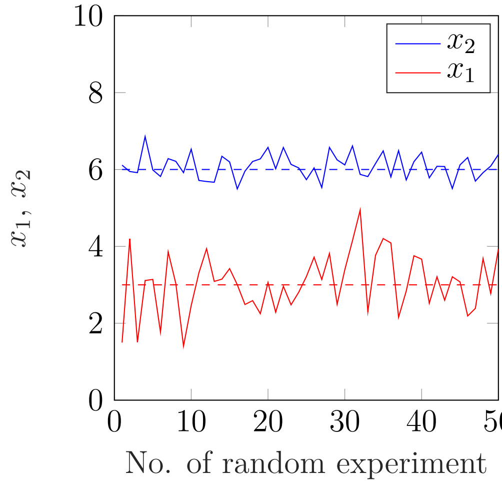


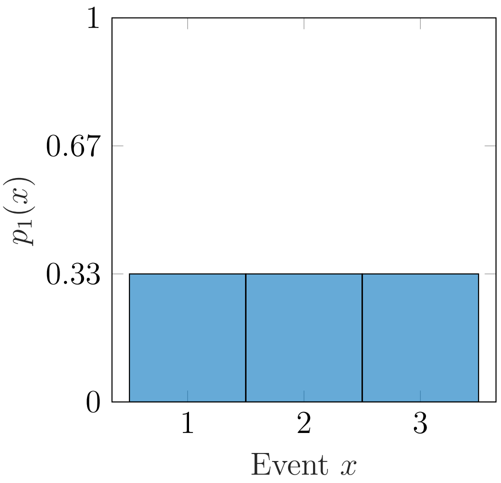


Answer:

3

b) A perfect six-sided dice is given. This dice contains the numbers 1, 2 and 3 on two faces each. Plot the discrete probability distribution $p_1(x)$ for the events '1', '2' and '3' in a graph. What is the name of this type of distribution?

Answer:Uniform distribution.



2

c) If you roll 2 of these dice and add the shown numbers for each roll, what are the possible events? Plot the discrete probability distribution $p_2(x)$ for the possible events in a graph. Describe differences and similarities in both distributions and state their mean values.

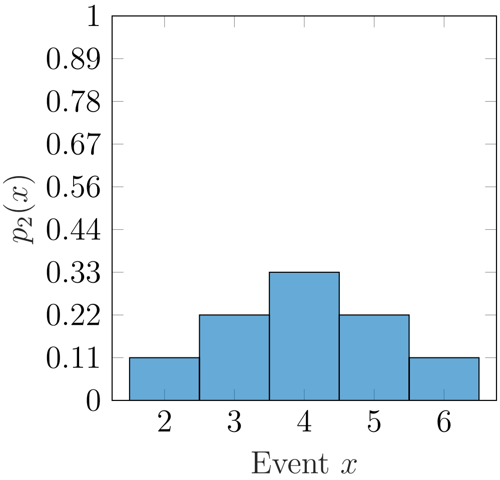
Answer:Possible events: 2, 3, 4, 5, 6. 1

$\mu_1 = 2$, $\mu_2 = 4$ 1

Combinations:

$$\begin{aligned} 1+1 &= 2 \\ 2+1|1+2 &= 3 \\ 1+3|2+2|3+1 &= 4 \\ 2+3|3+2 &= 5 \\ 3+3 &= 6 \end{aligned}$$

The second probability distribution is not a uniform distribution but a binomial one (word does not have to be known), therefore in the second distribution not all events have the same probability. Both distributions are symmetrical.



3

Task 6: Filters (17 Points)

The difference equation of a time discrete system is given

$$a_0 y(k) = b_1 u(k-1) + b_2 u(k-2) - a_2 y(k-2)$$

with $a_0 = 0.5$, $a_2 = 0.2$, $b_1 = 0.5$ and $b_2 = 0.6$.

a) Determine the corresponding transfer function in the z-domain.
Answer: The transfer function $G(z)$ of the given difference equation is:

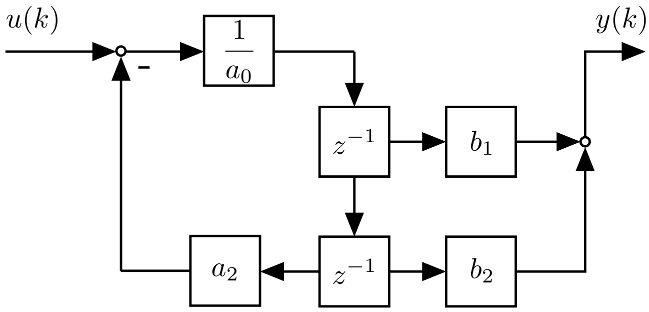
$$a_0 y(k) = b_1 u(k-1) + b_2 u(k-2) - a_2 y(k-2)$$

$$\circ\!\!-\!\!\bullet$$

$$\begin{aligned} a_0 Y(z) &= b_1 \cdot U(z) \cdot z^{-1} + b_2 \cdot U(z) \cdot z^{-2} - a_2 \cdot Y(z) \cdot z^{-2} \\ Y(z) \cdot \left(a_0 + a_2 \cdot z^{-2}\right) &= U(z) \cdot \left(b_1 \cdot z^{-1} + b_2 \cdot z^{-2}\right) \\ G(z) = \frac{Y(z)}{U(z)} &= \frac{b_1 \cdot z^{-1} + b_2 \cdot z^{-2}}{a_0 + a_2 \cdot z^{-2}} \end{aligned}$$

$\boxed{3}$

b) Draw the block diagram of the system.
Answer:



$\boxed{3}$

c) Calculate the poles and zeros of the transfer function with the given coefficients $a_0, ..., b_2$.
Answer:

$$\begin{aligned} G(z) = \frac{Y(z)}{U(z)} &= \frac{b_1 \cdot z^{-1} + b_2 \cdot z^{-2}}{a_0 + a_2 \cdot z^{-2}} \\ &= \frac{b_1 z + b_2}{a_0 z^2 + a_2} = \frac{\frac{b_1}{a_0} z + \frac{b_2}{a_0}}{z^2 + \frac{a_2}{a_0}} \\ &\rightarrow n_1 = -1.2;\ p_{1,2} = \pm 0.63i \end{aligned}$$

$\boxed{3}$

d) Calculate the z-transformed step response $H(z)$.
Answer:

$$H(z) = G(z) \frac{1}{1 - z^{-1}} = \frac{b_1 \cdot z^{-1} + b_2 \cdot z^{-2}}{(a_0 + a_2 \cdot z^{-2})(1 - z^{-1})}$$

$\boxed{2}$

e) What is the value of the step response in the time domain $h(k)$ for $k \to \infty$ with the given coefficients $a_0, ..., b_2$?
Answer: Endwert:

$$h(k \to \infty) = \lim_{z \to 1}(z-1)H(z) = \lim_{z \to 1} G(z) = \frac{b_1 + b_2}{a_0 + a_2} = 1.57.$$

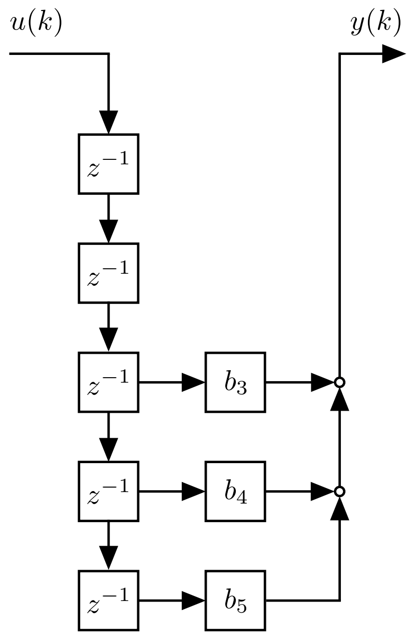
2

f) Now, an FIR filter with zeros at $n_1 = -1$, $n_2 = -2$, a sampling time $T_0 = 1\,\text{s}$ and a dead time of 3 s is given. Calculate the transfer function and draw the block diagram of the filter.
Answer:

$$\begin{aligned} G(z) = \frac{Y(z)}{U(z)} &= (z+1)(z+2)z^{-5} \\ &= (z^2 + 3z + 2)z^{-5} \\ &= z^{-3} + 3z^{-4} + 2z^{-5} \end{aligned}$$

Alternative solution:

$$\begin{aligned} G(z) &= (1 - z^{-1})(1 - 0.5z^{-1})z^{-3} \\ &= (1 - \frac{3}{2}z^{-1} + 0.5z^{-2})z^{-3} \\ &= z^{-3} - \frac{3}{2}z^{-4} + 0.5z^{-5} \end{aligned}$$



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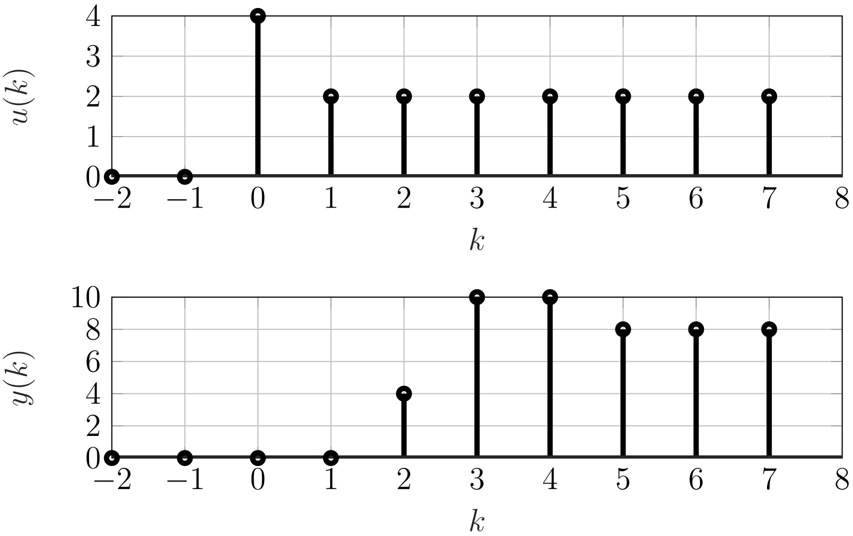
$\sum$ 17

Task 7: System Identification FIR (12 Points)

The following figure shows the response $y(k)$ of a unknown system to the input signal $u(k)$.



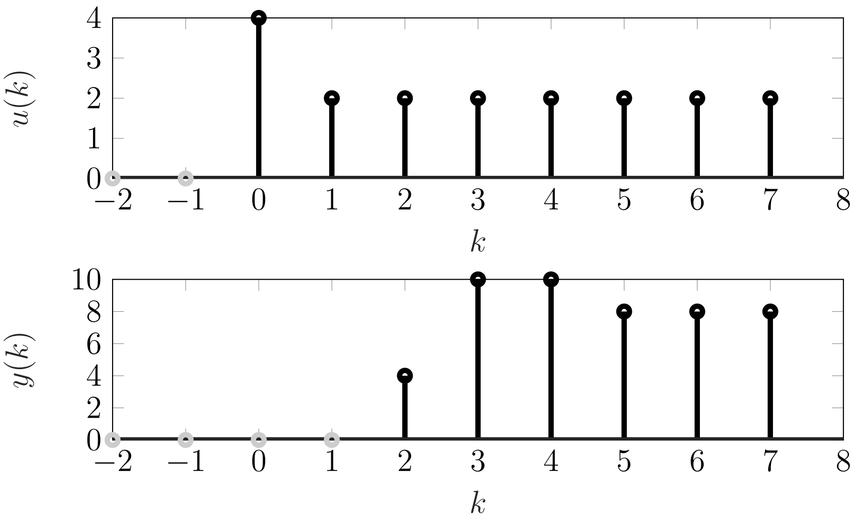
The system can be described by an FIR filter of finite order. Goal of the task is to determine the coefficients of the FIR filter using the given response.

a) The given system is causal and has no direct feedthrough. Additionally it contains a dead time. What is the dead time of the system? Explain your answer.

Answer: Whether a system has a direct feedthrough can be seen by the fact that the system reacts immediately to a change of the input signal at the same time step. This is not the case for the given signals. Furthermore the system even does not react at the second time step after the input signal is changed. The first change of the output signal takes place at the third time step after the input signal is changed. Therefore, the system contains a dead-time of one time step.

$$T_t = 1 \cdot T_0$$

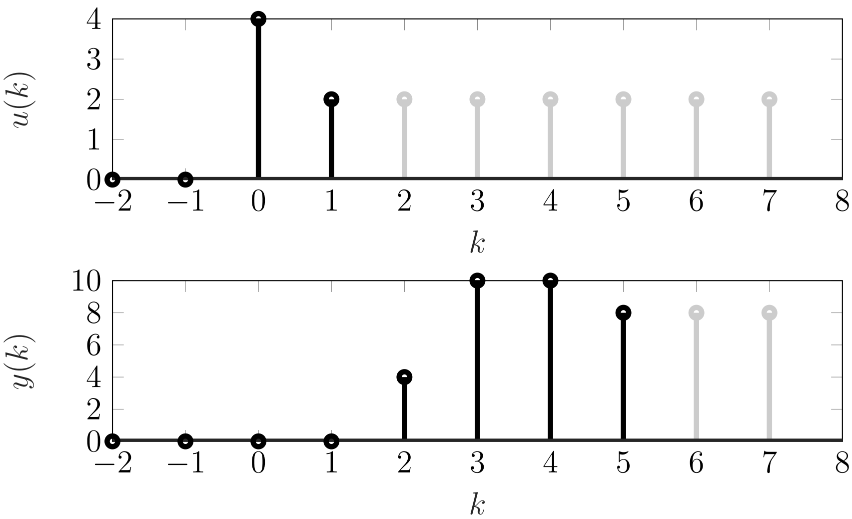
This can be seen from the following figure.



1

b) The system can be represented in the form of an FIR filter. Determine the order of the FIR filter leading to the system response shown. Explain your answer.

Answer: The order of the FIR filter describing the system can be determined by the time steps required for the system output to reach a constant value after the system input is no longer changed.



From the figure it can be seen that the system does not change its output $y(k = 5) = 10$ from time step $k = 5$. The input variable $u(k)$ is constant from time step $k = 1$ onward. Due to the dead-time of one time step, the FIR filter must have an order of $n_{FIR} = 5 - 1 - 1 = 3$.

1

c) Give the difference equation of an FIR filter with the order determined in part b). Calculate the coefficients of this filter based on the given system response.

Answer: The FIR filter, which describes the system, has order $n_{FIR} = 3$ and also has a dead-time of one time step. This results in the following difference equation of the FIR filter:
standard filter: $y(k) = b_0 u(k) + b_1 u(k-1) + b_2 u(k-2) + b_3 u(k-3) + b_4 u(k-4)$

No direct feedthrough: $b_0 = 0$

Dead time of one time step: $b_1 = 0$

$\Rightarrow y(k) = b_2 u(k-2) + b_3 u(k-3) + b_4 u(k-4)$
To determine the coefficients of the FIR filter, the values of the system response can be substituted into the difference equation. Since the values of the input variable $u(k < 0) = 0$, the coefficients can be determined step-wise.
For $k = 2$:

$$y(2) = 4 \qquad u(0) = 4 \qquad u(-1) = 0 \qquad u(-2) = 0$$

$$\begin{aligned} y(2) &= b_2u(0) + b_3u(-1) + b_4u(-2) \\ &= b_2 \cdot 4 + b_3 \cdot 0 + b_4 \cdot 0 \\ &= b_2 \cdot 4 \\ \rightarrow b_2 &= \frac{4}{4} = 1 \end{aligned}$$

For $k = 3$

$$y(3) = 10 \qquad u(1) = 2 \qquad u(0) = 4 \qquad u(-1) = 0$$

$$\begin{aligned} y(3) &= b_2u(1) + b_3u(0) + b_4u(-1) \\ &= 1 \cdot 2 + b_3 \cdot 4 + b_4 \cdot 0 \\ &= 2 + b_3 \cdot 4 \\ \rightarrow b_3 &= \frac{10-2}{4} = 2 \end{aligned}$$

For $k = 4$

$$y(4) = 10 \qquad u(2) = 2 \qquad u(1) = 2 \qquad u(0) = 4$$

$$\begin{aligned} y(2) &= b_2u(2) + b_3u(1) + b_4u(0) \\ &= 1 \cdot 2 + 2 \cdot 2 + b_2 \cdot 4 \\ &= 2 + 4 + b_4 \cdot 4 \\ \rightarrow b_4 &= \frac{10-2-4}{4} = 1 \end{aligned}$$

This results in the following difference equation:

$$y(k) = 1u(k-2) + 2u(k-3) + 1u(k-4)$$

$\boxed{5}$

d) Give the transfer function of the determined filter.
If you were unable to determine a filter, use the following difference equation to determine the transfer function:

$$y(k) = 3u(k) + 3u(k-1) + 5u(k-2) + 2u(k-3)$$

Answer: To determine the transfer function, the difference equation can be transformed into the z-domain.

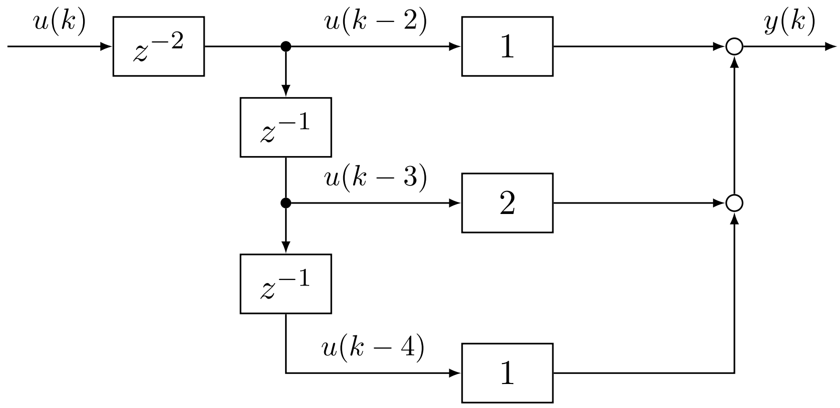
$$\begin{aligned} y(k) &= 1u(k-2) + 2u(k-3) + 1u(k-3) \\ &\circ\!\!-\!\!\bullet \quad Y(z) = U(z)z^{-2} + 2U(z)z^{-3} + U(z)z^{-4} \end{aligned}$$

By rearranging the transformed equation, the transfer function of the FIR filter is obtained.

$$\rightarrow G(z) = \frac{Y(z)}{U(z)} = z^{-2} + 2z^{-3} + z^{-4} = z^{-2}(1 + 2z^{-1} + 1z^{-2})$$

$\boxed{2}$

e) Draw the block diagram of the FIR filter determined.
Answer: A possible block diagram is shown in the following figure:



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$\sum$ 12