Sensorics Exam

Prof. Dr.-Ing. O. Nelles
Institute of Mechanics and Control Engineering - Mechatronics
University of Siegen

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Name:	
Mat.-No.:	
Grade:	

Task:	T1	T2	T3	T4	T5	T6	T7	T8	Sum
Scores:	13	17	24	10	21	12	13	10	120
Accomplished:									

Task 1: **Comprehension Questions**

Mark the correct answers clearly.
Every question has one or two correct answers!
For every correctly marked answer you will get one point. If there is one correct answer marked and one incorrect answer marked, you will get no point for that subtask.

a) The reactive power ...
- ☐ ... has a mean value equal to zero.
- ☐ ... is the part of the apparent power that can perform work.
- ☐ ... has mean value unequal to zero.

b) Assess the following statements regarding frequency analysis via DFT.
- ☐ The leakage effect and the picket fence effect eliminate each other.
- ☐ The leakage effect happens if the time signal has a discontinuity.
- ☐ The picket fence effect happens if the frequency of the given periodic signal does not exactly exist in the frequency discretization.

c) PTC resistance thermometers ...
- ☐ ... measure faster than thermocouples.
- ☐ ... have almost a linear characteristic.
- ☐ ... have an increasing resistance with decreasing temperature.

d) The piezoelectric force measurement ...

- ☐ ... can only be used to measure stationary forces.
- ☐ ... is used to measure forces on fluids to calculate the dynamic pressure.
- ☐ ... is well suited for fast dynamic measurement.

e) Torque ...

- ☐ ... and energy have identical units.
- ☐ ... and energy are the same.
- ☐ ... is one of the seven SI basic units.

f) The magnetic inductive flow measurement ...

- ☐ ... has a nonlinear characteristic.
- ☐ ... is not suitable for corrosive fluids and fluids that contain solids.
- ☐ ... requires no internal construction.

g) Assess the following statements regarding the Student's t-distribution.

- ☐ Its confidence interval $1 - \alpha$ is wider than the confidence interval of the corresponding normal distribution.
- ☐ If the size N of the regarded data set is big enough ($N \to \infty$), it converges to the cauchy distribution.
- ☐ It is used if the real standard deviation has to be estimated from data.

h) According to the Shannon theorem ...

- ☐ ... nonlinear filters can be approximated by linear filters by using feedback-control.
- ☐ ... the highest significant component of the signal f_{max} must not be higher than the Nyquist frequency $f_n = \frac{f_0}{2}$.
- ☐ ... the quantization error is reduced if the sampling time is increased.

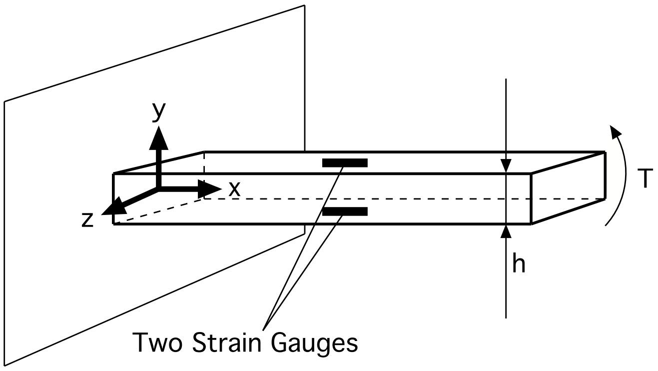
i) The median filter ...

- ☐ ... is commonly used to eliminate outliers.
- ☐ ... is a nonlinear filter.
- ☐ ... calculates the arithmetic average.

j) Assess the following statements regarding parametric methods.

- ☐ An FIR system is a parametric approach.
- ☐ Only a few number of parameters n are estimated using a relatively large number of data samples N ($n << N$).
- ☐ Their parameters have no direct physical meaning or interpretation.

Task 2: **Measurement of Torque**



In the picture above you can see a beam, that is loaded with a torque T on one end. The other end is fixed to a wall and two strain gauges are applied to the beam to measure the torque. The following equations are given:

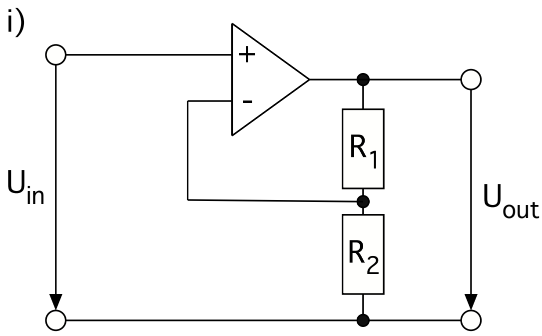
$$|\rho| = \frac{EI}{|T|} \quad ; \quad \Delta R = R_0 K \epsilon \quad ; \quad \epsilon = \frac{y}{|\rho|}$$

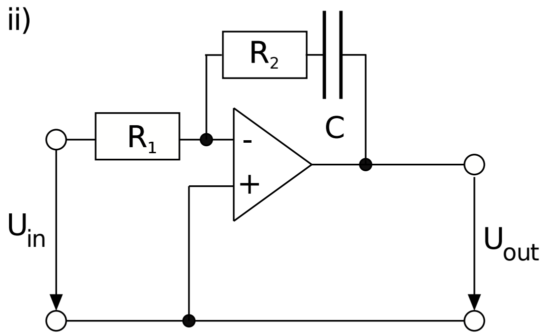
ρ: Curvature of the deflected beam; E: Young's modulus; I: Second moment of area; T: Torque (constant over the beam length); ΔR: The change in resistance of one strain gauge; R_0: The resistance of the strain gauge without any applied torque; K: Sensitivity of the strain gauges; ϵ: Deformation of the strain gauge, h: Beam height; y: Coordinate of the beam height, that starts exactly at the half of the beam height.

a) Sketch a bridge circuit that can be used to transform the change in resistance ΔR into a voltage U_d and derive the corresponding equation.

b) In what way can the sensitivity of the torque measurement be increased through a change in the bridge circuit and/or additional sensors?

c) The measured voltage of the bridge-circuit should be amplified with the help of an operational amplifier (OpAmp) circuit. Which of the following two circuits amplifies the input voltage U_{in} (assume ideal OpAmps)?





d) What relationship of R_1 and R_2 has to be fullfilled to achieve an amplification of the input voltage by a factor of four?

Task 3: Discrete-Time Systems

Two filter transfer functions are given

$$G_1(z) = \frac{(z^2 + z^1 + 1)}{3z^3} \quad \text{und}$$
$$G_2(z) = \frac{(z^5 + z^3 + z)}{3z^3} \ .$$

For the following investigations the sampling time is $T_0 = 1/2$ seconds.

a) Determine the difference equations for both filters $G_1(z)$ and $G_2(z)$.

b) Sketch one block-diagram for each of the two filters $G_1(z)$ and $G_2(z)$, that describes the corresponding system correctly.

c) Calculate the poles of the filters $G_1(z)$ and $G_2(z)$ in the z-domain.

d) Calculate the location of the poles found in subtask c) in the s-domain.

e) Calculate the gain for both filters with the help of the final value theorem. In general the final value theorem for a signal $x(k)$ is defined as follows:

$$x(k \to \infty) = (z-1) \lim_{z \to 1} X(z) \ .$$

f) State for each filter, if it is a causal or an acausal system. Explain your choices shortly.

g) Calculate each filter's phase depending on the frequency ω.

h) Sketch the phase-responses for both filters. Up to which maximum frequency ω_{max} should the phase-responses be sketched when dealing with discrete-time systems?

Task 4: Aliasing

The subtask f) can be solved independently of the remaining subtasks.

The two periodic signals $y_1(t) = \sin(2\pi \cdot 4\text{Hz} \cdot t)$ and $y_2(t) = \sin(2\pi \cdot 12\text{Hz} \cdot t)$ are given. Both signals are measured with the sampling frequency $f_0 = 10\text{Hz}$.

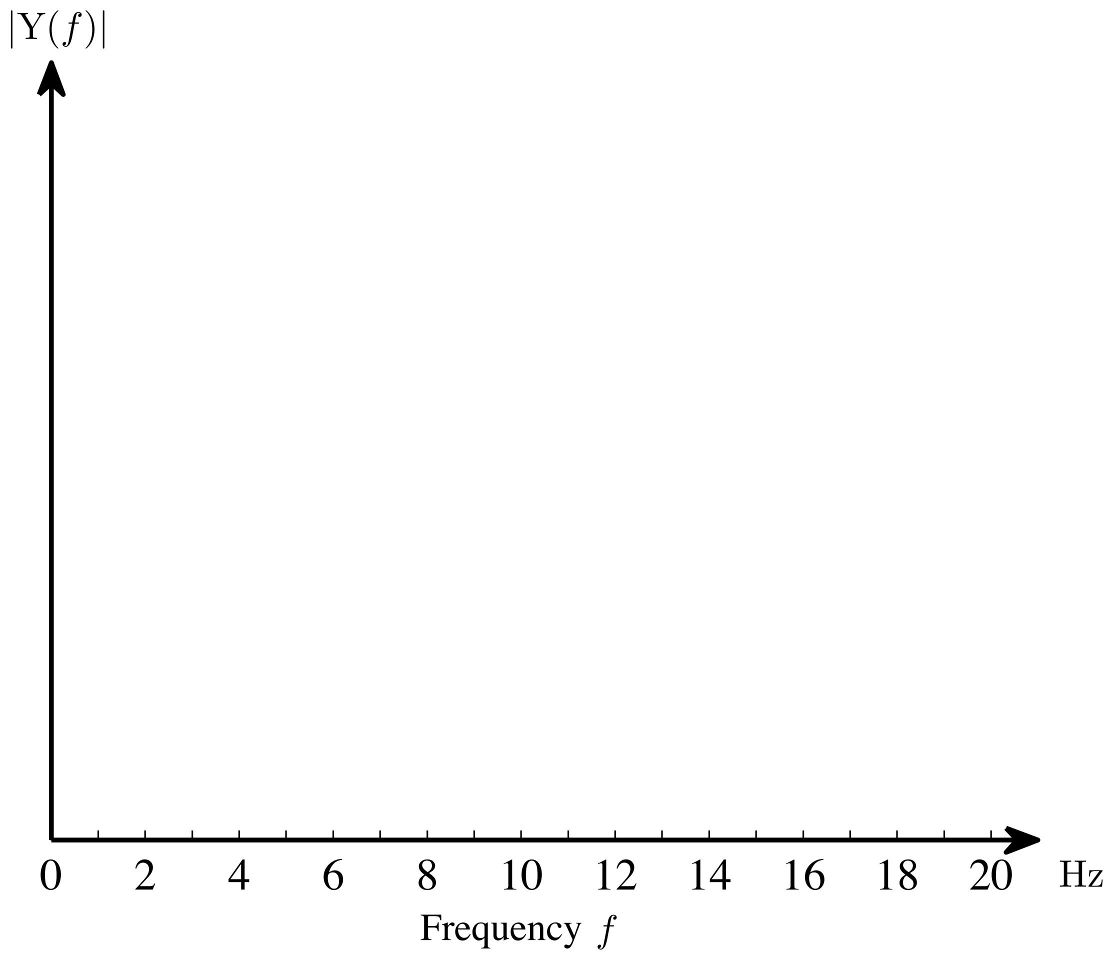
a) Sketch the amplitude spectrum of the given signals in the given diagram.

b) Highlight the Nyquist frequency (Shannon's sampling theorem) in the diagram below.

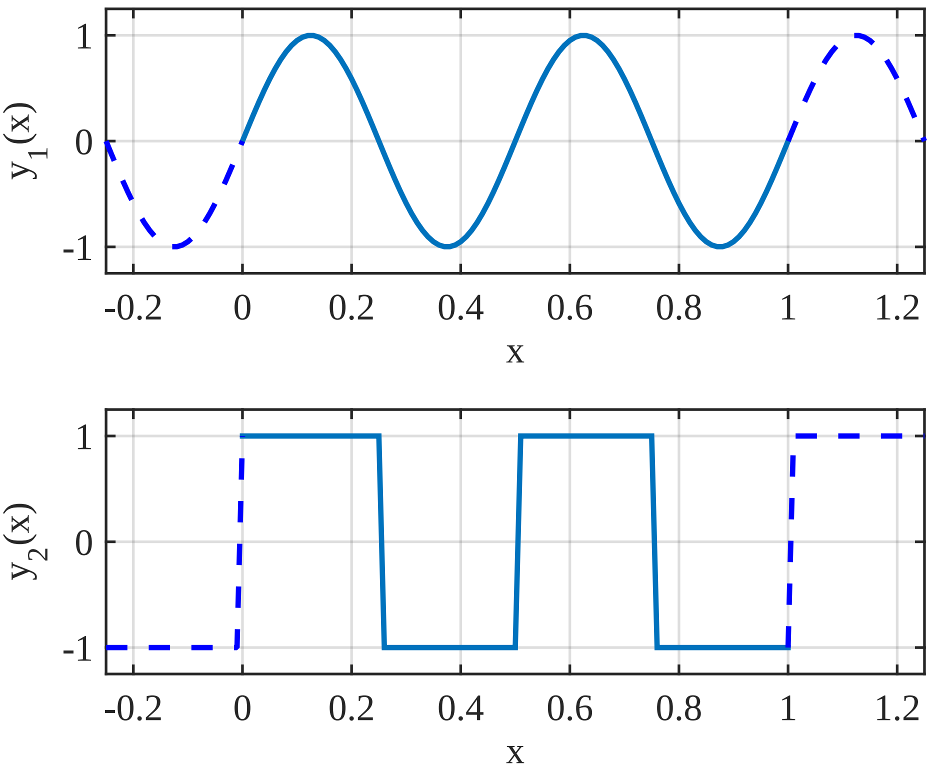
c) Does aliasing occur in one of the cases?

d) Sketch all shadow spectra resulting from sampling of $y_1(t)$ and $y_2(t)$ in the interval [0, 20] Hz.

e) What signal frequencies can be guessed from the sampled signals?



f) **This subtask can be solved independently from the others!**
Two periodic signals are given: One sine wave y_1 and one square wave y_2. Both have the same frequency $f = 2\text{Hz}$. Both signals are unlimited and continue to $\pm\infty$. The sampling frequency used for measurement is $f_0 = 10\text{Hz}$. Does aliasing occur in one of the cases?

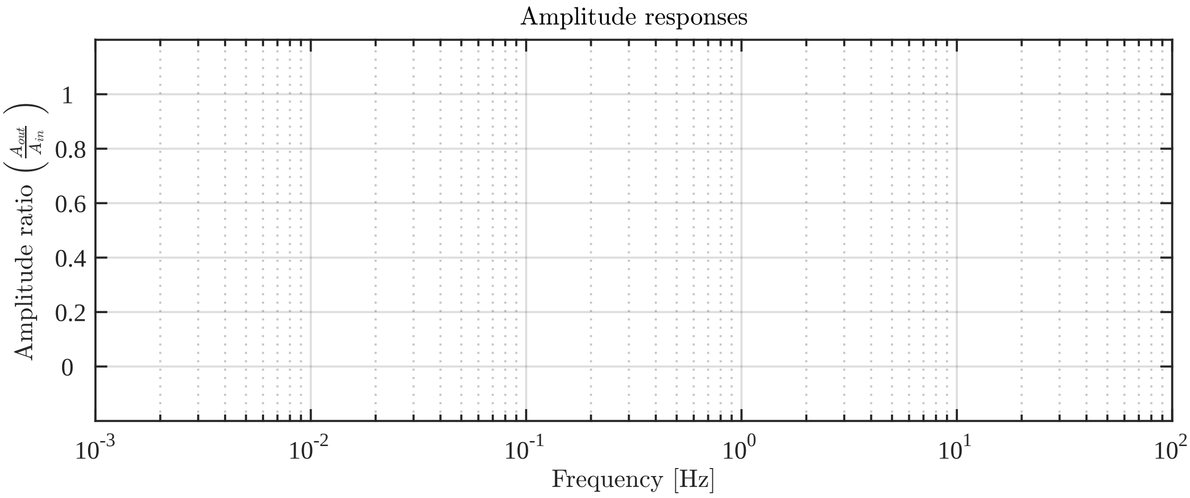


Task 5: Filter

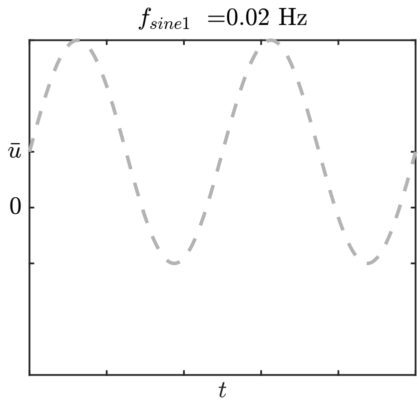
Note that subtask c) can be solved independently from subtasks a) and b).

a) There are two filters G_{LP} and G_{BP}, that should be compared. One filter is a low-pass G_{LP}, the other is a band-pass G_{BP}. The limit (cut-off) frequency of filter G_{LP} is $f_{LP} = 0.1$ Hz. The two limit (cut-off) frequencies of filter G_{BP} are $f_{BP1} = 6$ Hz and $f_{BP2} = 30$ Hz.

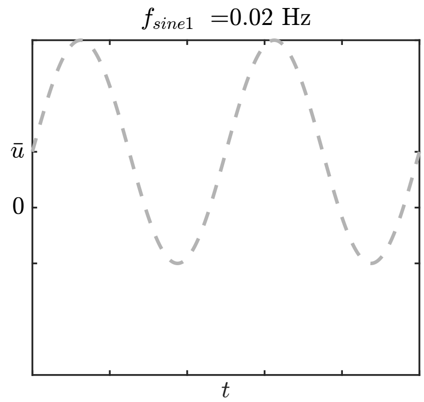
Sketch the ideal amplitude responses of both filters G_{LP} and G_{BP} in the figure below. Label your sketched amplitude responses with the correct transfer function.



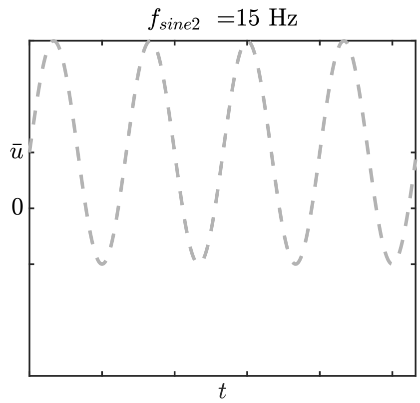
b) Now imagine you would filter sine waves with three different frequencies, $f_{sine1} = 0.02$ Hz, $f_{sine2} = 15$ Hz and $f_{sine3} = 80$ Hz through the ideal filters G_{LP} and G_{BP}. The sine waves are already shown in the sub-figures (a) to (f) below. Use the left columns to sketch the filtered output $y(t)$ **qualitatively**, when the filter G_{LP} is used and the right columns for the results of a filtering with G_{BP}. Assume that the responses have already reached the steady state. The value $\bar{u}$ labels the mean of the input signal $u(t)$.



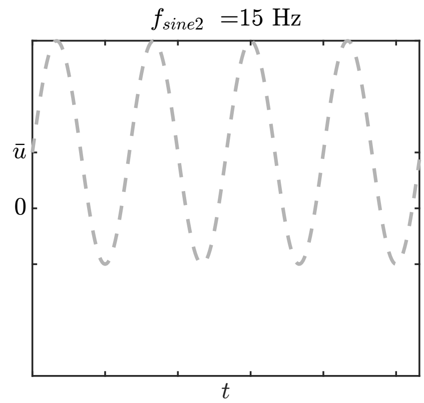
(a) Sketch the output $y(k)$ qualitatively, if this sine wave is sent through filter G_{LP}.



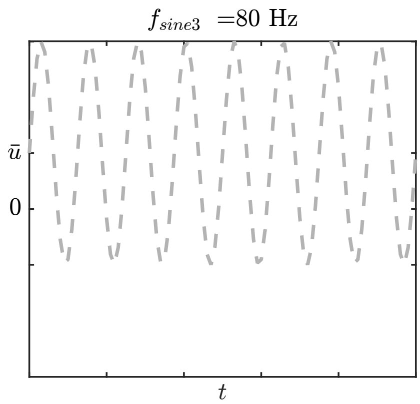
(b) Sketch the output $y(k)$ qualitatively, if this sine wave is sent through filter G_{BP}.



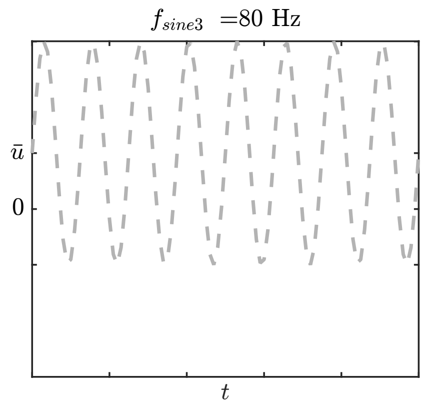
(c) Sketch the output $y(k)$ qualitatively, if this sine wave is sent through filter G_{LP}.



(d) Sketch the output $y(k)$ qualitatively, if this sine wave is sent through filter G_{BP}.



(e) Sketch the output $y(k)$ qualitatively, if this sine wave is sent through filter G_{LP}.



(f) Sketch the output $y(k)$ qualitatively, if this sine wave is sent through filter G_{BP}.

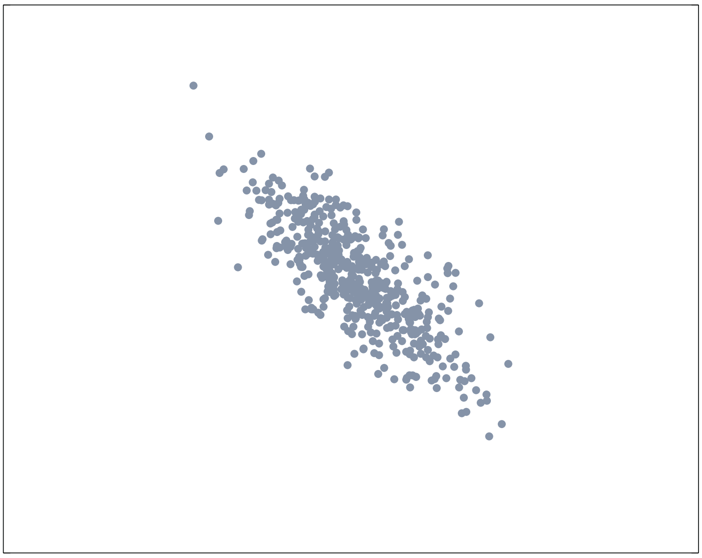
The following subtask is completely independent from subtasks a) and b).

c) The following list contains properties of filters. Assign which of these properties are valid for IIR (infinite impulse response) filters and which are valid for FIR (finite impulse response) filters.

- Typically low order
- Instability can not occur
- An equivalent time-continuous system exists
- A linear phase can be achieved exactly

Task 6: Principal Component Analysis (PCA) and Clustering

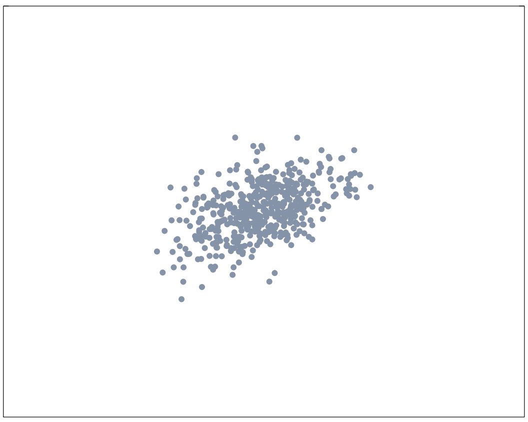
a) The following data distribution is given:



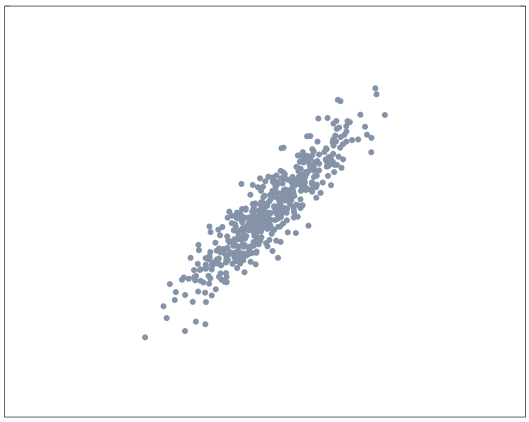
Sketch the first and second principal component **qualitatively correct** in the figure above and label **clearly** which of the two sketch compounds represents the first and the second principal component.

b) Now two data distributions are given:

a)



b)

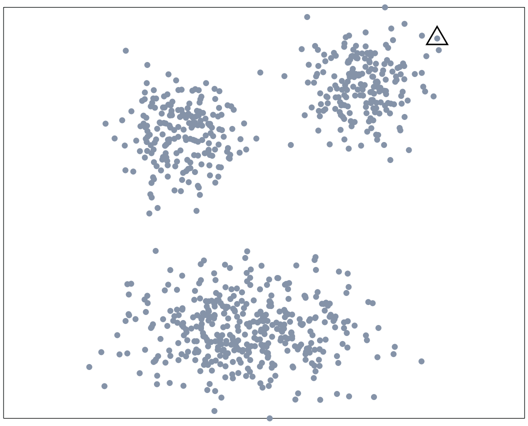


Which distribution is better suited for dimensionality reduction, a) or b) ? A short explanation is required.

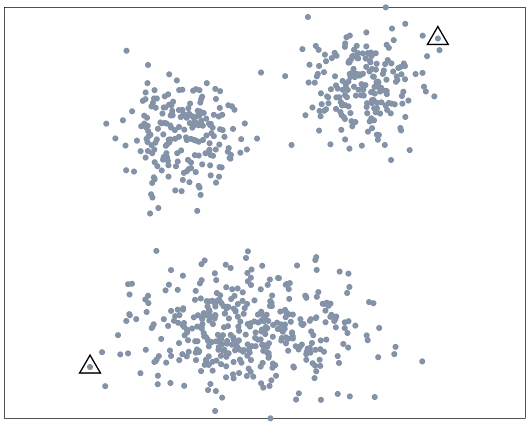
c) Another data distribution is given. It should be grouped via the k-means algorithm with different number of clusters. The initial centers of the clusters are marked by the triangles.

Draw the resulting centers of the clusters in the given figures. Why must the start values for the calculation be provided?

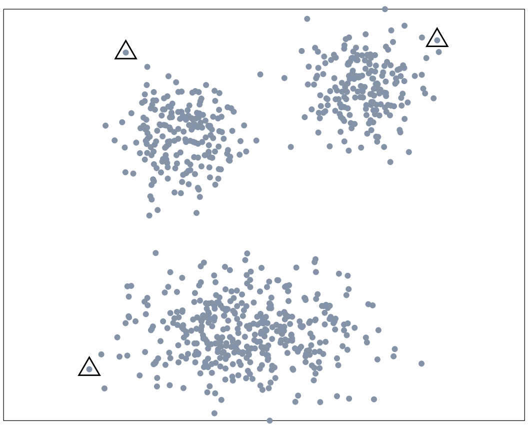
1)



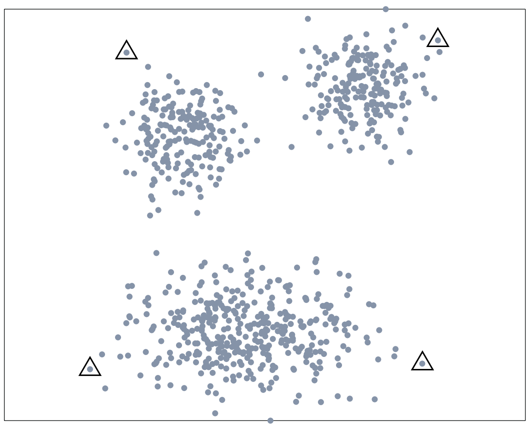
2)



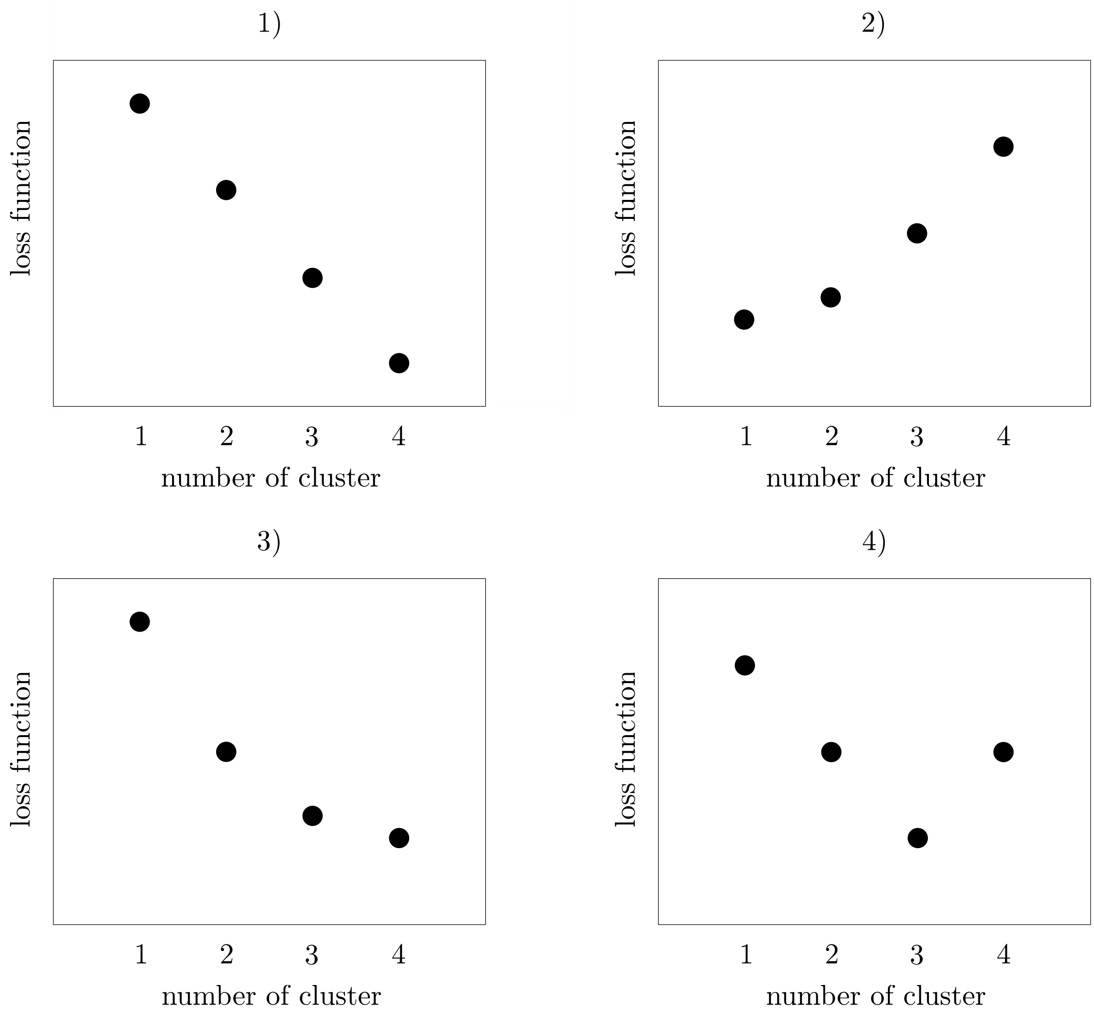
3)



4)



d) Four possible loss function curves of the k-means algorithm over the number of clusters are given. Choose which results from the example given in c).



Task 7: Probability Density Functions

a) Three probability density functions (pdf) with the following properties are given:

I) All values in the interval $[-2, 3]$ are equally probable.

II) All values in the interval $[-10, 0]$ are equally probable.

III) All values in the interval $[-8, 12]$ are equally probable.

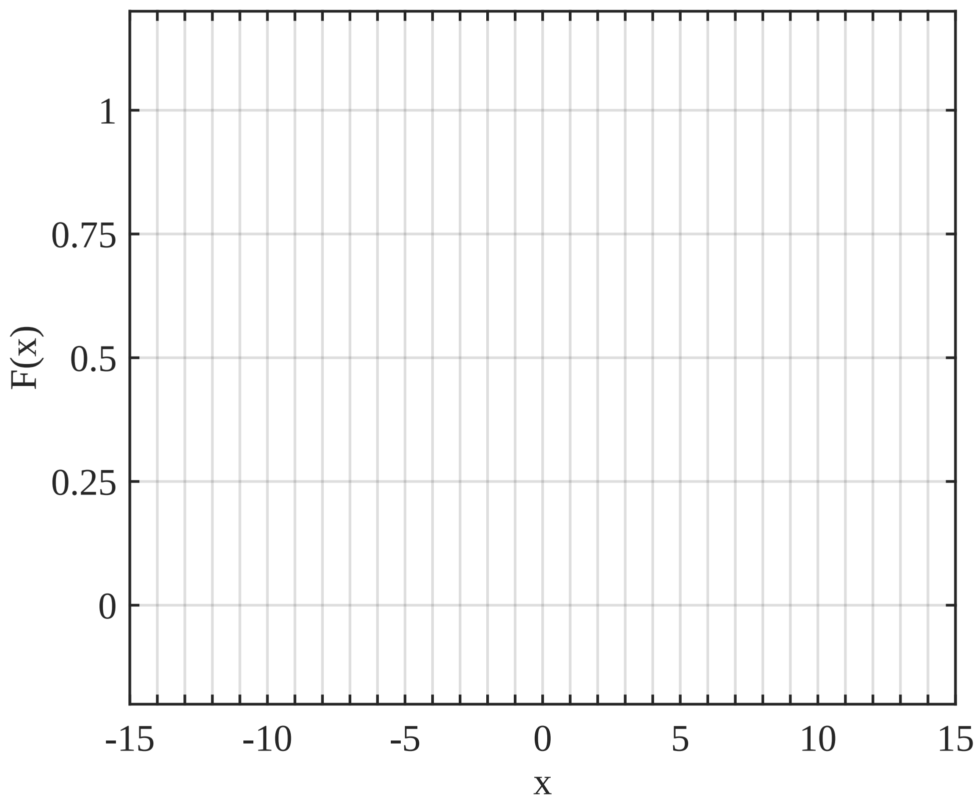
How do you call such a distribution?

b) Sketch the **three** probability density functions into **one** coordinate system. Assign each case (I, II and III) to the corresponding (sketched) distribution.

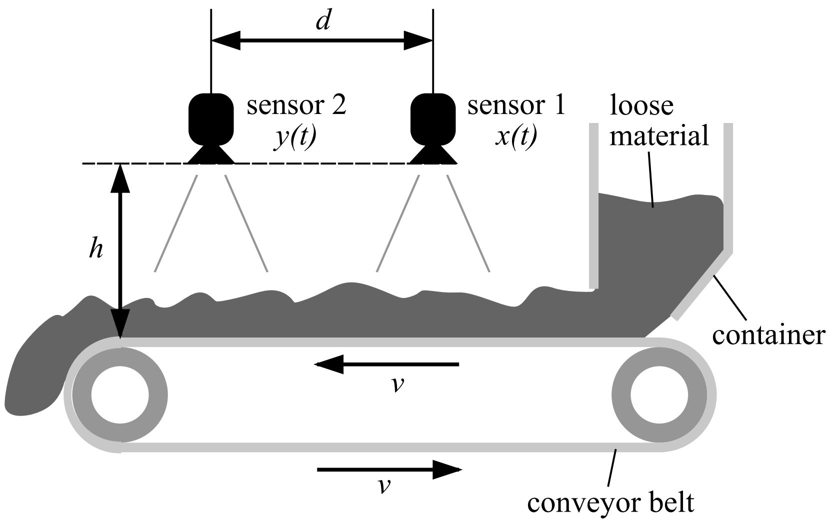
c) Sketch the cumulative distribution functions (cdf), that correspond to the probability density functions (pdf) defined in subtask a) I, II and III.

Hint: The cumulative distribution function $F(x)$ is the integral of the probability density function $f(x)$ with respect to the random variable x:

$$F(x) = \int_{-\infty}^{\infty} f(x)\mathrm{d}x \ .$$



Task 8: Measurement Errors and Correlation Analysis



The shown conveyor belt carries loose material from the container to the left. Two distance-sensors measure the hight of the loose material on the belt and store the hight together with the corresponding time, such that the signal $x(t)$ from sensor 1 and signal $y(t)$ from sensor 2 is available. The distance between the two sensors d is known as well as the hight h between the sensors and the surface of the belt. It is assumed, that the loose material is carried with the same constant velocity v the conveyor belt is running. A correlation analysis between $x(t)$ and $y(t)$ is used to determine the time interval Δt after which the same loose material traveled the distance d. With the help of d and Δt the velocity can be calculated.

Now some error scenarios should be investigated.

a) Assume sensor 2 is not correctly installed, such that the height above the conveyor belt's surface is different from the one of sensor 1. Does this affect the correct determination of the time interval Δt? Explain your answer shortly.

b) Now assume that the distance d varies due to inaccurate assembling, such that the real distance d_{real} between the two sensors is $d_{real} = d \pm \Delta d$. Additionally the time interval is not determined correctly, such that the measured value Δt_m differs from the real one Δt_{real}:
$\Delta t_m = \Delta t_{real} \pm \delta t$.

   I Derive the equation to calculate the Gaussian error propagation of the velocity Δv, if the sign of the deviations Δd and δt is unknown a priori.

   II What is the worst-case scenario, i.e. what leads to biggest error in the velocity measurement, considering the sign of Δd and δt.

c) Now assume that the wires of the sensors are interchanged, which leads to the result, that $x(t)$ is the signal from sensor 2 and $y(t)$ is the signal from sensor 1. What error will emerge in the velocity measurement?

Solutions:

<u>Task 1:</u> Comprehension Questions

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- [ ] ... is the part of the apparent power that can perform work.
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b) Assess the following statements regarding frequency analysis via DFT.

- [ ] The leakage effect and the picket fence effect eliminate each other.
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- [ ] ... nonlinear filters can be approximated by linear filters by using feedback-control.
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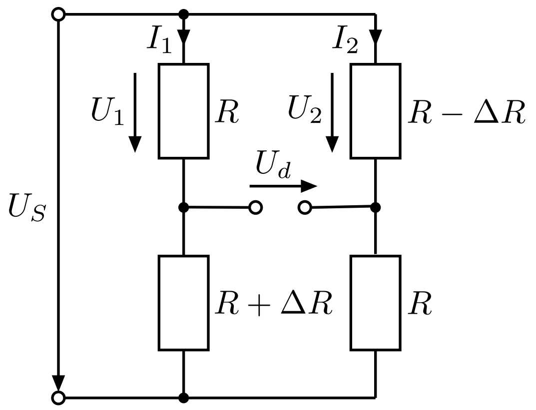
Task 2: Measurement of Torque



a) 6

$$U_1 - U_2 + U_d = 0 \quad (1)$$
$$U_1 = RI_1 \quad (2)$$
$$U_2 = (R - \Delta R)I_2 \quad (3)$$
$$I_1 = \frac{U_S}{2R + \Delta R} \quad (4)$$
$$I_2 = \frac{U_S}{2R - \Delta R} \quad (5)$$

(4) in (2) and (5) in (3) :

$$U_1 = \frac{RU_S}{2R + \Delta R} \quad (6)$$
$$U_2 = \frac{(R - \Delta R)U_S}{2R - \Delta R} \quad (7)$$

(6) and (7) in (1):
(...)
$U_d = -\frac{\Delta R^2}{4R^2 - \Delta R^2} U_S$ 4

b) If two additional strain gauges are applied to the beam, they can be used to realize a full bridge circuit, which leads to an increase of the sensitivity by nearly a factor of two. 2

c) The first one (i) amplifies the voltage. 1

$$U_{in} = \frac{R_2}{R_1 + R_2} U_{out}$$
$$\Leftrightarrow U_{out} = (\frac{R_1}{R_2} + 1) U_{in}$$

3

The second one (ii) is an integrator circuit.

d) $R_1 = 3 \cdot R_2$ (see equation above). 1

$\sum 17$

Task 3: Discrete-Time Systems

a) Determine the difference equations for both filters $G_1(z)$ and $G_2(z)$.

$$G_1(z) = \frac{(z^2 + z^1 + 1)}{3z^3}$$
$$\Leftrightarrow G_1(z) = \frac{1}{3}\left(z^{-1} + z^{-2} + z^{-3}\right)$$

$$\circ\!\!-\!\!\bullet$$

$$y_1(k) = \frac{1}{3}u(k-1) + \frac{1}{3}u(k-2) + \frac{1}{3}u(k-3)$$

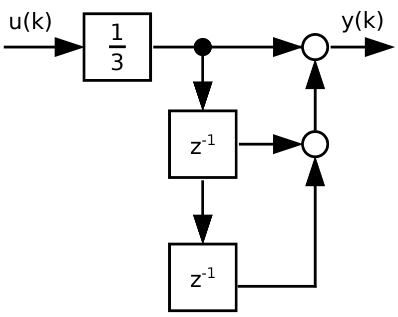
$$G_2(z) = \frac{(z^5 + z^3 + z)}{3z^3}$$
$$\Leftrightarrow G_2(z) = \frac{1}{3}\left(z^2 + 1 + z^{-2}\right)$$

$$\circ\!\!-\!\!\bullet$$

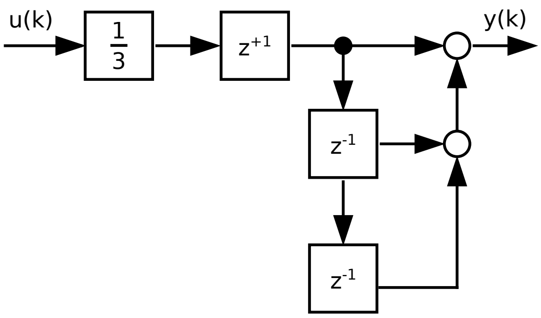
$$y_2(k) = \frac{1}{3}u(k+2) + \frac{1}{3}u(k) + \frac{1}{3}u(k-2)$$

$\boxed{4}$

b) Sketch one block-diagram for each of the two filters $G_1(z)$ and $G_2(z)$, that describes the corresponding system correctly.



(g) Block-diagram $G_1(z)$



(h) Block-diagram $G_2(z)$

$\boxed{4}$

c) Calculate the poles of the filters $G_1(z)$ and $G_2(z)$ in the z-domain.

$$\Leftrightarrow G_1(z) = \frac{(z^2 + z^1 + 1)}{3z^3}$$

$\Rightarrow$ Three poles at $z_{pG1} = 0$.

$$\Leftrightarrow G_2(z) = \frac{(z^5 + z^3 + z)}{3z^3}$$

$\Rightarrow$ Also three poles at $z_{pG2} = 0$. $\boxed{2}$

d) Calculate the location of the poles found in subtask c) in the s-domain.

$$\begin{aligned} z &= \mathrm{e}^{sT_0} \\ \Leftrightarrow s &= \frac{1}{T_0}\ln z \\ \Rightarrow s_{pG1} &= s_{pG2} = -\infty. \end{aligned}$$

$\boxed{1}$

e) Calculate the gain for both filters with the help of the final value theorem.

$$\begin{aligned} h_1(t\to\infty) &= \lim_{z\to 1}(z-1)G_1(z)\underbrace{\Sigma(z)}_{z\text{-transformierter Einheitssprung}} \\ &= \lim_{z\to 1}\cancel{(z-1)}\frac{(z^2+z^1+1)}{3z^3}\frac{z}{\cancel{z-1}} \\ &= 1 \end{aligned}$$

$$\begin{aligned} h_2(t\to\infty) &= \lim_{z\to 1}(z-1)G_2(z)\Sigma(z) \\ &= \lim_{z\to 1}\cancel{(z-1)}\frac{(z^5+z^3+z)}{3z^3}\frac{z}{\cancel{z-1}} \\ &= 1 \end{aligned}$$

$\boxed{2}$

f) State for each filter, if it is a causal or an acausal system. Explain your choices shortly.
$G_1(z)$: Causal filter, no future values are needed to calculate the system's current output.
$G_2(z)$: Acausal filter, future values are needed to calculate the system's current output. $\boxed{2}$

g) Calculate each filter's phase depending on the frequency ω.

Calculation of the phase of $G_1(z)$:

$$\begin{aligned} G_1(z) &= \frac{(z^2+z^1+1)}{3z^3} \\ \Leftrightarrow G_1(z) &= \frac{1}{3}\left(z^{-1}+z^{-2}+z^{-3}\right) \\ \Leftrightarrow G_1(z) &= \frac{1}{3}z^{-2}\left(z+1+z^{-1}\right) \\ \Leftrightarrow G_1(z) &= \frac{1}{3}z^{-2}\left(1+z+z^{-1}\right) \end{aligned} \tag{8}$$

For the phase calculation we set $s = i\omega$. The following equations arise: $\boxed{1}$

$$\begin{aligned} z^{\pm n} &= \mathrm{e}^{\pm nsT_0} \\ \Leftrightarrow z^{\pm n} &= \mathrm{e}^{\pm in\omega\cdot T_0} \\ \Leftrightarrow z^{\pm n} &= \cos(n\omega T_0) \pm i\sin(n\omega T_0)\ . \end{aligned}$$

$\boxed{1}$

With these relationships equation 8 becomes:

$$G_1(i\omega) = \frac{1}{3}\mathrm{e}^{-i2\omega T_0}\left(1 + \underbrace{\mathrm{e}^{i\omega T_0} + \mathrm{e}^{-i\omega T_0}}_{=2\cos(\omega T_0)}\right) .$$

The part of the equation above in the brackets has only a positive real part. Therefore, the system's phase can directly be read:

$$\varphi_1(\omega) = -2\omega T_0 .$$

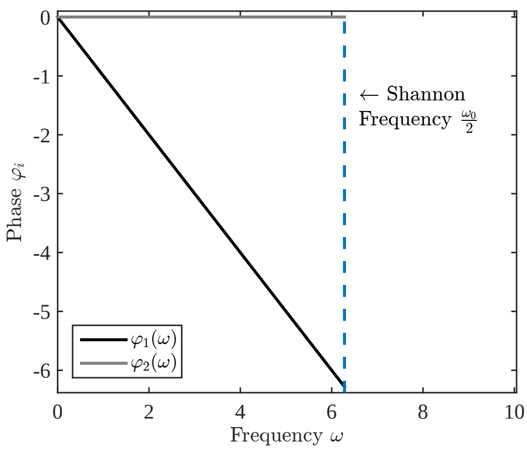
Calculation of the phase of $G_2(z)$: $\boxed{2}$

$$G_2(z) = \frac{(z^5 + z^3 + z)}{3z^3}$$
$$\Leftrightarrow G_2(z) = \frac{1}{3}\left(z^2 + 1 + z^{-2}\right)$$
$$\Leftrightarrow G_2(i\omega) = \frac{1}{3}\left(1 + \underbrace{\mathrm{e}^{i2\omega T_0} + \mathrm{e}^{-i2\omega T_0}}_{=2\cos(2\omega T_0)}\right)$$

In the equation above no imaginary part exists and therefore the phase is $\varphi_2(\omega) = 0$.

$\boxed{2}$

h) Sketch the phase-responses for both filters. Up to which maximum frequency ω_{max} should the phase-responses be sketched when dealing with discrete-time systems?



$\boxed{2}$

Be aware of the x-axis' scaling. Usually the x-axis is scaled logarithmically, whereas here, a linear scaling is chosen.

The phase-response (as well as the amplitude response) should only be sketched up to the Shannon-frequency $\omega_{\max} = \frac{\omega_0}{2}$. For higher frequencies no statements can be made. $\boxed{1}$

$\boxed{\sum 24}$

Task 4: Abtasttheorem

The periodic signale $y_1(t) = \sin(2\pi \cdot 4\text{Hz} \cdot t)$ and $y_1(t) = \sin(2\pi \cdot 12\text{Hz} \cdot t)$ are measured with the sampling frequency $f_0 = 10\text{Hz}$.

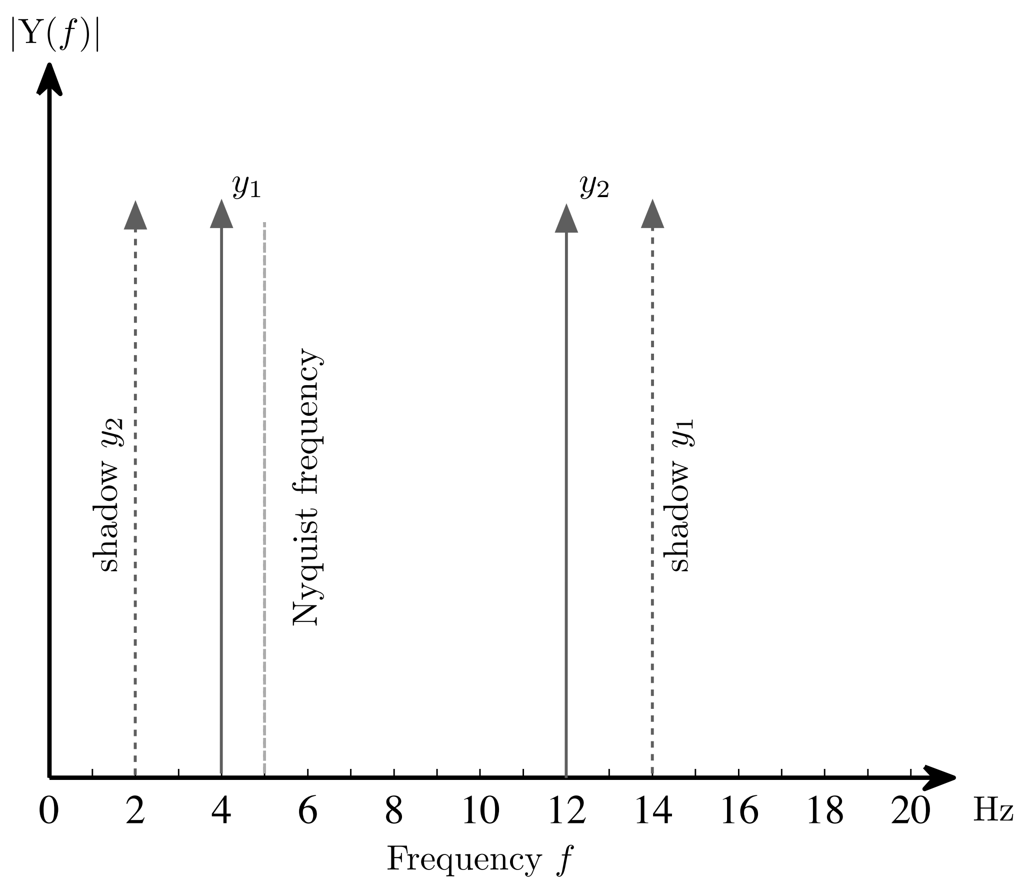
a) See figure below. $\boxed{1}$

b) Nyquist frequency (Shannon's sampling theorem) $= \frac{f_0}{2} = 5\text{Hz}$. See figure below. $\boxed{1}$

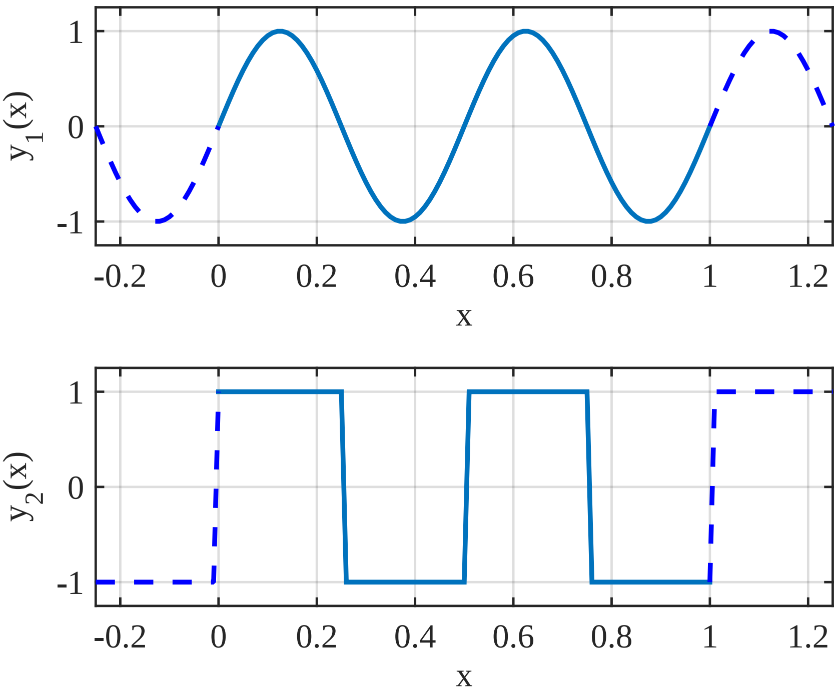
c) Aliasing occurs, if the highest frequency contained in the signal is greater than or equal the half of the sampling frequency. In case of signal y_1 the required frequency is 8 Hz and in case of signal y_2 the required frequency is 24 Hz. Here the sampling frequency is $f_0 = 5$ Hz, what is enough for signal y_1 but too small for signal y_2. Regarding signal y_1 no aliasing occurs, regarding signal y_2 aliasing occurs. $\boxed{2}$

d) See figure below. $\boxed{2}$

e) With respect to the sampling theorem signal can only be reconstructed without loss of informtation to $f = \frac{f_0}{2}$. In this case the shadow spectrum of signal y_2 with the frequency 2Hz would occur instead of the true signal frequency 12Hz. The signal frequency of signal y_1 lies beneath the Nyquist frequency and correctly measured. $\boxed{2}$



f) A sine wave and a square wave with the same frequency f are given.



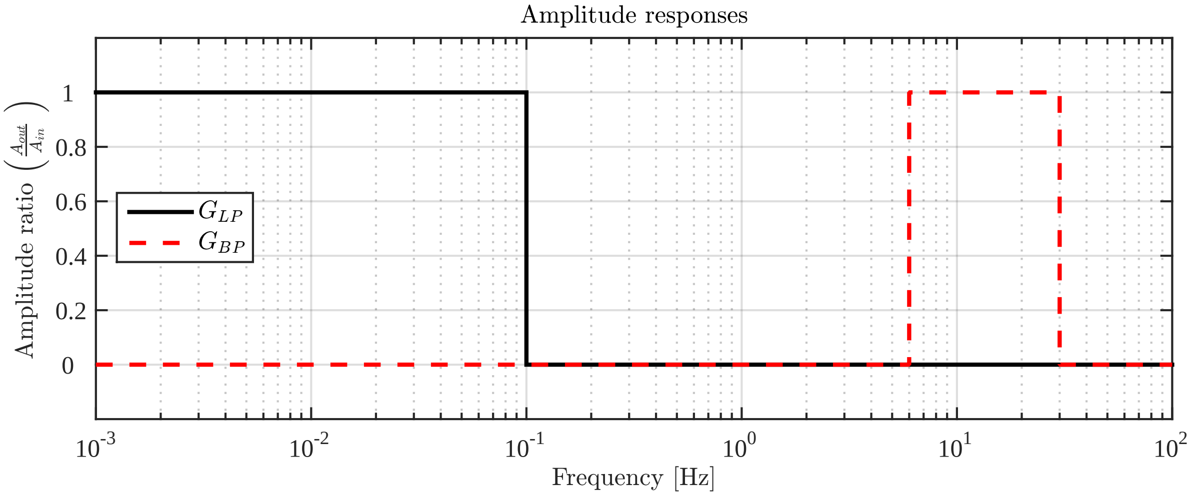
Both signals are unlimited and continue to $\pm\infty$. The sampling frequency used for measuring is $f_0 = 10\text{Hz}$. Does aliasing occur in one of the cases?

A sine wave whose frequency is $f = 2\text{Hz}$ that is measured with a sampling frequency $f_0 = 10\text{Hz}$ fulfills Shannon's sampling theorem. A square wave has multiple frequency in the amplitude spectrum, not only one like the sine wave. Because of the abrupt switching very high frequencies are excited. These high frequencies lies above the required frequency $f_{max} > 20\text{Hz}$ (Shannon's sampling theorem). Therefore aliasing occurs measuring the square wave. 2

$\sum 10$

Task 5: Filter

a) Sketch the ideal amplitude responses for both filters G_{LP} and G_{BP} in the figure below.



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b) The following list contains properties of filters. Assign which of these properties are valid for IIR (infinite impulse response) filters and which are valid for FIR (finite impulse response) filters.

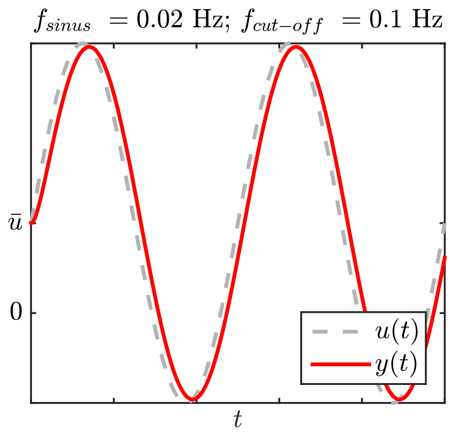
- Typically low order → IIR
- Instability can not occur → FIR
- An equivalent time-continuous system exists → IIR
- A linear phase can be achieved exactly → FIR

4

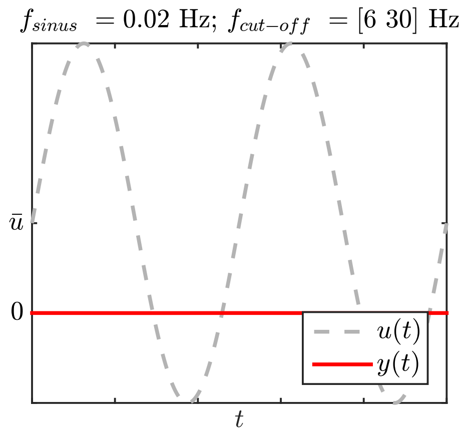
c) Sketch the signals after applying the ideal filters from subtask a).
The shown solution below is the exact result, if a first order Butterworth filter (low-pass) respectively a type 2 Chebyshev filter of order 5 (band-pass) is used. Notice that you get the full score for the correct oscillation (either the same frequency as the input or no oscillation after the filter) and the correct mean value after the filtering.

12

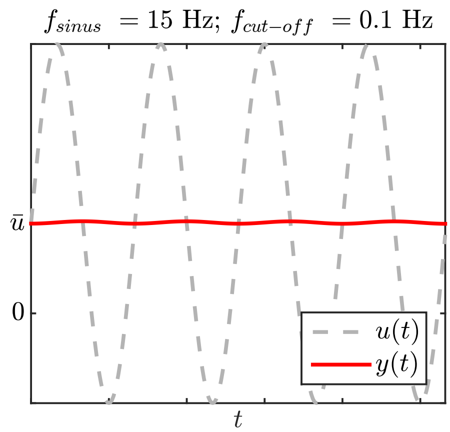
$\sum 21$



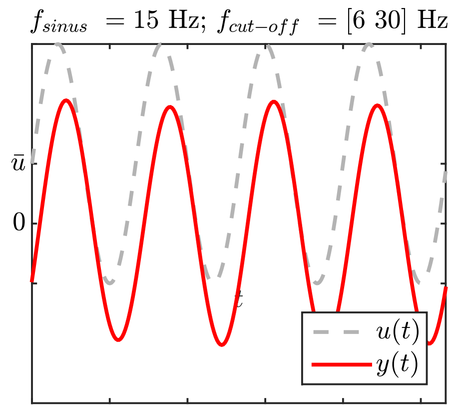
(a) Sketch the output $y(k)$ qualitatively, if this sine wave is sent through filter G_{LP}.



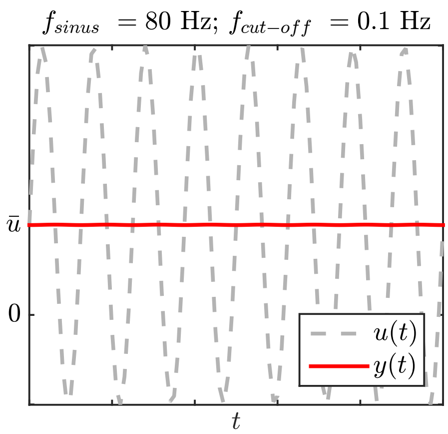
(b) Sketch the output $y(k)$ qualitatively, if this sine wave is sent through filter G_{BP}.



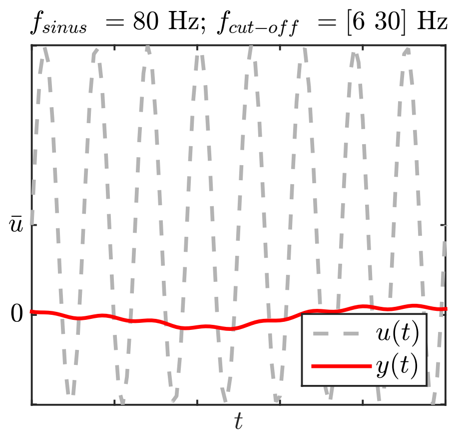
(c) Sketch the output $y(k)$ qualitatively, if this sine wave is sent through filter G_{LP}.



(d) Sketch the output $y(k)$ qualitatively, if this sine wave is sent through filter G_{BP}.



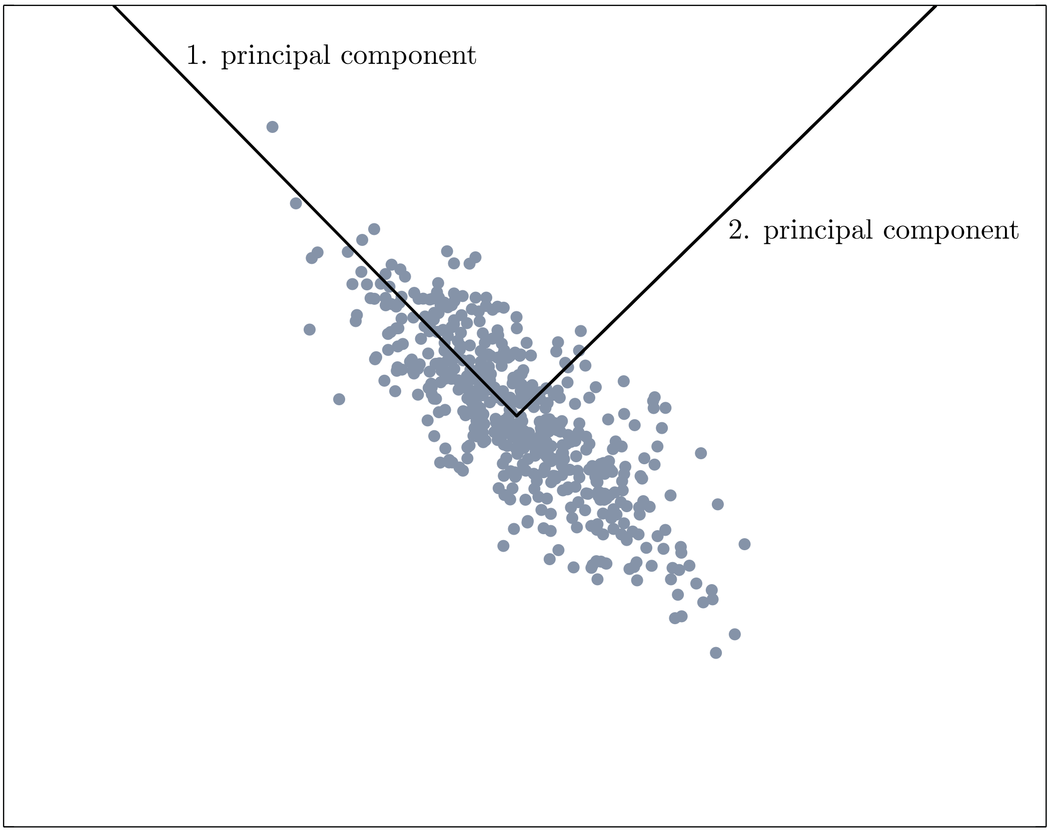
(e) Sketch the output $y(k)$ qualitatively, if this sine wave is sent through filter G_{LP}.



(f) Sketch the output $y(k)$ qualitatively, if this sine wave is sent through filter G_{BP}.

Task 6: Principal Component Analysis (PCA) and Clustering

a) The following is an exact solution. A qualitative correct solution is sufficient.

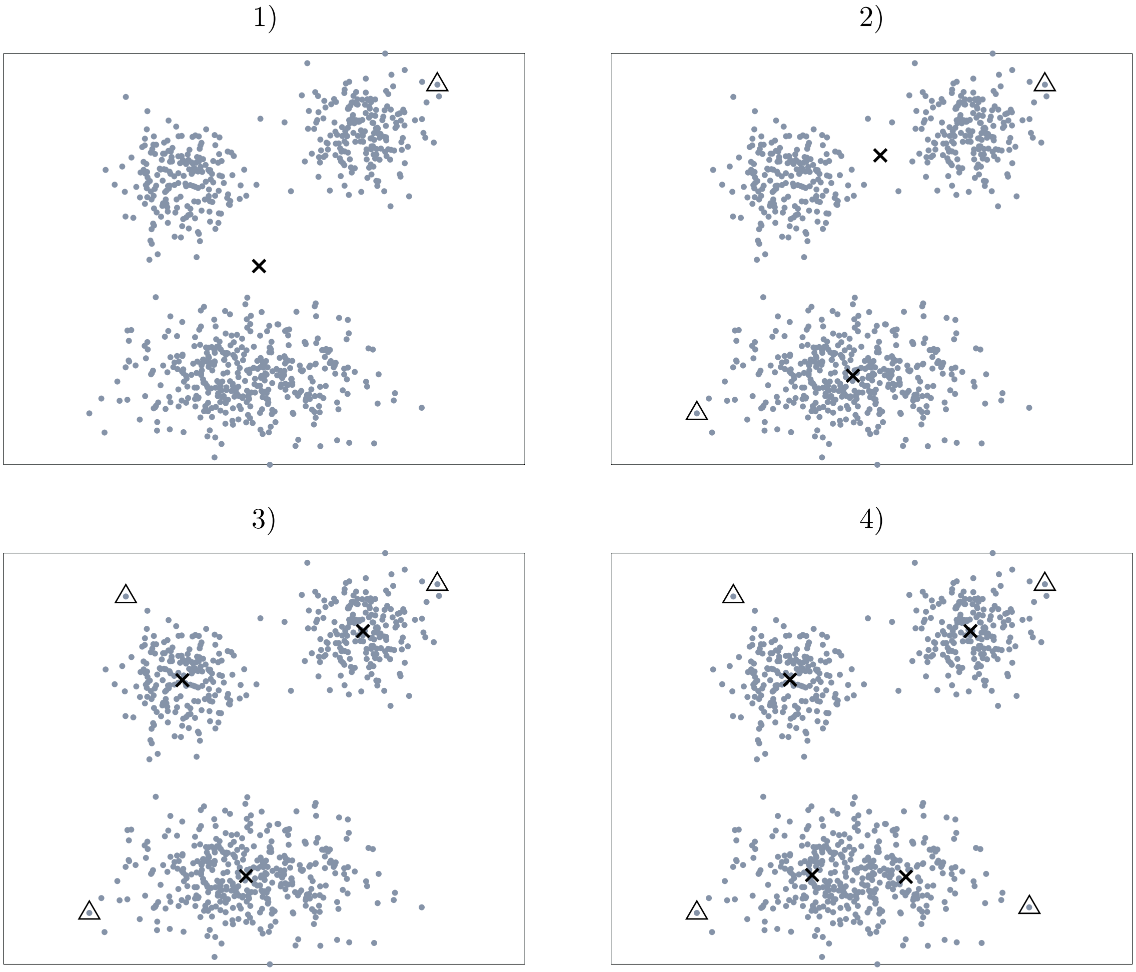


3

The angle und the order of the principal components are important. The principle components intersect in the mean value.

b) The data distribution in figure b) is better suited for dimensionality reduction, because the latitude along the first principal component is substantial bigger than the latitude of the second principal component. In figure a) the difference between the components is smaller. 2

c) The problem of k-means is its random initialization. It could happen, that the algorithm converges in a local optimum. Because of the given initial values, the solutions become qualitative unique. 1



4

The first figure has only one Cluster. Every data point belongs to it and therefore the center is the mean value. The second figure has two clusters. With respect to the given initial values one center is right between the two upper conglomerations and the other center is in the middle of the lower conglomeration. Three clusters with the given initial values lead to centers in the middle of the three conglomerations. Regarding four clusters with the given initial values, two centers are respectively in the middle of the two upper conglomerations and the other two are inside the lower conglomeration.

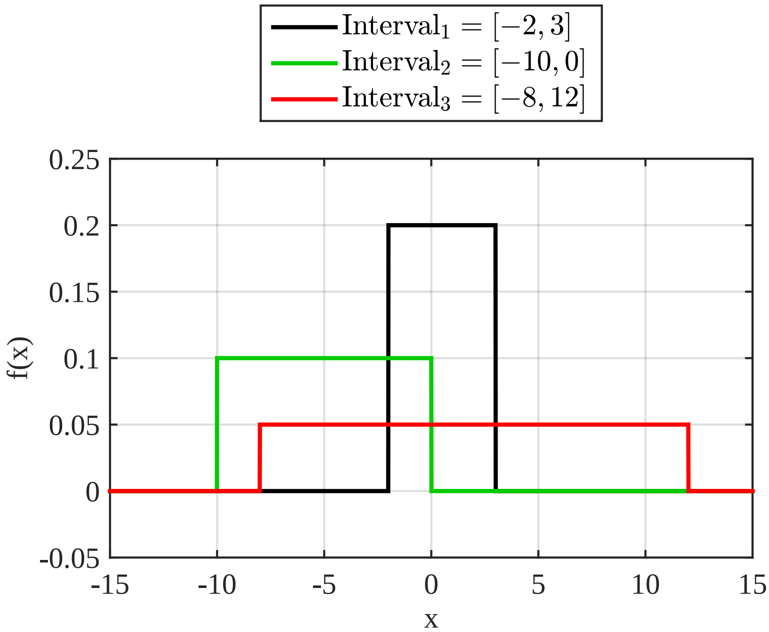
d) An ascent of the loss function over the number of clusters is impossible using the k-means algorithm. Therefore the progressions in figure 2) and 4) are improper. The progression in figure 1) decreases linear over the number of clusters. For the given data distribution this is not possible, because the improvement using four instead of three clusters must be smaller than the improvement using three instead of two clusters. The progression in figure 3) is suitable, because the improvement of the loss function decreases with the rising number of clusters. 2

$\sum 12$

Task 7: Stochastische Signale

a) uniform distribution 1

b) Sketch **three** distributions into **one** coordinate system.



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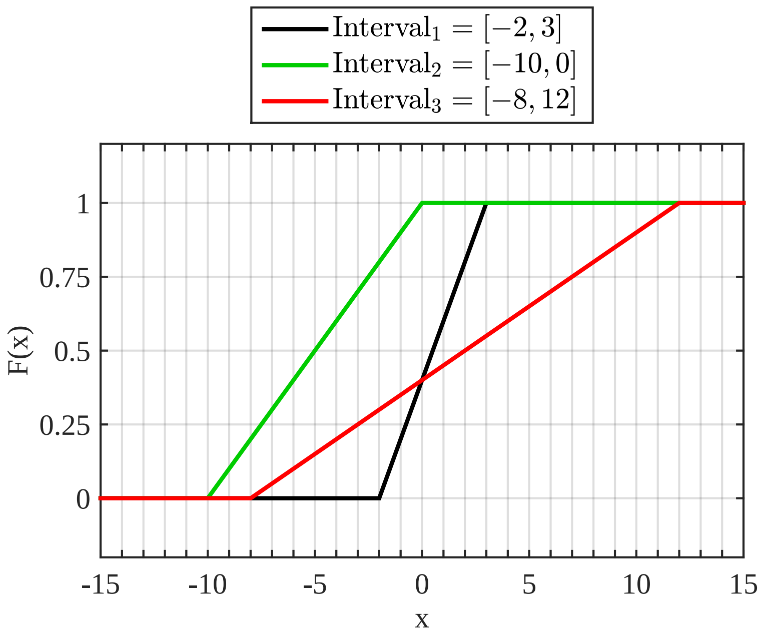
c) Sketch the cumulative distribution functions (cdf), that correspond to the probability density function (pdf) defined in subtask a) I, II and III.



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$\sum$ 13

Task 8: Measurement Errors and Correlation Analysis

a) Assume sensor 2 is not correctly installed, such that the height above the conveyor belt's surface is different from the one of sensor 1. Does this affect the correct determination of the time interval Δt?

Through the correlation analysis, the similarity of signal $x(t)$ and $y(t)$ for different time shifts Δt is calculated. The time shift where the maximum similarity is determined should not be affected by a different hight of sensor 2, because the signal's shape does not change through the different hight. Only the level, i.e. the absolute values of maxima and minima differ. $\boxed{4}$

b) Now assume that the distance d is corrupted, such that the real distance between the two sensors is $d_{real} = d \pm \Delta d$. Additionally the time interval is not determined correctly, such that the measured value Δt_m differs from the real one Δt_{real}: $\Delta t_m = \Delta t_{real} \pm \delta t$.

I Derive the equation to calculate the Gaussian error propagation of the velocity Δv, if the sign of the deviations Δd and δt is unknown a priori.

$$\begin{aligned} v &= \frac{d}{\Delta t} \\ \Rightarrow \Delta v &= \frac{\partial v}{\partial d}\Delta d + \frac{\partial v}{\partial \Delta t}\delta t \\ &= \frac{1}{\Delta t}\Delta d + (-\frac{d}{\Delta t^2}\delta t) \end{aligned}$$

$\boxed{3}$

II What is the worst-case scenario, i.e. what leads to biggest error in the velocity measurement, considering the sign of Δd and δt.

Worst case: Δd and δt have opposite signs. $\boxed{2}$

c) Now assume that the wires of the sensors are interchanged, which leads to the result, that $x(t)$ is the signal from sensor 2 and $y(t)$ is the signal from sensor 1. What error will emerge in the velocity measurement?

The velocity's direction will be measured wrongly due to the fact that the maximum correlation will appear for negative time shifts Δt. $\boxed{1}$

$\boxed{\sum 10}$