# Sensorics Exam

Prof. Dr.-Ing. O. Nelles
Institut für Mechanik und Regelungstechnik
Universität Siegen

27.08.2020

| | |
|---|---|
| Name: | |
| Mat.-No.: | |
| Grade: | |

| Task | T1 | T2 | T3 | T4 | T5 | T6 | T7 | Sum |
|---|---|---|---|---|---|---|---|---|
| Score: | 19 | 17 | 17 | 16 | 22 | 15 | 14 | 120 |
| Accomplished: | | | | | | | | |

Exam duration: 2 hours

Permitted Tools: Pocket Calculator and 4 pages of formula collection

## Task 1: Comprehension Questions (19 Points)

Mark the correct answers clearly.
**Every question has one, two or three correct answers!**
For every correctly marked answer you will get one point. For every wrong answer a point will be subtracted, but a question will never be rated with negative points.

a) Which statements are true regarding A/D Converters?

- ☐ The Flash Converter requires only two comparators for each $2^n - 1$ quantization level.
- ☐ The Weighting Method works faster than the Flash Converter (for the same resolution).
- ☐ The Dual Slope Converter works faster than the Flash Converter (for the same resolution).
- ☐ Conversion with the Dual Slope Principle is robust to unwanted influences (such as temperature).

b) Which statements are true regarding filters?

- ☐ A first type Chebyshef filter of 4rth order has a phase shift of -360° for $\omega \to \infty$.
- ☐ Butterworth filters have a monotone amplitude response.
- ☐ For identical orders, a Cauer filter has a steeper transition band than a Bessel, Butterworth or Chebyshef filter.
- ☐ Even though the Elliptic filter has a smoother amplitude response than the Chebyshef filter, the step response of a Cheychef filter (of same order) oscillates stronger.

c) What is the meaning of the terms *dead zero* and *life zero*?

- ☐ If a measurement of a signal $\neq 0$ has a value $= 0$, it is called a dead zero.
- ☐ Live zero is a measurement technique which allows detection of a broken wire.
- ☐ Live zero means a measurement device is broken and has to be replaced.
- ☐ If a measurement of a signal $= 0$ has a value $= 0$, it is called a dead zero.

d) Which statements are true regarding time discrete systems?

- ☐ Moving average filters are FIR filters.
- ☐ Autoregressive moving average filters are more advanced moving average filters that don't use feedback anymore.
- ☐ Moving average filters can become unstable if the coefficients are not chosen carefully.
- ☐ The impulse response of an autoregressive system never reaches a constant value.

e) Which statements are true regarding aliasing?

- ☐ Increasing the sampling frequency is a common way to deal with aliasing effects.
- ☐ Upsampling signals should always be carried out with anti aliasing filters to make sure no new frequency components above $f_0/2$ appear in the signal.
- ☐ Aliasing happens, when shadow spectra appear in the frequency response of a system.
- ☐ If a periodic signal is sampled at the same rate as it repeats, it appears to be a constant.

f) Which statements are true regarding clustering?

- ☐ Clustering tries to find data points that are grouped together.
- ☐ Clustering is usually a supervised learning method.
- ☐ The number of clusters has not to be known in advance, but is determined through the clustering algorithm.
- ☐ Different clustering algorithms exist, that differ in how the distances of data points from the cluster center are determined.

g) How large should the internal resistance of a voltage meter be?

- ☐ Very small, in the ideal case zero.
- ☐ Very large, in the ideal case infinitely large.
- ☐ It depends on the electrical circuit, what internal resistance is ideal.
- ☐ The internal resistance has no influence on the measurement.

h) Assess following statements regarding errors:

- ☐ The quantization error is half as large if a 16 bit A/D converter is used instead of an 8 bit converter.
- ☐ The absolute error of a measurement is the difference between the measured value and the true value, divided by the true value.
- ☐ By averaging measurements, random errors are typically reduced with $1/\sqrt{N}$, where $N$ is the number of measurements.
- ☐ Random errors cannot be reduced by averaging over several measurements.

i) Assess the following statements regarding the Student's t-distribution.

- ☐ The t-distribution is identical to the normal distribution.
- ☐ Its confidence interval $1 - \alpha$ is wider than the confidence interval of the corresponding normal distribution.
- ☐ If the size $N$ of the regarded data set is big enough ($N \to \infty$), it converges to the cauchy distribution.
- ☐ It is used if the real standard deviation has to be estimated from data.

j) Operational amplifiers ...

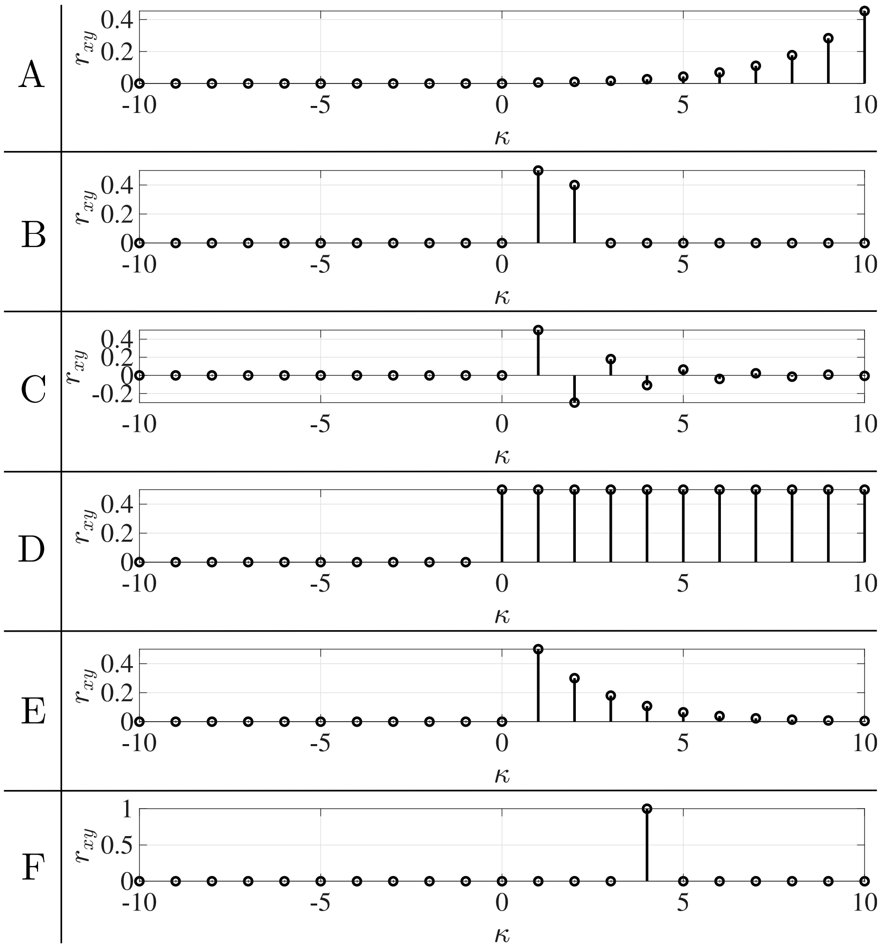
- ☐ ... are active components (an external energy source is needed).
- ☐ ... have a very high **output** resistance.
- ☐ ... are amplifiers with a very high gain.
- ☐ ... have a very high **input** resistance.

## Task 2: Correlation function (17 Punkte)

a) Three systems are given

$$G_1(z) = \frac{0.5z^{-1}}{1 - 0.6z^{-1}}$$
$$G_2(z) = 0.5z^{-1} + 0.4z^{-2}$$
$$G_3(z) = z^{-4}.$$

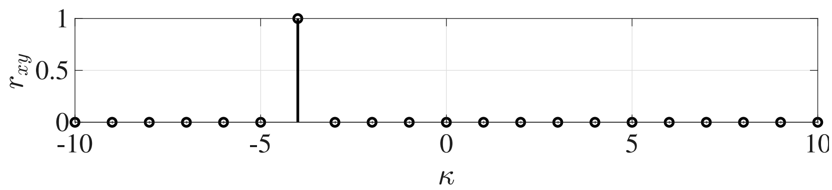
The systems were excited with an Kronecker delta $x(k)$ and the resulting output $y(k)$ was recorded. Subsequently, the cross-correlation function $r_{xy}(\kappa)$ was calculated. Assign the correct cross-correlation functions $A, B, ..., F$ to the right system $G_1, G_2, G_3$.




| System | Cross-correlation function |
|---|---|
| 1 | |
| 2 | |
| 3 | |

b) Another cross-correlation function $H$ is given, calculated exactly as in the previous task. What is the transfer function of the underlying system?

H

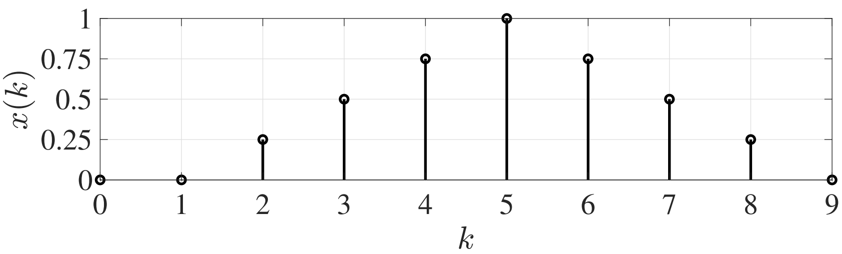


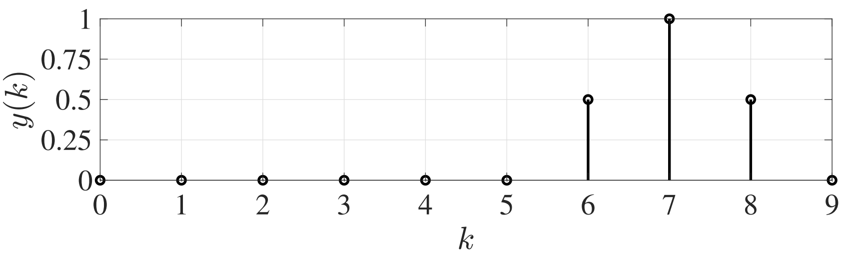

c) How does the system from task b) differ from the systems in task a)?

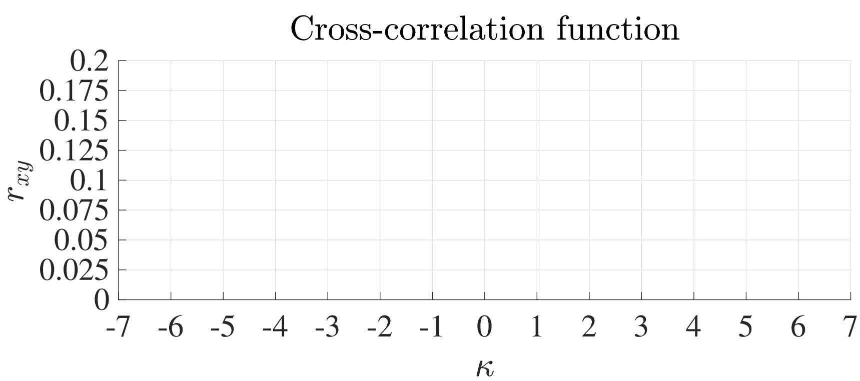
d) Is it possible to represent a real pysical system with the transfer function from task b)?

e) The time-discrete signals $x(k)$ and $y(k)$ are given. Calculate the cross-correlation function $r_{xy}$ for $\kappa = -7, -6, ..., 7$ and sketch the cross-correlation function in the given diagram.
Hint:

$$r_{xy}(\kappa) = \begin{cases} \frac{1}{N} \sum\limits_{k=0}^{N-1-\kappa} x(k) \cdot y(k+\kappa), & \text{for } \kappa \geq 0 \\ \frac{1}{N} \sum\limits_{k=-\kappa}^{N-1} x(k) \cdot y(k+\kappa), & \text{else.} \end{cases}$$

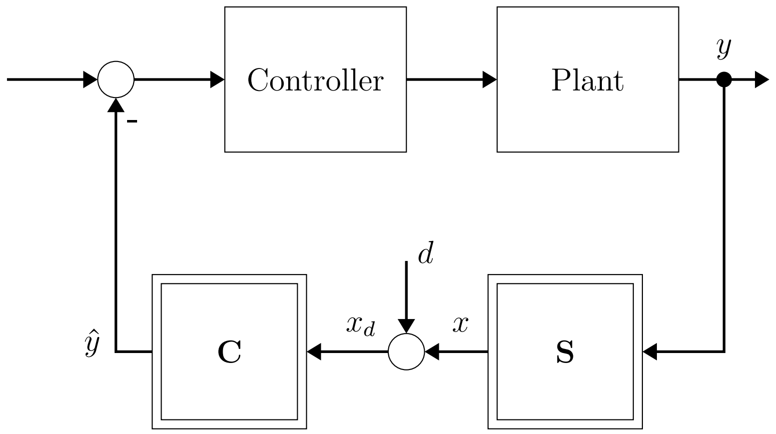


## Task 3: Compensator (17 Points)

In order to control a plant, the output of the system has to be measured. For many sensors (e.g. Thermocouples) the measurement of the output results in a voltage value. From this voltage value, the physical value (e.g. a temperature) can be calculated.
In this example we have a sensor in the feedback loop which measures the output $y$ of the plant. The output of the sensor $S$ is $x$. This value can suffer from additional disturbance $d$. $x_d$ is then fed into the compensator $C$. The Compenstor $C$ does not compensate the added noise $d$ it just reconstructs $y$ by $\hat{y}$ from $x_d$ with the inverse characteristic from $S$. So with $d = 0$ the statement $\hat{y} = y$ is valid.




The equation of the sensor is

$$f_S(y) = \sqrt{2e^y - 2} = x$$

and it is only operating in the positive range $y > 0$.

a) Find the equation for the Compensator $f_C(x_d) = \hat{y}$.
*Hint: Leave the disturbance $d$ out of the equation.*

b) Find the sensitivity of the Compensator $\dfrac{d\hat{y}}{dx_d}$.

c) At which value of $x_d = x_{set}$ the sensitivity is the highest? Provide your answer with a calculation or explanation.

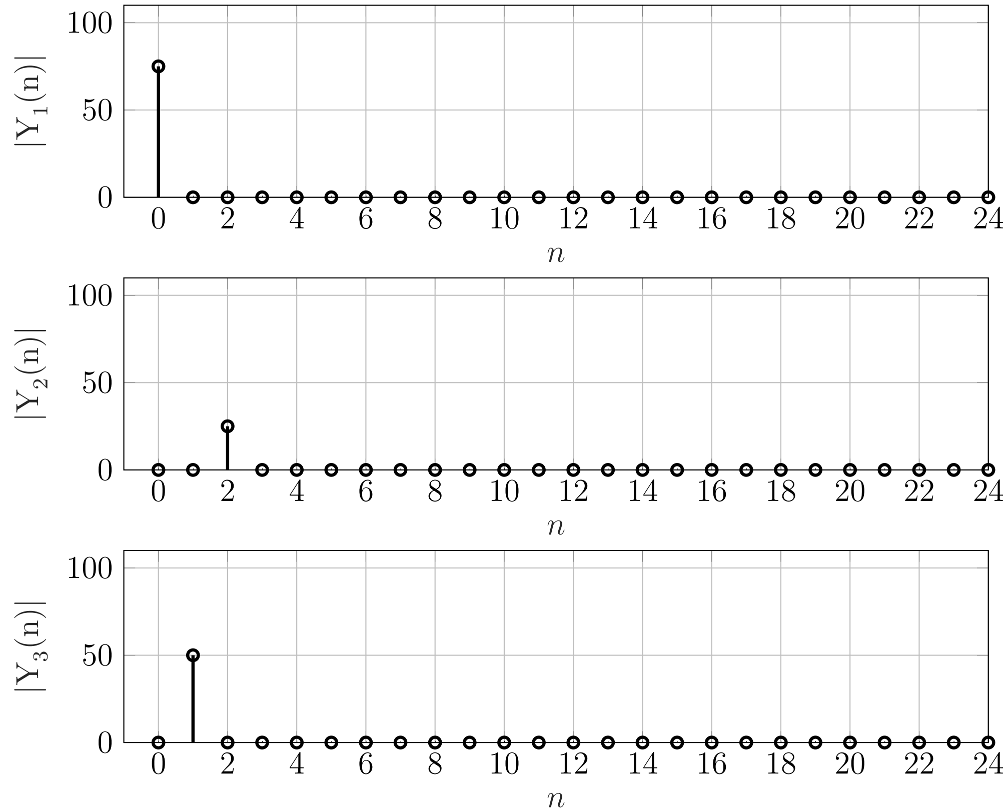
d) We now have systematic disturbance of $d = 0.1$. What are the values for $\hat{y}$, $x_d$ and $y$ if $x = x_{set}$?

e) What is the relative error $e_{rel}$ and what is the absolute error $e_{abs}$ with the disturbance $d = 0.5$ if $x = x_{set}$ in your measurement of $y$?

**Task 4:** **DFT (16 Points)**

In the following you can see the DFT results (left half of the spectrum) for different signals.

*Hint:* $|Y(n)| = \frac{N}{2} \cdot A$ *if $A$ is the Amplitude of an oscillation.*
*At $\omega = 0$ holds:* $|Y(n)| = N \cdot A$.

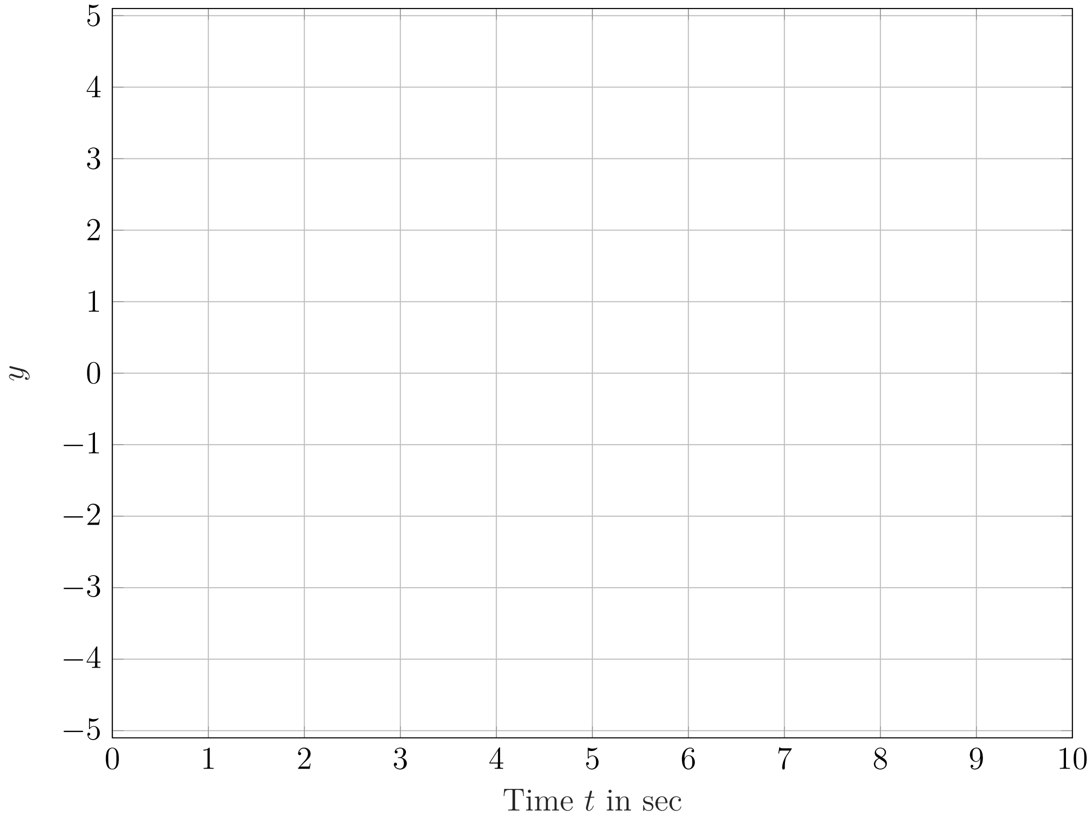



The signal were recorded with a sampling time of 0.2 sec in the time frame from 0 to 9.8 sec (first sample at t=0 sec).

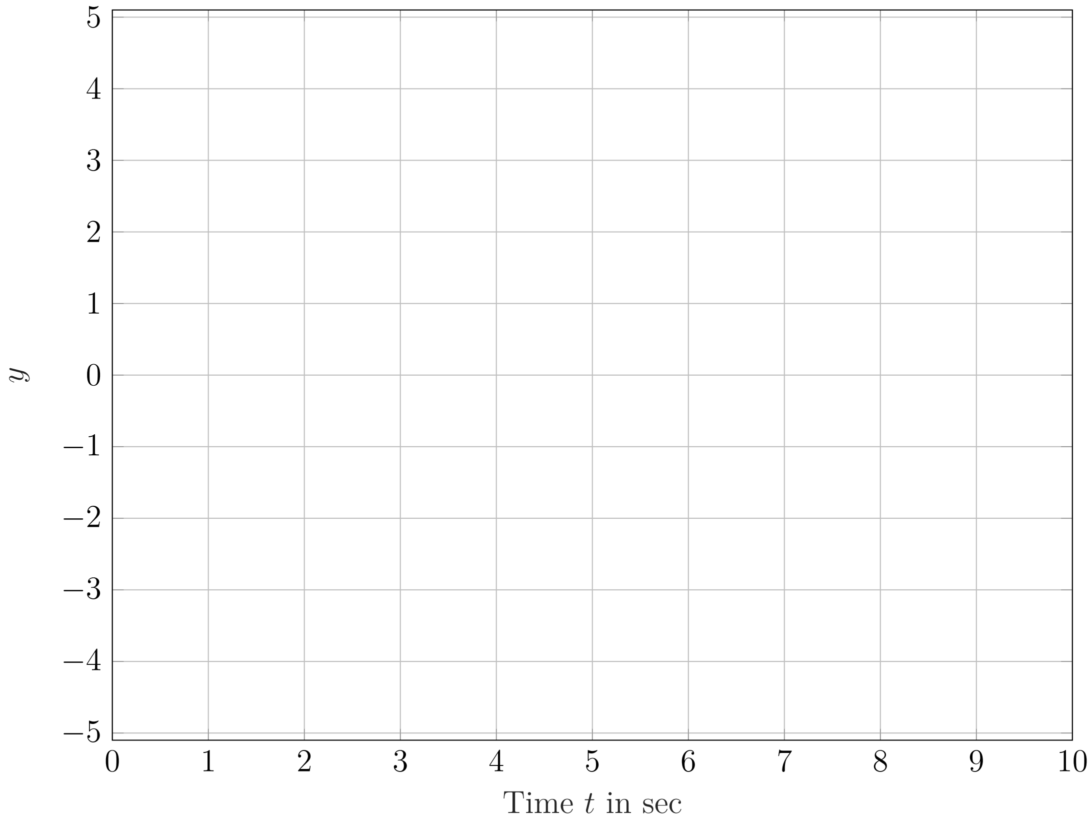
a) What is the sampling frequency $f_0$?

b) Determine $N$ when $N$ is the number of samples per signal.

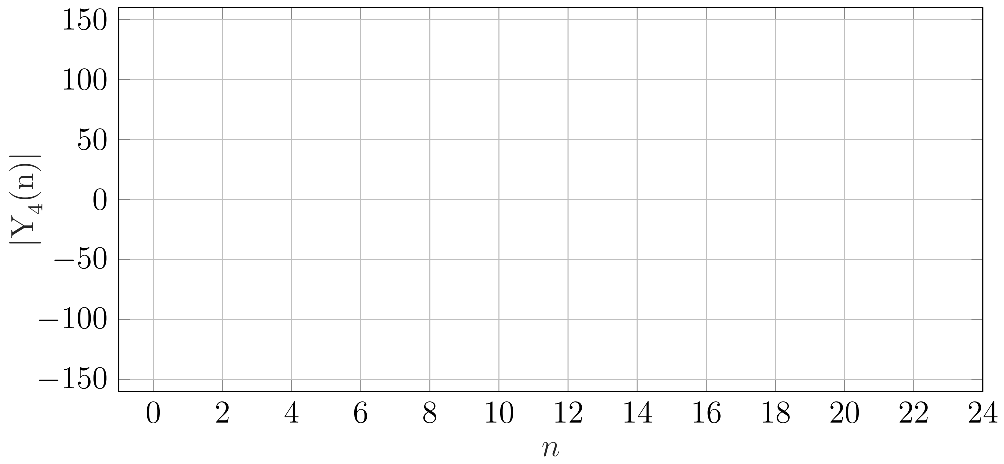
c) Draw the signals $y_1(t)$, $y_2(t)$ and $y_3(t)$ according to the shown DFT plots into the empty diagram. Make sure that one can determine which line represents which signal.



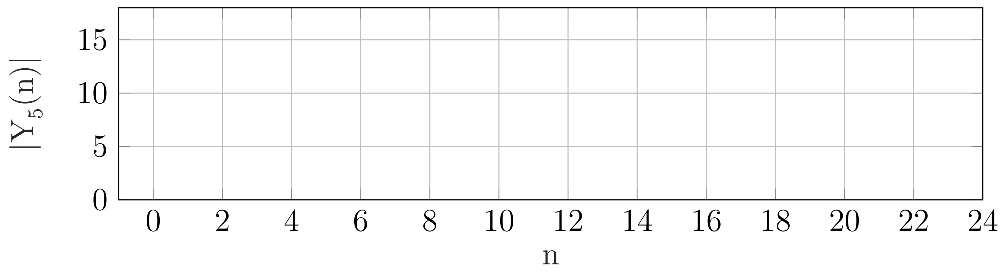

d) Draw $y_4(t) = y_1(t) - y_2(t) - y_3(t)$ into the following empty diagram.




e) Plot the output of the DFT $|Y_4(n)|$ of your previously drawn signal $y_4(t)$ into the following empty diagram.



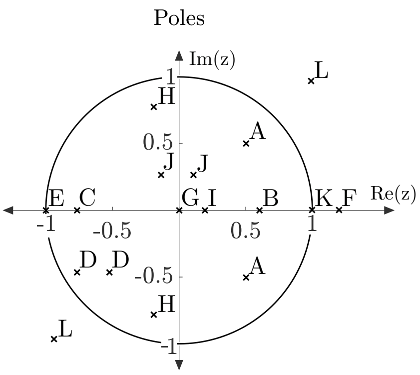

f) Sketch qualitatively (do not calculate!) the output of the DFT $|Y_5(n)|$ for $y_5\,(t) = \sin\left(2\pi t \frac{25}{20}\text{Hz}\right)$ into the following diagram.



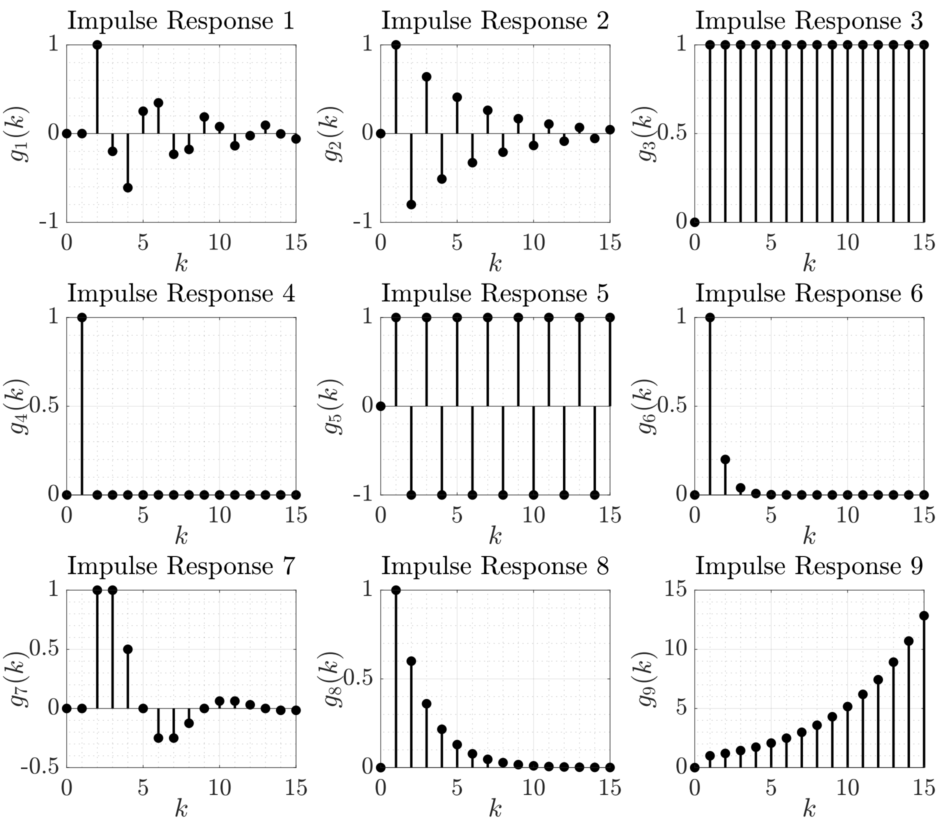
## Task 5: Impulse Responses (22 Punkte)

Given are the pole locations for 12 different systems (A-L) in the pole zero map and 9 different impulse responses (1-9).

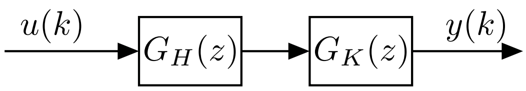
a) Match the pole locations to the impulse responses. Note down your answers in the given table. (Note: Three pole locations do not belong to any impulse response. All transfer functions have the structure $G(z) = \frac{1}{A(z)}$ with $A(z) = \prod_{i=1}^{n}(z - p_i)$.)




| Impulse Response | Poles |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |




b) Assume that the poles of system H are at $p_{H,1} = -0.1 + 0.8i$ and $p_{H,2} = -0.1 - 0.8i$ and the pole of system K at $p_K = 1$. System H and K are now connected in series to form the new overall system $G_{HK}$ (see diagram below). Calculate the resulting transfer function for $G_{HK}$ and apply the final value theorem to the system's response with input signal $u(k) = 2 \cdot \delta_K(k)$.



## Task 6: Quantization (15 Points)

Two different AD-converters are available for a quantization task. Both are rounding the signal values *down* to the next quantization step. The parameters of the converters are as follows:

| | 10-bit AD-converter | 12-bit AD-converter |
|---|---|---|
| sampling frequency: | $f_0 = 100$ Hz | $f_0 = 100$ Hz |
| resolution: | 10 bit | 12 bit |
| measurement range: | $U_{\text{range}} = 0 \dots 1024$ mV | $U_{\text{range}} = 0 \dots 1024$ mV |

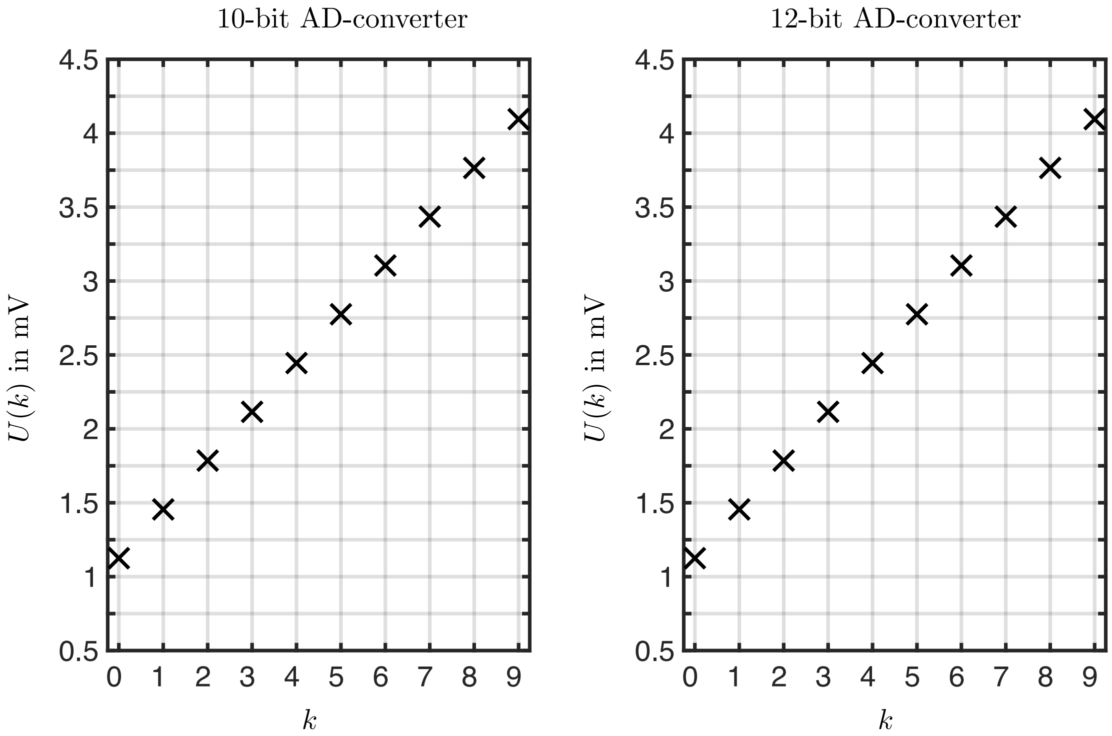
a) Calculate the step size $\Delta U$ and the maximal quantization error $e_{\text{Q max}}$ of both AD-converters.

b) How do you call the error signal $(U(k) - U_{\text{Q}}(k),\ k = 1, \dots, N)$ resulting from the quantization error?

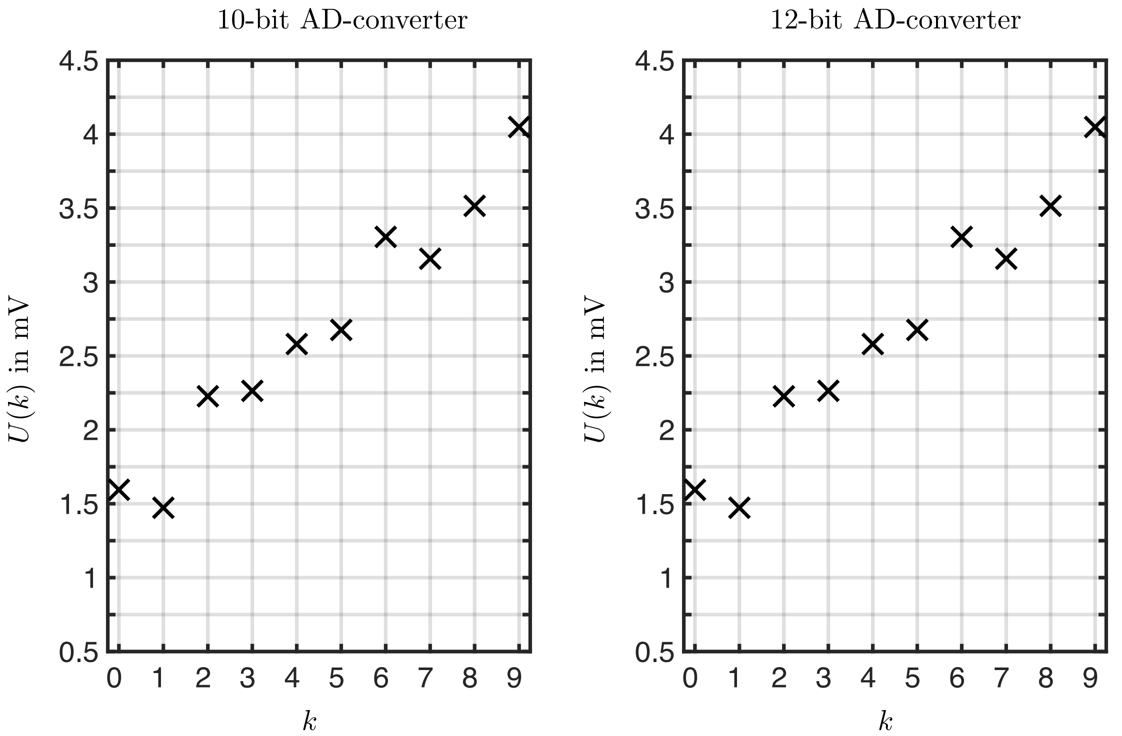
c) What type of error occurs from rounding the values down? Can this error be corrected and if so, how?

d) Assuming uniformly distributed signal, which has to be quantized. What is the probability distribution of the quantization error?

e) The values of an *ideal* ramp $U(k)$ shall be quantized using both AD-converters. Sketch the resulting signal $U_{\text{Q}}(k)$ in the given diagram.




f) The values of a *noisy* ramp $U(k)$ shall be quantized using both AD-converters. Sketch the resulting signal $U_{\text{Q}}(k)$ in the given diagram.




g) The quantized signal of the 12-bit AD-converter is closer to the real values. Therefore, the 12-bit AD-converter reconstructs the noise even better. State one way to get rid of the high frequency noise.

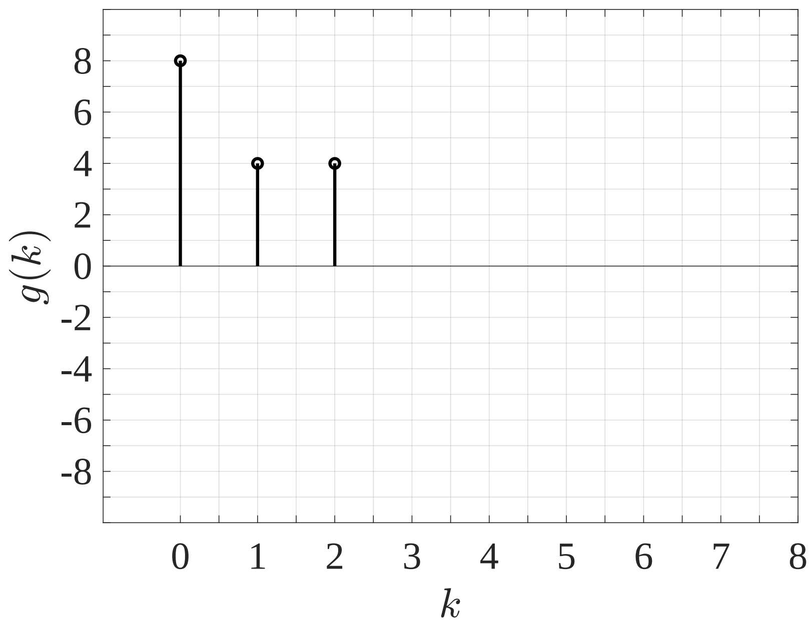
**Task 7: Linear Filter (14 Points)**

a) The linear transfer function of a filter

$$G_1(z) = \frac{b_0 z + b_1}{z + a_1} \tag{1}$$

is given. Calculate the corresponding difference equation.

b) From the part of the impulse response shown in the figure the coefficients $b_0$, $b_1$ and $a_1$ should be calculated.



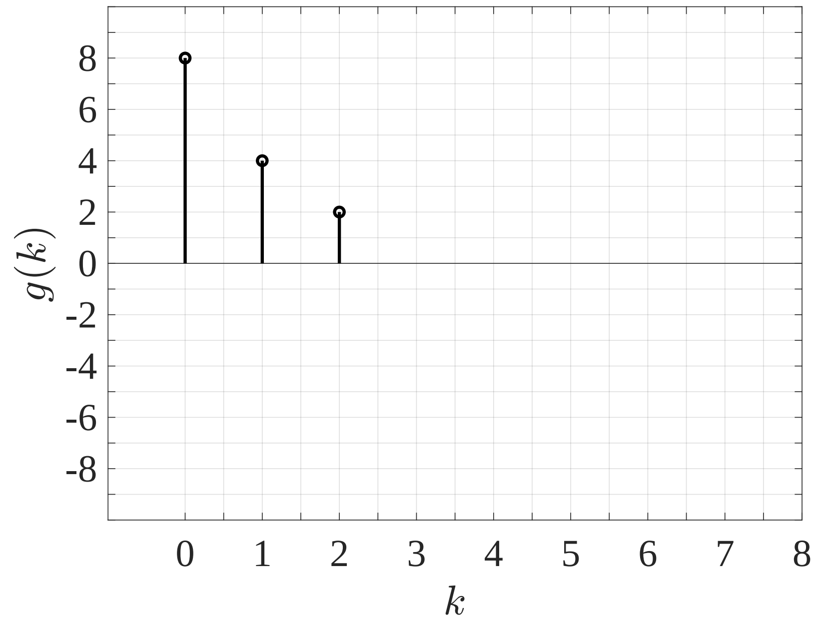

c) Draw the next 4 values (time steps $k = 3, \ldots, 6$) of the impulse response of $G_1(z)$ into the diagram.

d) Now, let the transfer function

$$G_2(z) = \frac{b_0 z^2}{z^2 + a_1 z + a_2} \tag{2}$$

be given. Calculate the corresponding difference equation.

e) Calculate from the shown part of the impulse response the coefficients $b_0$, $a_1$ and $a_2$ of $G_2(z)$.




f) Draw the next 4 values of the impulse response (time steps $k = 3\ldots,6$) of the impulse response of $G_2(z)$ into the diagram.

# Solution:

## Task 1: Comprehension Questions (19 Points)

Mark the correct answers clearly.
**Every question has one, two or three correct answers!**
For every correctly marked answer you will get one point. For every wrong answer a point will be subtracted, but a question will never be rated with negative points.

a) Which statements are true regarding A/D Converters?

- ☐ The Flash Converter requires only two comparators for each $2^{n} - 1$ quantization level.
- ☐ The Weighting Method works faster than the Flash Converter (for the same resolution).
- ☐ The Dual Slope Converter works faster than the Flash Converter (for the same resolution).
- ■ Conversion with the Dual Slope Principle is robust to unwanted influences (such as temperature).

b) Which statements are true regarding filters?

- ■ A first type Chebyshef filter of 4rth order has a phase shift of -360° for $\omega \to \infty$.
- ■ Butterworth filters have a monotone amplitude response.
- ■ For identical orders, a Cauer filter has a steeper transition band than a Bessel, Butterworth or Chebyshef filter.
- ☐ Even though the Elliptic filter has a smoother amplitude response than the Chebyshef filter, the step response of a Cheychef filter (of same order) oscillates stronger.

c) What is the meaning of the terms *dead zero* and *life zero*?

- ☐ If a measurement of a signal $\neq 0$ has a value $= 0$, it is called a dead zero.
- ■ Live zero is a measurement technique which allows detection of a broken wire.
- ☐ Live zero means a measurement device is broken and has to be replaced.
- ■ If a measurement of a signal $= 0$ has a value $= 0$, it is called a dead zero.

d) Which statements are true regarding time discrete systems?

- ■ Moving average filters are FIR filters.
- ☐ Autoregressive moving average filters are more advanced moving average filters that don't use feedback anymore.
- ☐ Moving average filters can become unstable if the coefficients are not chosen carefully.
- ■ The impulse response of an autoregressive system never reaches a constant value.

e) Which statements are true regarding aliasing?

- [x] Increasing the sampling frequency is a common way to deal with aliasing effects.
- [ ] Upsampling signals should always be carried out with anti aliasing filters to make sure no new frequency components above $f_0/2$ appear in the signal.
- [ ] Aliasing happens, when shadow spectra appear in the frequency response of a system.
- [x] If a periodic signal is sampled at the same rate as it repeats, it appears to be a constant.

f) Which statements are true regarding clustering?

- [x] Clustering tries to find data points that are grouped together.
- [ ] Clustering is usually a supervised learning method.
- [ ] The number of clusters has not to be known in advance, but is determined through the clustering algorithm.
- [x] Different clustering algorithms exist, that differ in how the distances of data points from the cluster center are determined.

g) How large should the internal resistance of a voltage meter be?

- [ ] Very small, in the ideal case zero.
- [x] Very large, in the ideal case infinitely large.
- [ ] It depends on the electrical circuit, what internal resistance is ideal.
- [ ] The internal resistance has no influence on the measurement.

h) Assess following statements regarding errors:

- [ ] The quantization error is half as large if a 16 bit A/D converter is used instead of an 8 bit converter.
- [ ] The absolute error of a measurement is the difference between the measured value and the true value, divided by the true value.
- [x] By averaging measurements, random errors are typically reduced with $1/\sqrt{N}$, where $N$ is the number of measurements.
- [ ] Random errors cannot be reduced by averaging over several measurements.

i) Assess the following statements regarding the Student's t-distribution.

- [ ] The t-distribution is identical to the normal distribution.
- [x] Its confidence interval $1 - \alpha$ is wider than the confidence interval of the corresponding normal distribution.
- [ ] If the size $N$ of the regarded data set is big enough ($N \to \infty$), it converges to the cauchy distribution.
- [x] It is used if the real standard deviation has to be estimated from data.

j) Operational amplifiers ...

- [x] ... are active components (an external energy source is needed).
- [ ] ... have a very high **output** resistance.
- [x] ... are amplifiers with a very high gain.
- [x] ... have a very high **input** resistance.

## **Task 2: Correlation function (17 Punkte)**

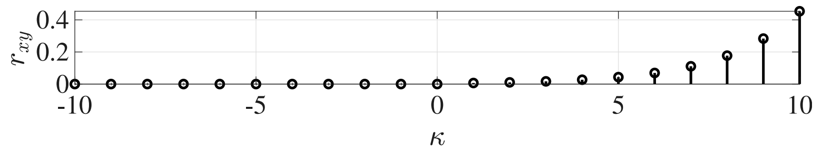
a) Three systems are given

$$
\begin{aligned}
G_1(z) &= \frac{0.5z^{-1}}{1-0.6z^{-1}}\\
G_2(z) &= 0.5z^{-1} + 0.4z^{-2}\\
G_3(z) &= z^{-4}.
\end{aligned}
$$

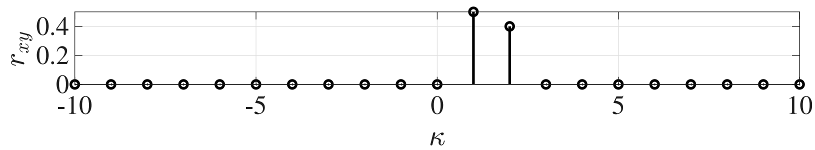
The systems were excited with an Kronecker delta $x(k)$ and the resulting output $y(k)$ was recorded. Subsequently, the cross-correlation function $r_{xy}(\kappa)$ was calculated. Assign the correct cross-correlation functions $A, B, ..., F$ to the right system $G_1, G_2, G_3$.

**Answer**:

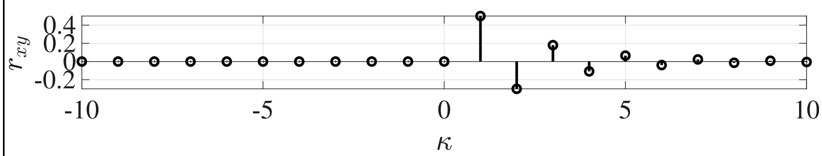
A



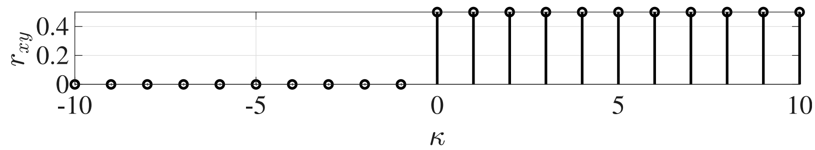

B



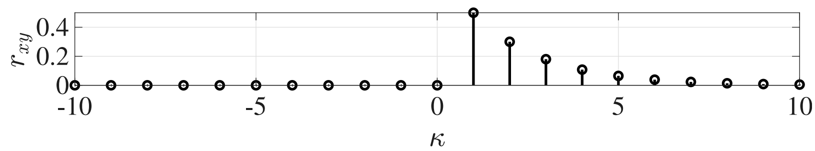

C



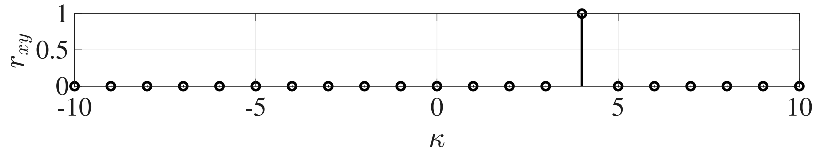

D




E



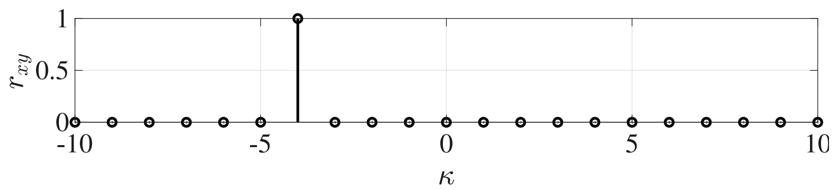

F




| System | Cross-correlation-function |
|---|---|
| 1 | E |
| 2 | B |
| 3 | F |

6

$H$




b) Another cross-correlation function $H$ is given, calculated exactly as in the previous task. What is the transfer function of the underlying system?

**Answer**:

$$G_H = z^4$$

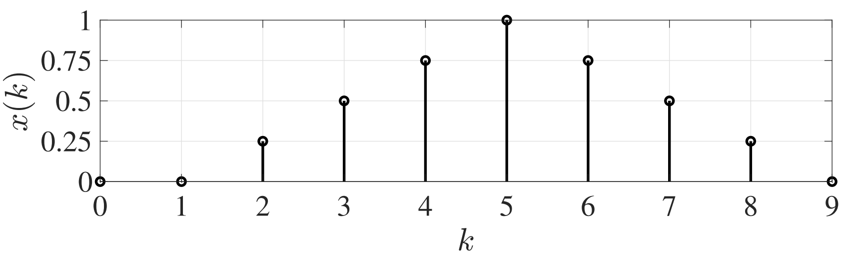
1

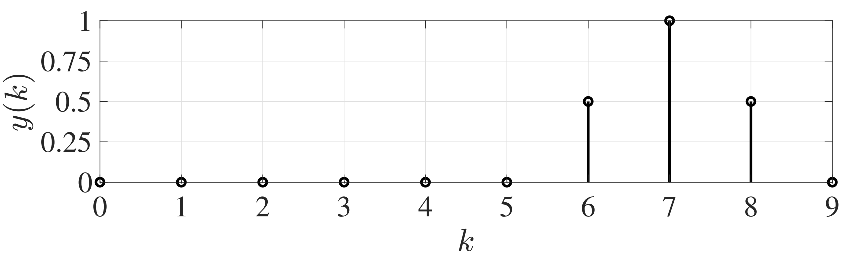
c) How does the system from task b) differ from the systems in task a)?
**Answer**: The system is acausal. 1

d) Is it possible to represent a real pysical system with the transfer function from task b)? **Answer**:No. 1

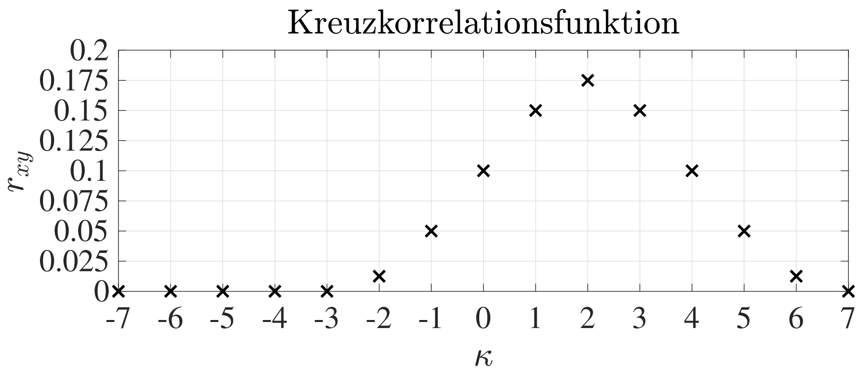
e) The time-discrete signals $x(k)$ and $y(k)$ are given. Calculate the cross-correlation function $r_{xy}$ for $\kappa = -7, -6, ..., 7$ and sketch the cross-correlation function in the given diagram.
Hint:

$$r_{xy}(\kappa) = \begin{cases} \frac{1}{N} \sum\limits_{k=0}^{N-1-\kappa} x(k) \cdot y(k+\kappa), & \text{for } \kappa \geq 0 \\ \frac{1}{N} \sum\limits_{k=-\kappa}^{N-1} x(k) \cdot y(k+\kappa), & \text{else.} \end{cases}$$




**Answer**: 3



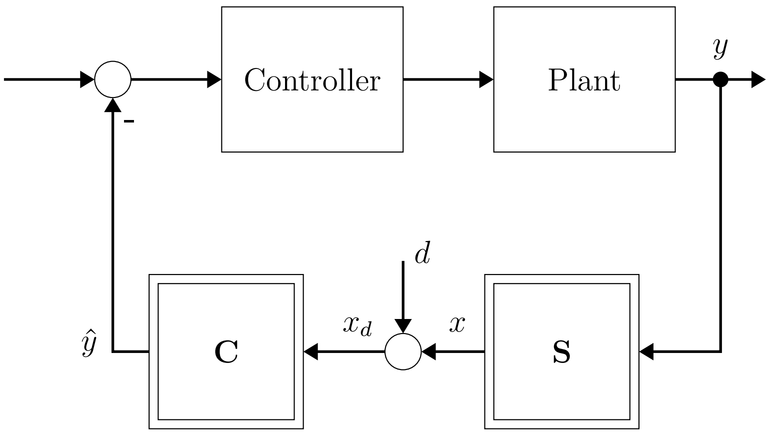

$$
\begin{aligned}
r_{xy}(-7) &= \frac{1}{10}(0.5 \cdot 0 + 0.25 \cdot 0 + 0 \cdot 0) = 0 \\
r_{xy}(-6) &= \frac{1}{10}(0.75 \cdot 0 + 0.5 \cdot 0 + 0.25 \cdot 0 + 0 \cdot 0) = 0 \\
r_{xy}(-5) &= \frac{1}{10}(1 \cdot 0 + 0.75 \cdot 0 + 0.5 \cdot 0 + 0.25 \cdot 0 + 0 \cdot 0) = 0 \\
r_{xy}(-4) &= \frac{1}{10}(0.75 \cdot 0 + 1 \cdot 0 + 0.75 \cdot 0 + 0.5 \cdot 0 + 0.25 \cdot 0 + 0 \cdot 0) = 0 \\
r_{xy}(-3) &= 0 \\
r_{xy}(-2) &= \frac{1}{10}(0.25 \cdot 0.5) = 0.0125 \\
r_{xy}(-1) &= \frac{1}{10}(0.5 \cdot 0.5 + 1 \cdot 0.25) = 0.05 \\
r_{xy}(0) &= \frac{1}{10}(0.5 \cdot 0.75 + 1 \cdot 0.5 + 0.5 \cdot 0.25) = 0.1 \\
r_{xy}(1) &= \frac{1}{10}(0.5 \cdot 1 + 1 \cdot 0.75 + 0.5 \cdot 0.5) = 0.15 \\
r_{xy}(2) &= \frac{1}{10}(0.5 \cdot 0.75 + 1 \cdot 1 + 0.5 \cdot 0.75) = 0.175 \\
r_{xy}(3) &= \frac{1}{10}(0.5 \cdot 1 + 1 \cdot 0.75 + 0.5 \cdot 0.5) = 0.15 \\
r_{xy}(4) &= \frac{1}{10}(0.5 \cdot 0.75 + 1 \cdot 0.5 + 0.5 \cdot 0.25) = 0.1 \\
r_{xy}(5) &= \frac{1}{10}(0.5 \cdot 0.5 + 1 \cdot 0.25) = 0.05 \\
r_{xy}(6) &= \frac{1}{10}(0.25 \cdot 0.5) = 0.0125 \\
r_{xy}(7) &= 0
\end{aligned}
$$

$\boxed{5}$

$\boxed{\sum 17}$

## Task 3: Compensator (17 Points)

In order to control a plant, the output of the system has to be measured. For many sensors (e.g. Thermocouples) the measurement of the output results in a voltage value. From this voltage value, the physical value (e.g. a temperature) can be calculated.
In this example we have a sensor in the feedback loop which measures the output $y$ of the plant. The output of the sensor $S$ is $x$. This value can suffer from additional disturbance $d$. $x_d$ is then fed into the compensator $C$. The Compenstor $C$ does not compensate the added noise $d$ it just reconstructs $y$ by $\hat{y}$ from $x_d$ with the inverse characteristic from $S$. So with $d = 0$ the statement $\hat{y} = y$ is valid.




The equation of the sensor is

$$f_S(y) = \sqrt{2e^y - 2} = x$$

and it is only operating in the positive range $y > 0$.

a) Find the equation for the Compensator $f_C(x_d) = \hat{y}$.
*Hint: Leave the disturbance $d$ out of the equation.*
**Answer**: $f_C(x_d) = \hat{y} = \ln(0.5x^2 + 1)$ 3

b) Find the sensitivity of the Compensator $\frac{d\hat{y}}{dx_d}$.
**Answer**: $\frac{d\hat{y}}{dx_d} = \frac{2x_d}{x_d^2 + 2}$ 2

c) At which value of $x_d = x_{set}$ the sensitivity is the highest? Provide your answer with a calculation or explanation.
**Answer**:
One way to find the maximum: Take the derivative and set it to 0.

$$\frac{d^2\hat{y}}{dx^2} = \frac{-2x^2 + 4}{(x^2 + 2)^2} = 0$$

$$x^2 = 2$$

$$x_{d1} = \sqrt{2}; x_{d2} = -\sqrt{2}$$

The maximum must be $x_{d1} = \sqrt{2} = x_{set}$ since the operating range is for positive values of $y$ which also means, that all values of $x$ must be positive. 6

d) We now have systematic disturbance of $d = 0.1$. What are the values for $\hat{y}$, $x_d$ and $y$ if $x = x_{set}$?
**Answer**:

$$x_d = 1.51$$
$$\hat{y} = 0.764$$
$$y = 0.693$$

3

e) What is the relative error $e_{rel}$ and what is the absolute error $e_{abs}$ with the disturbance $d = 0.5$ if $x = x_{set}$ in your measurement of $y$?
**Answer**:

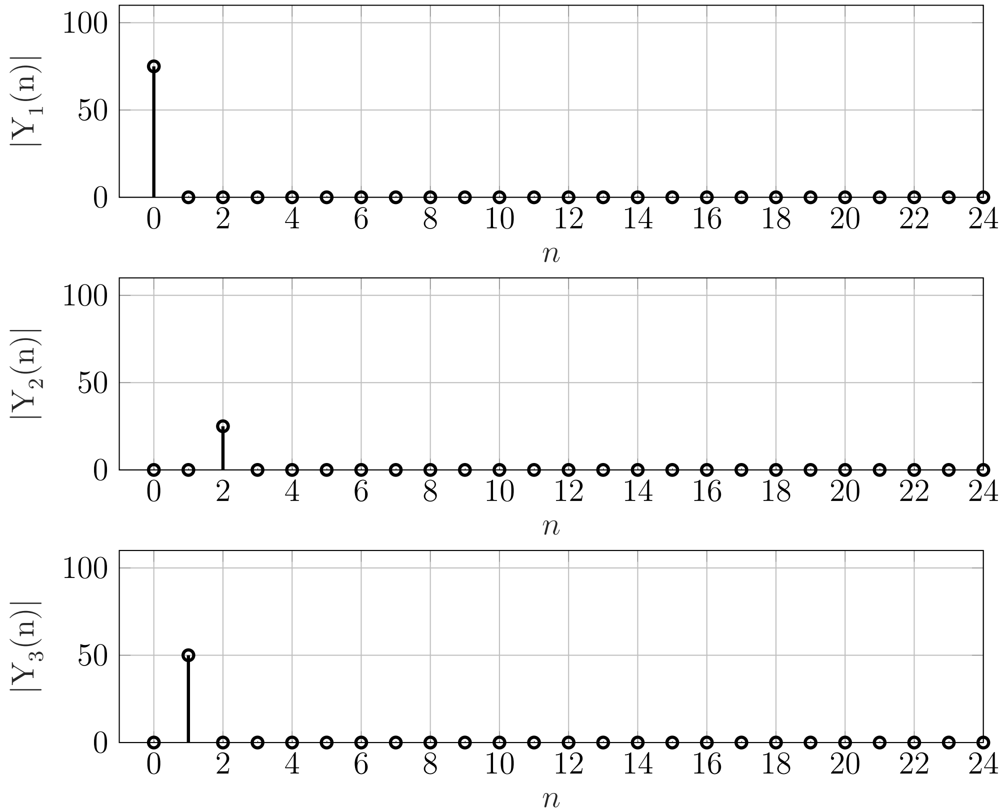
$$\begin{aligned} y &= 0.69 \\ x_d &= \sqrt{2} + 0.5 = 1.91 \\ \hat{y} &= \ln\left(0.5 \cdot (1.91)^2 + 1\right) = 1.04 \\ e_{abs} &= |\hat{y} - y| = 0.35 \\ e_{rel} &= \frac{e_{abs}}{y} = 0.50 \end{aligned}$$

3

$\sum 17$

## Task 4: DFT (16 Points)

In the following you can see the DFT results (left half of the spectrum) for different signals.
*Hint*: $|Y(n)| = \frac{N}{2} \cdot A$ *if $A$ is the Amplitude of an oscillation.*
*At $\omega = 0$ holds: $|Y(n)| = N \cdot A$.*

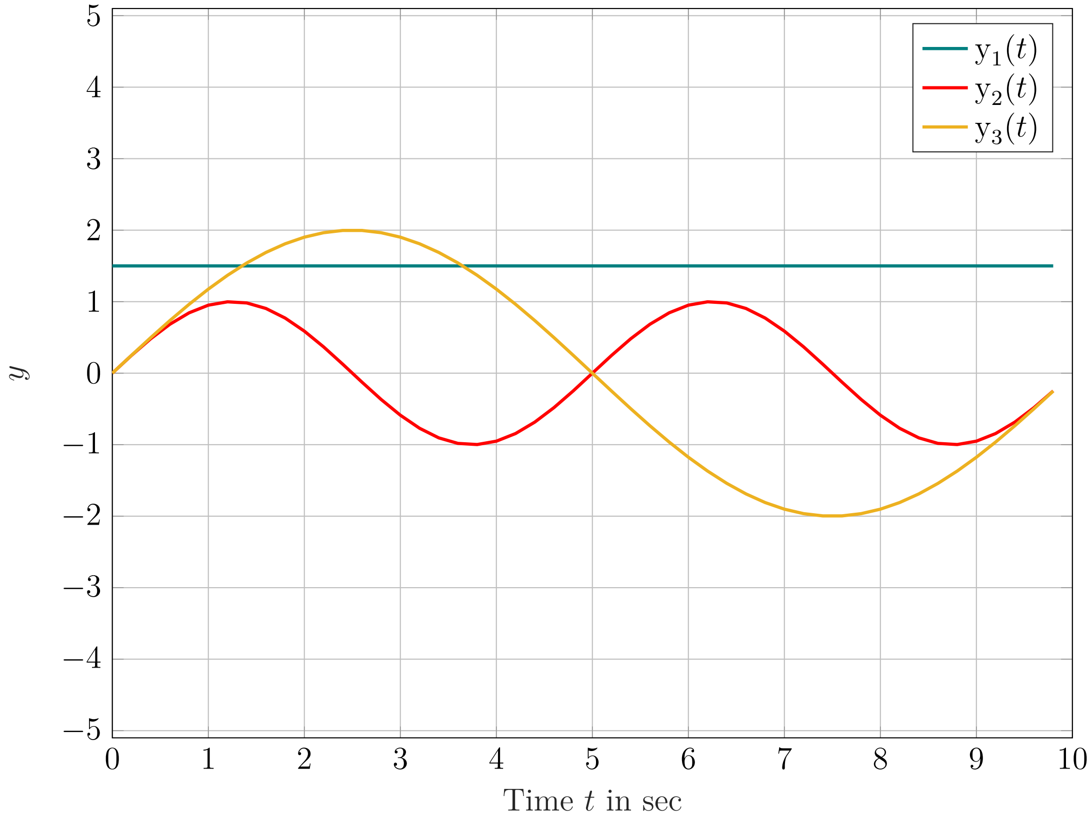



The signal were recorded with a sampling time of 0.2 sec in the time frame from 0 to 9.8 sec (first sample at t=0 sec).

a) What is the sampling frequency $f_0$?
**Answer**: $f_0 = \frac{1}{0.2 \text{ sec}} = 5$ Hz 1

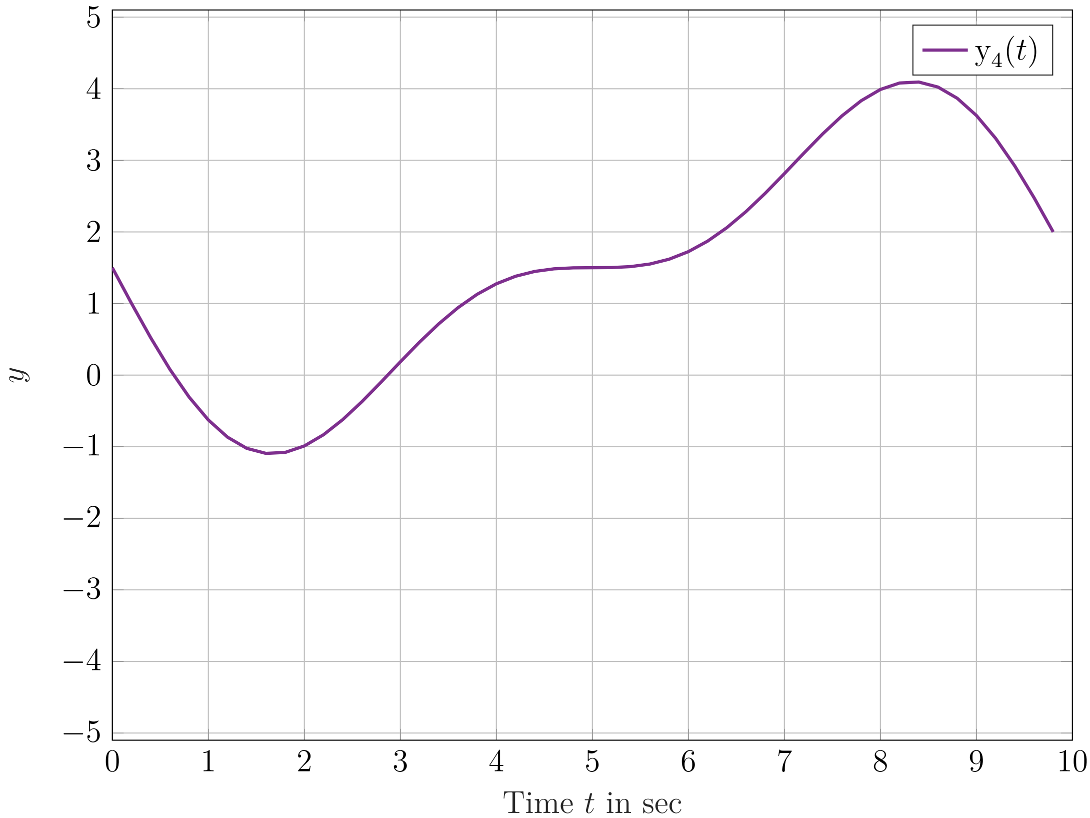
b) Determine $N$ when $N$ is the number of samples per signal.
**Answer**: $N = 50 = \frac{9.8}{0.2} + 1$ 1

c) Draw the signals $y_1(t)$, $y_2(t)$ and $y_3(t)$ according to the shown DFT plots into the empty diagram. Make sure that one can determine which line represents which signal.
**Answer**:
$y_1(t)$ is a constant with value 1.5.
$y_2(t)$ is a sine wave with the frequency $\frac{2}{50} \cdot 5$ Hz and an amplitude of 1.
$y_3(t)$ is a sine wave with the frequency $\frac{1}{50} \cdot 5$ Hz and an amplitude of 2.




6

d) Draw $y_4(t) = y_1(t) - y_2(t) - y_3(t)$ into the following empty diagram.

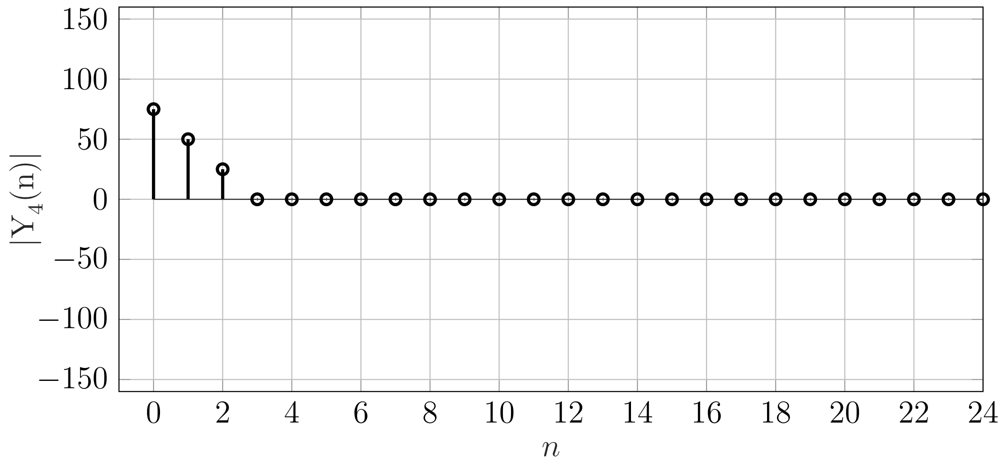



2

e) Plot the output of the DFT $|Y_4(n)|$ of your previously drawn signal $y_4(t)$ into the following empty diagram.

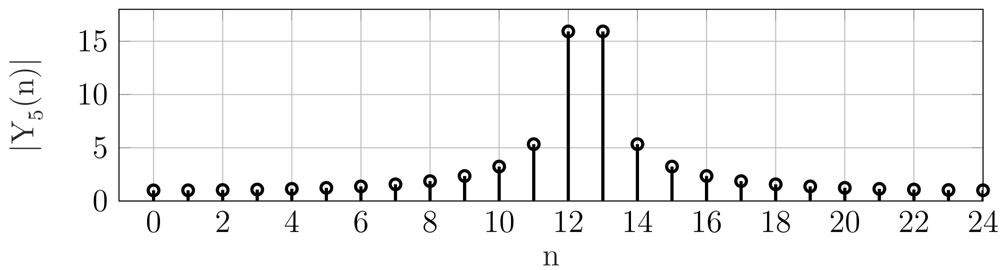
**Answer**:

The spectrum $|Y_4(N)| = |Y_1(N)| + |Y_2(N)| + |Y_3(N)|$ is a sum of the first 3 spectra, since the DFT is a linear transformation. The sign of the single components does not matter since we only look at the magnitude of the DFT.




3

f) Sketch qualitatively (do not calculate!) the output of the DFT $|Y_5(n)|$ for $y_5(t) = \sin\left(2\pi t \frac{25}{20}\text{Hz}\right)$ into the following diagram.



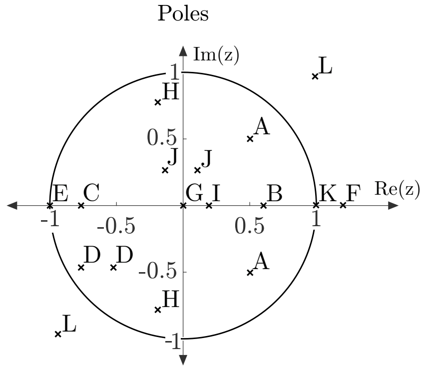

3

$\sum$ 16

## Task 5: Impulse Responses (22 Punkte)

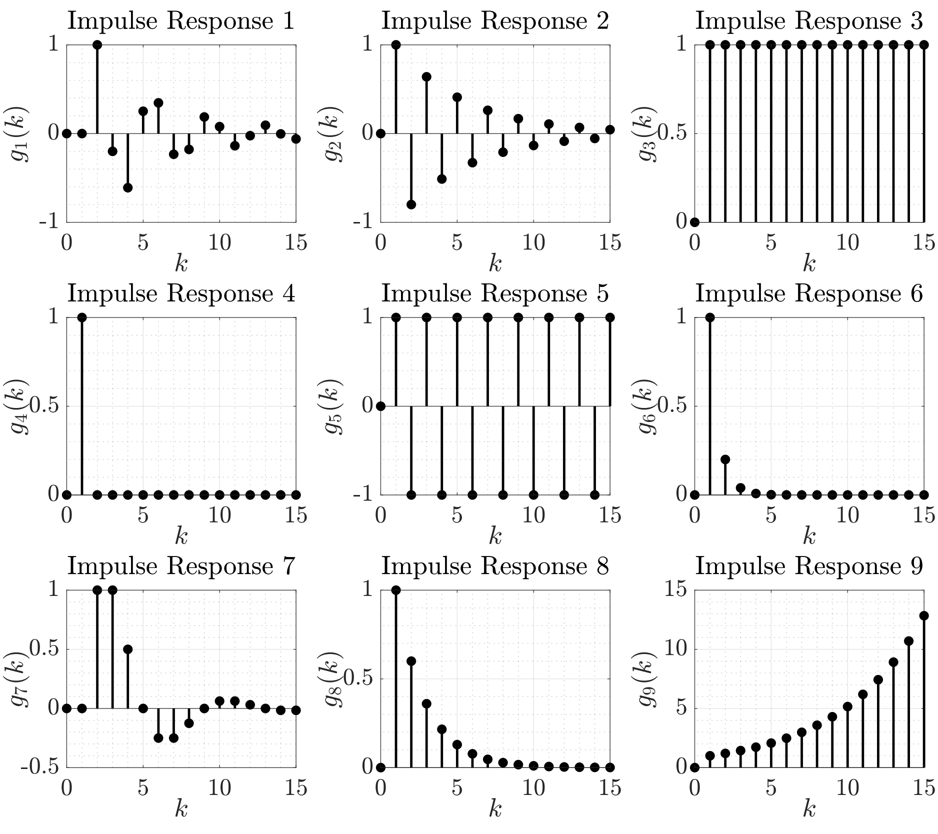
Given are the pole locations for 12 different systems (A-L) in the pole zero map and 9 different impulse responses (1-9).

a) Match the pole locations to the impulse responses. Note down your answers in the given table. (Note: Three pole locations do not belong to any impulse response. All transfer functions have the structure $G(z) = \frac{1}{A(z)}$ with $A(z) = \prod_{i=1}^{n}(z - p_i)$.)

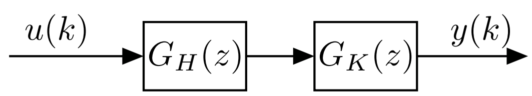



| Impulse Response | Poles |
|---|---|
| 1 | H |
| 2 | C |
| 3 | K |
| 4 | G |
| 5 | E |
| 6 | I |
| 7 | A |
| 8 | B |
| 9 | F |

2
2
2
2
2
2
2
2
2




b) Assume that the poles of system H are at $p_{H,1} = -0.1 + 0.8i$ and $p_{H,2} = -0.1 - 0.8i$ and the pole of system K at $p_K = 1$. System H and K are now connected in series to form the new overall system $G_{HK}$ (see diagram below). Calculate the resulting transfer function for $G_{HK}$ and apply the final value theorem to the system's response with input signal $u(k) = 2 \cdot \delta_K(k)$.



$$G_{HK}(z) = \frac{1}{(z-(-0.1+0.8i))(z-(-0.1-0.8i))(z-1)} = \frac{1}{(z^2+0.2z+0.65)(z-1)}$$

$$= \frac{z^{-2}}{(1+0.2z^{-1}+0.65z^{-2})(z-1)}$$

$$U(z) = \mathcal{Z}\left\{u(k)\right\} = 2 \cdot 1$$

$$\lim_{z\to 1}(z-1)Y(z) = \lim_{z\to 1}(z-1)G_{HK}(z)U(z)$$

$$= \lim_{z\to 1}\cancel{(z-1)} \cdot \frac{1}{(z^2+0.2z+0.65)\cancel{(z-1)}} \cdot 2 = \frac{40}{37} \approx 1.081$$

$\boxed{4}$

$\boxed{\sum 22}$

## Task 6: Quantization (15 Points)

Two different AD-converters are available for a quantization task. Both are rounding the signal values *down* to the next quantization step. The parameters of the converters are as follows:

| | **10-bit AD-converter** | | **12-bit AD-converter** | |
|---|---|---|---|---|
| sampling frequency: | $f_0 = 100$ | Hz | $f_0 = 100$ | Hz |
| resolution: | 10 | bit | 12 | bit |
| measurement range: | $U_{\text{range}} = 0 \dots 1024$ | mV | $U_{\text{range}} = 0 \dots 1024$ | mV |

a) Calculate the step size $\Delta U$ and the maximal quantization error $e_{\text{Q max}}$ of both AD-converters.
The step size $\Delta U$ is calculated by partitioning the measurement range in $2^n$ equidistant parts:

$$\begin{aligned}
\text{In general:} \quad \Delta U &= \frac{U_{\max} - U_{\min}}{2^n} \\
\Delta U_{\text{10-bit}} &= \frac{1024\,\text{mV} - 0\,\text{mV}}{2^{10}} \\
\Delta U_{\text{10-bit}} &= \frac{1024\,\text{mV}}{1024} \\
\Delta U_{\text{10-bit}} &= 1\,\text{mV} \\
\Delta U_{\text{12-bit}} &= \frac{1024\,\text{mV} - 0\,\text{mV}}{2^{12}} \\
\Delta U_{\text{12-bit}} &= \frac{1024\,\text{mV}}{4096} \\
\Delta U_{\text{12-bit}} &= \frac{1}{4}\,\text{mV}\,.
\end{aligned}$$

The maximal quantization error is identical to the step size, when rounding the real values down:

$$e_{\text{Q max}} = \frac{U_{\max} - U_{\min}}{2^n}$$ 2

$$e_{\text{Q max 10-bit}} = 1\,\text{mV}$$

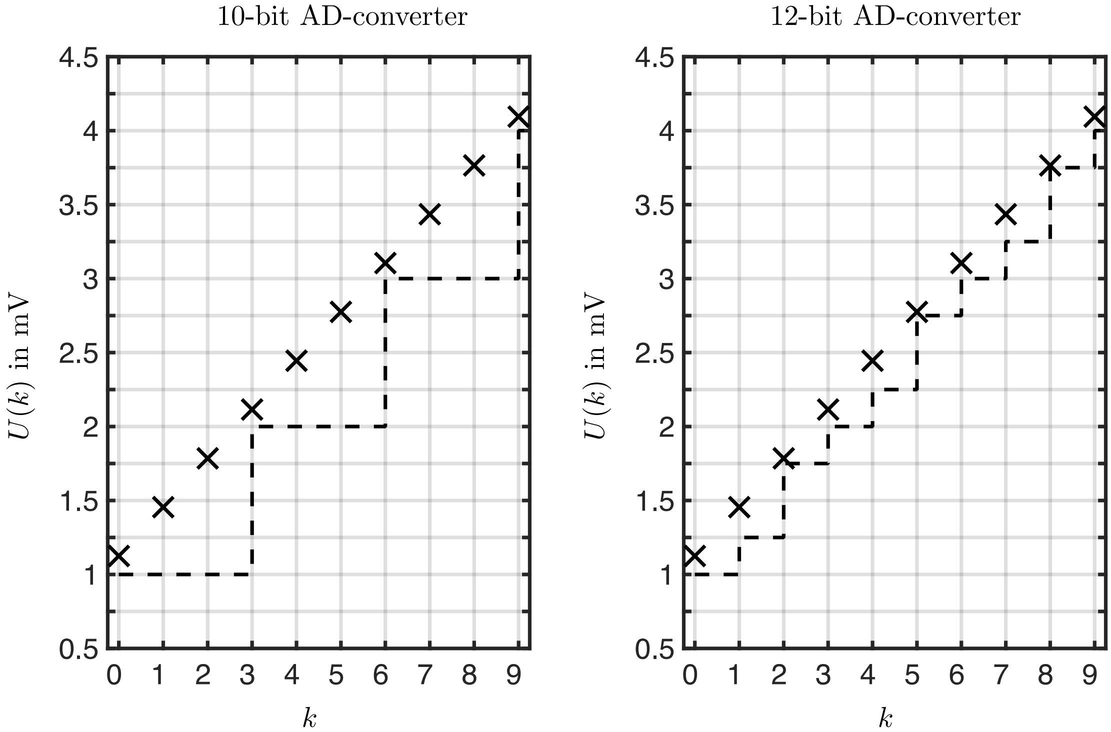
$$e_{\text{Q max 12-bit}} = \frac{1}{4}\,\text{mV}$$ 2

b) How do you call the error signal ($U(k) - U_{\text{Q}}(k)$, $k = 1, \dots, N$) resulting from the quantization error?
Quantization noise. 1

c) What type of error occurs from rounding the values down? Can this error be corrected and if so, how?
This is a systematic error. 1
The signal values are corrected by adding the half step size $\Delta U/2$. 1

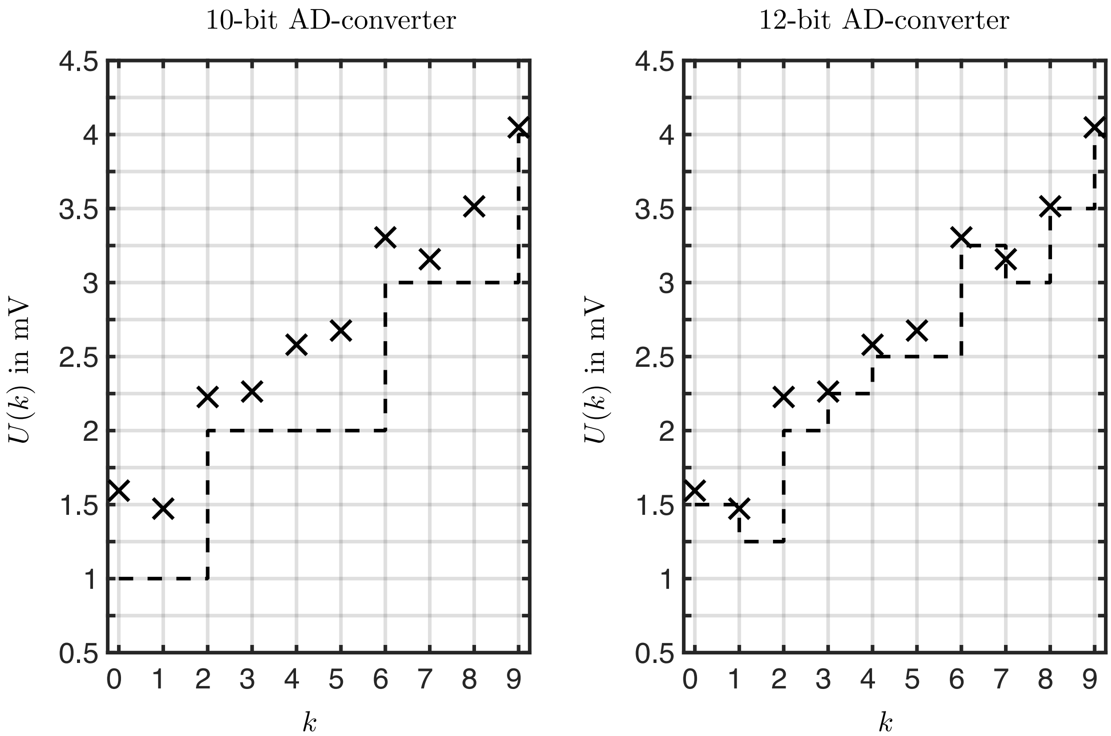
d) Assuming uniformly distributed signal, which has to be quantized. What is the probability distribution of the quantization error?
The quantization error is uniformly distributed as well. 1

e) The values of an *ideal* ramp $U(k)$ shall be quantized using both AD-converters. Sketch the resulting signal $U_{\mathrm{Q}}(k)$ in the given diagram.




3

f) The values of a *noisy* ramp $U(k)$ shall be quantized using both AD-converters. Sketch the resulting signal $U_{\mathrm{Q}}(k)$ in the given diagram.




3

g) The quantized signal of the 12-bit AD-converter is closer to the real values. Therefore, the 12-bit AD-converter reconstructs the noise even better. State one way to get rid of the high frequency noise.
The high frequent parts of the noise can be filtered out by using an low-pass filter. $\boxed{1}$

$\boxed{\sum 15}$

**<u>Task 7:</u> Linear Filter (14 Points)**

a) The linear transfer function of a filter

$$G_1(z) = \frac{b_0 z + b_1}{z + a_1} \tag{3}$$

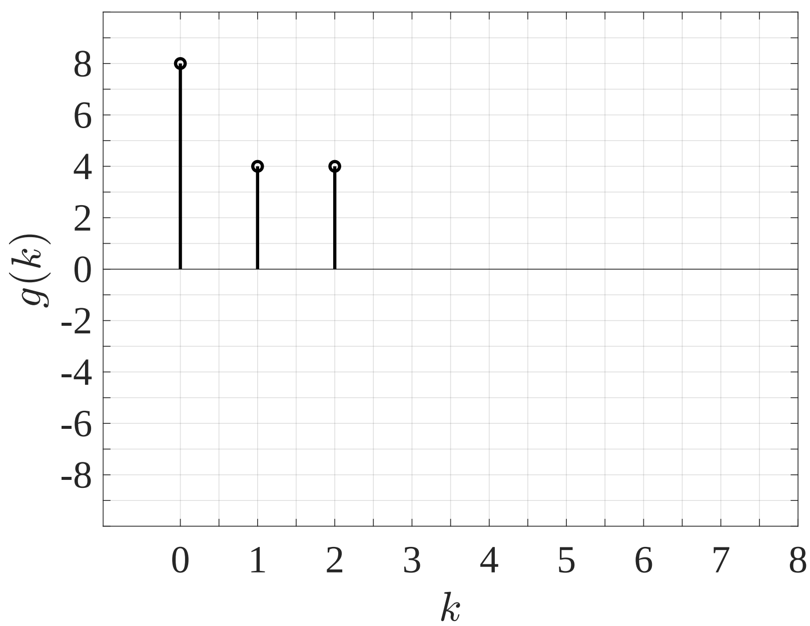
is given. Calculate the corresponding difference equation.

The corresponding difference equation is

$$y(k) = b_0 u(k) + b_1 u(k-1) - a_1 y(k-1). \tag{4}$$

$\boxed{1}$

b) From the part of the impulse response shown in the figure the coefficients $b_0$, $b_1$ and $a_1$ should be calculated.

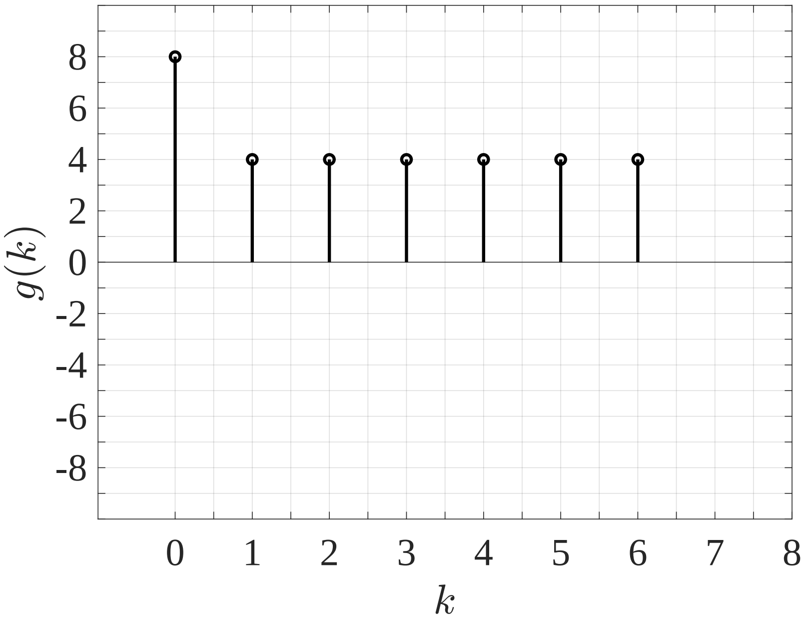



It holds that

$$\begin{aligned} g(0) &= 8 = b_0 \cdot 1 + b_1 \cdot 0 - a_1 \cdot 0 \rightarrow b_0 = 8 && (5) \\ g(2) &= 4 = b_0 \cdot 0 + b_1 \cdot 0 - a_1 \cdot 4 \rightarrow a_1 = -1 && (6) \\ g(1) &= 4 = b_0 \cdot 0 + b_1 \cdot 1 + 1 \cdot 8 \rightarrow b_1 = -4. && (7) \end{aligned}$$

$\boxed{4}$

c) Draw the next 4 values (time steps $k = 3, \ldots, 6$) of the impulse response of $G_1(z)$ into the diagram.




2

d) Now, let the transfer function

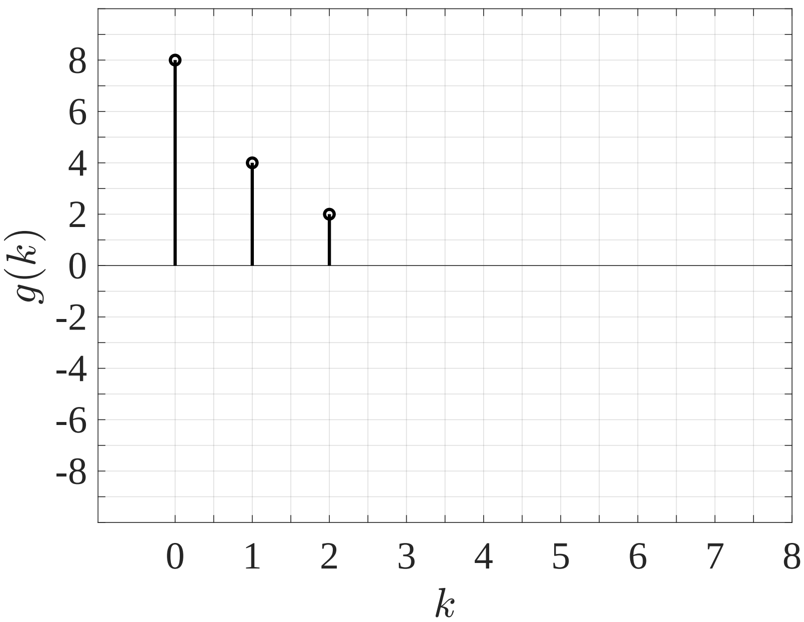
$$G_2(z) = \frac{b_0 z^2}{z^2 + a_1 z + a_2} \tag{8}$$

be given. Calculate the corresponding difference equation. The corresponding difference equation is

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) + b_0 u(k). \tag{9}$$

1

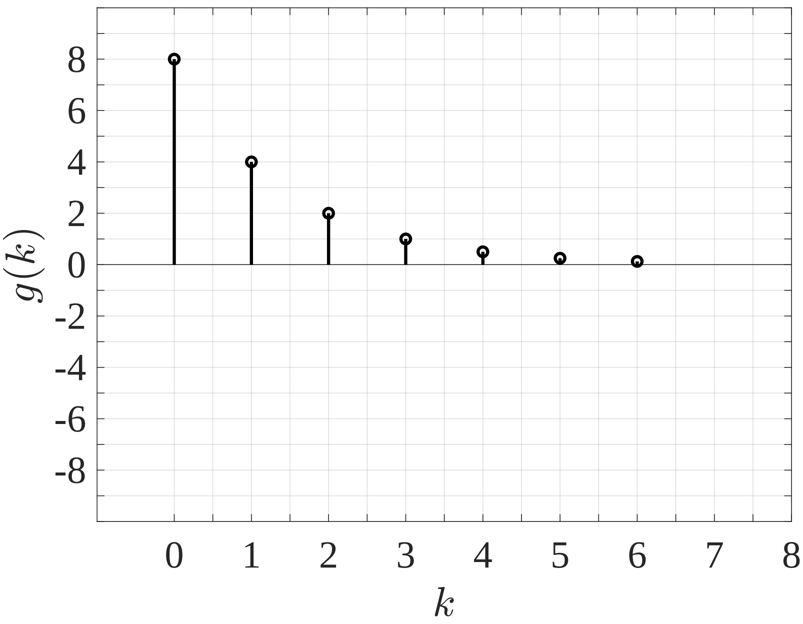
e) Calculate from the shown part of the impulse response the coefficients $b_0$, $a_1$ and $a_2$ of $G_2(z)$.



$$g(0) = 8 = b_0 \cdot 1 - a_1 \cdot 0 - a_2 \cdot 0 \rightarrow b_0 = 8 \quad (10)$$
$$g(1) = 4 = b_0 \cdot 0 - a_1 \cdot 8 - a_2 \cdot 0 \rightarrow a_1 = -0.5 \quad (11)$$
$$g(2) = 2 = b_0 \cdot 0 + 4 \cdot 0.5 - a_2 \cdot 8 \rightarrow a_2 = 0 \quad (12)$$

4

f) Draw the next 4 values of the impulse response (time steps $k = 3\ldots,6$) of the impulse response of $G_2(z)$ into the diagram.




2

$\sum 14$