Sensorics Exam

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12th of September 2014

Name:	
Mat.-No.:	
Grade:	

Task:	T1	T2	T3	T4	T5	T6	T7	T8	Sum
Scores:	13	17	24	10	20	12	14	10	120
Accomplished:									

Task 1: **Comprehension Questions**

Mark the correct answers clearly.
Every question has one or two correct answers!
For every correctly marked answer you will get one point. If there is one correct answer marked and one incorrect answer marked, you will get no point for that subtask.

a) Torque . . .
- ☐ . . . and energy have identical units.
- ☐ . . . and energy are the same.
- ☐ . . . is one of the seven SI basic units.

b) The reactive power . . .
- ☐ . . . has a mean value equal to zero.
- ☐ . . . has mean value unequal to zero.
- ☐ . . . is the part of the apparent power that can perform work.

c) PTC resistance thermometers . . .
- ☐ . . . measure faster than thermocouples.
- ☐ . . . have an increasing resistance with decreasing temperature.
- ☐ . . . have almost a linear characteristic.

d) The piezoelectric force measurement ...
- ☐ ... is well suited for fast dynamic measurement.
- ☐ ... can only be used to measure stationary forces.
- ☐ ... is used to measure forces on fluids to calculate the dynamic pressure.

e) The magnetic inductive flow measurement ...
- ☐ ... has a nonlinear characteristic.
- ☐ ... is not suitable for corrosive fluids and fluids that contain solids.
- ☐ ... requires no internal construction.

f) Assess the following statements regarding the Student's t-distribution.
- ☐ It is used if the real standard deviation has to be estimated from data.
- ☐ Its confidence interval $1 - \alpha$ is wider than the confidence interval of the corresponding normal distribution.
- ☐ If the size N of the regarded data set is big enough ($N \to \infty$), it converges to the cauchy distribution.

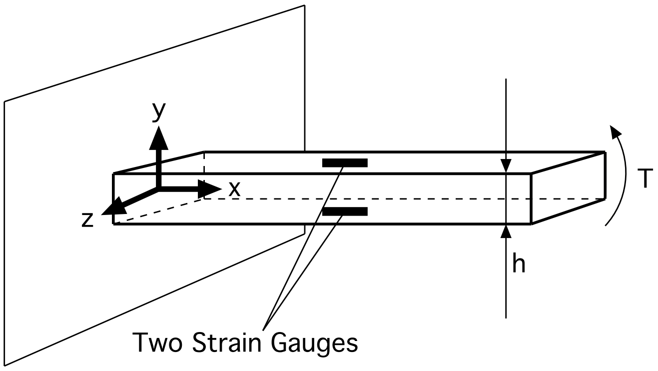
g) The median filter ...
- ☐ ... is commonly used to eliminate outliers.
- ☐ ... is a nonlinear filter.
- ☐ ... calculates the arithmetic average.

h) According to the Shannon theorem ...
- ☐ ... nonlinear filters can be approximated by linear filters by using feedback-control.
- ☐ ... the highest significant component of the signal f_{max} must not be higher than the Nyquist frequency $f_n = \frac{f_0}{2}$.
- ☐ ... the quantization error is reduced if the sampling time is increased.

i) Assess the following statements regarding parametric methods.
- ☐ Only a few number of parameters n are estimated using a relatively large number of data samples N ($n << N$).
- ☐ An FIR system is a parametric approach.
- ☐ Their parameters have no direct physical meaning or interpretation.

j) Assess the following statements regarding frequency analysis via DFT.
- ☐ The leakage effect happens if the time signal has a discontinuity.
- ☐ The leakage effect and the picket fence effect eliminate each other.
- ☐ The picket fence effect happens if the frequency of the given periodic signal does not exactly exist in the frequency discretization.

Task 2: Measurement of Torque



In the picture above you can see a beam, that is loaded with a torque T on one end. The other end is fixed to a wall and two strain gauges are applied to the beam to measure the torque. The following equations are given:

$$|\rho| = \frac{EI}{|T|} \quad ; \quad \Delta R = R_0 K \epsilon \quad ; \quad \epsilon = \frac{y}{|\rho|}$$

ρ: Curvature of the deflected beam; E: Young's modulus; I: Second moment of area; T: Torque (constant over the beam length); ΔR: The change in resistance of one strain gauge; R_0: The resistance of the strain gauge without any applied torque; K: Sensitivity of the strain gauges; ϵ: Deformation of the strain gauge, h: Beam height; y: Coordinate of the beam height, that starts exactly at the half of the beam height.

a) Evaluate the equation that describes the change of the resistances ΔR depending on the applied torque T. What relationship does the change of the resistance of the upper and the lower strain gauge have?

b) Sketch a bridge circuit that can be used to transform the change in resistance ΔR into a voltage U_d and derive the corresponding equation (Just use ΔR for the change of resistance - **NOT** the equation from a!).

c) In what way can the sensitivity of the torque measurement be increased through a change in the bridge circuit?

Task 3: Discrete-Time Systems

Two filter transfer functions are given

$$G_1(z) = \frac{(z^2 + z^1 + 1)}{3z^2} \quad \text{and}$$
$$G_2(z) = \frac{(z^2 + z^1 + 1)}{3z} \ .$$

For the following investigations the sampling time is $T_0 = 1/4$ seconds.

a) Determine the difference equations for both filters $G_1(z)$ and $G_2(z)$.

b) Sketch one block-diagram for each of the two filters $G_1(z)$ and $G_2(z)$, that describes the corresponding system correctly.

c) Calculate the poles of the filters $G_1(z)$ and $G_2(z)$ in the z-domain.

d) Calculate the location of the poles found in subtask c) in the s-domain.

e) Calculate the gain for both filters with the help of the final value theorem. In general the final value theorem for a signal $x(k)$ is defined as follows:

$$x(k \to \infty) = (z-1) \lim_{z \to 1} X(z) \ .$$

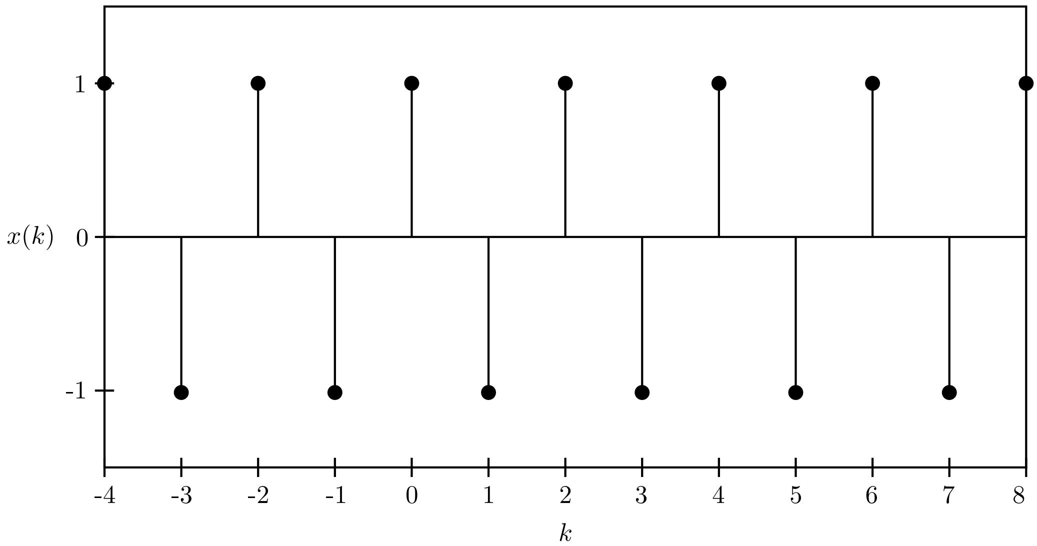
f) State for each filter, if it is a causal or an acausal system. Explain your choices shortly.

g) Calculate each filter's phase depending on the frequency ω.

h) Sketch the phase-responses for both filters. Up to which maximum frequency ω_{max} should the phase-responses be sketched when dealing with discrete-time systems?

Task 4: Discrete Fourier-Transformation

The following periodic sequence $x(k)$ is given.



a) After how many values N does the series repeat?

b) First derive the equation system, that has to be solved to achieve the discrete Fourier-Transform X(n), and than express the equation system with matrix-vector notation $\underline{X} = \underline{F}\,\underline{x}$.
Use in both cases the abbreviation $W_N = e^{-i\frac{2\pi}{N}}$.
For the equation system use a general signal $x(k)$ and for the matrix-vector notation use a general signal vector $\underline{x}$.

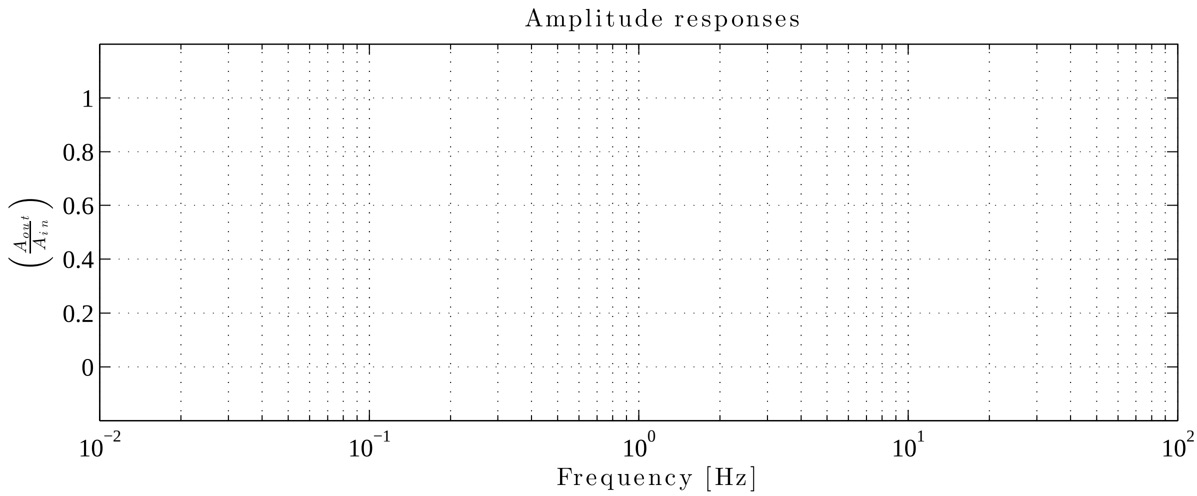
c) Calculate all required powers of W_N and plot them in the complex plane.

d) Now use the given signal and the calculated powers of W_N to calculate the discrete amplitude spectrum $|X(n)|$.

Task 5: Filter

a) There are two filters G_{LP} and G_{HP}, that should be compared. One filter is a low-pass G_{LP}, the other is a high-pass G_{HP}. The limit (cut-off) frequency of filter G_{LP} is $f_{LP} = 2$ Hz, and the limit (cut-off) frequency of filter G_{HP} is $f_{HP} = 1$ Hz.

Sketch the ideal amplitude responses of both filters G_{LP} and G_{HP} in the figure below. Label your sketched amplitude responses with the correct transfer function.



The following subtasks are completely independent from subtask a), i.e. the filters that will be defined in the following subtasks have nothing to do with the ones from subtask a).

b) Two filter transfer functions are given:

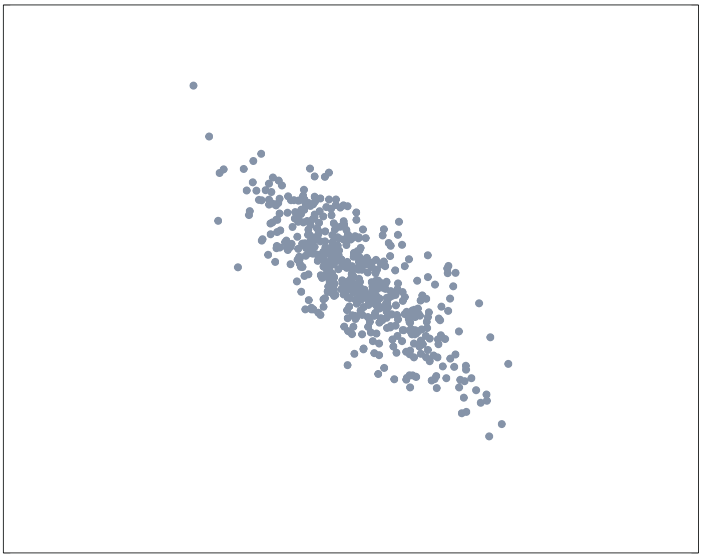
$$G_1(z) = \frac{1}{3}\left(z^{-2} + z^{-5} + z^{-8}\right) \text{ and}$$
$$G_2(z) = \frac{1}{3}\left(z^{-2} - \frac{1}{2}z^{-5} - \frac{1}{2}z^{-8}\right) .$$

Calculate and sketch the step responses of both filters $G_1(z)$ and $G_2(z)$ for a sampling time $T_S = 1/8$ seconds.

c) Assign the correct designation to each of the two filters $G_1(z)$ and $G_2(z)$. Choose from the following designations: High-pass and low-pass.

<u>Task 6:</u> Principal Component Analysis (PCA) and Clustering

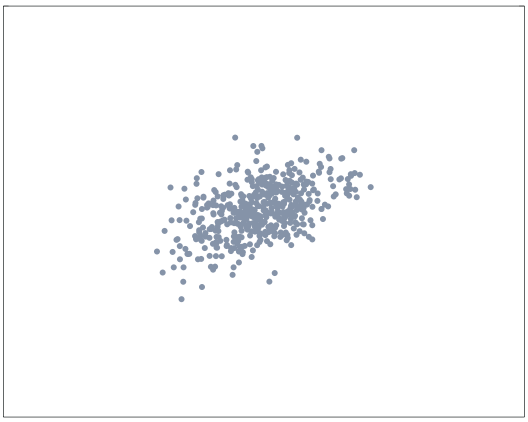
a) The following data distribution is given:



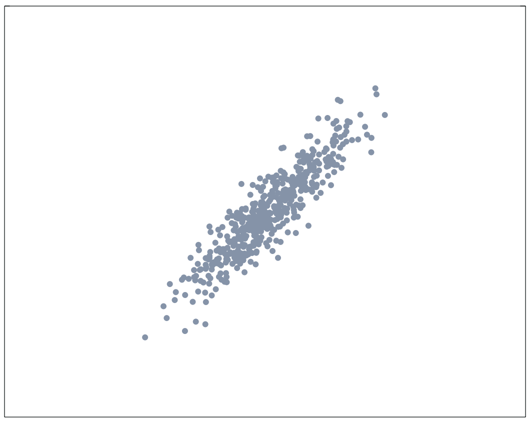
Sketch the first and second principal component **qualitatively correct** in the figure above and label **clearly** which of the two sketch compounds represents the first and the second principal component.

b) Now two data distributions are given:

a)



b)

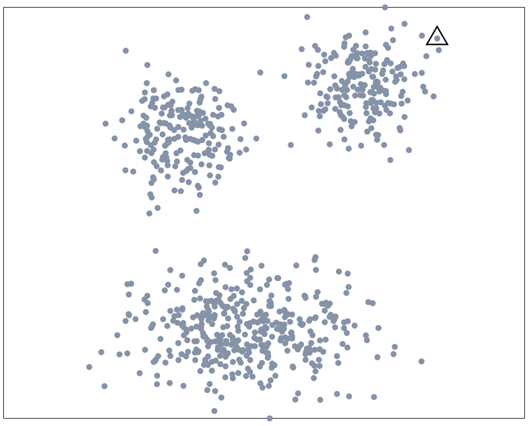


Which distribution is better suited for dimensionality reduction, a) or b) ? A short explanation is required.

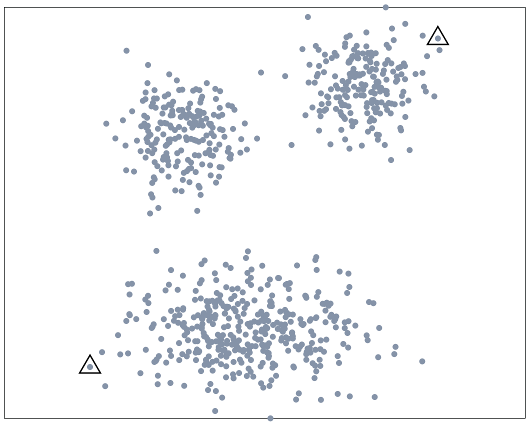
c) Another data distribution is given. It should be grouped via the k-means algorithm with different number of clusters. The initial centers of the clusters are marked by the rectangles.

Draw the resulting centers of the clusters in the given figures. Why must the start values for the calculation be provided?

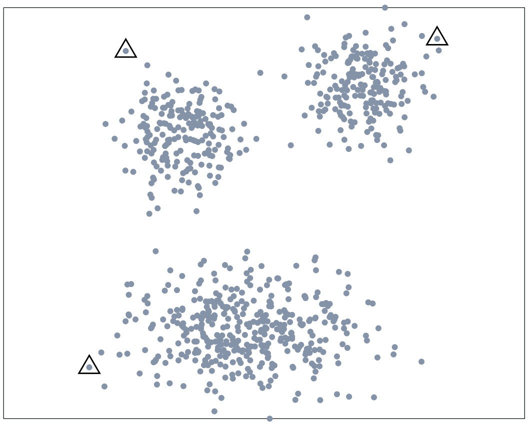
1)



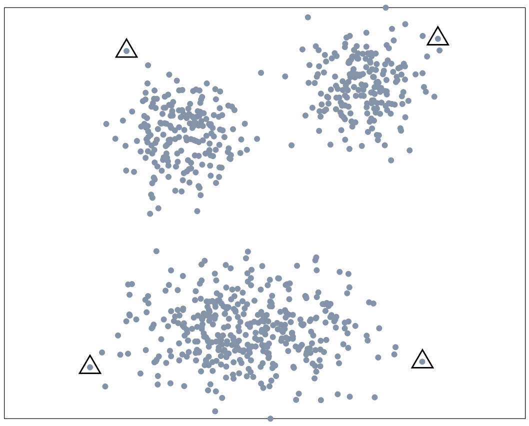
2)



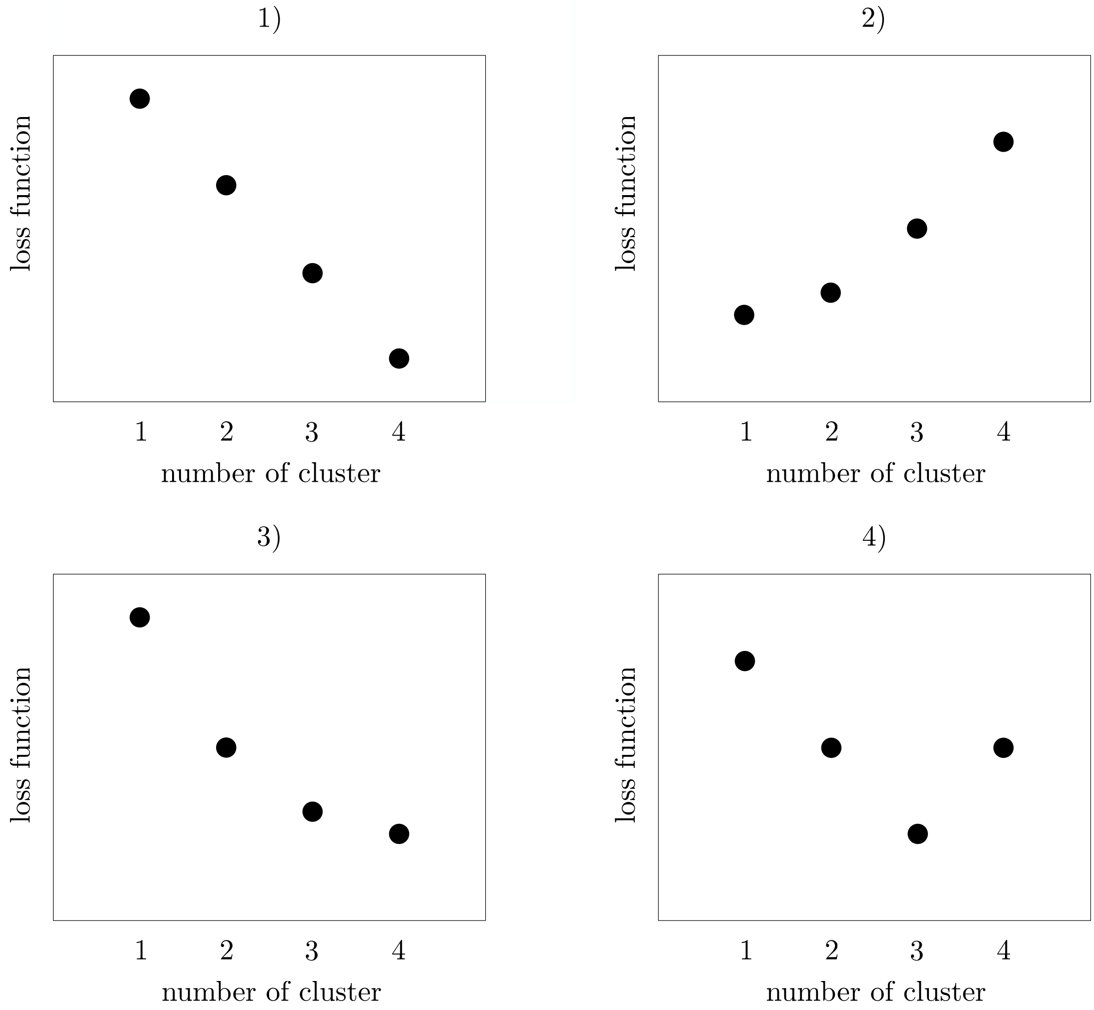
3)



4)



d) Four possible loss function curves of the k-means algorithm over the number of clusters are given. Choose which results from the example given in c).



Task 7: Probability Density Functions

a) Sketch **three** normal distributions into **one** coordinate system, that have the following properties:

I True mean value $\mu_1 = 0$, Standard deviation $\sigma_1 = 1$.

II True mean value $\mu_2 = -5$, Standard deviation $\sigma_2 = 2$.

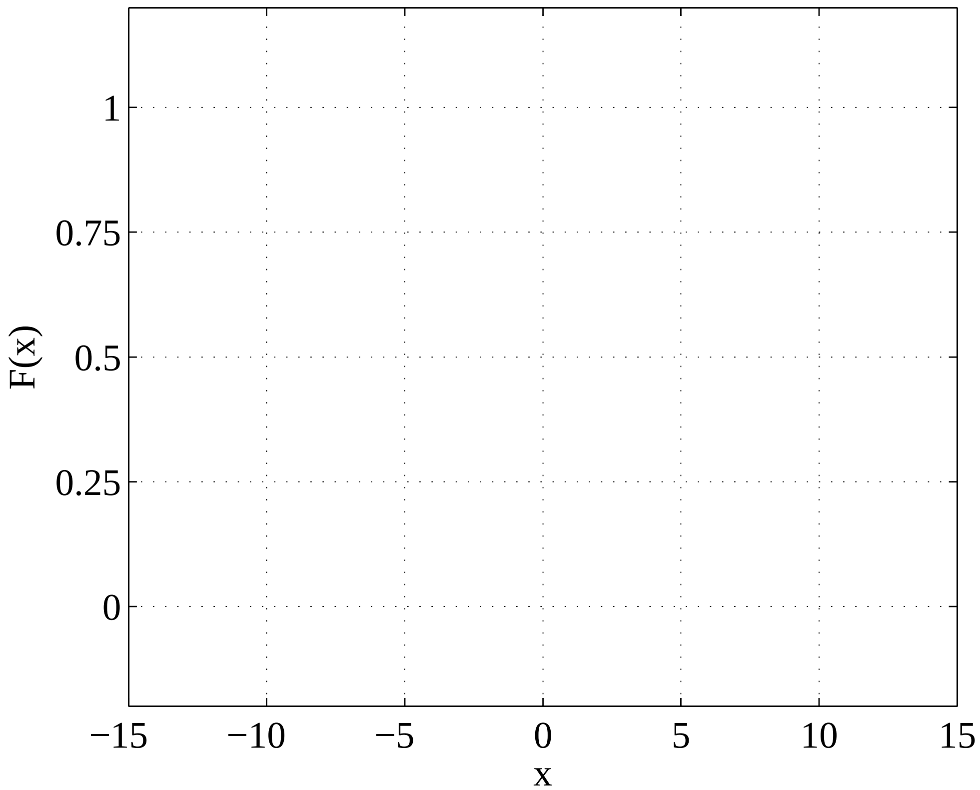
III True mean value $\mu_3 = 5$, Standard deviation $\sigma_3 = 4$.

Assign each case (I, II and III) to the corresponding (sketched) normal distribution.

b) Sketch the cumulative distribution functions (cdf), that correspond to the probability density functions (pdf) defined in subtask a) I, II and III.

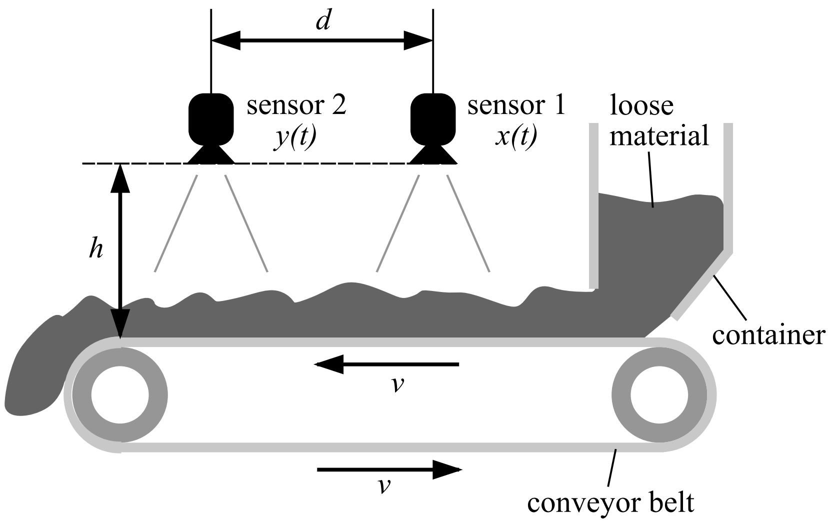
Hint: The cumulative distribution function $F(x)$ is the integral of the probability density function $f(x)$ with respect to the random variable x:

$$F(x) = \int_{-\infty}^{\infty} f(x)\mathrm{d}x \ .$$



Mostly, sketches that are qualitatively correct are sufficient to achieve the maximum score for this task, but characteristic points should be as exact as possible. Furthermore relative differences between the three functions should be clear.

Task 8: **Measurement Errors and Correlation Analysis**



The shown conveyor belt carries loose material from the container to the left. Two distance-sensors measure the hight of the loose material on the belt and store the hight together with the corresponding time, such that the signal $x(t)$ from sensor 1 and signal $y(t)$ from sensor 2 is available. The distance between the two sensors d is known as well as the hight h between the sensors and the surface of the belt. It is assumed, that the loose material is carried with the same constant velocity v the conveyor belt is running. A correlation analysis between $x(t)$ and $y(t)$ is used to determine the time interval Δt after which the same loose material traveled the distance d. With the help of d and Δt the velocity can be calculated.

Now some error scenarios should be investigated.

a) Assume sensor 2 is not correctly installed, such that the height above the conveyor belt's surface is different from the one of sensor 1. Does this affect the correct determination of the time interval Δt? Explain your answer shortly.

b) Now assume that the distance d varies due to inaccurate assembling, such that the real distance d_{real} between the two sensors is $d_{real} = d \pm \Delta d$. Additionally the time interval is not determined correctly, such that the measured value Δt_m differs from the real one Δt_{real}:
$\Delta t_m = \Delta t_{real} \pm \delta t$.

   I Derive the equation to calculate the Gaussian error propagation of the velocity Δv, if the sign of the deviations Δd and δt is unknown a priori.

   II What is the worst-case scenario, i.e. what leads to biggest error in the velocity measurement, considering the sign of Δd and δt.

c) Now assume that the wires of the sensors are interchanged, which leads to the result, that $x(t)$ is the signal from sensor 2 and $y(t)$ is the signal from sensor 1. What error will emerge in the velocity measurement?

Solutions:

Task 1: **Comprehension Questions**

a) Torque ...
- ■ ... and energy have identical units.
- ☐ ... and energy are the same.
- ☐ ... is one of the seven SI basic units.

b) The reactive power ...
- ■ ... has a mean value equal to zero.
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c) PTC resistance thermometers ...
- ☐ ... measure faster than thermocouples.
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■ … is well suited for fast dynamic measurement.

□ … can only be used to measure stationary forces.

□ … is used to measure forces on fluids to calculate the dynamic pressure.

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□ … has a nonlinear characteristic.

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■ It is used if the real standard deviation has to be estimated from data.

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■ … is commonly used to eliminate outliers.

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h) According to the Shannon theorem …

□ … nonlinear filters can be approximated by linear filters by using feedback-control.

■ … the highest significant component of the signal f_{max} must not be higher than the Nyquist frequency $f_n = \frac{f_0}{2}$.

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■ The leakage effect happens if the time signal has a discontinuity.

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■ The picket fence effect happens if the frequency of the given periodic signal does not exactly exist in the frequency discretization.

$\boxed{\sum 15}$

<u>Task 2:</u> Measurement of Torque

a) Evaluate the equation that describes the change of the resistances ΔR depending on the applied torque T. What relationship does the change of the resistance of the upper and the lower strain gauge have?

$$\Delta R = R_0 K \epsilon \tag{1}$$

$$|\rho| = \frac{EI}{|T|} \tag{2}$$

$$\epsilon = \frac{y}{|\rho|} \tag{3}$$

(2) in (3):

$$\epsilon = \frac{y|T|}{EI} \tag{4}$$

(4) in (1):

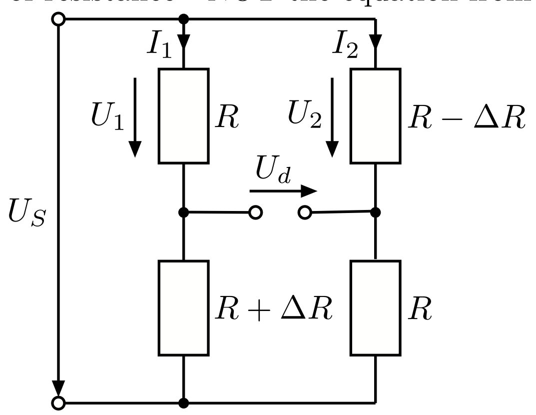
$$\Delta R = \frac{R_0 K y |T|}{EI} \tag{5}$$

4

Upper strain gauge ($y = +\frac{h}{2}$): $\Delta R_U = \frac{R_0 K h |T|}{2EI} \widehat{=} \Delta R$
Lpper strain gauge ($y = -\frac{h}{2}$): $\Delta R_L = -\frac{R_0 K h |T|}{2EI} \widehat{=} - \Delta R$

Relationship: $\Delta R_U = -\Delta R_L$ 2

b) Sketch a bridge circuit that can be used to transform the change in resistance ΔR into a voltage U_d and derive the corresponding equation (Just use ΔR for the change of resistance - NOT the equation from a!).



6

$$U_1 - U_2 + U_d = 0 \tag{6}$$

$$U_1 = R I_1 \tag{7}$$

$$U_2 = (R - \Delta R) I_2 \tag{8}$$

$$I_1 = \frac{U_S}{2R + \Delta R} \tag{9}$$

$$I_2 = \frac{U_S}{2R - \Delta R} \tag{10}$$

(9) in (7) and (10) in (8) :

$$U_1 = \frac{RU_S}{2R + \Delta R} \tag{11}$$

$$U_2 = \frac{(R - \Delta R)U_S}{2R - \Delta R} \tag{12}$$

(11) and (12) in (6):
(...)
$U_d = -\frac{\Delta R^2}{4R^2 - \Delta R^2} U_S$ $\boxed{4}$

c) In what way can the sensitivity of the torque measurement be increased through a change in the bridge circuit?

If two additional strain gauges are applied to the beam, they can be used to realize a full bridge circuit, which leads to an increase of the sensitivity by nearly a factor of two. $\boxed{1}$

$\boxed{\sum 17}$

Task 3: Discrete-Time Systems

a) Determine the difference equations for both filters $G_1(z)$ and $G_2(z)$.

$$G_1(z) = \frac{(z^2 + z^1 + 1)}{3z^2}$$
$$\Leftrightarrow G_1(z) = \frac{1}{3}\left(1 + z^{-1} + z^{-2}\right)$$

$$\circ\!\!-\!\!\bullet$$

$$y_1(k) = \frac{1}{3}u(k) + \frac{1}{3}u(k-1) + \frac{1}{3}u(k-2)$$

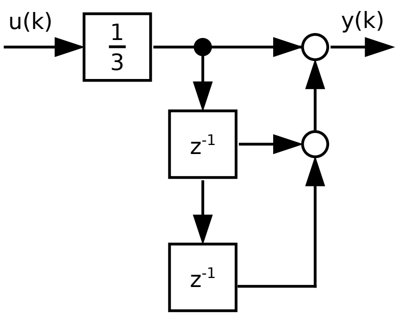
$$G_2(z) = \frac{(z^2 + z^1 + 1)}{3z}$$
$$\Leftrightarrow G_2(z) = \frac{1}{3}\left(z + 1 + z^{-1}\right)$$

$$\circ\!\!-\!\!\bullet$$

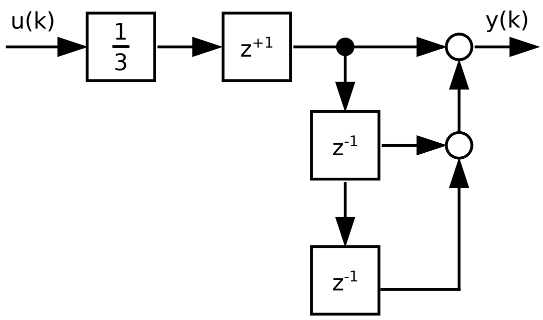
$$y_2(k) = \frac{1}{3}u(k+1) + \frac{1}{3}u(k) + \frac{1}{3}u(k-1)$$

$\boxed{4}$

b) Sketch one block-diagram for each of the two filters $G_1(z)$ and $G_2(z)$, that describes the corresponding system correctly.



(a) Block-diagram $G_1(z)$



(b) Block-diagram $G_2(z)$

$\boxed{4}$

c) Calculate the poles of the filters $G_1(z)$ and $G_2(z)$ in the z-domain.

$$\Leftrightarrow G_1(z) = \frac{(z^2 + z^1 + 1)}{3z^2}$$

$\Rightarrow$ Two poles at $z_{pG1} = 0$.

$$\Leftrightarrow G_2(z) = \frac{(z^2 + z^1 + 1)}{3z}$$

$\Rightarrow$ One pole at $z_{pG2} = 0$. $\boxed{2}$

d) Calculate the location of the poles found in subtask c) in the s-domain.

$$
\begin{aligned}
& z = \mathrm{e}^{sT_0} \\
\Leftrightarrow\ & s = \frac{1}{T_0} \ln z
\end{aligned}
$$

$$\Rightarrow s_{pG1} = s_{pG2} = -\infty.$$ 1

e) Calculate the gain for both filters with the help of the final value theorem.

$$
\begin{aligned}
h_1(t \to \infty) &= \lim_{z\to 1} (z-1) G_1(z) \qquad \underbrace{\Sigma(z)}_{z\text{-transform of an unit-step}} \\
&= \lim_{z\to 1} \cancel{(z-1)} \frac{(z^2 + z^1 + 1)}{3z^2} \frac{z}{\cancel{z-1}} \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
h_2(t \to \infty) &= \lim_{z\to 1} (z-1) G_2(z) \Sigma(z) \\
&= \lim_{z\to 1} \cancel{(z-1)} \frac{(z^2 + z^1 + 1)}{3z} \frac{z}{\cancel{z-1}} \\
&= 1
\end{aligned}
$$

2

f) State for each filter, if it is a causal or an acausal system. Explain your choices shortly.
$G_1(z)$: Causal filter, no future values are needed to calculate the system's current output.
$G_2(z)$: Acausal filter, future values are needed to calculate the system's current output. 2

g) Calculate each filter's phase depending on the frequency ω.

Calculation of the phase of $G_2(z)$:

$$
\begin{aligned}
& G_1(z) = \frac{(z^2 + z^1 + 1)}{3z^2} \\
\Leftrightarrow\ & G_1(z) = \frac{1}{3} \left(1 + z^{-1} + z^{-2}\right) \\
\Leftrightarrow\ & G_1(z) = \frac{1}{3} z^{-1} \left(z + 1 + z^{-1}\right) \\
\Leftrightarrow\ & G_1(z) = \frac{1}{3} z^{-1} \left(1 + z + z^{-1}\right)
\end{aligned}
\tag{13}
$$

For the phase calculation we set $s = i\omega$. The following equations arise: 1

$$
\begin{aligned}
& z^{\pm n} = \mathrm{e}^{\pm n s T_0} \\
\Leftrightarrow\ & z^{\pm n} = \mathrm{e}^{\pm i n \omega \cdot T_0} \\
\Leftrightarrow\ & z^{\pm n} = \cos\left(n\omega T_0\right) \pm i \sin\left(n\omega T_0\right) .
\end{aligned}
$$

1

With these relationships equation 13 becomes:

$$G_1(i\omega) = \frac{1}{3}\mathrm{e}^{-i\omega T_0}\left(1 + \underbrace{\mathrm{e}^{i\omega T_0} + \mathrm{e}^{-i\omega T_0}}_{=2\cos(\omega T_0)}\right) .$$

The part of the equation above in the brackets has only a positive real part. Therefore, the system's phase can directly be read:

$$\varphi_1(\omega) = -\omega T_0 .$$

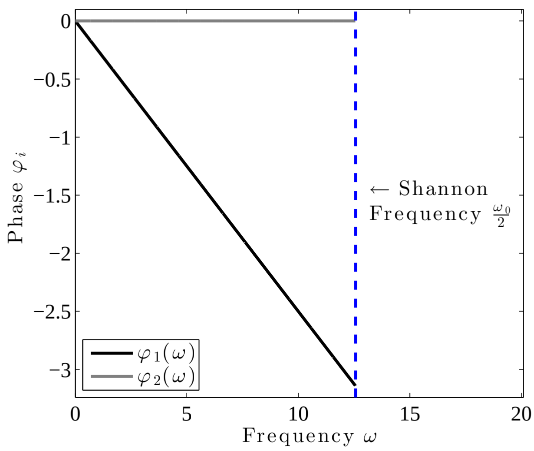
Calculation of the phase of $G_2(z)$: $\boxed{2}$

$$G_2(z) = \frac{(z^2 + z^1 + 1)}{3z}$$
$$\Leftrightarrow G_2(z) = \frac{1}{3}\left(z + 1 + z^{-1}\right)$$
$$\Leftrightarrow G_2(i\omega) = \frac{1}{3}\left(1 + \underbrace{\mathrm{e}^{i\omega T_0} + \mathrm{e}^{-i\omega T_0}}_{=2\cos(\omega T_0)}\right)$$

In the equation above no imaginary part exists and therefore the phase is $\varphi_2(\omega) = 0$.

$\boxed{2}$

h) Sketch the phase-responses for both filters. Up to which maximum frequency ω_{max} should the phase-responses be sketched when dealing with discrete-time systems?



$\boxed{2}$

Be aware of the x-axis' scaling. Usually the x-axis is scaled logarithmically, whereas here, a linear scaling is chosen.

The phase-response (as well as the amplitude response) should only be sketched up to the Shannon-frequency $\omega_{\max} = \frac{\omega_0}{2}$. For higher frequencies no statements can be made. $\boxed{1}$

$\boxed{\sum 24}$

Task 4: Discrete Fourier-Transformation

In general: $\text{DFT}\{x(k)\} = X(n) = \sum_{k=0}^{N-1} x(k) \cdot W_N^{nk}$ with $W_N = e^{-i\frac{2\pi}{N}}$.

a) The periodic, time discrete signal is $N = 2$ long: 1

$$\underline{\text{x}} = \begin{bmatrix} x(0) \\ x(1) \end{bmatrix} .$$

b) Therefore the equation for the discrete Fourier transformation is:

$$\text{DFT}\{x(k)\} = X(n) = \sum_{k=0}^{1} x(k) \cdot W_2^{nk} .$$

The equation system for achieving the Fourier transform $X(n)$ is:

$$X(0) = W_2^0 \cdot x(0) + W_2^0 \cdot x(1)$$
$$X(1) = W_2^0 \cdot x(0) + W_2^1 \cdot x(1) .$$ 2

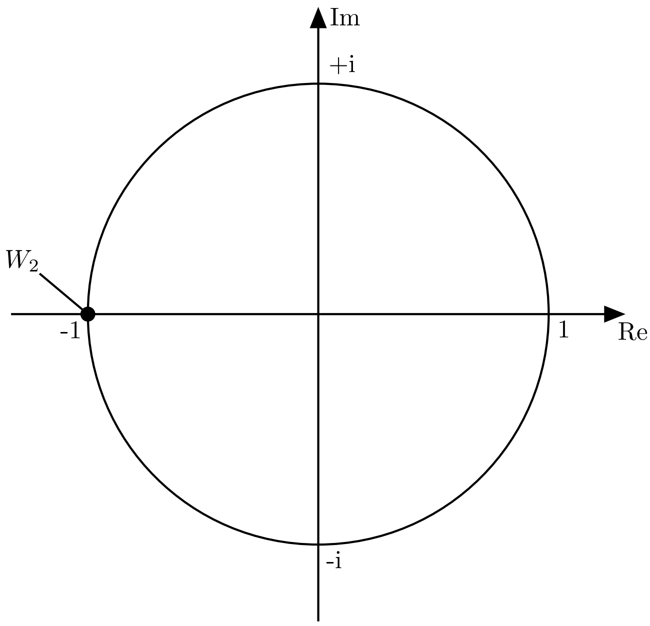
The same in matrix-vector notation:

$$\underbrace{\begin{bmatrix} X(0) \\ X(1) \end{bmatrix}}_{\underline{X}} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & W_2 \end{bmatrix}}_{\underline{F}} \cdot \underbrace{\begin{bmatrix} x(0) \\ x(1) \end{bmatrix}}_{\underline{x}} . \qquad (14)$$ 1

c) The only power of W_N to calculate is W_2:

$$W_2 = e^{-i\frac{2\pi}{2}1} = e^{-i\pi} = \cos(-\pi) + i \cdot \sin(-\pi) = -1 .$$ 1

Plot of them in the complex plane.



3

d) The given signal with periodic time $N = 2$ is:

$$\underline{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} .$$

Used in equation **??** it with $W_2 = -1$ leads to $\underline{X}$:

$$\underline{X} = \begin{bmatrix} 1 & 1 \\ 1 & (-1) \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1-1 \\ 1-(-1) \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} .$$ $\boxed{1}$

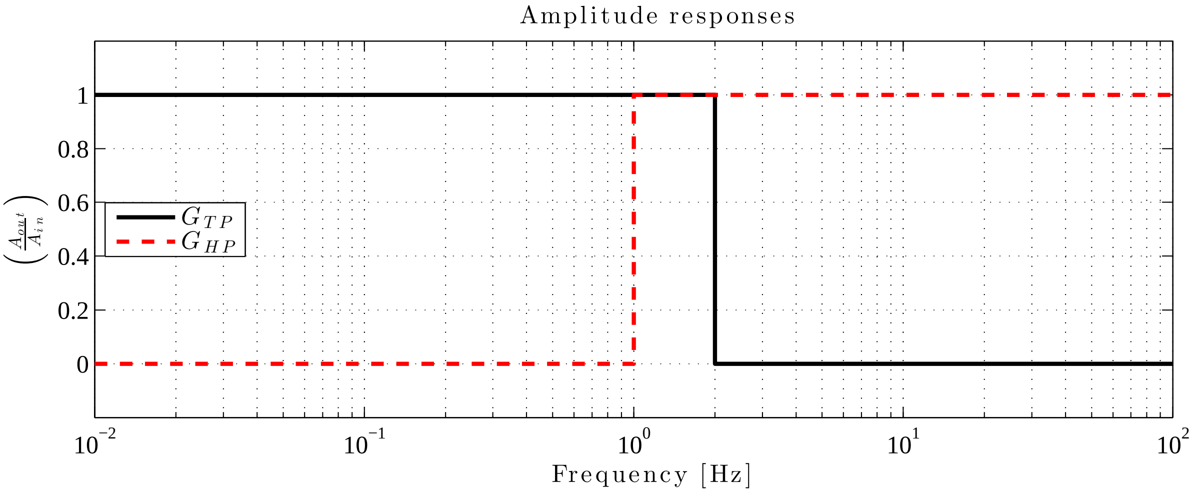
The discrete amplitude spectrum $|\underline{X}|$ is:

$$|\underline{X}| = \begin{bmatrix} |1| \\ |2| \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} .$$ $\boxed{1}$

$\boxed{\sum 10}$

Task 5: Filter

a) Sketch the ideal amplitude responses for both filters G_{LP} and G_{HP} in the figure below.



4

b) Calculate and sketch the step responses of both filters $G_1(z)$ and $G_2(z)$ for a sampling time $T_S = 1/8$ seconds.
Step: $\sigma(k) = 1$, für $k \geq 0$.
Calculation of the step response for $G_1(z)$:

$$G_1(z) = \frac{1}{3}\left(z^{-2} + z^{-5} + z^{-8}\right)$$

$$\bullet\!\!-\!\!\circ$$

$$y_1(k) = \frac{1}{3}\left(u(k-2) + u(k-5) + u(k-8)\right)$$

$$\Rightarrow h_1(k=0) = \frac{1}{3}\left(\sigma(-2) + \sigma(-5) + \sigma(-8)\right)$$

$$= 0$$

$$h_1(k = 2 \text{ bis } k = 4) = \frac{1}{3}$$

$$h_1(k = 4 \text{ bis } k = 7) = \frac{2}{3}$$

$$h_1(k = 8 \text{ bis } k = \infty) = 1$$

4

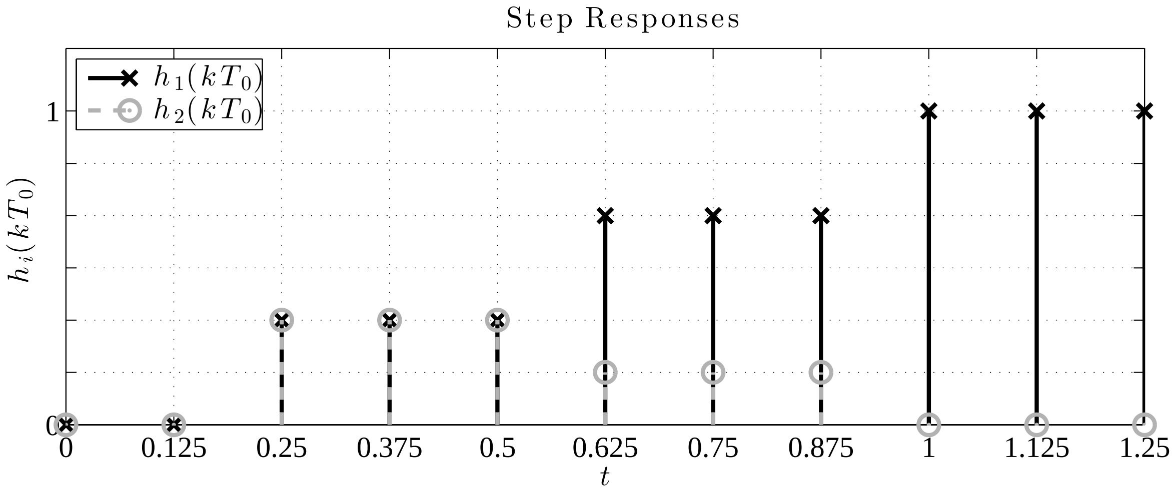
Calculation of the step response for $G_2(z)$:

$$G_2(z) = \frac{1}{3}z^{-2} - \frac{1}{6}z^{-5} - \frac{1}{6}z^{-8}$$

$$\begin{aligned}
y_1(k) &= \frac{1}{3}u(k-2) - \frac{1}{6}u(k-5) - \frac{1}{6}u(k-8) \\
\Rightarrow h_2(k=0) &= \frac{1}{3}\sigma(-2) - \frac{1}{6}\sigma(-5) - \frac{1}{6}\sigma(-8) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
h_2(k = 2 \text{ bis } k = 4) &= \frac{1}{3} \\
h_2(k = 4 \text{ bis } k = 7) &= \frac{1}{6} \\
h_2(k = 8 \text{ bis } k = \infty) &= 0
\end{aligned}$$

4



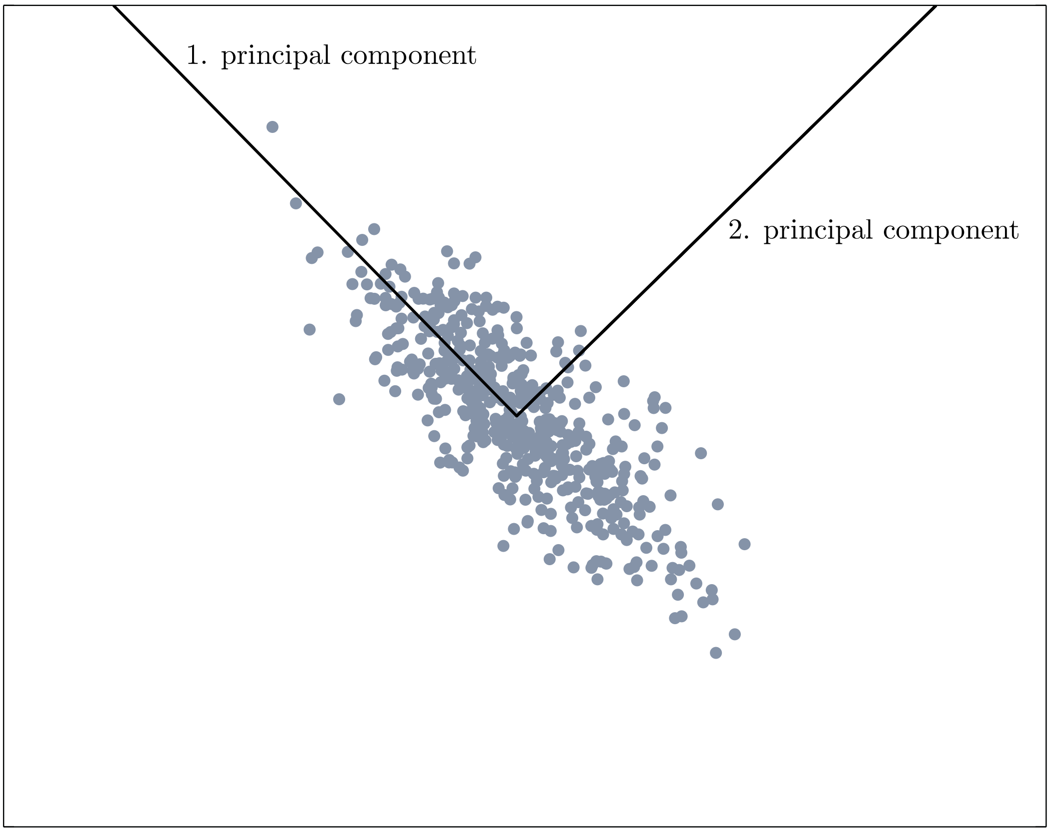
4

c) Assign the correct designation to each of the two filters $G_1(z)$ and $G_2(z)$.
$G_1(z)$: Low-pass $\Rightarrow$ low frequencies can pass the filter, i.e. a constant is maintained if a step is applied.
$G_2(z)$: High-pass $\Rightarrow$ only high frequencies can pass the filter, i.e. only where there is a change in the input signal, the filter's output is different from zero. 4

$\sum 20$

Task 6: Principal Component Analysis (PCA) and Clustering

a) The following is an exact solution. A qualitative correct solution is sufficient.

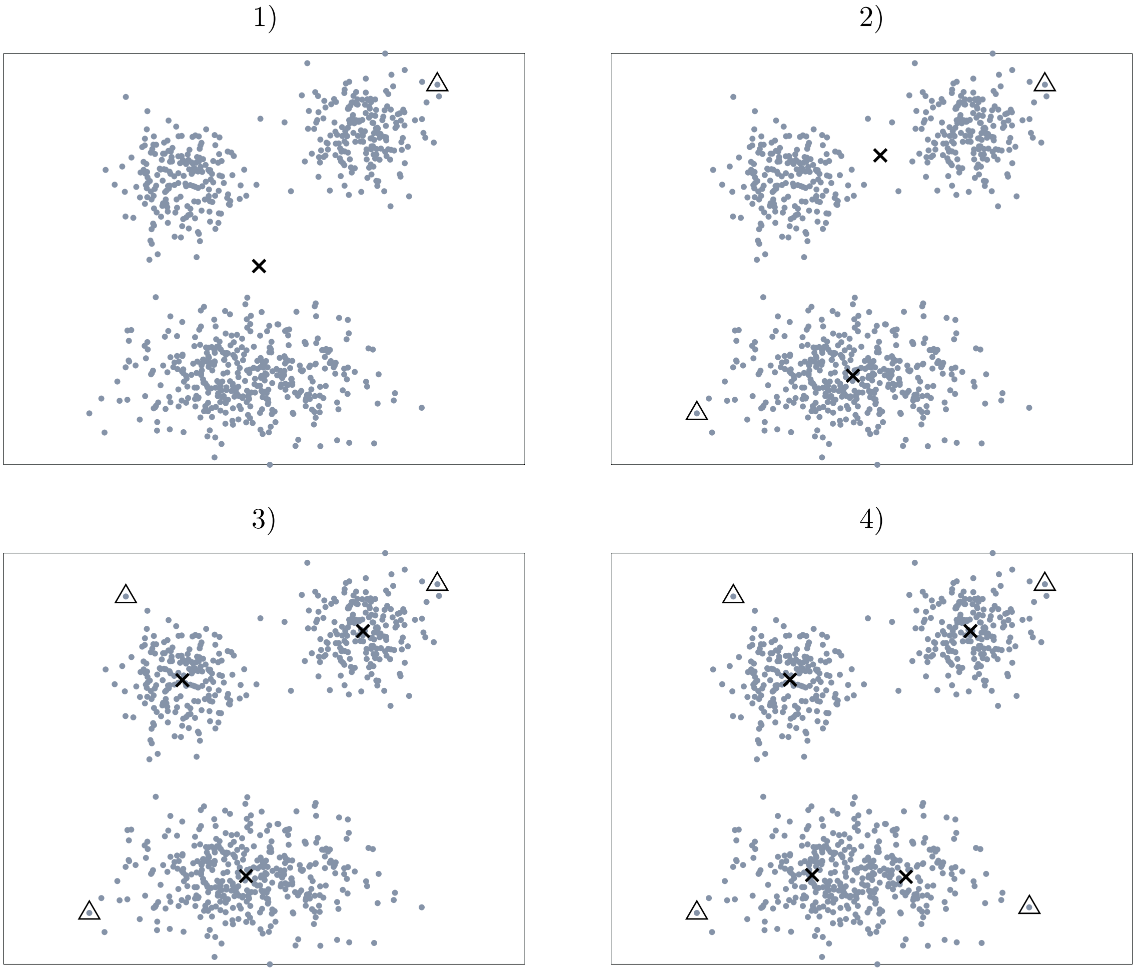


3

The angle und the order of the principal components are important. The principle components intersect in the mean value.

b) The data distribution in figure b) is better suited for dimensionality reduction, because the latitude along the first principal component is substantial bigger than the latitude of the second principal component. In figure a) the difference between the components is smaller. 2

c) The problem of k-means is its random initialization. It could happen, that the algorithm converges in a local optimum. Because of the given initial values, the solutions become qualitative unique. 1



4

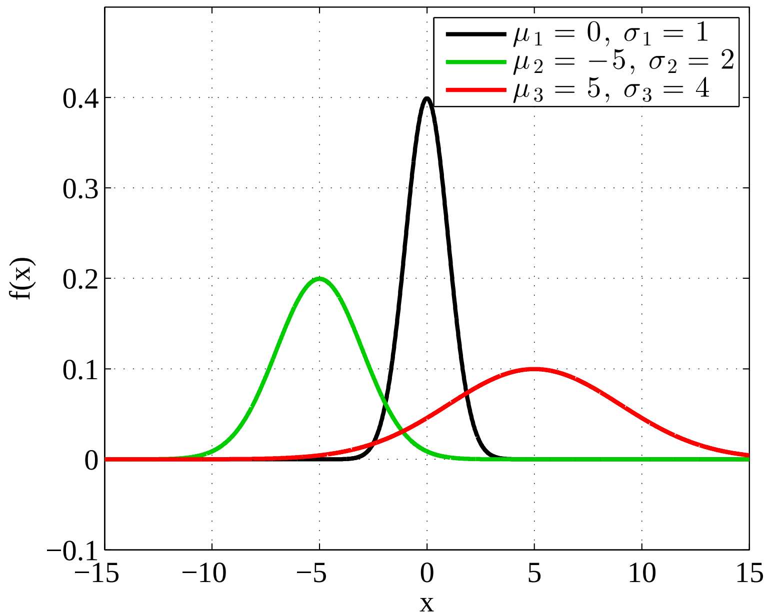
The first figure has only one Cluster. Every data point belongs to it and therefore the center is the mean value. The second figure has two clusters. With respect to the given initial values one center is right between the two upper conglomerations and the other center is in the middle of the lower conglomeration. Three clusters with the given initial values lead to centers in the middle of the three conglomerations. Regarding four clusters with the given initial values, two centers are respectively in the middle of the two upper conglomerations and the other two are inside the lower conglomeration.

d) An ascent of the loss function over the number of clusters is impossible using the k-means algorithm. Therefore the progressions in figure 2) and 4) are improper. The progression in figure 1) decreases linear over the number of clusters. For the given data distribution this is not possible, because the improvement using four instead of three clusters must be smaller than the improvement using three instead of two clusters. The progression in figure 3) is suitable, because the improvement of the loss function decreases with the rising number of clusters. 2

$\sum$ 12

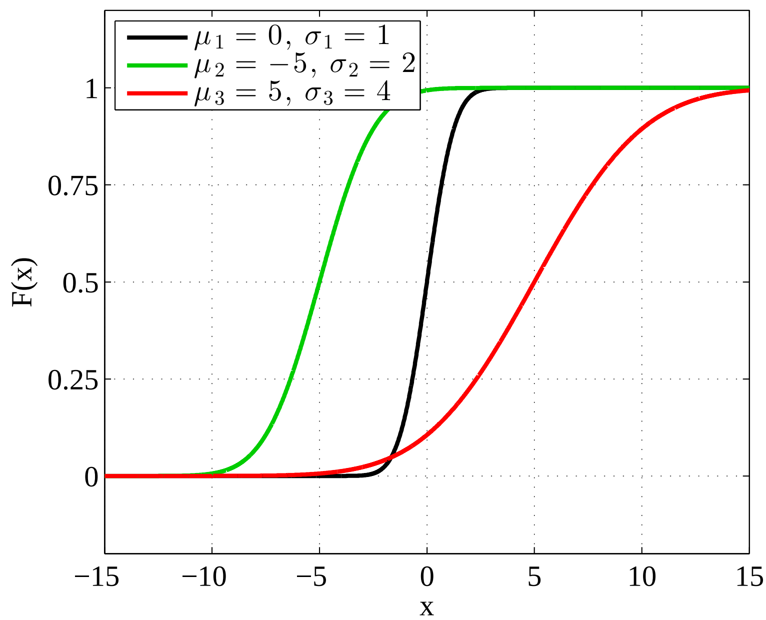
Task 7: **Stochastische Signale**

a) Sketch **three** normal distributions into **one** coordinate system.



7

b) Sketch the cumulative distribution functions (cdf), that correspond to the probability density function (pdf) defined in subtask a) I, II and III.



7

$\sum 14$

Task 8: Measurement Errors and Correlation Analysis

a) Assume sensor 2 is not correctly installed, such that the height above the conveyor belt's surface is different from the one of sensor 1. Does this affect the correct determination of the time interval Δt?

Through the correlation analysis, the similarity of signal $x(t)$ and $y(t)$ for different time shifts Δt is calculated. The time shift where the maximum similarity is determined should not be affected by a different hight of sensor 2, because the signal's shape does not change through the different hight. Only the level, i.e. the absolute values of maxima and minima differ. $\boxed{4}$

b) Now assume that the distance d is corrupted, such that the real distance between the two sensors is $d_{real} = d \pm \Delta d$. Additionally the time interval is not determined correctly, such that the measured value Δt_m differs from the real one Δt_{real}: $\Delta t_m = \Delta t_{real} \pm \delta t$.

I Derive the equation to calculate the Gaussian error propagation of the velocity Δv, if the sign of the deviations Δd and δt is unknown a priori.

$$\begin{aligned} v &= \frac{d}{\Delta t} \\ \Rightarrow \Delta v &= \left|\frac{\partial v}{\partial d}\Delta d\right| + \left|\frac{\partial v}{\partial \Delta t}\delta t\right| \\ &= \left|\frac{1}{\Delta t}\Delta d\right| + \left|-\frac{d}{\Delta t^2}\delta t\right| \end{aligned}$$

$\boxed{3}$

II What is the worst-case scenario, i.e. what leads to biggest error in the velocity measurement, considering the sign of Δd and δt.

Worst case: Δd and δt have opposite signs. $\boxed{2}$

c) Now assume that the wires of the sensors are interchanged, which leads to the result, that $x(t)$ is the signal from sensor 2 and $y(t)$ is the signal from sensor 1. What error will emerge in the velocity measurement?

The velocity's direction will be measured wrongly due to the fact that the maximum correlation will appear for negative time shifts Δt. $\boxed{1}$

$\boxed{\sum 10}$