Sensorics Exam

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31st of July 2018

Name:	
Mat.-No.:	
Grade:	

Task:	T1	T2	T3	T4	T5	T6	T7	T8	Sum
Scores:	14	14	16	16	14	14	15	17	120
Accomplished:									

Task 1: Comprehension Questions (14 points)

Mark the correct answers clearly.
Every question has one or two correct answers!
For every correctly marked answer you will get one point. If there is one correct answer marked and one incorrect answer marked, you will get no point for that subtask.

a) K-means clustering ...
- ☐ ... always converges, even with different initializations, to the same solution for one set of data.
- ☐ ... belongs to the supervised learning algoritms.
- ☐ ... can operate on input data without output data.

b) Filters with linear phase ...
- ☐ ... can only be achieved with IIR filters.
- ☐ ... can still have a nonlinear phase shift.
- ☐ ... are known as systems with a pure dead time.

c) PCA ...
- ☐ ... utilizes singular value decomposition.
- ☐ ... represents a nonlinear transformation.
- ☐ ... selects axes with smallest variance first.

d) Systematic errors ...
- [ ] ... cause a bias.
- [ ] ... can be compensated.
- [ ] ... occur randomly.

e) A median filter ...
- [ ] ... is the same as a mean filter.
- [ ] ... filters outliers out of the signal.
- [ ] ... can only be causal.

f) The moving coil mechanism ...
- [ ] ... can directly be used to measure AC quantities.
- [ ] ... can be used to measure voltage.
- [ ] ... can be used to measure current.

g) Assess the following statements regarding AC quantities.
- [ ] Apparent power and active power are never the same.
- [ ] Apparent power and active power are independent of the phase-shift between current and voltage.
- [ ] Reactive power cannot perform any work.

h) Assess the following statements regarding speed measurement. ...
- [ ] The doppler effect works with acoustic signals only.
- [ ] Speed cannot be obtained by integration of acceleration measurements.
- [ ] A generator can be used for speed measurement.

i) Assess the following statements regarding measurement of temperature.
- [ ] In PTC resistance thermometers (metal) the resistance shrinks with rising temperature.
- [ ] NTC Resistance themometers have a highly nonlinear chracteristic.
- [ ] Thermocouples are less accurate than PTC resistance thermometers.

j) Aliasing can occur...
- [ ] if an analogue signal is sampled too slow.
- [ ] if a digital signal is downsampled.
- [ ] if a digital signal is upsampled

Task 2: Temperature Measurement(14 points)

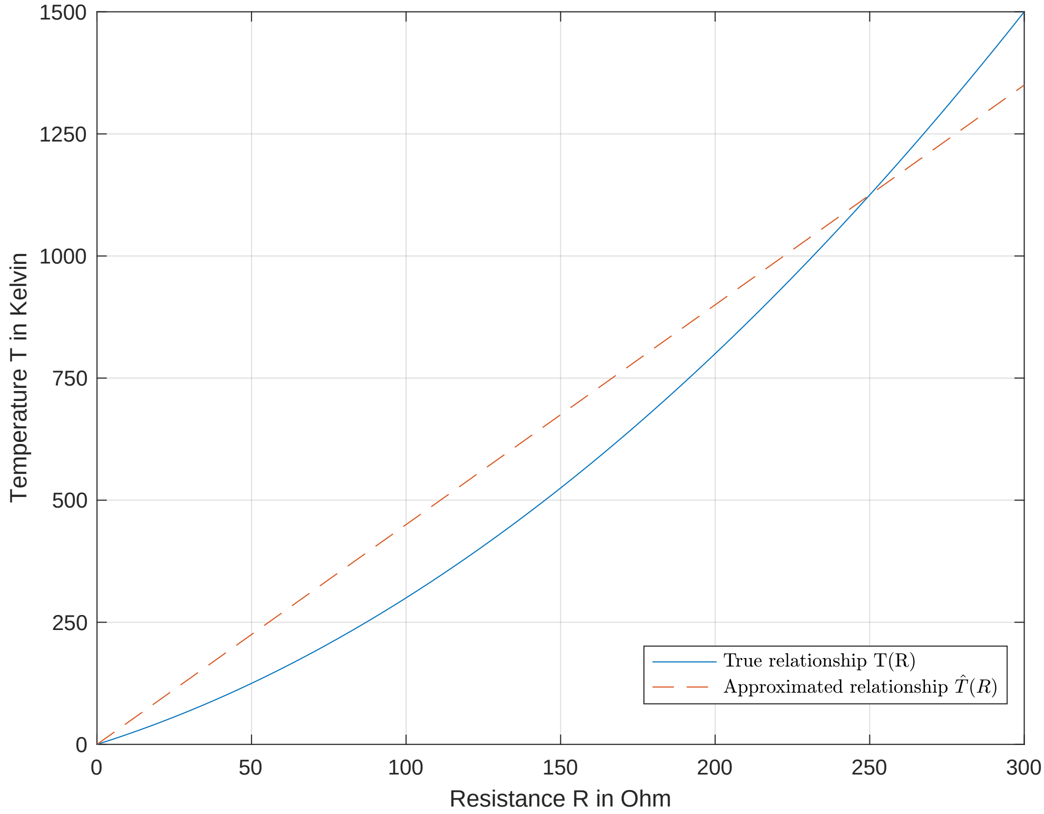


Figure 1: Relationship of the temperature of a sensor

The true relationship between the temperature T and the measured resistance R of a sensor are described by following equation:

$$T(R) = 2R + 0.01R^2 \tag{1}$$

One of the easiest simplifications to such equations is a linear approximation. The chosen approximation is:

$$T_a(R) = 4.5R \tag{2}$$

The graphs of both functions can be seen in Fig. 1.

a) What type of error occurs when using the approximated relationship instead of the real one?

b) At what value of the resistance R does the biggest absolute error occur (in the range $0\Omega < R < 300\Omega$)? Show your calculations (graphical estimation is not enough)!

c) What is the absolute relative Error at this point?

d) Now make an approximation around the operating point by utilizing the Taylor series expansion. Take the point of the biggest absolute error from subtask *b*) as operating point. If you were not able to identify that point, pick $R_0 = 200\Omega$ as operating point. Truncate the Taylor series expansion after the linear/affine part.
Hint: In general the Taylor series for a function $f(x)$ around an operating point x_0 is given by

$$Tf(x; x_0) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n \tag{3}$$

Here, $n!$ denotes the factorial of n and $f^{(n)}(x_0)$ denotes the n-th derivative of $f(x)$ evaluated at x_0.

e) Draw the graph of the Taylor approximation into Fig. 6.

f) In what case is this Taylor approximation better suited than the given approximation?

Task 3: Statistics

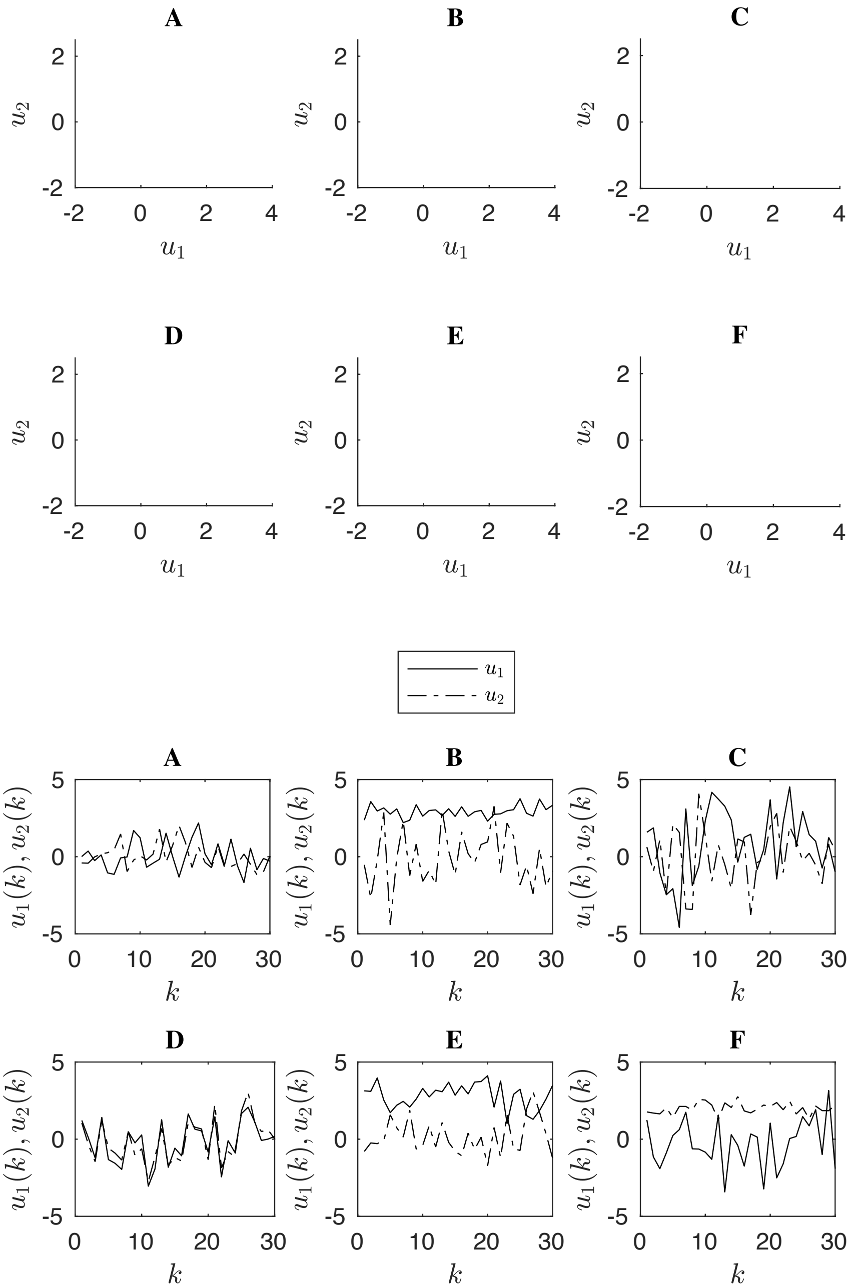


Figure 2: Signals u_1 and u_2 from different distributions.

a) Draw the two-dimensional normal distributions for the given signals into Fig. 2.

b) A two-dimensional normal distribution has perfectly circular shaped contour lines. What properties does the distribution posses?

c) This subtask is about the students distribution, also known as the t-distribution. Figure 3 shows a normal distribution for a set of data with mean $\mu = 0$ and standard deviation $\sigma^2 = 1$.

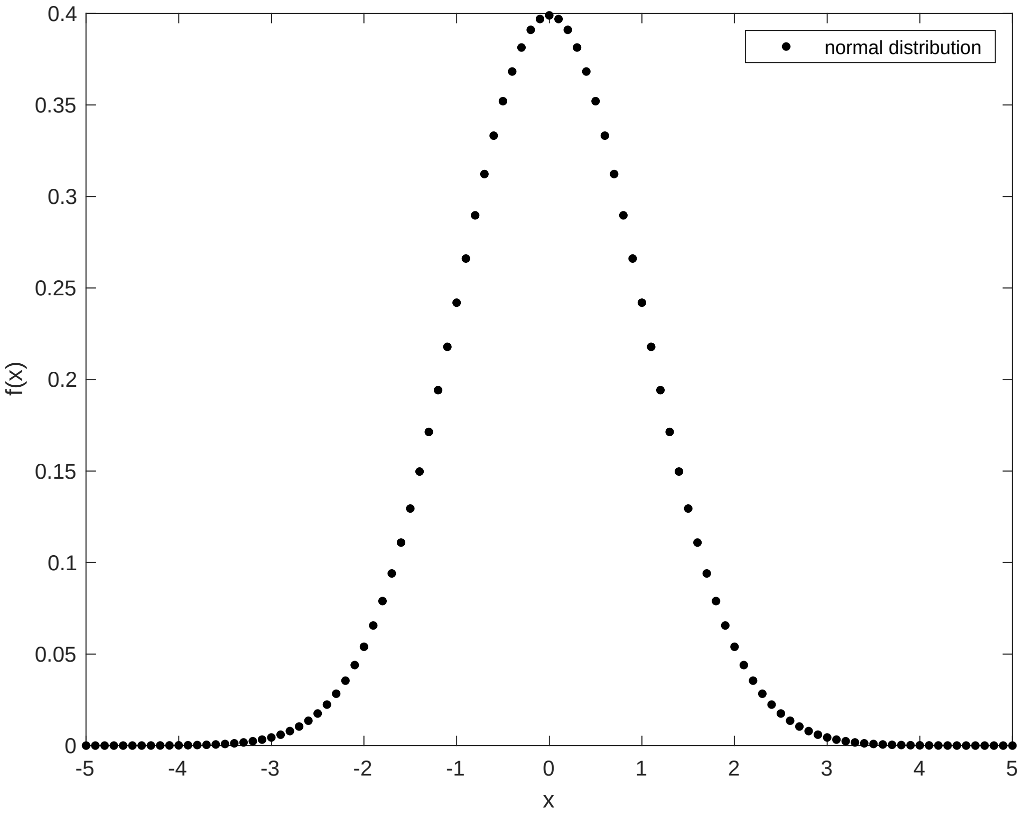


Figure 3: Students- and normal distribution

Draw qualitatively a students distribution for a very low number of degress of freedom in Fig. 3. Name the line according to the task.

d) Draw qualitatively a students distribution for a very high number of degress of freedom in Fig. 3. Name the line according to the task.

e) Besides the number of degrees of freedom, which two parameters need to be estimated in order to calculate the Student's distribution?

f) When should you use the students distribution instead of the normal distribution?

Task 4: Block diagram (16 points)

Two different systems are given. The transfer function of system (1) is

$$G_1(z) = \frac{1.5}{z} + \frac{0.7}{z^2} + \frac{2}{z^{-1}} \ ,$$

while system (2) is described by the difference equation

$$y(k+2) + 0.6y(k+1) = 2u(k+1) + 0.8u(k) \ .$$

a) Carry out a z-transform for the difference equation of system (2) and form the transfer function in z. Shift in time, so that only powers ≤ 0 of z remain.

b) How can systems (1) and (2) be characterized? Mark the right answer in the table.

	causal	non-causal	IIR	FIR
System (1)				
System (2)				

c) Draw block diagrams of systems (1) and (2). Herefore, use only z^{-1}-, z^1-blocks and blocks for the coefficients.

d) The step response of which system is constant for $k = 4$? Give a reason why. Explain in one sentence.

e) Imagine you have a non-causal system.

1) Give one example for an application, where you should not use a non-causal system.

2) Name an extension for the previous application that enables you to use the non-causal system.

3) Give one example for an application, where non-causal systems are usually used.

Task 5: Mean Filter

Given is following input/ output relation in discrete time:

$$y\left(k\right)) = \frac{1}{5}\left(u\left(k+2\right) + u\left(k+1\right) + u\left(k\right) + u\left(k-1\right) + u\left(k-2\right)\right) \tag{4}$$

a) Use the given relationship to calculate G(z).

b) What is the phase shift of the system? Give an explanation or calculation for your answer.
Hint: You can use follwoing relationship: $e^{i\phi} = \cos\left(\phi\right) + i\sin\left(\phi\right)$

c) Draw the step response $y(k)$ of the step input $u(k)$ from Fig. 4 into the same graph.

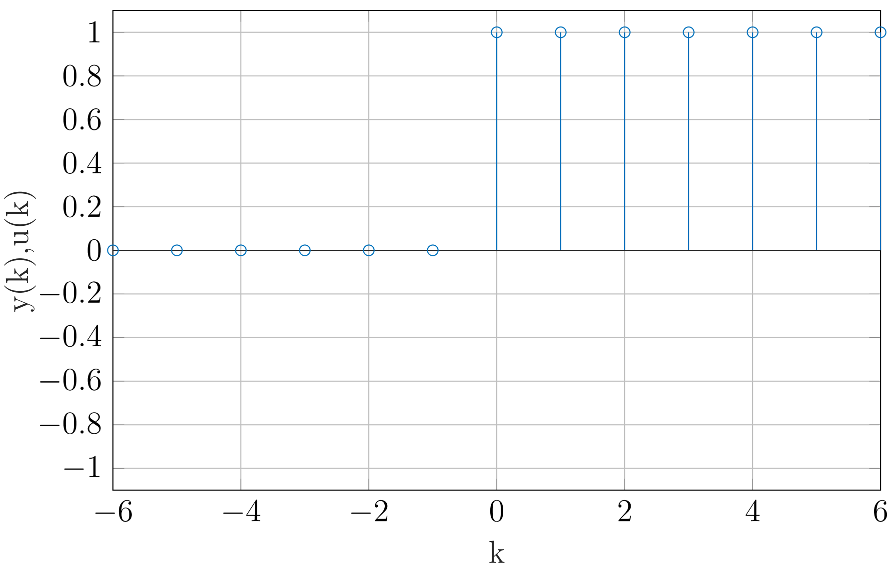


Figure 4: Step input u(k): (○)

d) Create the transfer function $G_d(z)$ by shifting $G(z)$ so that $G_d(z)$ is causal. The shift should be as small as possible.

e) Draw the impulse response $y_d(k)$ (which corresponds to the transfer function $G_d(Z)$) of the impulse input $u(k)$ from Fig. 5 into the same graph.

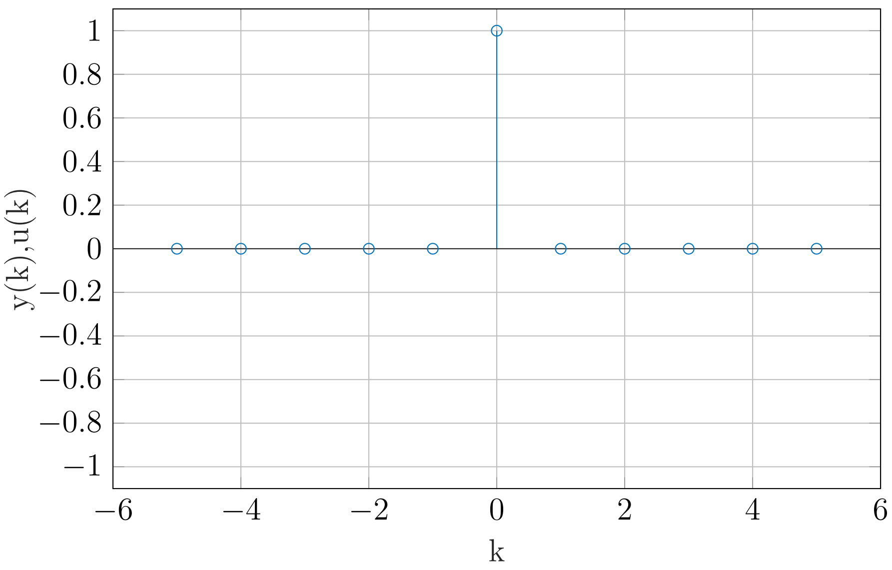
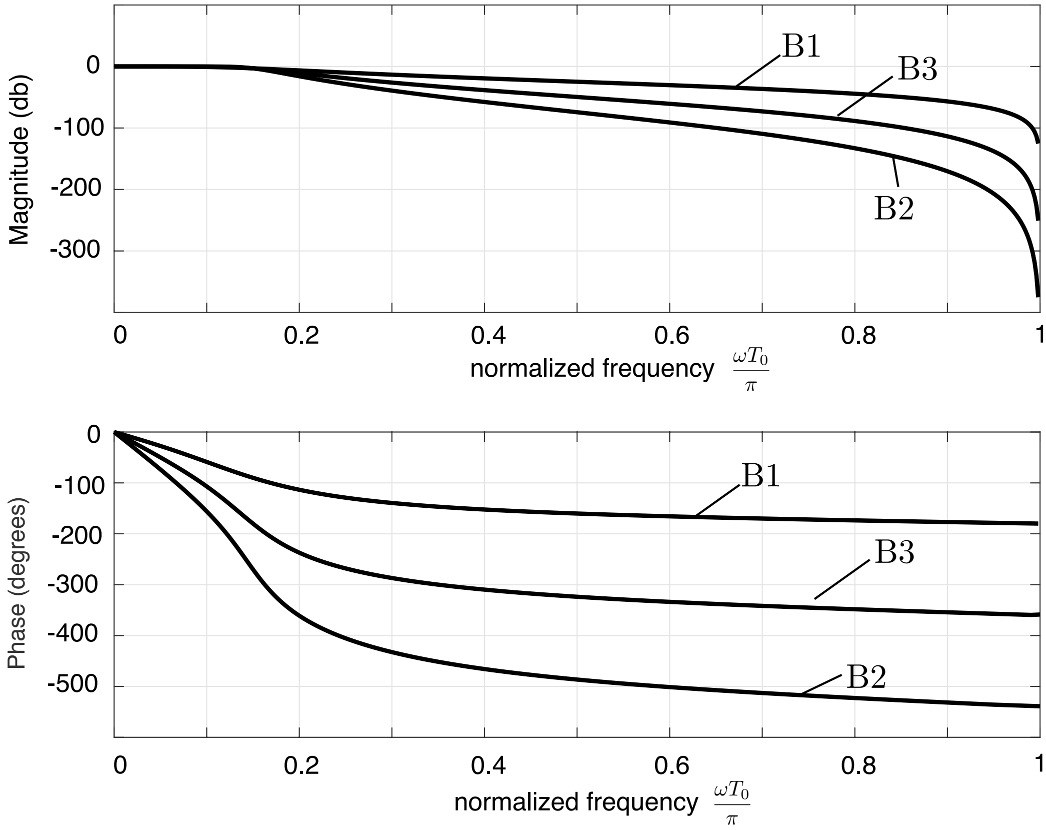


Figure 5: Impulse input u(k): (○)

Task 6: Filter (14 points)

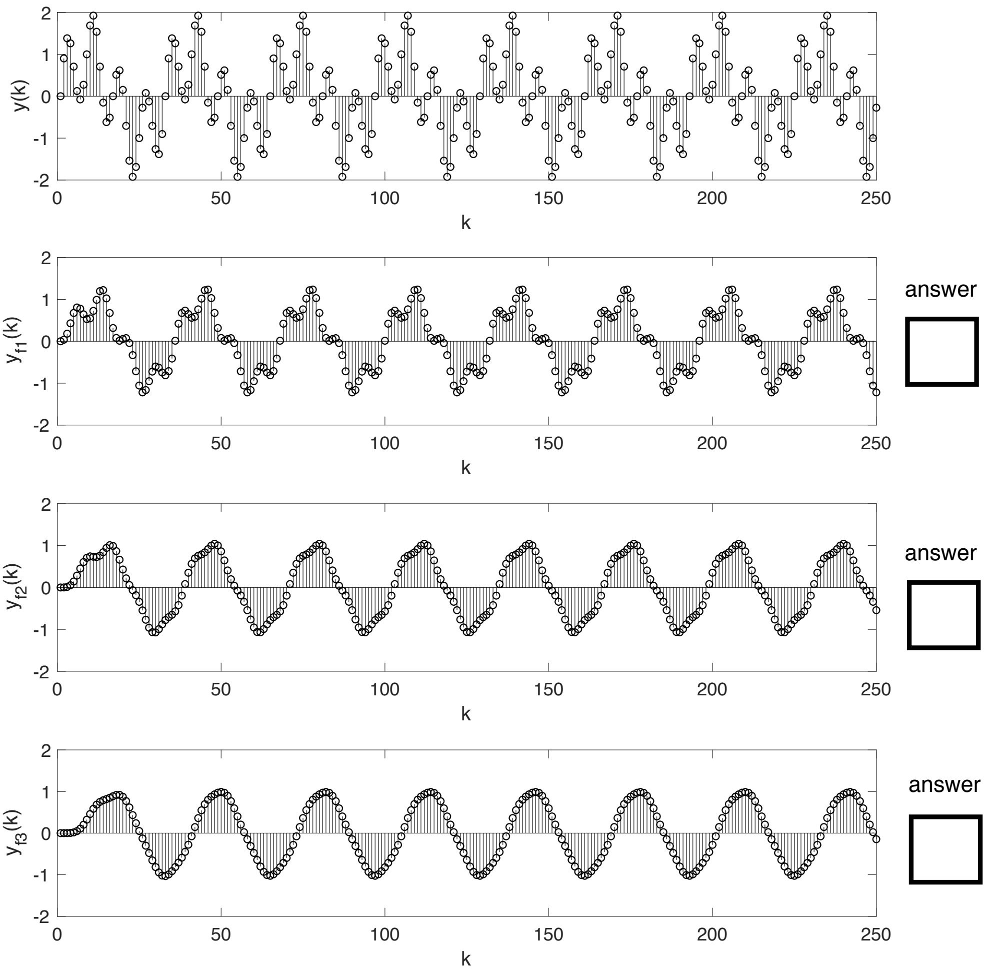
Consider three Butterworth filters B1-B3 with the shown frequency responses given.



a) What are the types of the filters B1-B3? (high-pass, band-pass, ...)

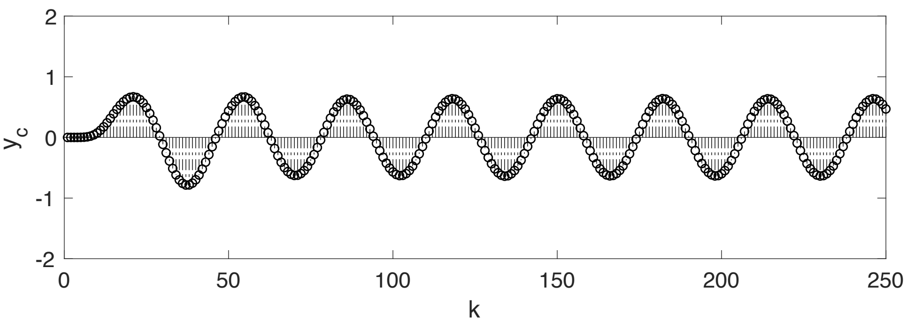
b) What are the orders of the filters B1-B3?

c) Now, the input $y(k) = \sin(\pi/4 \cdot k) + \sin(\pi/16 \cdot k)$ is applied to all three filters. In the top subplot the unfiltered signal is shown. Write the name of the filter (B1, B2 or B3) which is used to obtain the filtered signal plotted below the unfiltered signal in the answer box.



d) Why is the first part of the filter response ($k < 30$) different compared to the periodic response afterwards?

e) In the figure below the signal $y(k)$ is filtered with a Chebyshev filter type I of sixth order with the same cut-off frequency. What is different compared to the Butterworth-filters and what causes this behavior?



Task 7: Discrete-Time Systems / Amplitude Response (15 points)

Let the following transfer function be given in the z-domain

$$G(z) = \frac{1 - 0.5z}{0.5 - z}. \tag{5}$$

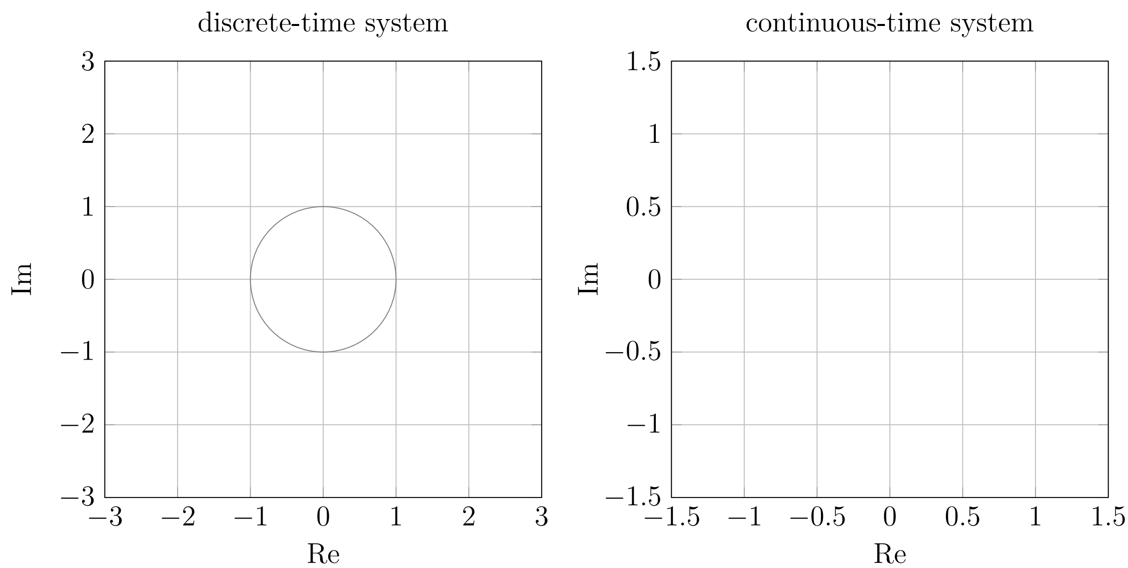
a) Calculate the pole and the zero of the transfer function. Draw them in the following diagram.

b) Calculate the amplitude response of the system, if the sampling time is given by $T_0 = 1$.

c) How is a system with this amplitude response called?

d) Determine the continuous-time system which has the same poles and zeros in the s-domain.

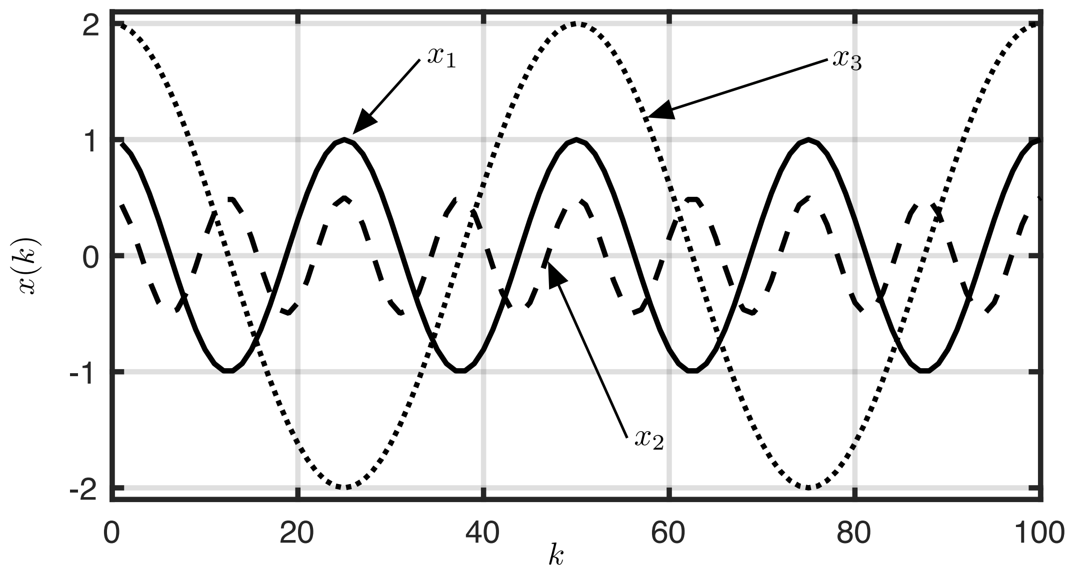
e) Draw the poles and zeros of the continuous system in the corresponding diagram. What is the difference for pole and zero locations between continuous time and discrete-time system?



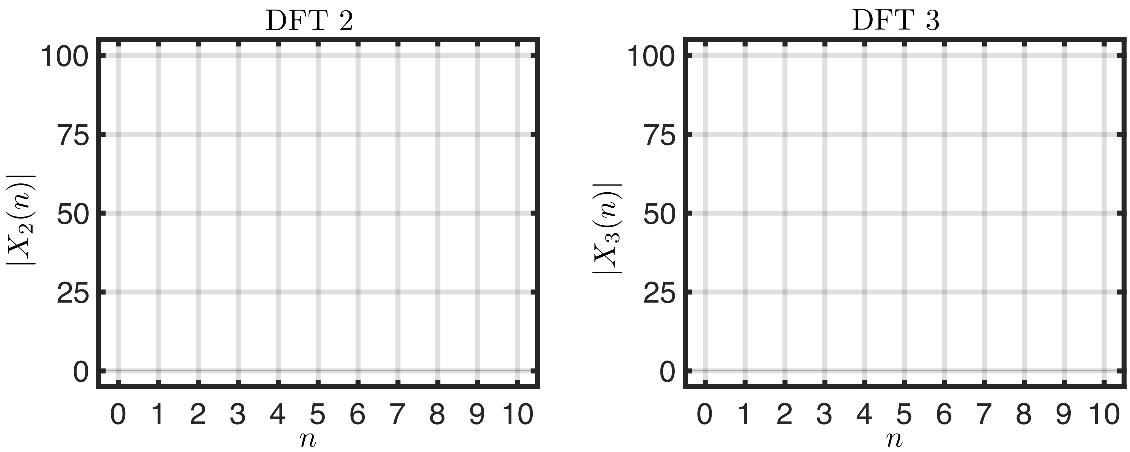
Task 8: Signal analysis (17 points)

In the diagram below, three sinusoidal signals (x_1, x_2 and x_3) are shown. Each contains $N = 100$ data points. For signal x_1 the absolute value of the DFT is given:

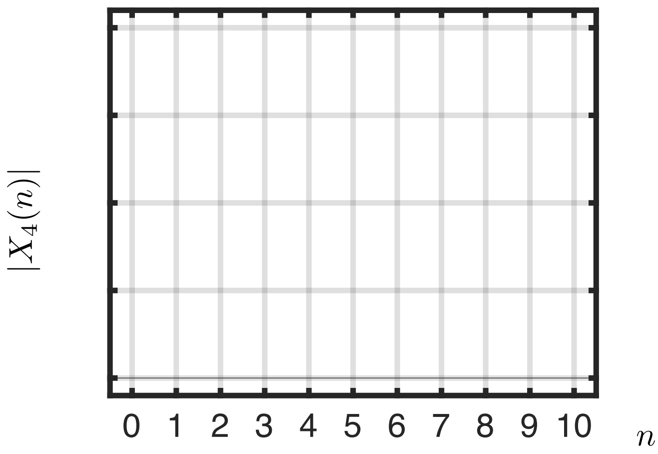
$$|X_1(n)| = \begin{cases} 50 & \text{for} \quad n = 4 \\ 0 & \text{for} \quad n = 0, 1, 2, 3, 5, 6, 7, ..., 49\,. \end{cases}$$



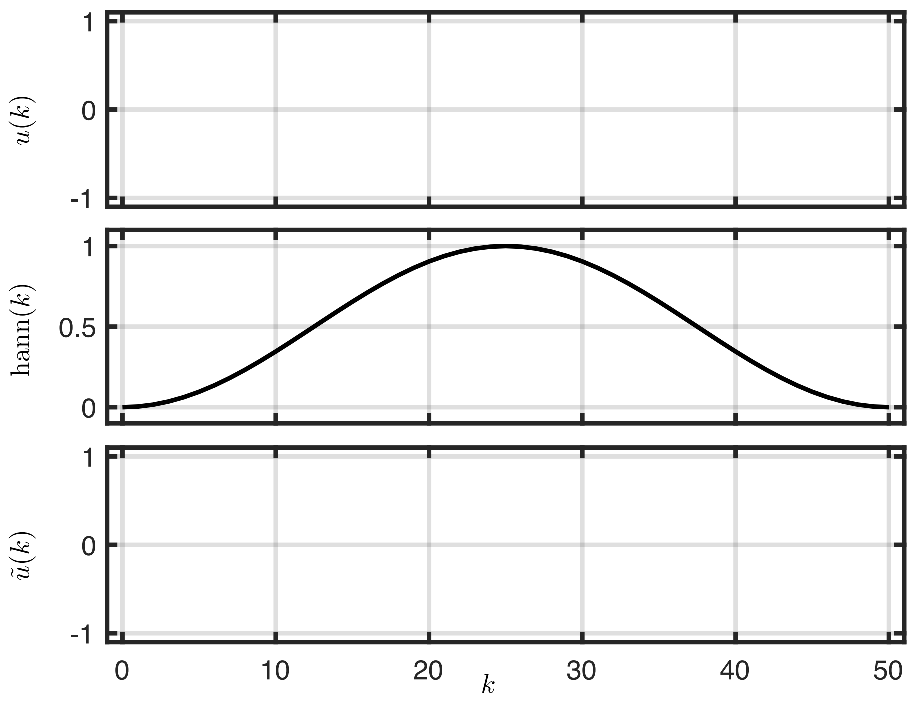
a) Sketch the absolute values of the DFTs ($|X_2(n)|$ and $|X_3(n)|$) corresponding to the signals x_2 and x_3 in the following diagram. Sketch only the values for $n = 0, 1, 2, \ldots, 10$. Note: No leakage appears for the chosen signals.



b) Sketch the absolute values of the DFT $|X_4(n)|$ for the signal $x_4(k) = -x_2(k) - x_3(k)$ in the given diagram. Sketch only the values for $n = 0, 1, 2, \ldots, 10$. Complete the missing values on the vertical axis.



c) Sketch in the following diagram a cosine-shaped signal where the leakage effect appears significantly. Besides this, the signal must fulfill the following properties: (1) $k = 0, 1, 2, \ldots 50$, (2) only one frequency, (3) a maximum of 3 oscillations, and (4) amplitude equal to one.



d) Now, apply the given Hann-window to the signal from the previous subtask. Sketch the resulting signal $\tilde{u}(k)$ in the given diagram.

e) The signal $u(t) = \sin(2\pi f t)$ with $f = 2\,\mathrm{Hz}$ is sampled using a frequency $f_0 = 100\,\mathrm{Hz}$. How many samples N should be recorded, to cover the correct frequency of the signal in a DFT?

f) Two arbitrary signals $x_5(k) = -x_6(k)$ are given. What is the difference between the corresponding DFTs $X_5(n)$ and $X_6(n)$? What is the difference between the absolute values of the DFTs $|X_5(n)|$ and $|X_6(n)|$? Give short answers!

Solutions:

<u>Task 1:</u> Comprehension Questions

Mark the correct answers clearly.
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a) K-means clustering ...
- ☐ ... always converges, even with different initializations, to the same solution for one set of data.
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- ☐ ... can only be achieved with IIR filters.
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- ■ ... utilizes singular value decomposition.
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- ☐ ... is the same as a mean filter.
- ■ ... filters outliers out of the signal.
- ☐ ... can only be causal.

f) The moving coil mechanism ...
- ☐ ... can directly be used to measure AC quantities.
- ■ ... can be used to measure voltage.
- ■ ... can be used to measure current.

g) Assess the following statements regarding AC quantities.
- ☐ Apparent power and active power are never the same.
- ☐ Apparent power and active power are independent of the phase-shift between current and voltage.
- ■ Reactive power cannot perform any work.

h) Assess the following statements regarding speed measurement. . . .

- [ ] The doppler effect works with acoustic signals only.
- [ ] Speed cannot be obtained by integration of acceleration measurements.
- [x] A generator can be used for speed measurement.

i) Assess the following statements regarding measurement of temperature.

- [ ] In PTC resistance thermometers (metal) the resistance shrinks with rising temperature.
- [x] NTC Resistance themometers have a highly nonlinear chracteristic.
- [x] Thermocouples are less accurate than PTC resistance thermometers.

j) Aliasing can occur. . .

- [x] if an analogue signal is sampled too slow.
- [x] if a digital signal is downsampled.
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$\sum$ 14

Task 2: Temperature Measurement(21 points)

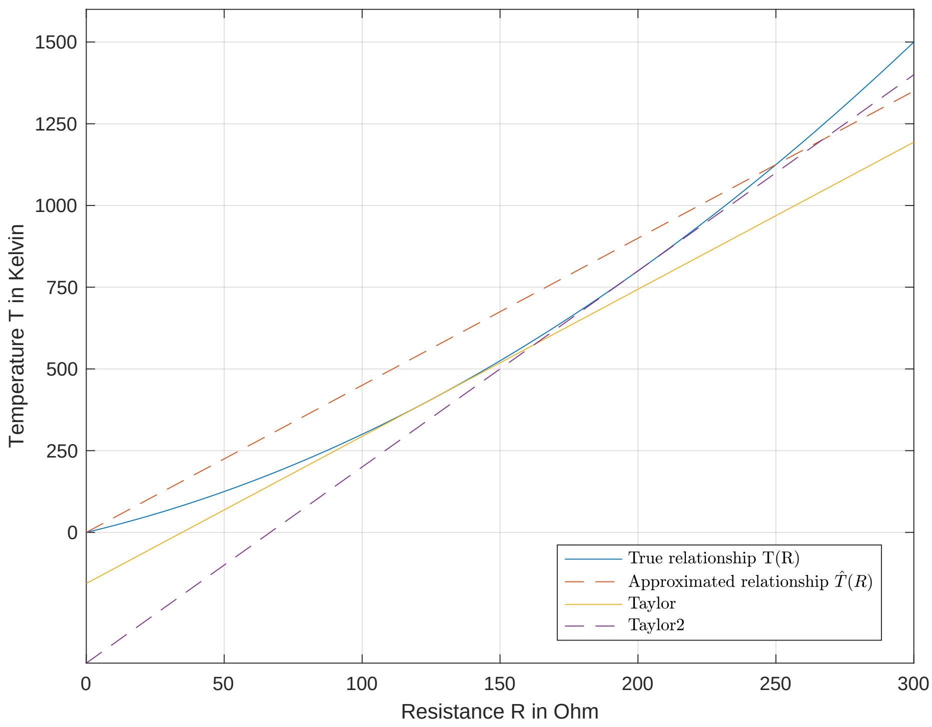


Figure 6: Relationship of the temperature of a sensor

The true relationship between the temperature T and the measured resistance R of a sensor are described by following equation:

$$T(R) = 2R + 0.01R^2 \tag{6}$$

One of the easiest simplifications to such equations is a linear approximation. The chosen approximation is:

$$T_a(R) = 4.5R \tag{7}$$

The graphs of both functions can be seen in Fig. 6.

a) What type of error occurs when using the approximated relationship instead of the real one?
Answer: A systematic, static error.

2

b) At what value of the resistance R does the biggest absolute error occur (in the range $0\Omega < R < 300\Omega$)? Show your calculations (graphical estimation is not enough)!

Answer:

$$E = T - T_a = -2.5R + 0.01R^2 \tag{8}$$

$$E' = \frac{dE}{dR} = -2.5 + 0.02R \tag{9}$$

$$maxError : \frac{dE}{dR} = 0 \tag{10}$$

$$2.5 = 0.02R \tag{11}$$

$$R = 125 \tag{12}$$

4

c) What is the absolute relative Error at this point?
Answer:

$$E_{rel} = \frac{T(R) - T_a(R)}{T(R)} = \frac{-2.5 \cdot 125 + 0.01 \cdot 125^2}{2 \cdot 125 + 0.01 \cdot 125^2} = -0.385 \tag{13}$$

$$|E_{rel}| = 0.385 \tag{14}$$

2

d) Now make an approximation around the operating point by utilizing the Taylor series expansion. Take the point of the biggest absolute error from subtask *b*) as operating point. If you were not able to identify that point, pick $R_0 = 200\Omega$ as operating point. Truncate the Taylor series expansion after the linear/affine part.
Hint: In general the Taylor series for a function $f(x)$ around an operating point x_0 is given by

$$Tf(x; x_0) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n \tag{15}$$

Here, $n!$ denotes the factorial of n and $f^{(n)}(x_0)$ denotes the n-th derivative of $f(x)$ evaluated at x_0.

Answer:

$$T_f(R, R_0) \approx \frac{T^0 R_0}{0!}(R - R_0)^0 + \frac{T^1 R_0}{1!}(R - R_0)^1 \tag{16}$$

$$\approx \frac{(2R_0 + 0.01R_0^2)}{1}(R - R_0)^0 + \frac{(2 + 0.02R_0)}{1}(R - R_0)^1 \tag{17}$$

$$\approx 2R_0 + 0.01R_0^2 + (2 + 0.02R_0)\,R - (2 + 0.02R_0)\,R_0 \tag{18}$$

$$\approx 4.5R - 156.25 \tag{19}$$

For $R_0 = 200$:

$$T_f(R, R_0) \approx 6R - 400 \tag{20}$$

4

e) Draw the graph of the Taylor approximation into Fig. 6.
Answer: Yellow line (lower affine function). Purple line (dashed) for $R_0 = 200$. 1

f) In what case is this Taylor approximation better suited than the given approximation?
Answer: When the characteristics around an operating are more important than an approximation with a small error in general. 1

$\sum$ 14

Task 3: Statistics

a) Draw the two-dimensional normal distributions for the given signals into Fig. 7.

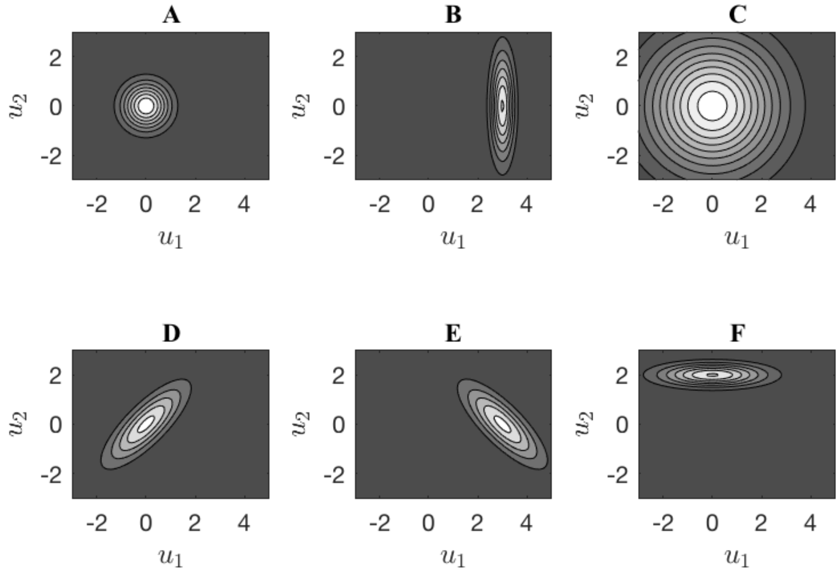


Figure 7: Correct pdf countour plots

6

b) A two-dimensional normal distribution has perfectly circular shaped contour lines. What properties does the distribution posses?
Answer:
The probabilities are independent of one another, thus the signals are uncorrelated. $\rho_{u_1,u_2} = 0$ 2

c) Figure 8 shows a normal distribution for a set of data with mean $\mu = 0$ and standard deviation $\sigma^2 = 1$. This subtask is about the students distribution, also known as the t-distribution.
Draw qualitatively a students distribution for a very low number of degress of freedom in Fig. 8. Name the line according to the task.
Answer: Flat mid, fat tails. Green line 2

d) Draw qualitatively a students distribution for a very high number of degress of freedom in Fig. 8. Name the line according to the task.
Answer: Same /almost the same as normal distribution. Dashed blue line. 2

e) Besides the number of degrees of freedom, which two parameters need to be estimated in order to calculate the Student's distribution?
Answer: Mean and variance/standard deviation. 2

f) When should you use the students distribution instead of the normal distribution?
Answer: When there are more unknown uncertainties to account for, like measurement uncertainties. Variance is unkown. 2

$\sum 16$

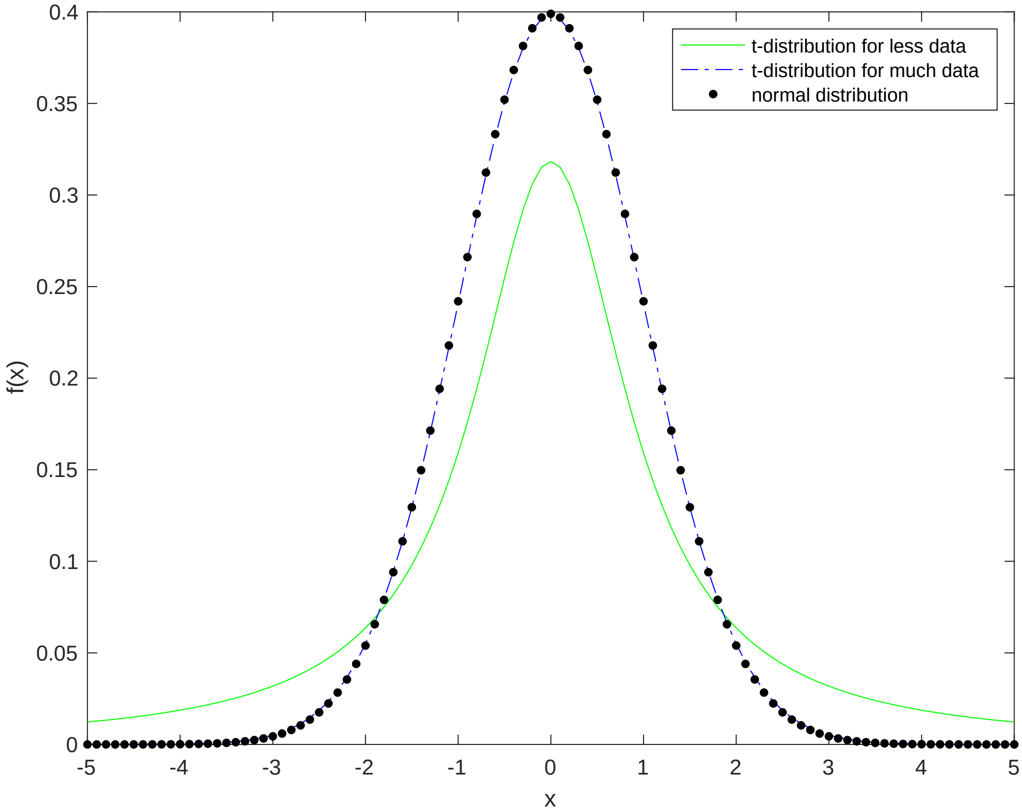


Figure 8: Students- and normal distribution

Task 4: Block diagram (14 points)

Two different systems are given. The transfer function of system (1) is

$$G_1(z) = \frac{1.5}{z} + \frac{0.7}{z^2} + \frac{2}{z^{-1}} ,$$

while system (2) is described by the difference equation

$$y(k+2) + 0.6y(k+1) = 2u(k+1) + 0.8u(k) .$$

a) Carry out a z-transform for the difference equation of system (2) and form the transfer function in z. Shift in time, so that only powers ≤ 0 of z remain.
Answer:

$$\begin{aligned}
y(k+2) + 0.6y(k+1) &= 2u(k+1) + 0.8u(k) \\
y(k) + 0.6y(k-1) &= 2u(k-1) + 0.8u(k-2) \\
Y(z) + 0.6z^{-1}Y(z) &= 2U(z)z^{-1} + 0.8U(z)z^{-2} \\
Y(z) \cdot (1 + 0.6z^{-1}) &= U(z) \cdot (2z^{-1} + 0.8z^{-2}) \\
\Rightarrow G(z) = \frac{Y(z)}{U(z)} &= \frac{2z^{-1} + 0.8z^{-2}}{1 + 0.6z^{-1}}
\end{aligned}$$

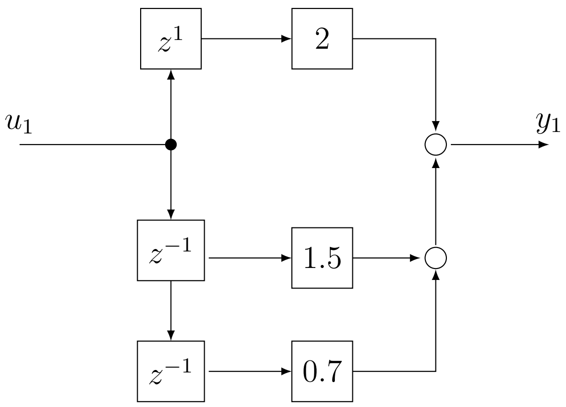
$\boxed{4}$

b) How can systems (1) and (2) be characterized? Mark the right answer in the table.
Answer:

	causal	non-causal	IIR	FIR
System (1)		x		x
System (2)	x		x	

2

c) Draw block diagrams of systems (1) and (2). Herefore, use only z^{-1}-, z^{1}-blocks and blocks for the coefficients.
Answer:



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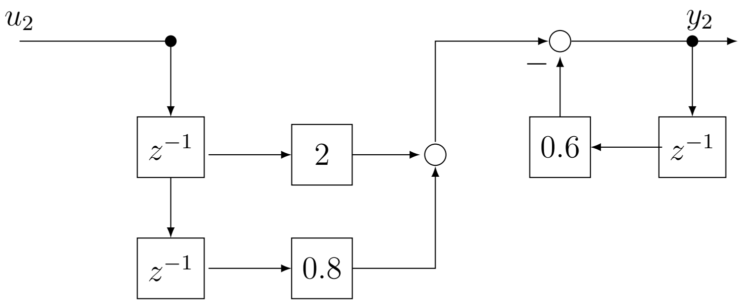


2

d) The step response of which system is constant for $k = 4$? Give a reason why. Explain in one sentence.
Answer:
The step response of the FIR-system. (System (1))
No feedback. All delayed inputs contribute to the output at this time. 3

e) Imagine you have a non-causal system.

1) Give one example for an application, where you should not use a non-causal system.

Answer:
Online Application, or feedback control: filtering control variable.

2) Name an extension for the previous application that enables you to use the non-causal system.

Answer:
Add buffer.

3) Give one example for an application, where non-causal systems are usually used.

Answer:
Image processing / every processing of recorded data.

$\boxed{3}$

$\boxed{\sum 16}$

Task 5: Mean Filter

Given is following input/ output relation in discrete time:

$$y\left(k\right)) = \frac{1}{5}\left(u\left(k+2\right) + u\left(k+1\right) + u\left(k\right) + u\left(k-1\right) + u\left(k-2\right)\right) \tag{21}$$

a) Use the given relationship to calculate G(z).
Answer:

$$Y\left(z\right) = \frac{1}{5}\left(U\left(z\right)z^{-2} + U\left(z\right)z^{-1} + U\left(z\right)z^{0} + U\left(z\right)z^{1} + U\left(z\right)z^{2}\right) \tag{22}$$

$$G\left(z\right) = \frac{1}{5}\left(z^{2} + z^{1} + z^{0} + z^{-1} + z^{-2}\right) \tag{23}$$

2

b) What is the phase shift of the system? Give an explanation or calculation for your answer.
Hint: You can use follwoing relationship: $e^{i\phi} = \cos\left(\phi\right) + i\sin\left(\phi\right)$
Answer:
No phase shift. Imaginary parts cancel each other.

$$G\left(i\omega\right) = \frac{1}{5}\left(e^{-2i\omega T_0} + e^{-1i\omega T_0} + e^{0i\omega T_0} + e^{1i\omega T_0} + e^{2i\omega T_0}\right) \tag{24}$$

$$= \frac{1}{5}\left(\cos\left(2\omega T_0\right) + i\sin\left(2\omega T_0\right) + \cos\left(\omega T_0\right) + i\sin\left(\omega T_0\right) + 1 + \cos\left(-\omega T_0\right)\right. \tag{25}$$

$$\left. + i\sin\left(-\omega T_0\right) + \cos\left(-2\omega T_0\right) + i\sin\left(-2\omega T_0\right)\right)$$

$$G\left(i\omega\right) = \frac{1}{5}\left(2\cos\left(2\omega T_0\right) + 2\cos\left(\omega T_0\right) + 1\right) \tag{26}$$

4

c) Draw the step response $y(k)$ of the step input $u(k)$ from Fig. 9 into the same graph.
Answer: 3

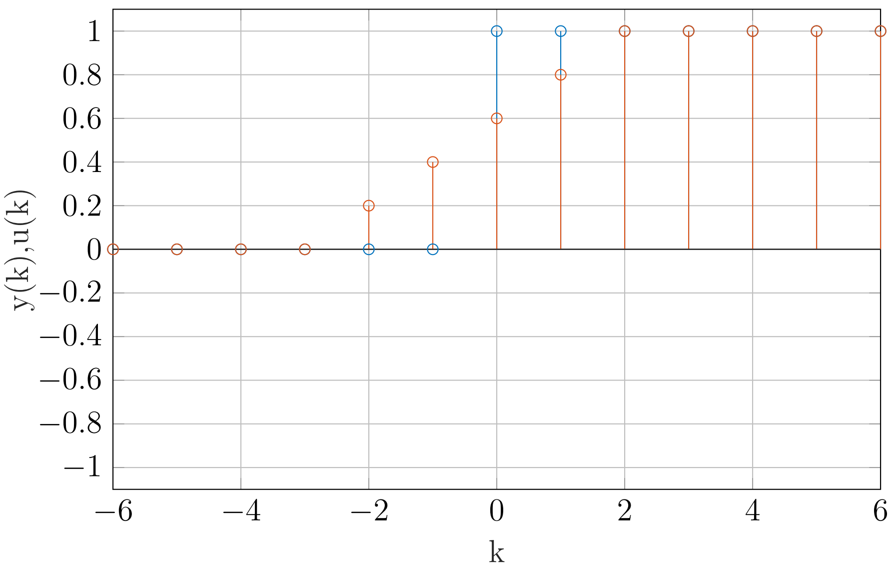


Figure 9: Step input u(k): (○), step response y(k): (○)

d) Create the transfer function $G_d(z)$ by shifting $G(z)$ so that $G_d(z)$ is causal. The shift should be as small as possible.
Answer:

$$G_d\left(z\right) = \frac{1}{5} z^{-2} \left(z^2 + z^1 + z^0 + z^{-1} + z^{-2}\right) \tag{27}$$

2

e) Draw the impulse response $y_d(k)$ (which corresponds to the transfer function $G_d(Z)$) of the impulse input $u(k)$ from Fig. 10 into the same graph.
Answer:

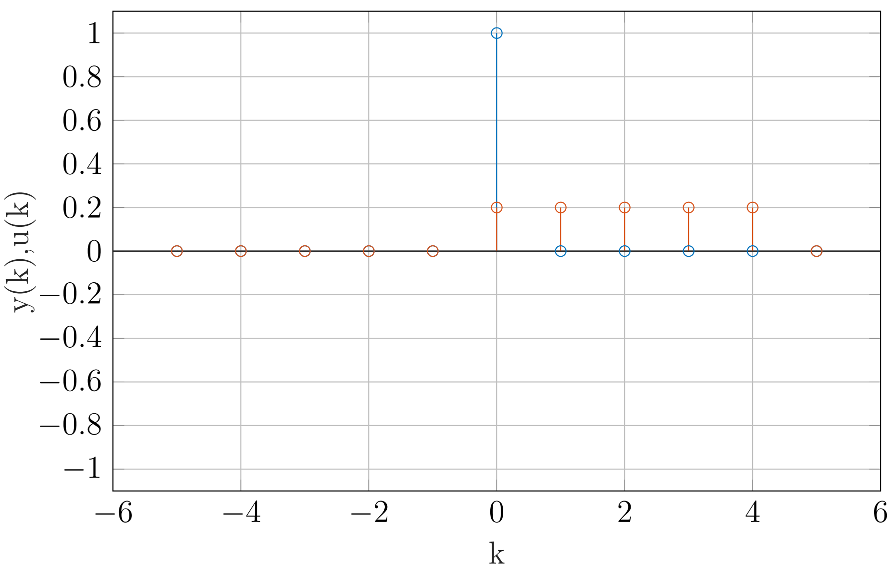


Figure 10: Impulse input u(k): (○), impulse response y(k): (○)

$\boxed{3}$

$\boxed{\sum 14}$

Task 6: Filter

a) What are the types of the filters B1-B3? (high-pass, band-pass, ...)
Answer:
B1-B3: low-pass filter 1

b) What are the orders of the filters B1-B3?
Answer:
B1: second order, B2: sixth order, B3: fourth order 3

c) Now, the input $y(k) = \sin(\pi/4 \cdot k) + \sin(\pi/16 \cdot k)$ is applied to all three filters. In the top subplot the unfiltered signal is shown. Write the name of the filter (B1, B2 or B3) which is used to obtain the filtered signal plotted below the unfiltered signal in the answer box.
Answer:
$y_{f1} : B1$, $y_{f2} : B3$ and $y_{f3} : B2$ 3

d) Why is the first part of the filter response ($k < 30$) different compared to the periodic response afterwards?
Answer:
the behavior is different due to the initial conditions 2

e) In the figure below the signal $y(k)$ is filtered with a Chebyshev filter type I of sixth order with the same cut-off frequency. What is different compared to the Butterworth-filters and what causes this behavior?
Answer:
The amplitude is different. This is caused by the ripples of the amplitude response within the pass band. 2

$\sum 11$

<u>Task 7:</u> Discrete-Time System/ Amplitude Response (15 Points)

a) Calculate the pole and the zero of the transfer function. Draw them in the following diagram.
Answer:
The zero is $n = 2$, the pole $p = 0.5$. 2

b) Calculate the amplitude response of the system, if the sampling time is given by $T_0 = 1$.
Answer:
For the amplitude response it holds that

$$\begin{aligned} |G(i\omega)| &= \frac{|1 - 0.5\cos(\omega) - 0.5i\sin(\omega)|}{|0.5 - \cos(i\omega) - i\sin(\omega)|} \\ &= \sqrt{\frac{(1 - 0.5\cos(\omega))^2 + 0.25\sin^2(\omega)}{(0.5 - \cos(\omega))^2 + \sin^2(\omega)}} \\ &= \sqrt{\frac{1 - \cos(\omega) + 0.25}{0.25 - \cos(\omega) + 1}} \\ &= 1 \end{aligned}$$

5

c) How is a system with this amplitude response called?
Answer:
allpass 1

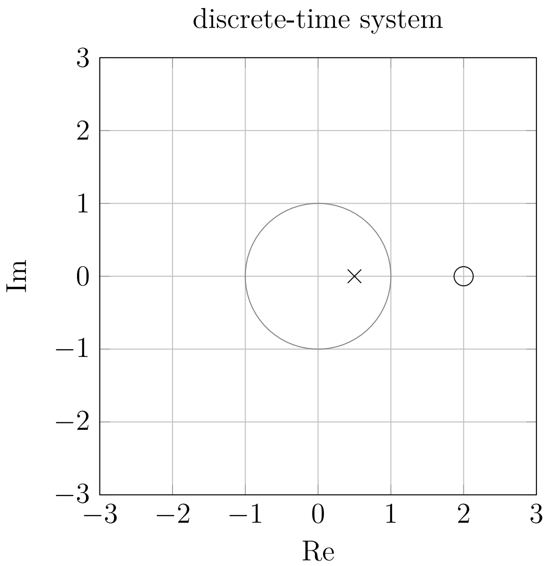
d) Determine the continuous-time system which has the same poles and zeros in the s-domain.
Answer:
It holds that $z = e^{iT_s s}$ and thus $s = \frac{1}{T_s}\ln(z)$. Therefore it holds for the zero that $n_c = \ln(0.5) \approx -0.69$ and for the pole $p_c = \ln\left(\frac{1}{0.5}\right) \approx= 0.69$. The transfer function thus is

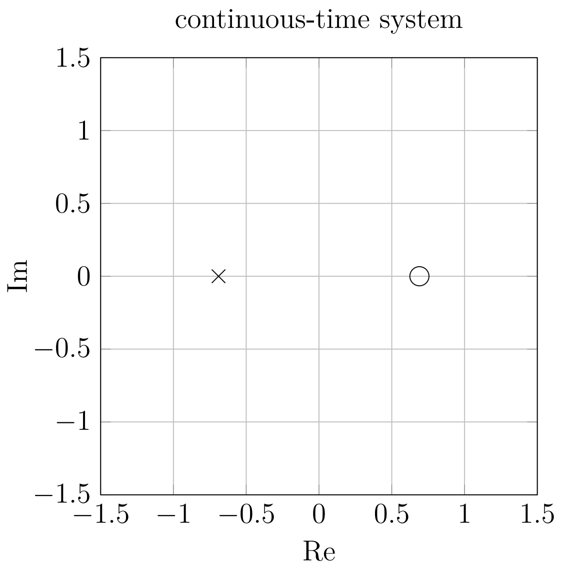
$$G(s) = \frac{s - 0.69}{s + 0.69}. \qquad (28)$$

4

e) Draw the poles and zeros of the continuous system in the corresponding diagram. What is the difference for pole and zero locations between continuous time and discrete-time system?
Answer:
For the continuous system the poles are mirrored at the imaginary axis. For the discrete system it holds that the zero is $\frac{1}{p}$. 3

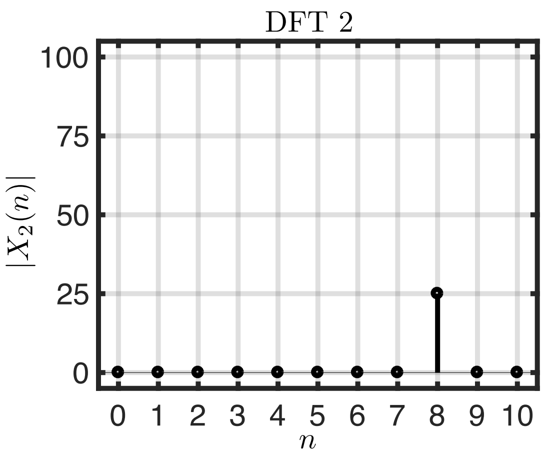
$\boxed{\sum 15}$

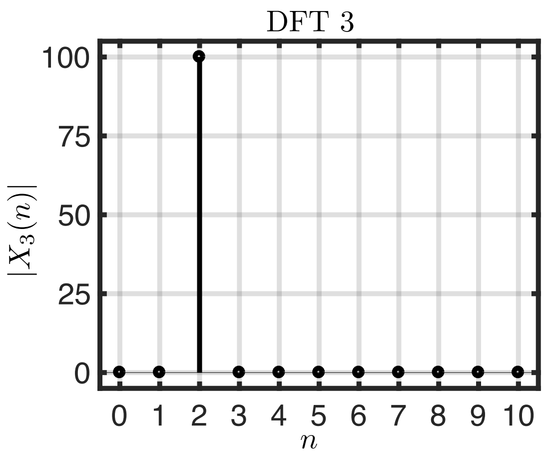




Task 8: Signal analysis (17 points)

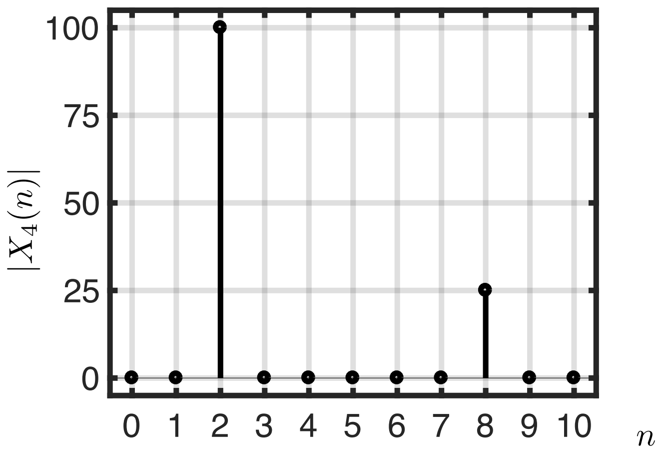
a) Sketch the absolute values of the DFTs ($|X_2(n)|$ and $|X_3(n)|$) corresponding to the signals x_2 and x_3 in the following diagram. Sketch only the values for $n = 0, 1, 2, \ldots, 10$. Note: No leakage appears for the chosen signals.





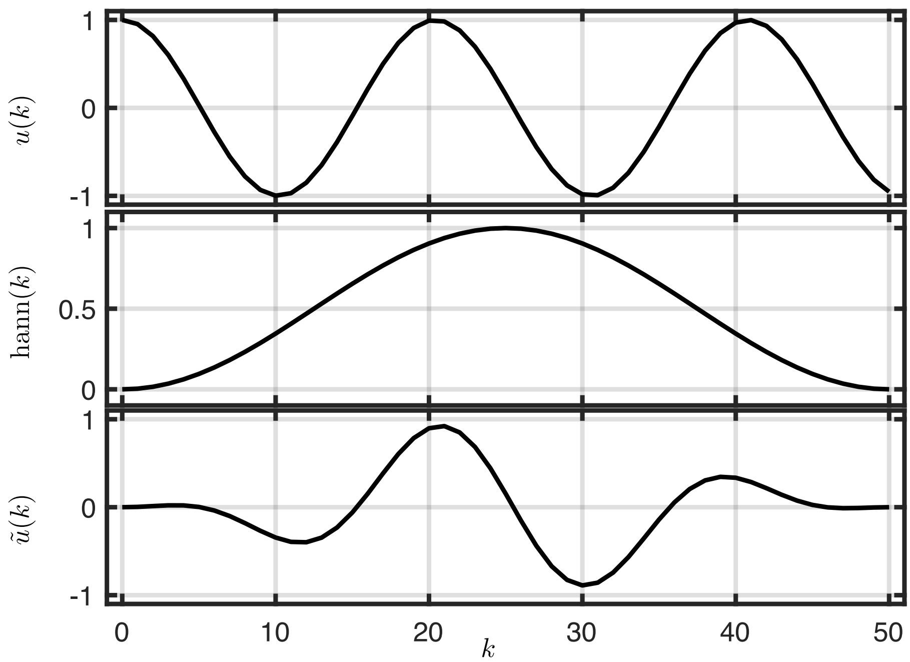
The signal x_2 has twice the frequency and half of the amplitude compared to x_1. Thus, the absolute value of the DFT of x_2 has its peak at the frequency index $n = 8$ with half of the height compared to the DFT of x_1. In contrast, signal x_3 has half of the frequency and twice the amplitude compared to x_1. Thus the absolute value of the DFT has its peak at the frequency index $n = 2$ with twice the height compared to the DFT of x_1. $\boxed{6}$

b) Sketch the absolute values of the DFT $|X_4(n)|$ for the signal $x_4(k) = -x_2(k) - x_3(k)$ in the given diagram. Sketch only the values for $n = 0, 1, 2, \ldots, 10$. Complete the missing values on the vertical axis.



3

c) Sketch in the following diagram a cosine-shaped signal where the leakage effect appears significantly.



2

d) Now, apply the given Hann-window to the signal from the previous subtask. Sketch the resulting signal $\tilde{u}(k)$ in the given diagram. 2

e) The signal $u(t) = \sin(2\pi f t)$ with $f = 2\,\text{Hz}$ is sampled using a frequency $f_0 = 100\,\text{Hz}$. How many samples N should be recorded, to cover the correct frequency of the signal in a DFT?
The represented frequencies in a DFT are given by:

$$f_n = n f_0/N, \quad n = 0, 1, 2, \ldots, (N-1)\,. \tag{29}$$

for $f_n = 2\,\text{Hz}$ and $f_0 = 100\,\text{Hz}$ it follows:

$$2\,\text{Hz} = \frac{100\,\text{Hz} \cdot n}{N} \tag{30}$$

$$\text{For } N \neq 0 \;\Rightarrow\; N = \frac{100\,\text{Hz} \cdot n}{2\,\text{Hz}} \tag{31}$$

$$N = 50n \tag{32}$$

For $n \in \mathbb{N}$ the correct answers are $N = 50, 100, 150, \ldots$ $\boxed{2}$

f) Two arbitrary signals $x_5(k) = -x_6(k)$ are given. What is the difference between the corresponding DFTs $X_5(n)$ and $X_6(n)$? What is the difference between the absolute values of the DFTs $|X_5(n)|$ and $|X_6(n)|$? Give short answers!
The DFT-matrices are equal for both signals ($\underline{W}_{N,5} = \underline{W}_{N,6}$).These matrices contain complex numbers thus, a multiplication with a real valued signal usually gives a vaector with complex values. The given signals differ only in the sign, thus the DFTs $X_5(n)$ and $X_6(n)$ differ only in the sign too. Thus, the complex pointer of $X_5(n)$ points always to the opposite direction compared to $X_6(n)$. But the absolute values of $|X_5(n)|$ and $|X_6(n)|$ are identical ($|a + ib| = \sqrt{a^2 + b^2}$ and $|-a - ib| = \sqrt{(-a)^2 + (-b)^2} = \sqrt{a^2 + b^2}$). $\boxed{2}$

$\boxed{\sum 17}$