Sensorics Exam

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31st of July 2015

Name:	
Mat.-No.:	
Grade:	

Task:	T1	T2	T3	T4	T5	T6	T7	T8	T9	Sum
Scores:	11	19	20	7	6	7	10	14	31	125
Accomplished:										

Duration of examination: 2 hours

You are allowed to use a calculator and four pages of notes

Task 1: Comprehension Questions

Mark the correct answers clearly.
Every question has one or two correct answers!
For every correctly marked answer you will get one point. If there is one correct answer marked and one incorrect answer marked, you will get no point for that subtask.

a) Which statements are true for strain gauges?
- ☐ Environmental influences can easily be compensated through a clever arrangement of the strain gauges.
- ☐ Strain gauges utilize the resistance change caused by a change in length and a change of the cross section area of a conductor for the measurement.
- ☐ Strain gauges utilize the change of its capacity for the measurement.

b) How can speed be measured?
- ☐ Via the measurement of the acceleration and a subsequent differentiation.
- ☐ With the help of the Doppler-effect.
- ☐ Via the measurement of the rotational speed and a subsequent division by the radius.

c) How large should the internal resistance of a voltage meter be?
- ☐ Very large, in the ideal case infinite large.
- ☐ Very small, in the ideal case zero.
- ☐ It depends on the electrical circuit, what internal resistance is ideal.

d) Which statements are true for the measurement of temperatures?
- ☐ Thermocouples are more accurate than resistance thermometer.
- ☐ Thermocouples are suitable for point-wise measurements.
- ☐ Thermocouples have a smaller time constant than resistance thermometer.

e) The discrete Fourier transform is periodic ...
- ☐ ... only in time.
- ☐ ... only in frequency.
- ☐ ... in time and frequency.

f) A temporal sequence of N measurements that is transformed with the DFT results in a number of ...
- ☐ ... N discrete frequencies.
- ☐ ... $N/2$ discrete frequencies.
- ☐ ... 2^N discrete frequencies.

g) An increase in the number of measurements has the following effect on the frequency resolution:

☐ It becomes finer.

☐ It becomes coarser.

☐ It remains unaffected.

h) In order to make a meaningful statement about the contained frequencies in a non-stationary signal, ...

☐ the signal can be examined with a short-time DFT.

☐ the signal can not be examined with a short-time DFT.

☐ the signal can be examined with a wavelet transform.

Task 2: Displacement Measurement

Figure 1 shows two methods to measure displacements with capacitors. The mass m is attached to one spring with spring stiffness c_s and one damper with viscous damping d. In each of the two cases, one capacitor plate is attached to the mass m. The variable x_0 denotes the distance of two capacitor plates when the equilibrium is reached. The displacement x is measured relatively to the equilibrium. The permittivity of the medium between the capacitor plates is ϵ. The area of each capacitor plate is A.

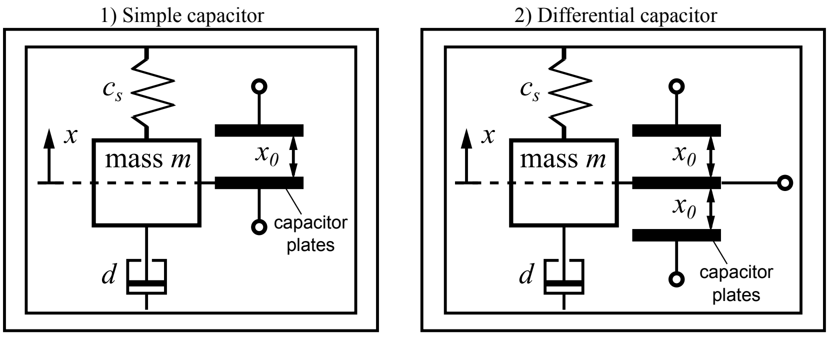


Fig. 1: Displacement measurement with simple capacitor and differential capacitor.

a) Determine the relationship between the capacitance $C(x)$ and the displacement x for the simple capacitor (case 1).

b) Determine the relationship between the upper capacitance $C_u(x)$ and the lower capacitance $C_l(x)$ depending on the displacement x for the differential capacitor (case 2).

c) Derive the equation that describes the bridge voltage U_d depending on the capacitance $C(x)$ for the bridge circuit shown in Fig. 13. C_{fix} is a constant capacitance, U_0 is the input voltage and R are resistances. Hint: The complex impedance of a capacitor is $Z_C = 1/(j\omega C)$.

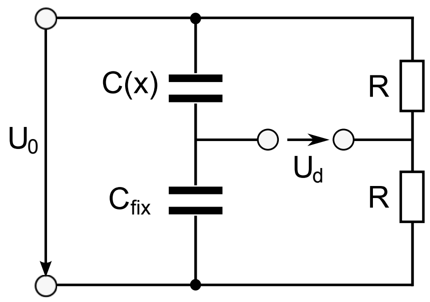


Fig. 2: Bridge circuit to measure the capacitance.

d) Substitute $C(x)$ with the results from subtask a), such that one equation shows the relationship between the bridge voltage U_d depending on x for the simple capacitor.

e) Now assume, that the fixed capacitance from Fig. 13 is replaced by $C(-x)$ according to the differential capacitor. Derive the equation that describes the bridge voltage U_d depending on the **displacement** x in case of the differential capacitor. Sketch qualitatively the curve of the bridge voltage U_d over the displacement x for the differential capacitor.

f) Compare the relationship between the bridge voltage U_d and the displacement x for both cases, the simple and the differential capacitor. What relationship would you prefer and why?

Task 3: Quantization

Fig. 3 shows an analogue signal (black line) together with its sampled version (gray bars and dots). The sampled signal is not yet quantized.

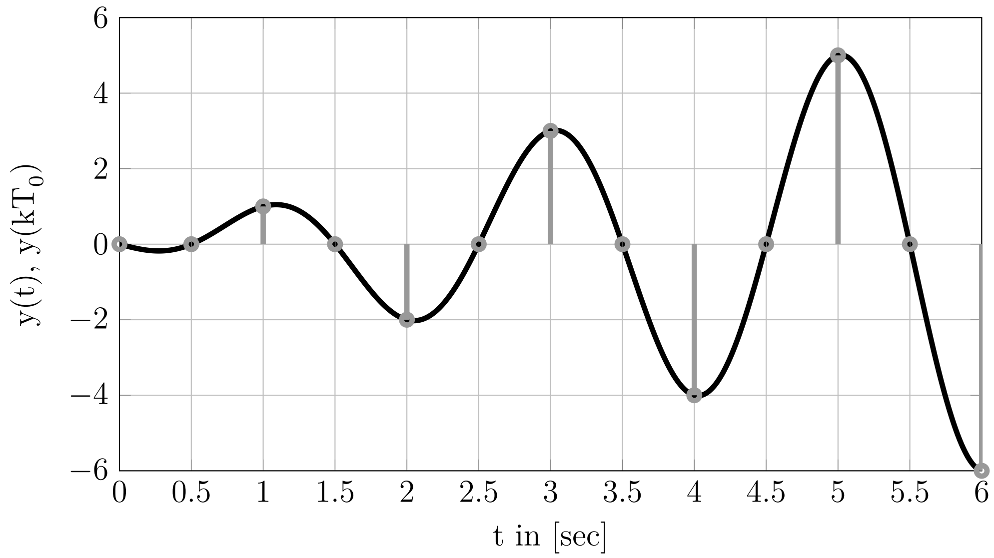


Fig. 3: Sampled, not yet quantized signal

a) Determine the signal frequency f_S of the analogue signal. Hint: The signal's frequency is not time-dependent and therefore constant.

b) Determine the sampling frequency f_0.

c) The signal should be quantized by a 2-bit A/D converter. Determine the corresponding integer number for each of the 13 samples (gray) from Fig. 3 and draw the digital signal into Fig. 4. Use minimum and maximum values as limits for the A/D conversion.

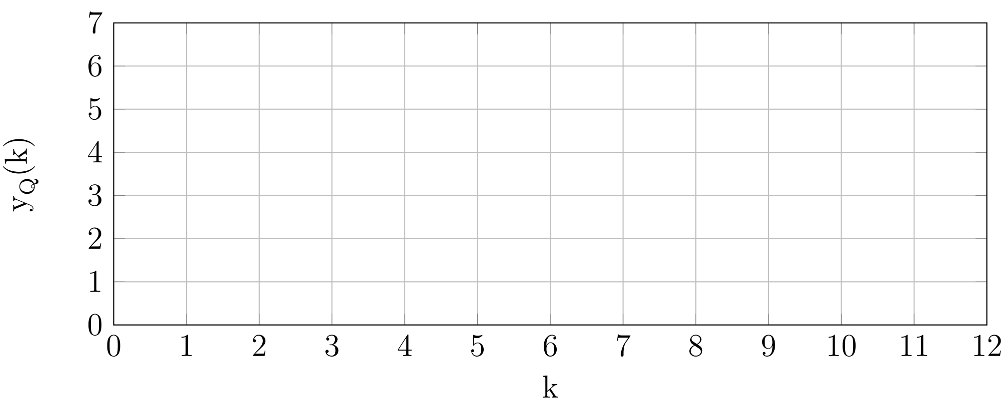


Fig. 4: Quantization with 2-bit A/D converter

d) Now the signal should be quantized by a 3-bit A/D converter. Determine the corresponding integer number for each of the 13 samples (gray) from Fig. 3 and draw the digital signal into Fig. 5. Use minimum and maximum values as limits for the A/D conversion.

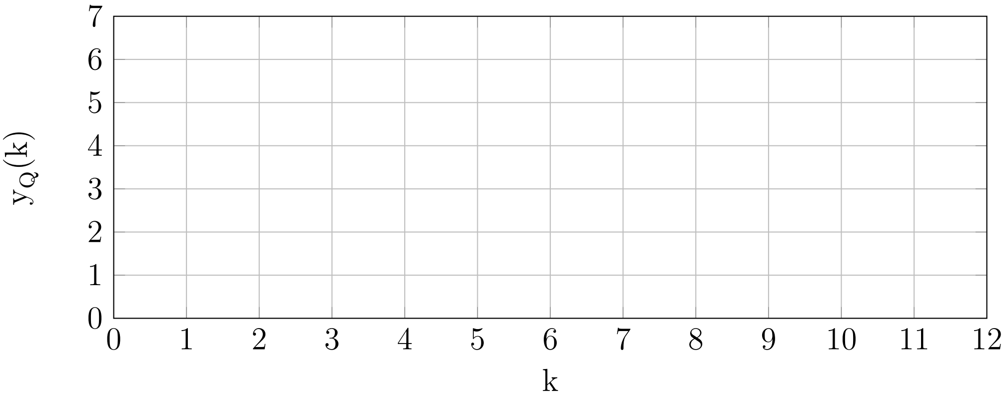


Fig. 5: Quantization with 3-bit A/D converter

e) Determine the maximum quantization error for the 2-bit A/D converter as well as for the 3-bit A/D converter.

Task 4: Probability Density

In Fig. 6 four different discrete signals are shown.

a) On what probability density distribution are the signals presumably based upon?

b) Assign the signals from Fig. 6 to a probability density function (PDF) from Fig. 7 (each signal follows one PDF). Describe your choices in a short sentence:

Signal	A	B	C	D
PDF				

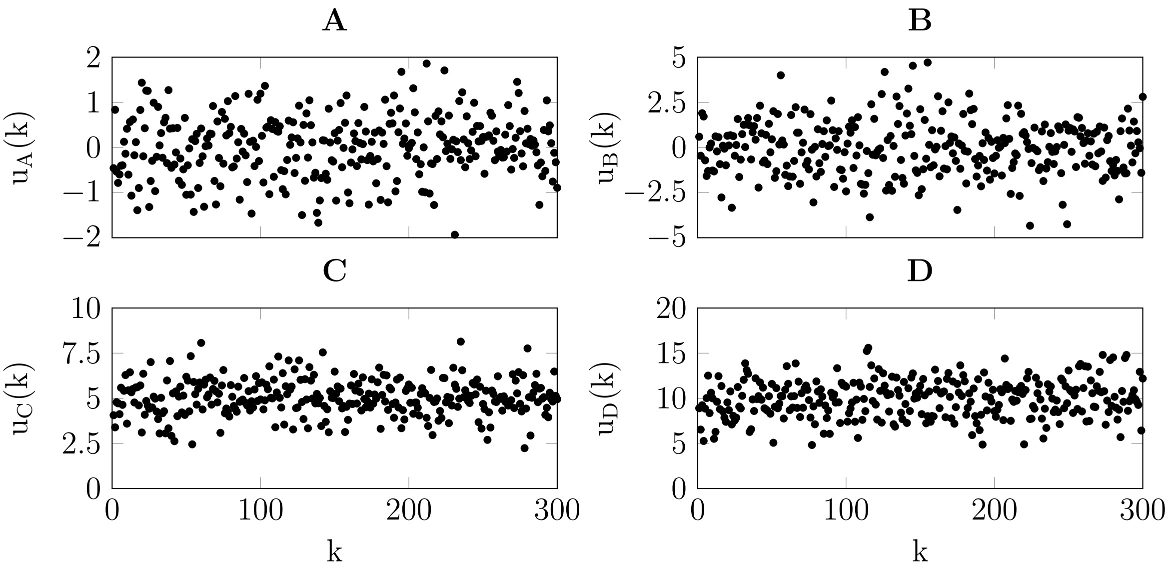


Fig. 6: Discrete signals

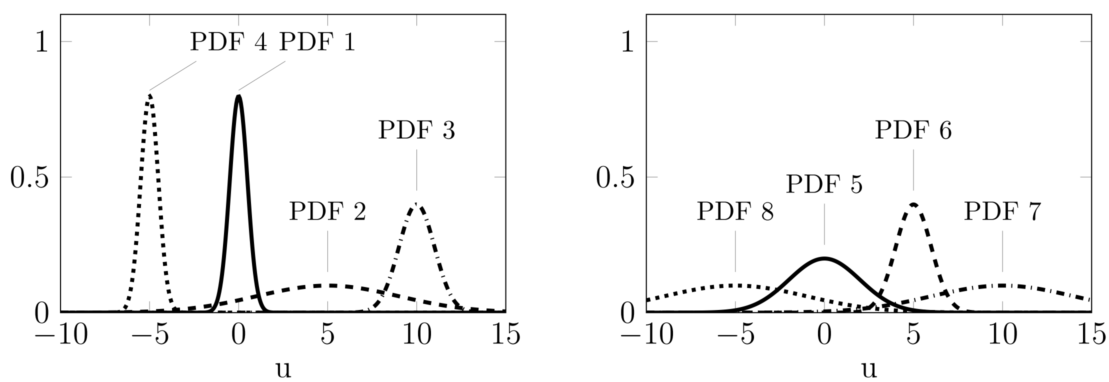


Fig. 7: Probability density functions

Task 5: PCA

From different inputs u_1 and u_2, the multivariate distributions A, B and C arise. Assign the correct singular values to each data distribution, that result from a principal component analysis (PCA). For each data distribution only one singular value combination from SV1 up to SV9 is correct. Give reasons for your choices (short explanation).

Data distribution	A	B	C
Singular value			

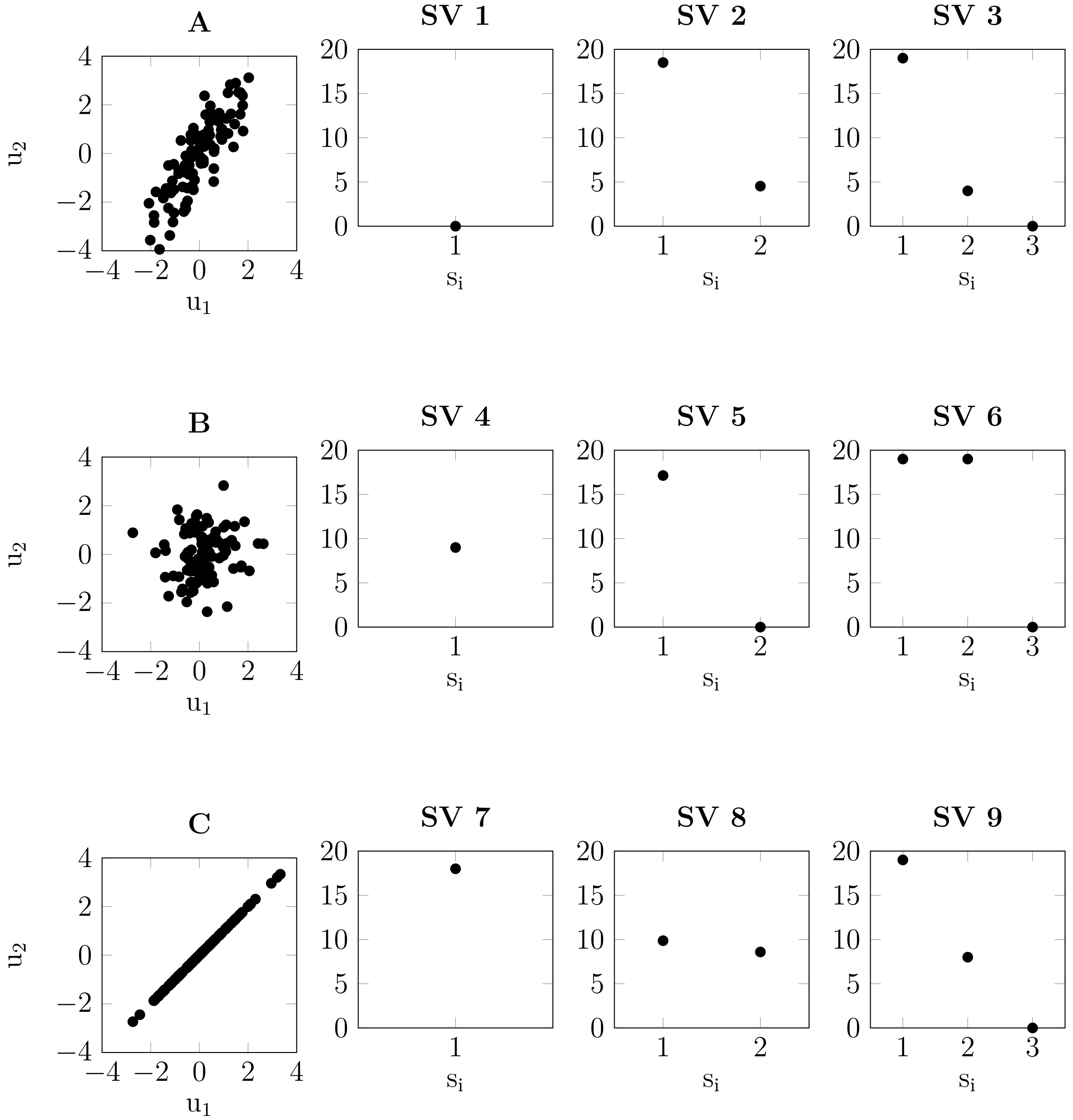


Fig. 8: Data distributions and singular value combinations

Task 6: Downsampling

The time-discrete signal $u(k)$ with sampling time T_0 is shown in Fig 9. The signal $u(k)$ should be compressed by downsampling.

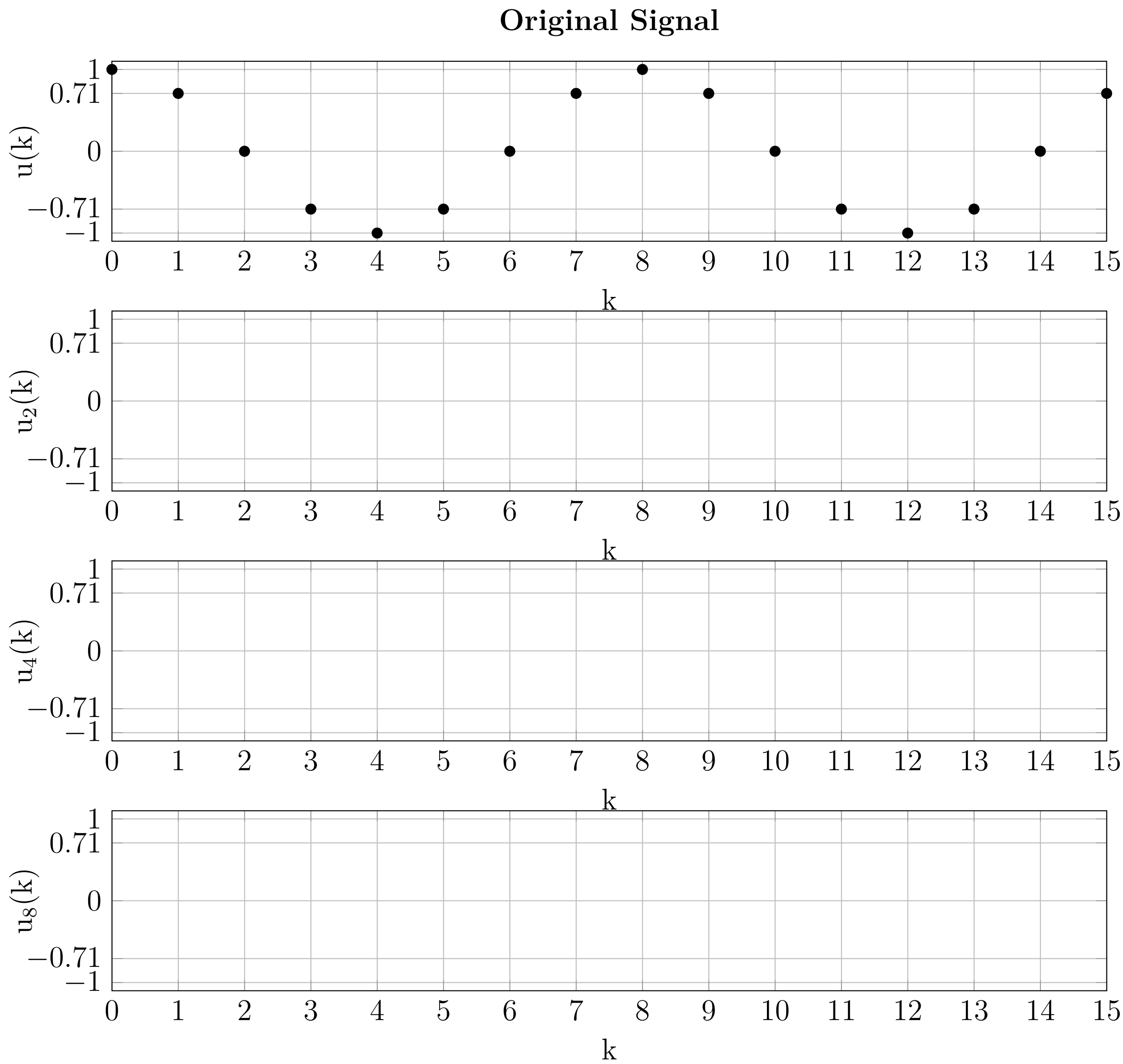


Fig. 9: Time-discrete signal

a) Compress the signal $u(k)$ by factors of 2, 4 and 8. In what way changes the sampling time T_0? Sketch the results ($u_2(k)$,$u_4(k)$ and $u_8(k)$) into Fig. 9. Hint: The first value is always retained. Think about the meaning of k for different sampling times.

b) For which of the compressions do problems arise? How is the appearing phenomenon called?

c) How can the problem from subtask b) be prevented?

d) How would the signal look like, if the solution from subtask c) would be applied?

Task 7: Dynamic System

The following time-discrete system is given:

$$G(z) = \frac{Y(z)}{U(z)} = \frac{1}{z - 0.5} \tag{1}$$

a) How is a discrete impulse defined in the z-domain (equation)?

b) How does the signal $Y(z)$ of the given system $G(z)$ look like in the z-domain, if an impulse is applied.

c) Determine the final value of the impulse response for $G(z)$ with the help of the final value theorem.

d) Transform the signal $Y(z)$ from b) to the discrete-time domain and calculate $y(k)$ for $k = \{0, 1, \ldots, 5\}$. ($y(k) = 0$ for $k < 0$).

e) Specify the transfer function of a common FIR filter of order 4 in the z-domain.

f) Specify coefficients of the FIR filter from subtask e, if it should be used to approximate the transfer function $G(z)$ from Eq. 1.

Task 8: Filter

In Fig. 10 two different filters are shown.

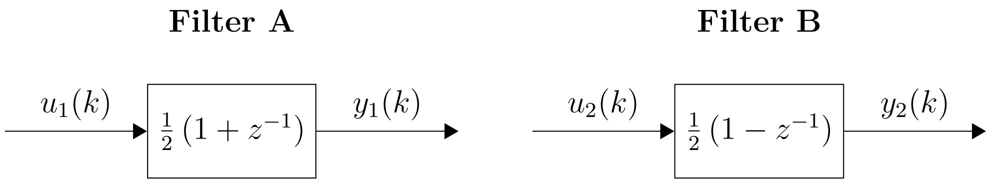
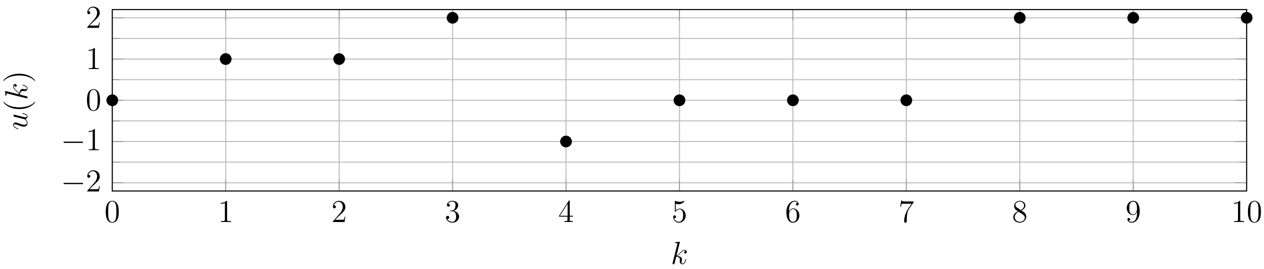


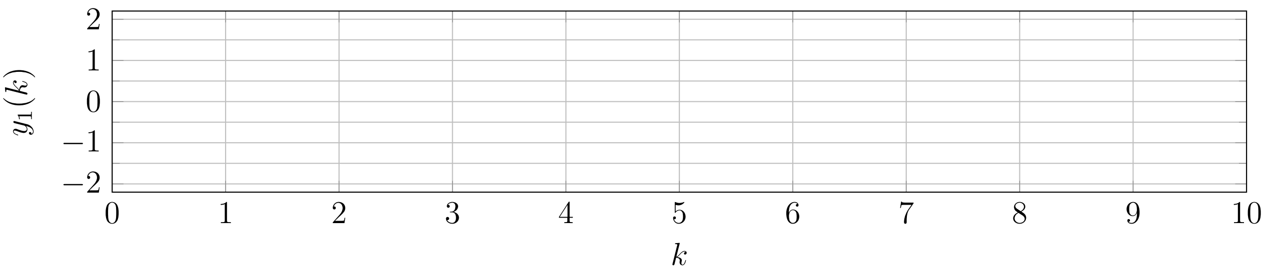
Fig. 10: Filter transfer functions

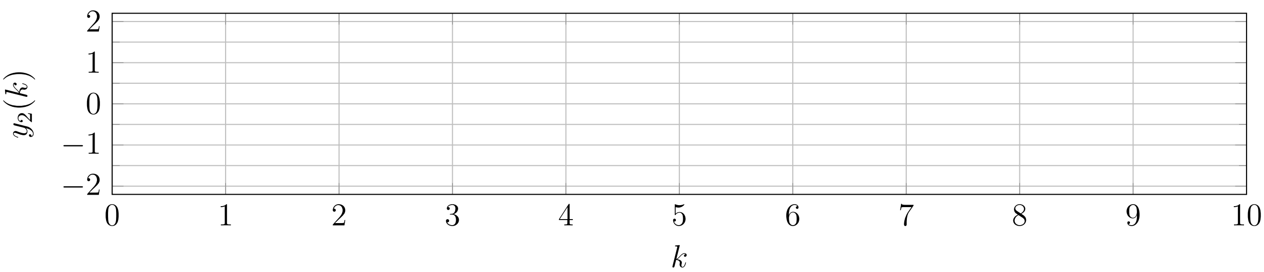
a) Calculate the amplitude response for each filter.

b) Assign the correct filter type to each of the two filters. Choose from the following types: High-pas, low-pass, band-pass or band-stop. Hint: Set $\omega = 0$ and $\omega = \frac{\pi}{T_0}$ in the solution from subtask 1).

c) The signal $u(k)$ shown below is applied to both filters ($u(k) = u_1(k) = u_2(k)$). Sketch the filter's responses $y_1(k)$ and $y_2(k)$ into the corresponding diagrams ($u(k) = 0$ for $k < 0$).







d) Both filters are now connected in parallel as shown in Fig. 11. The input signal $u(k)$ from subtask 3) is applied to this system. Sketch the system response $y_3(k)$ into the corresponding diagram. Give reasons for the resulting behavior based on the overall transfer function.

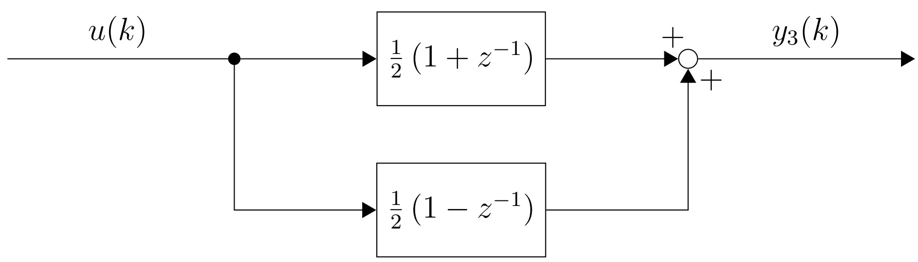
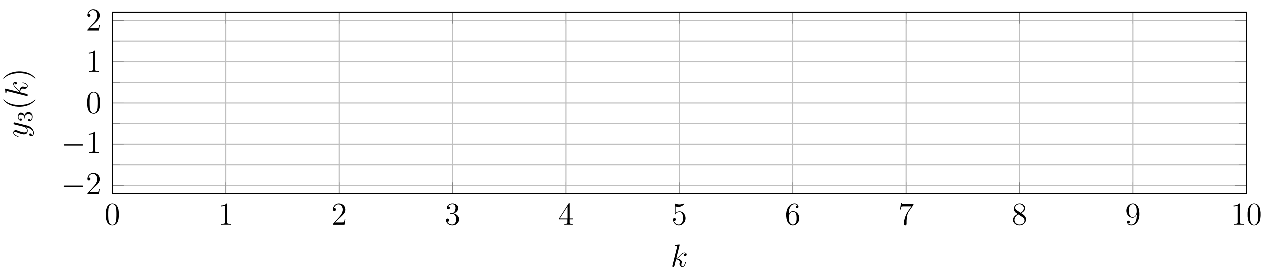


Fig. 11: Filters A and B connected in parallel



Task 9: Block Diagram

The following difference equation is given:

$$y(k) + a_1 y(k-1) + a_2 y(k-2) \quad = \quad b_1 u(k-1) + b_3 u(k-3). \tag{2}$$

a) Sketch the direct form I block diagram (first $B(z)$, then $\frac{1}{A(z)}$) of Eq. 2.

b) Sketch the non-redundant direct form II block diagram (first $\frac{1}{A(z)}$, then $B(z)$) of Eq. 2.

c) Determine the transfer function $G(z)$ of Eq. 2.

d) The following table lists several possible coefficient combinations for Eq. 2. Assign the correct step response **A** to **H** (see Fig. 12 next page) to each coefficient combination. Explain your choices shortly. Hint: It is not necessary to calculate the poles for any of the given coefficient combinations in order to solve this subtask.

a_1	a_2	b_1	b_3	Step Response
-0.5	-0.1	0	0.4	
-0.5	0.5	0	0.4	
-0.3	0.4	3.5	2	
0	-0.6	0.4	0	

e) Assume to approximate the IIR system from Eq. 2 by an FIR system. If step response in Fig. 12 **G** would be the step response $H_{IIR}(z)$ of the IIR system, what FIR system order do you need, if the absolute maximum error between the FIR step response $H_{FIR}(z)$ and H_{IIR} should be less than 0.05 at any time. Explain the choice of the FIR system order shortly.

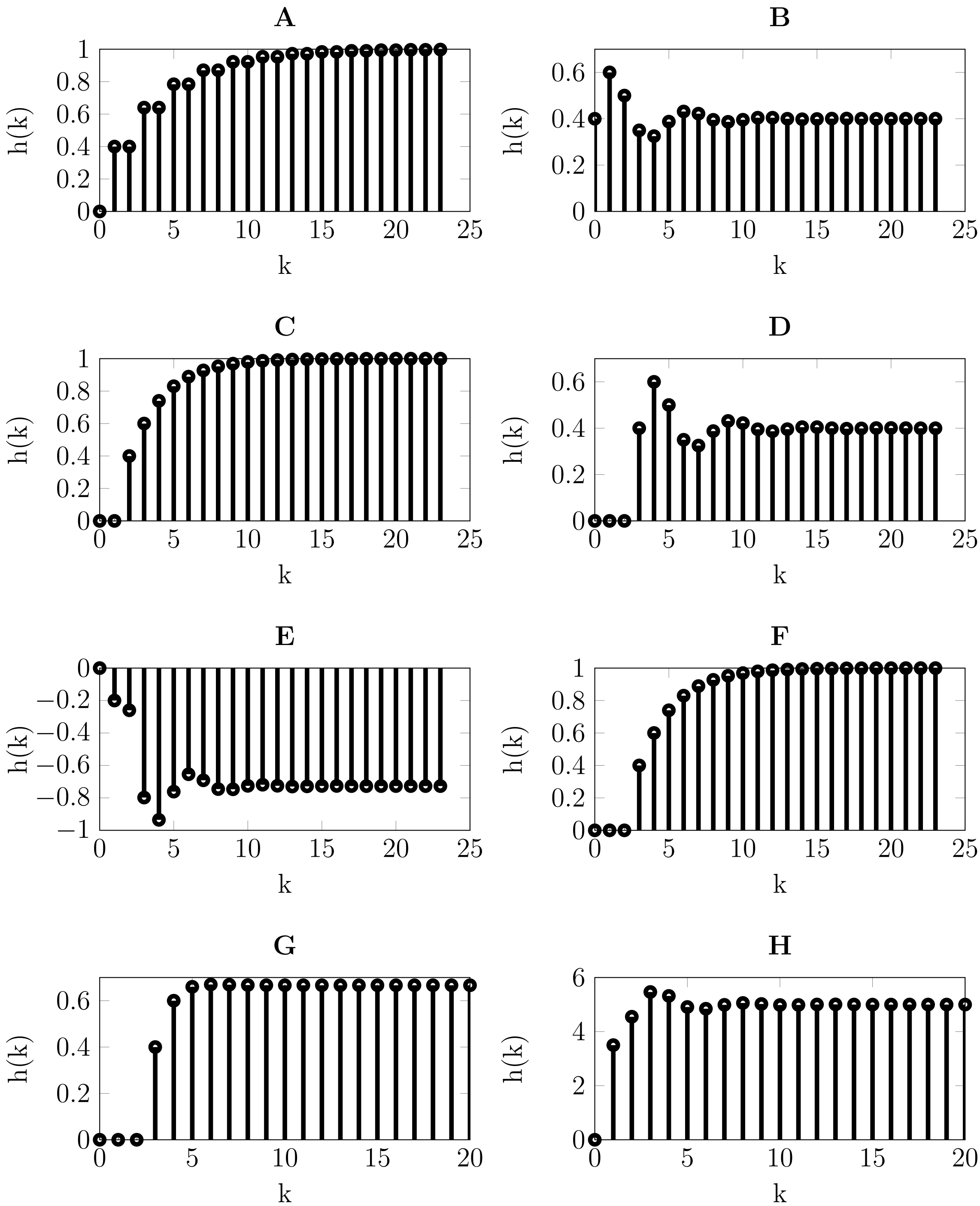


Fig. 12: Step responses **A** to **H**

Solutions:

Task 1: Comprehension Questions

a) Which statements are true for strain gauges?

- [x] Environmental influences can easily be compensated through a clever arrangement of the strain gauges.
- [x] Strain gauges utilize the resistance change caused by a change in length and a change of the cross section area of a conductor for the measurement.
- [ ] Strain gauges utilize the change of its capacity for the measurement.

b) How can speed be measured?

- [ ] Via the measurement of the acceleration and a subsequent differentiation.
- [x] With the help of the Doppler-effect.
- [ ] Via the measurement of the rotational speed and a subsequent division by the radius.

c) How large should the internal resistance of a voltage meter be?

- [x] Very large, in the ideal case infinite large.
- [ ] Very small, in the ideal case zero.
- [ ] It depends on the electrical circuit, what internal resistance is ideal.

d) Which statements are true for the measurement of temperatures?

- [ ] Thermocouples are more accurate than resistance thermometer.
- [x] Thermocouples are suitable for point-wise measurements.
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e) The discrete Fourier transform is periodic ...

- [ ] ... only in time.
- [ ] ... only in frequency.
- [x] ... in time and frequency.

f) A temporal sequence of N measurements that is transformed with the DFT results in a number of ...

- [x] ... N discrete frequencies.
- [ ] ... $N/2$ discrete frequencies.
- [ ] ... 2^N discrete frequencies.

g) An increase in the number of measurements has the following effect on the frequency resolution:

- [x] It becomes finer.
- [ ] It becomes coarser.
- [ ] It remains unaffected.

h) In order to make a meaningful statement about the contained frequencies in a non-stationary signal, ...

■ the signal can be examined with a short-time DFT.

□ the signal can not be examined with a short-time DFT.

■ the signal can be examined with a wavelet transform.

$\sum$ 11

Task 2: Displacement Measurement

a) Determine the relationship between the capacitance $C(x)$ and the displacement x for the simple capacitor (case 1).

$$\begin{aligned} C(x) &= \epsilon_0\epsilon\frac{A}{d(x)} \\ &= \epsilon_0\epsilon\frac{A}{x_0 - x} \end{aligned}$$

$\boxed{2}$

b) Determine the relationship between the upper capacitance $C_u(x)$ and the lower capacitance $C_l(x)$ depending on the displacement x for the differential capacitor (case 2).

$$\begin{aligned} C_u(x) &= \epsilon_0\epsilon\frac{A}{x_0 - x} \\ C_l(x) &= \epsilon_0\epsilon\frac{A}{x_0 + x} \end{aligned}$$

$\boxed{2}$

c) Derive the equation that describes the bridge voltage U_d depending on the capacitance $C(x)$ for the bridge circuit shown in Fig. 13. C_{fix} is a constant capacitance, U_0 is the input voltage and R are resistances. Hint: The complex impedance of a capacitor is $Z_C = 1/(j\omega C)$.

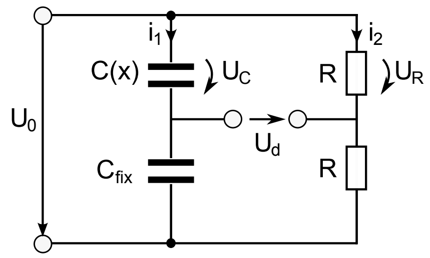


Fig. 13: Bridge circuit to measure the capacitance.

$$\begin{aligned} U_d &= \frac{R}{2R}U_0 - \frac{Z_{C(x)}}{Z_{C(x)} + Z_{C_{fix}}}U_0 \\ &= \frac{1}{2}U_0 - \frac{1/(j\omega C(x))}{1/(j\omega C(x)) + 1/(j\omega C_{fix})}U_0 \\ &= \frac{1}{2}U_0 - \frac{\frac{1}{C(x)}}{\frac{1}{C(x)} + \frac{1}{C_{fix}}}U_0 \\ &= \frac{1}{2}U_0 - \frac{C_{fix}}{C(x) + C_{fix}}U_0 \end{aligned}$$

$\boxed{4}$

d) Substitute $C(x)$ with the results from subtask a).

$$
\begin{aligned}
U_d &= \frac{1}{2}U_0 - \frac{C_{fix}}{\epsilon_0\epsilon\frac{A}{x_0-x} + C_{fix}}U_0 \\
&= \frac{1}{2}U_0 - \frac{C_{fix}(x_0-x)}{\epsilon_0\epsilon A + C_{fix}(x_0-x)}U_0
\end{aligned}
$$

2

e) Now assume, that the fixed capacitance from Fig. 13 is replaced by $C(-x)$ according to the differential capacitor. Derive the equation that describes the bridge voltage U_d depending on the **displacement** x in case of the differential capacitor.

We just have to replace C_{fix} by $C(-x)$:

$$
U_d = \frac{1}{2}U_0 - \frac{C(-x)}{C(x)+C(-x)}U_0
$$

1

Now we substitute $C(x)$ by $C_l(x)$ and $C(-x)$ by $C_u(x)$ from subtask b).

$$
\begin{aligned}
U_d &= \frac{1}{2}U_0 - \frac{\epsilon_0\epsilon\frac{A}{x_0-x}}{\epsilon_0\epsilon\frac{A}{x_0+x} + \epsilon_0\epsilon\frac{A}{x_0-x}}U_0 \\
&= \frac{1}{2}U_0 - \frac{\frac{1}{x_0-x}}{\frac{1}{x_0+x} + \frac{1}{x_0-x}}U_0 \\
&= \frac{1}{2}U_0 - \frac{x_0+x}{x_0-x+x_0+x}U_0 \\
&= \frac{1}{2}U_0 - \frac{x_0+x}{2x_0}U_0 \\
&= \left(\frac{1}{2} - \frac{x_0}{2x_0} - \frac{x}{2x_0}\right)U_0 \\
&= -\frac{x}{2x_0}U_0
\end{aligned}
$$

4

As a result of the differential capacitor, the relationship between the bridge voltage U_d and the displacement x is linear. Therefore any sketch is correct, if the linear dependency is recognizable, i.e. a straight line through the origin with negative slope.

2

f) Compare the relationship between the bridge voltage U_d and the displacement x for both cases, the simple and the differential capacitor. What relationship would you prefer and why?

As can be seen by the results of the upper subtasks, the relationship between the bridge voltage U_d and the displacement x is nonlinear in case of the simple capacitor and linear for the differential capacitor. The linear relationship is preferable, since the sensitivity of the system is the same for all displacements x. Furthermore the behavior is easy to understand and to handle.

2

$\sum 19$

Task 3: Quantization

a) Determine the signal frequency f_S of the analogue signal. Hint: The signal's frequency is not time-dependent and therefore constant.

$f_S = 0.5$ Hz 1

b) Determine the sampling frequency f_0.

$f_0 = 2$ Hz 1

c) The solution is shown in Fig. 14.

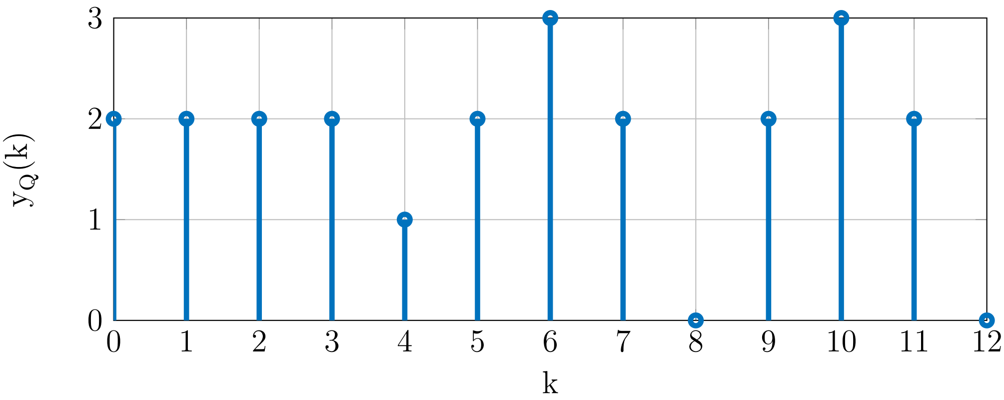


Fig. 14: Quantization with 2-bit A/D converter

6

If the minimum and maximum amplitudes are in the medium values of the intervals, Fig. 15 shows the correct solution.

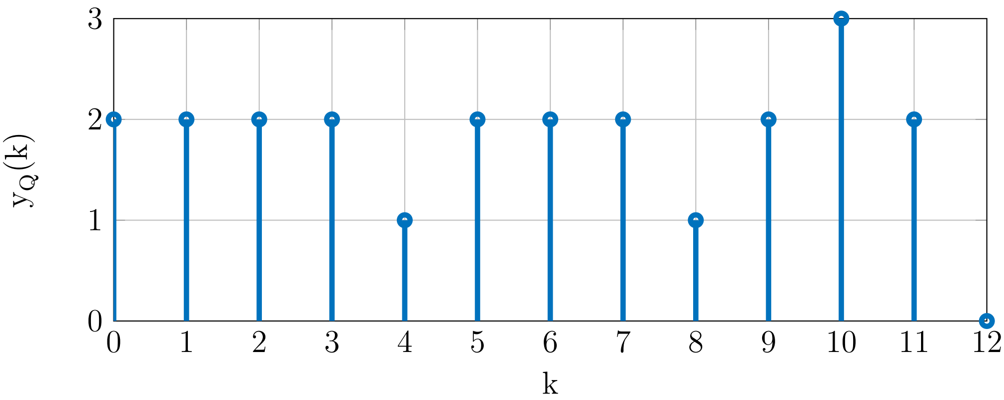


Fig. 15: Quantization with 2-bit A/D converter, alternative solution

d) The solution is shown in Fig. 16. 10

If the minimum and maximum amplitudes are in the medium values of the intervals, Fig. 17 shows the correct solution.

e) Determine the maximum quantization error for the 2-bit A/D converter as well as for the 3-bit A/D converter.

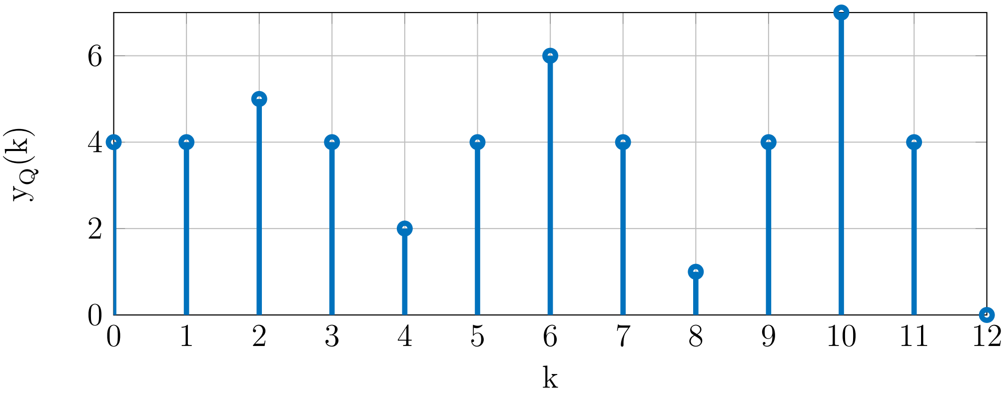


Fig. 16: Quantization with 3-bit A/D converter

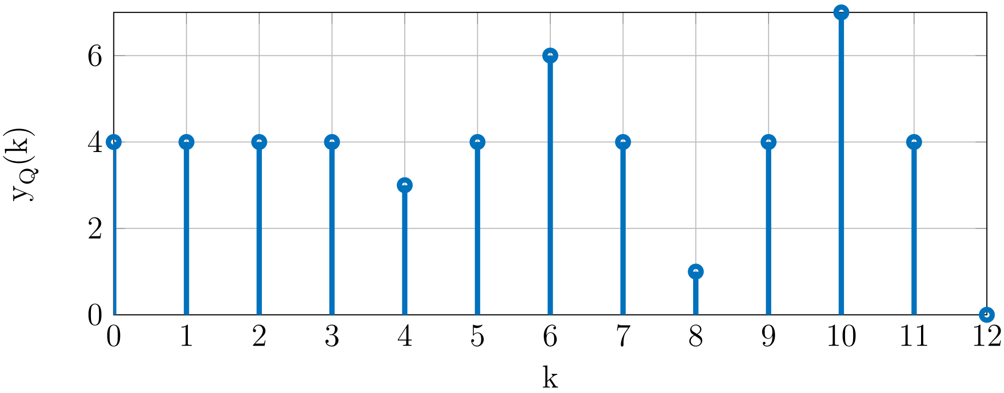


Fig. 17: Quantization with 3-bit A/D converter, alternative solution

Depending on the quantization method, the maximum errors differ. If the minimum and maximum amplitudes are in the medium values of the intervals, the maximum quantization error can be calculated as:

$$e_{Qmax}^{med} = \frac{1}{2}\frac{y_{max}(t) - y_{min}(t)}{2^n - 1}$$

for an n-bit converter. In case the maximum and minimum amplitudes mark the limits of the intervals, the maximum quantization error can be calculated as:

$$e_{Qmax}^{lim} = \frac{y_{max}(t) - y_{min}(t)}{2^n}$$

To achieve the full score for this task, the maximum quantization error for only one of these two methods has to be calculated.

Maximum quantization error for the 2-bit A/D converter is either $e_{Qmax}^{med} = 1.83$ or $e_{Qmax}^{lim} = 2.75$.

Maximum quantization error for the 3-bit A/D converter is either $e_{Qmax}^{med} = 0.79$ or $e_{Qmax}^{lim} = 1.38$.

$\boxed{2}$

$\boxed{\sum 20}$

Task 4: Probability Density

a) Normal distribution. 1

b) With the help of the mean values, each signal can be assigned to 2 probability density function at maximum. Based on the signal's variance, each Gaussian can be assigned uniquely.

Example: Signal A has a mean value of $\mu = 0$. With this information, only PDF 1 and 5 can be correct. The signal scatters roughly from -1.5 up to 1.5. PDF 5 is more likely to spread wider, such that PDF 1 belongs to Signal A.

Signal	A	B	C	D
PDF	PDF 1	PDF 5	PDF 6	PDF 7

6

$\sum 7$

Task 5: PCA

The shown data distributions are described by two inputs. It follows, that only two singular values exist for each data distribution, such that SV 1, SV 3, SV 4, SV 6, SV 7 and SV 9 can't be correct.

For data distribution B, knowing the value of u_1 contains almost no information about the corresponding value of u_2. Both inputs are equally important. The singular values represent the importance of each input. It follows, that SV8 belongs to data distribution 8.

In data distribution C a linear dependency between u_1 and u_2 exists. If one value of one input is known, the value of the other input can be determined exactly. Therefore one singular value has to be zero → SV5 is correct.

For data distribution A the range in which u_2 values might occur given a value for u_1 is small. This leads to a big difference between the two singular values, but no singular value is exactly zero → SV2. 3

Data distribution	A	B	C
Singular values	SV 2	SV 8	SV 5

3

$\sum 6$

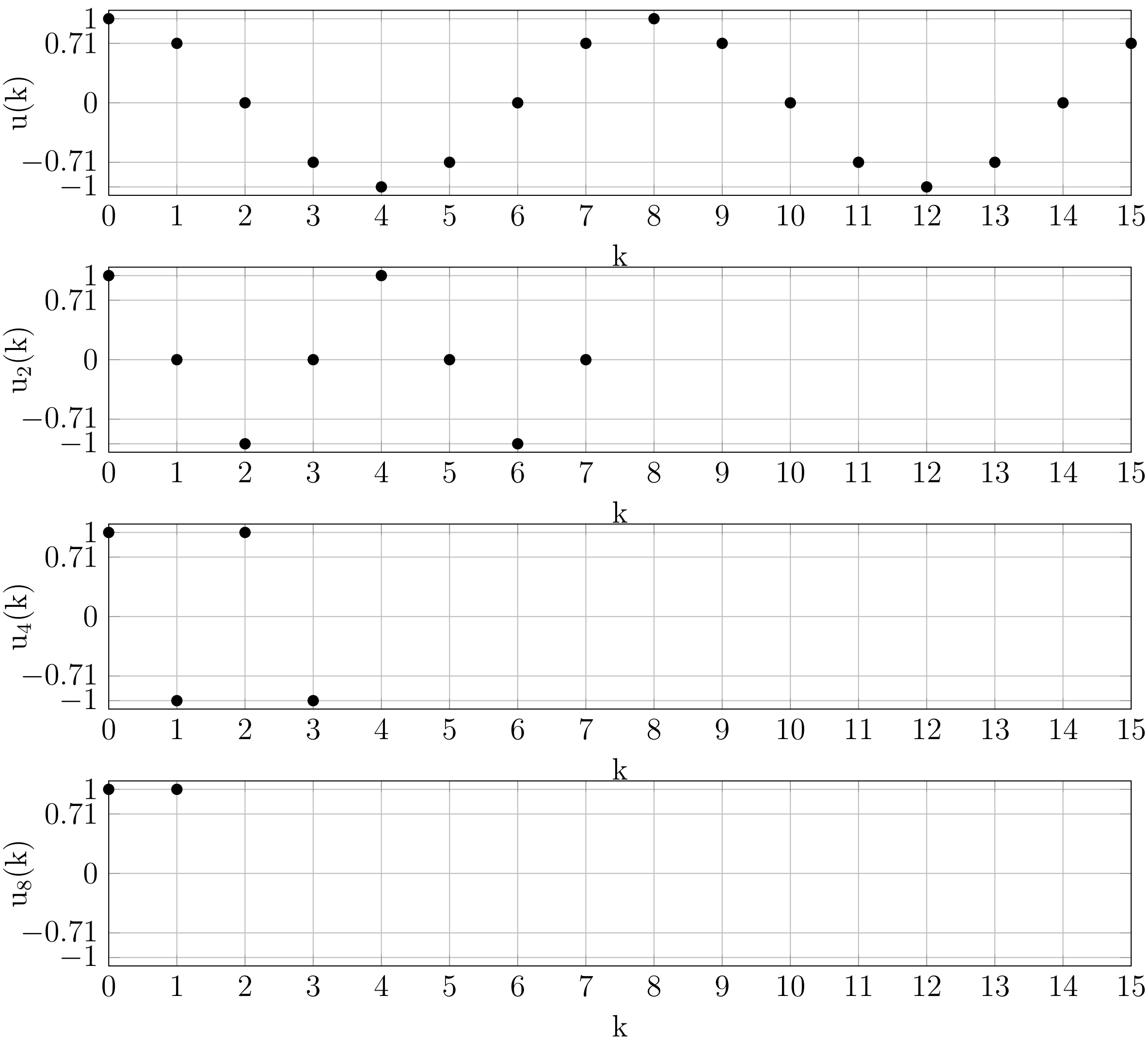
Task 6: Downsampling

a) The sampling times are:

$$T_{0,2} = 2 \cdot T_0 \quad ; \quad T_{0,4} = 4 \cdot T_0 \quad ; \quad T_{0,8} = 8 \cdot T_0 \tag{3}$$

The correct solutions for the compressed signals are shown below. 1

Original Signal



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b) For a compression factor of 8, the signal's frequency is no longer correctly described. The sampling theorem is violated, aliasing occurs. 1

c) Because of the violation of the sampling theorem, a constant signal appears with mean value $\mu = 1$. In contrast, the original signal has a mean value of $\mu = 0$. To prevent the occurring aliasing a low-pass filter should be used before the downsampling (anti aliasing filter).

d) Ideally the result would be $u_8(k) = 0$. 2

$\sum 7$

Task 7: Dynamic System

a) An impulse in the z-domain is defined as follows:

$$U(z) = 1 \tag{4}$$

$\boxed{1}$

b) The impulse response in z corresponds to the multiplication of the system with an impulse:

$$Y(z) = G(z) \cdot U(z) \tag{5}$$
$$Y(z) = \frac{1}{z - 0.5} \cdot 1 \tag{6}$$
$$Y(z) = \frac{1}{z - 0.5} \tag{7}$$

$\boxed{1}$

c) The final value theorem in general is defined as:

$$y(k \to \infty) = \lim_{z \to 1} ((z - 1) \cdot G(z) \cdot U(z)) \tag{8}$$
$$y(k \to \infty) = \lim_{z \to 1} ((z - 1) \cdot Y(z)) \tag{9}$$

If follows for the given system:

$$y(k \to \infty) = \lim_{z \to 1} \left((z - 1) \cdot \frac{1}{z - 0.5} \right) \tag{10}$$
$$y(k \to \infty) = 0 \cdot \frac{1}{z - 0.5} \tag{11}$$
$$\Rightarrow y(k \to \infty) = 0 \tag{12}$$

$\boxed{2}$

d) The transformation is as follows:

$$Y(z) = \frac{1}{z - 0.5} \tag{13}$$
$$Y(z) = \frac{z^{-1}}{1 - 0.5 \cdot z^{-1}} \tag{14}$$
$$\Leftrightarrow \quad Y(z) - 0.5 \cdot z^{-1} \cdot Y(z) = z^{-1} \tag{15}$$
$$\bullet\!\!-\!\!\circ \tag{16}$$
$$y(k) = 0.5y(k - 1) + \delta_k(k - 1) \tag{17}$$

The first 6 discrete values of the signal are: $\boxed{2}$

$$\begin{aligned}
y(k=0) &= 0.5y(k=-1) + \delta_k(k=-1) && (18)\\
&= 0.5 \cdot 0 + 0 = 0 && (19)\\
y(k=1) &= 0.5y(k=0) + \delta_k(k=0) && (20)\\
&= 0.5 \cdot 0 + 1 = 1 && (21)\\
y(k=2) &= 0.5y(k=1) + \delta_k(k=1) && (22)\\
&= 0.5 \cdot 1 + 0 = 0.5 && (23)\\
y(k=3) &= 0.5y(k=2) + \delta_k(k=2) && (24)\\
&= 0.5 \cdot 0.5 + 0 = 0.25 && (25)\\
y(k=4) &= 0.5y(k=3) + \delta_k(k=3) && (26)\\
&= 0.5 \cdot 0.25 + 0 = 0.125 && (27)\\
y(k=5) &= 0.5y(k=4) + \delta_k(k=4) && (28)\\
&= 0.5 \cdot 0.125 + 0 = 0.0625 && (29)
\end{aligned}$$

$\boxed{2}$

e) In general the transfer function is defined as:

$$G(z) = b_0 + b_1 \cdot z^{-1} + b_2 \cdot z^{-2} + b_3 \cdot z^{-3} + b_4 \cdot z^{-4} \quad (30)$$

$\boxed{1}$

f) The impulse response values of the IIR system are utilized as coefficients for the FIR filter:

$$\begin{aligned}
&b_i = y(k=i) && (31)\\
&b_0 = y(k=0) \quad ; \quad b_1 = y(k=1) \quad ; \quad b_2 = y(k=2) && (32)\\
&b_3 = y(k=3) \quad ; \quad b_4 = y(k=4) && (33)
\end{aligned}$$

$\boxed{1}$

$\boxed{\sum 10}$

Task 8: Filter

a) Calculation of the amplitude response for **Filter A:**

$$\begin{aligned} G_1(z) &= \frac{1}{2}\left(1+z^{-1}\right) && (34)\\ G_1(i\omega) &= \frac{1}{2}\left(1+e^{-i\omega T_0}\right) && (35)\\ G_1(i\omega) &= \frac{1}{2}\left(1+\cos(\omega T_0)-i\sin\left(\omega T_0\right)\right) && (36)\\ \|G_1(i\omega)\| &= \sqrt{\left(\frac{1}{2}+\frac{1}{2}\cos\left(\omega T_0\right)\right)^2+\frac{1}{4}\sin^2\left(\omega T_0\right)} && (37)\\ &= \sqrt{\frac{1}{2}+\frac{1}{2}\cos\left(\omega T_0\right)} && (38) \end{aligned}$$

for **Filter B:** $\boxed{3}$

$$\begin{aligned} G_2(z) &= \frac{1}{2}\left(1-z^{-1}\right) && (39)\\ G_2(i\omega) &= \frac{1}{2}\left(1-e^{-i\omega T_0}\right) && (40)\\ G_2(i\omega) &= \frac{1}{2}\left(1-\cos(\omega T_0)-i\sin\left(\omega T_0\right)\right) && (41)\\ \|G_2(i\omega)\| &= \sqrt{\left(\frac{1}{2}-\frac{1}{2}\cos\left(\omega T_0\right)\right)^2+\frac{1}{4}\sin^2\left(\omega T_0\right)} && (42)\\ &= \sqrt{\frac{1}{2}-\frac{1}{2}\cos\left(\omega T_0\right)} && (43) \end{aligned}$$

$\boxed{3}$

b) The filters are of type:
Filter A: Low-pass, because

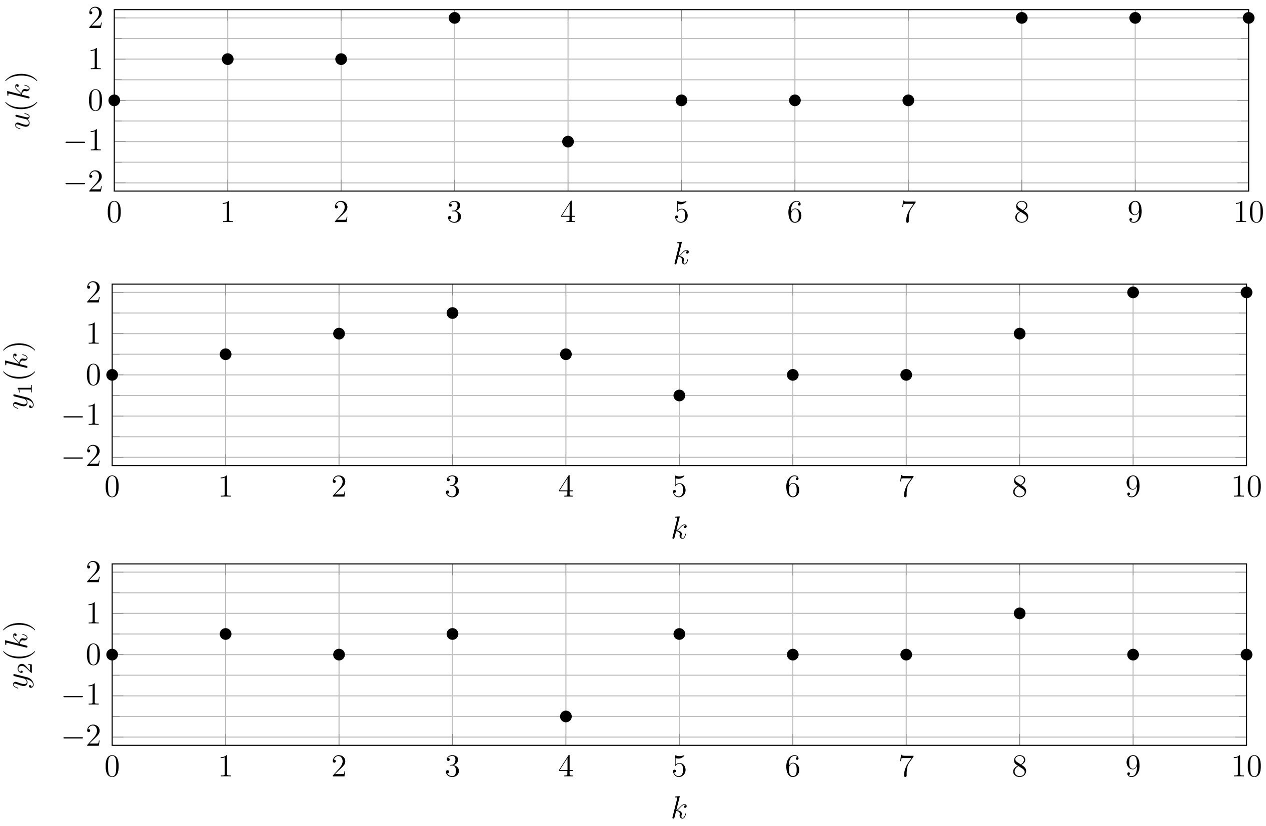
$$\|G_1(0)\| = 1 \qquad \left\|G_1\left(i\frac{\pi}{T_0}\right)\right\| = 0 \tag{44}$$

Filter B: High-pass, because

$$\|G_2(0)\| = 0 \qquad \left\|G_2\left(i\frac{\pi}{T_0}\right)\right\| = 1 \tag{45}$$

$\boxed{2}$

c) The correct signals are shown below.



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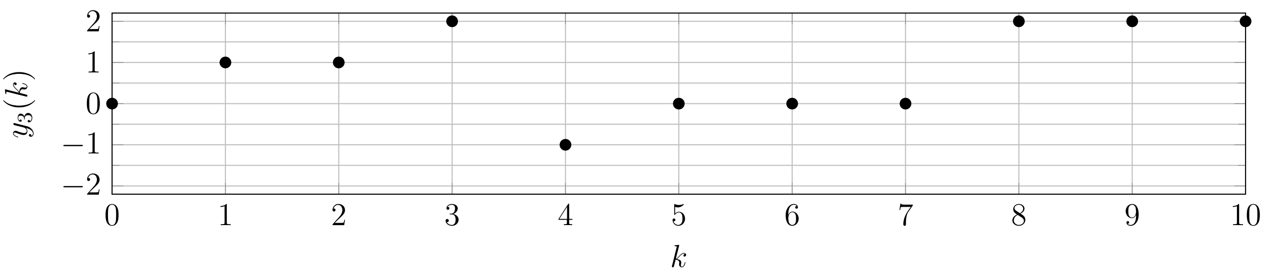
d) The sum of both transfer functions is 1. Therefore the output of the filters connected in parallel equals exactly the input signal $u(k)$.



2

$\sum 14$

Task 9: Block Diagram

a) Sketch the direct form I block diagram (first $B(z)$, then $\frac{1}{A(z)}$) of Eq. 2.

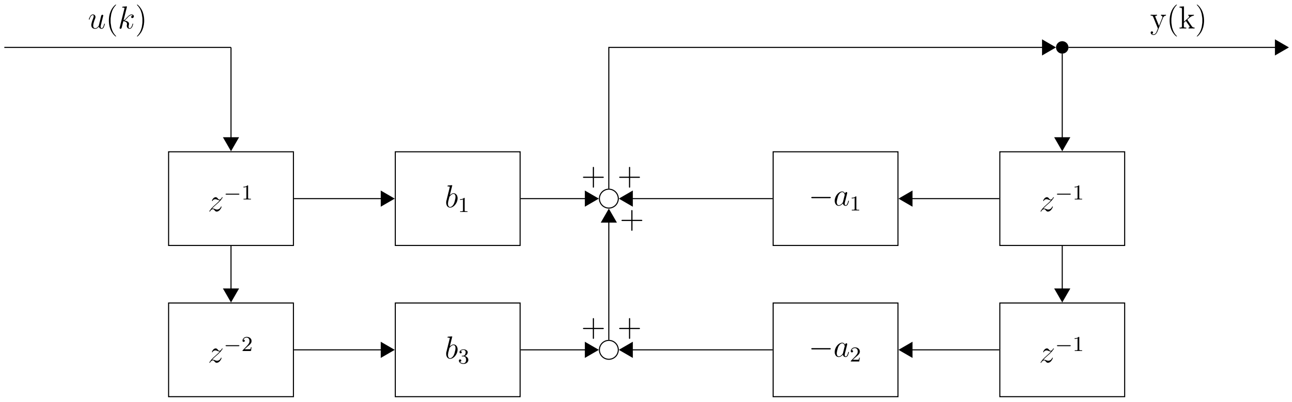


Fig. 18: Direct form I block diagram.

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b) Sketch the non-redundant direct form II block diagram (first $\frac{1}{A(z)}$, then $B(z)$) of Eq. 2.

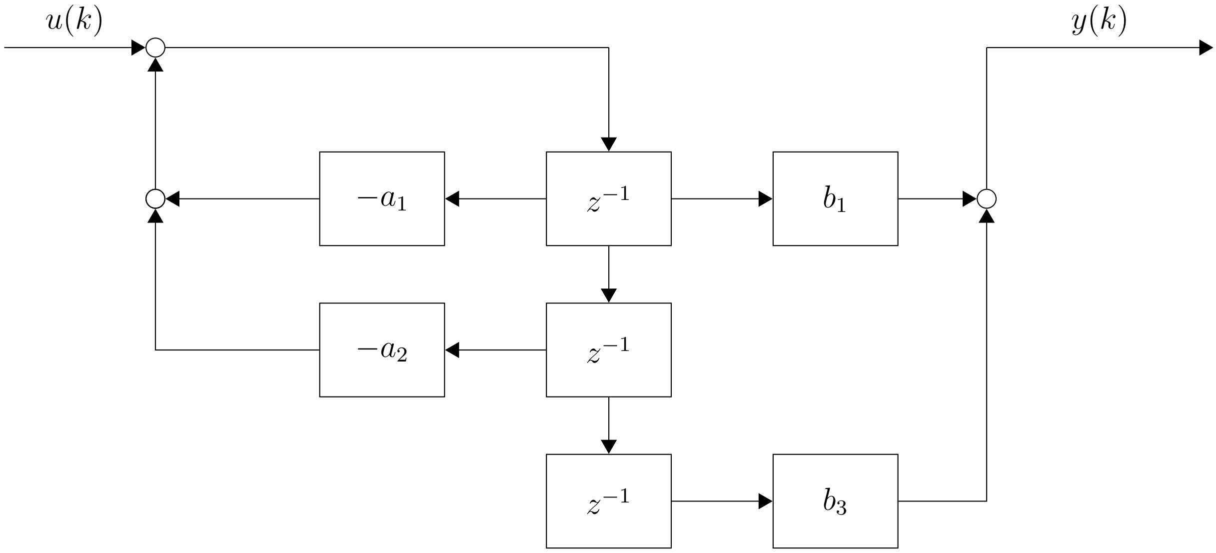

Fig. 19: Non-redundant direct form II block diagram.

5

c) Determine the transfer function $G(z)$ of Eq. 2.

$$G_(z) = \frac{b_1 z^{-1} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

1

d) The following table lists several possible coefficient combinations for Eq. 2. Assign the correct step response **A** to **H** (see Fig. 12 next page) to each coefficient combination. Explain your choices shortly.

a_1	a_2	b_1	b_3	Step Response
-0.5	-0.1	0	0.4	**F**
-0.5	0.5	0	0.4	**D**
-0.3	0.4	3.5	2	**H**
0	-0.6	0.4	0	**A**

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F: The only step response, that is delayed by three time steps in combination with a gain of 1 0.5

D: The only oscillating step response with correct delay (3 time steps) 0.5

H: Only step response with correct gain 0.5

A: Only system with $a_1 = 0$ and $a_2 \neq 0$ leading to pairs of equal values, e.g. $h(2) = h(3)$. 0.5

e) Assume to approximate the IIR system from Eq. 2 by an FIR system. The sampling frequency is kept constant. If step response **G** would be the step response $H_{IIR}(z)$ of the IIR system, what FIR system order do you need, if the absolute maximum error between the FIR step response $H_{FIR}(z)$ and H_{IIR} should be less than 0.05 at any time. Explain the choice of the FIR system order shortly.

The order should be at least 5. After 5 discrete time steps the final value of the step response is almost reached. The FIR approximation of order n is exact until the n-th time step, after that the final value of the FIR system is exactly reached and the difference between $H_{FIR}(z)$ and $H_{IIR}(z)$ will stay constant. 1 1

$\sum 31$