Sensorics Exam

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1st of March 2016

Name:	
Mat.-No.:	
Grade:	

Task:	T1	T2	T3	T4	T5	T6	T7	T8	T9	Sum
Scores:	19	16	13	8	6	7	10	14	27	120
Accomplished:										

Duration of examination: 2 hours

You are allowed to use a calculator and four pages of notes

Task 1: Comprehension Questions

Mark the correct answers clearly.
Every question has one or two correct answers!
For every correctly marked answer you will get one point. If there is one correct answer marked and one incorrect answer marked, you will get no point for that subtask.

a) Which statements are true for strain gauges?
- ☐ Environmental influences can easily be compensated through a clever arrangement of the strain gauges.
- ☐ Strain gauges utilize the resistance change caused by a change in length and a change of the cross section area of a conductor for the measurement.
- ☐ Strain gauges utilize the change of its capacity for the measurement.

b) How can speed be measured?
- ☐ Via the measurement of the acceleration and a subsequent differentiation.
- ☐ With the help of the Doppler-effect.
- ☐ Via the measurement of the rotational speed and a subsequent division by the radius.

c) How large should the internal resistance of a voltage meter be?
- ☐ Very large, in the ideal case infinite large.
- ☐ Very small, in the ideal case zero.
- ☐ It depends on the electrical circuit, what internal resistance is ideal.

d) Which statements are true for the measurement of temperatures?
- ☐ Thermocouples are more accurate than resistance thermometer.
- ☐ Thermocouples are suitable for point-wise measurements.
- ☐ Thermocouples have a smaller time constant than resistance thermometer.

e) The discrete Fourier transform is periodic ...
- ☐ ... only in time.
- ☐ ... only in frequency.
- ☐ ... in time and frequency.

f) A temporal sequence of N measurements that is transformed with the DFT results in a number of ...
- ☐ ... N discrete frequencies.
- ☐ ... $N/2$ discrete frequencies.
- ☐ ... 2^N discrete frequencies.

g) An increase in the number of measurements has the following effect on the frequency resolution:
- ☐ It becomes finer.
- ☐ It becomes coarser.
- ☐ It remains unaffected.

h) In order to make a meaningful statement about the contained frequencies in a non-stationary signal, ...
- ☐ the signal can be examined with a short-time DFT.
- ☐ the signal can not be examined with a short-time DFT.
- ☐ the signal can be examined with a wavelet transform.

i) The median filter ...
- ☐ ...is commonly used to eliminate outliers.
- ☐ ...is a nonlinear filter.
- ☐ ...calculates the arithmetic average.

j) Assess the following statements regarding parametric methods.
- ☐ Only a few number of parameters n are estimated using a relatively large number of data samples N ($n << N$).
- ☐ An FIR system is a parametric approach.
- ☐ Their parameters have no direct physical meaning or interpretation.

k) According to the Shannon theorem ...
- ☐ ...nonlinear filters can be approximated by linear filters by using feedback-control.
- ☐ ...the highest significant component of the signal f_{max} must not be higher than the Nyquist frequency $f_n = \frac{f_0}{2}$.
- ☐ ...the quantization error is reduced if the sampling time is increased.

l) Assess the following statements regarding measurement techniques.
- ☐ Voltage meters should have an internal resistance as small as possible.
- ☐ The ideal operational amplifier has an infinite input resistance.
- ☐ The ideal operational amplifier has an infinite output resistance.

m) The direct measurement of speed can be done by ...
- ☐ ...using the Doppler effect of acoustic or electromagnetic waves.
- ☐ ...using the Combination of 2 cameras and correlation analysis.
- ☐ ...using the piezoelectric effect.

n) If you derive a noisy signal with respect to time t ...
- ☐ ...the gain of the transfer function changes.
- ☐ ...the noise is amplified.
- ☐ ...the system becomes unstable.

<u>Task 2:</u> Displacement Measurement

Figure 1 shows two methods to measure displacements with the help of inductances. The mass m is attached to one spring with spring stiffness c and one damper with viscous damping d. In each of the two cases, an iron plunger is attached to the mass m. The variable s_0 denotes the length of the coil that is filled with air in the equilibrium state. The displacement x is measured relatively to the equilibrium. The following equation to calculate the inductance L of one coil should be used for this task:

$$L = \frac{N^2 \mu_0 A}{s} \ , \tag{1}$$

with the length s inside the coil *not* filled with iron, the vacuum permeability μ_0, the number of coil windings N and the area A where the flux lines pass through. All shown coils have identical properties (equal N, A and μ_0).

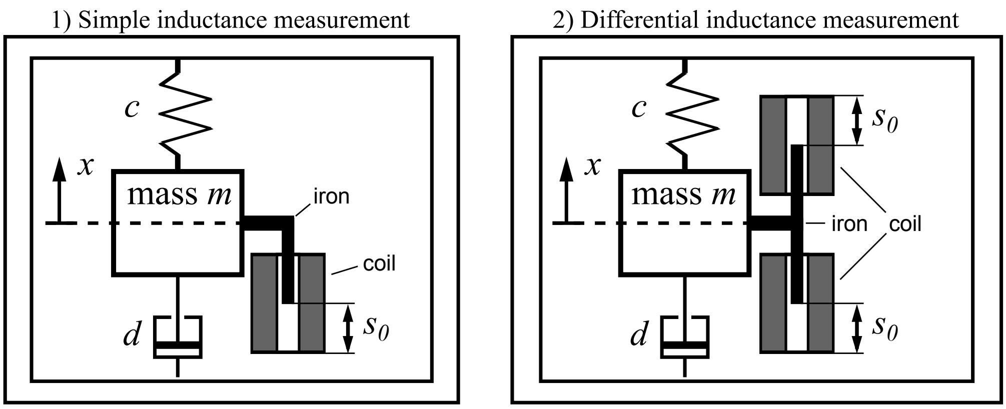


Fig. 1: Displacement measurement with simple inductance and differential inductance.

a) Determine the relationship between the inductance $L(x)$ and the displacement x for the simple inductance measurement (case 1).

b) Determine the equation for the calculation the upper inductance $L_u(x)$ and the lower inductance $L_l(x)$ depending on the displacement x for the differential inductance measurement (case 2).

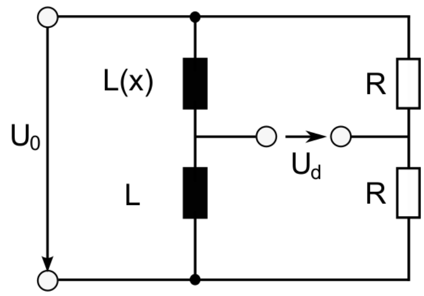


Fig. 2: Bridge circuit to measure the inductance.

c) Derive the equation that describes the bridge voltage U_d depending on the inductance $L(x)$ for the bridge circuit shown in Fig. 2. L is a constant inductance, U_0 is the input voltage and R are resistances.

d) Substitute $L(x)$ with the results from subtask a), such that one equation shows the relationship between the bridge voltage U_d depending on x for the simple inductance measurement.

e) Now assume, that the fixed inductance from Fig. 2 is replaced by $L(-x)$ according to the differential inductance measurement. Derive the equation that describes the bridge voltage U_d depending on the **displacement** x in case of the differential inductance measurement. Sketch qualitatively the curve of the bridge voltage U_d over the displacement x for the differential inductance measurement.

f) Compare the relationship between the bridge voltage U_d and the displacement x for both cases, the simple and the differential inductance measurement. What relationship would you prefer and why?

Task 3: Quantization

Figure 3 shows an analogue signal (gray line), that should be quantized within this task with a 2-bit A/D converter. No sampling is required throughout the whole task. Therefore the time maintains continuous!

a) How many quantization levels exist for the 2-bit A/D converter?

b) In order to determine the intervals for the quantization, use the signal's minimum and maximum values as outer limits. Determine the intermediate interval boundaries between all quantization levels.

c) Sketch the quantized signal into Fig. 3, if you **round off** all values to the corresponding quantization interval floor.

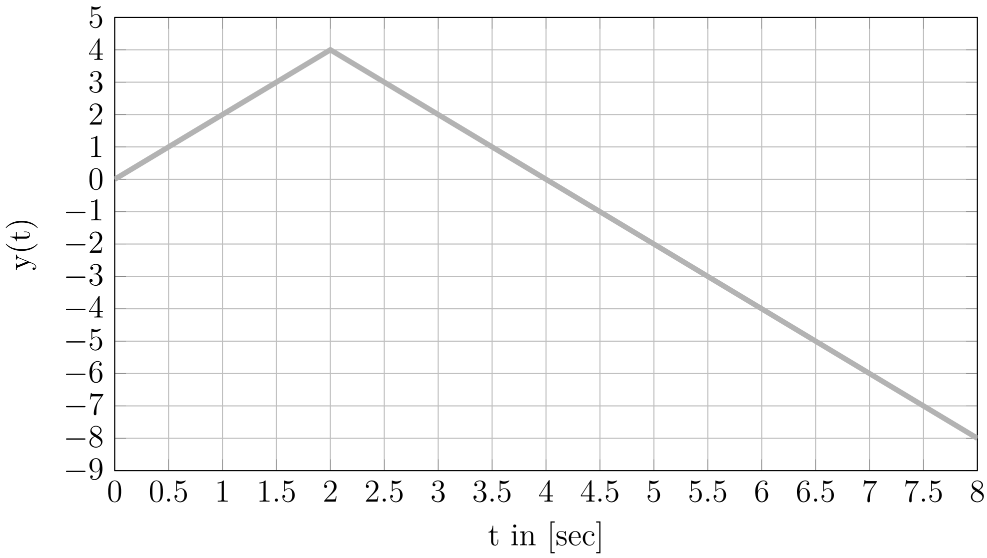


Fig. 3: Quantization with 2-bit A/D converter

d) Sketch the quantized signal into Fig. 4, if you **round up** all values to the corresponding quantization interval ceiling.

e) Now quantize the signal shown in Fig. 5 using the same quantization levels as determined in subtask b). Round up all values to the corresponding quantization interval ceiling. Sketch the solution into Fig. 5.

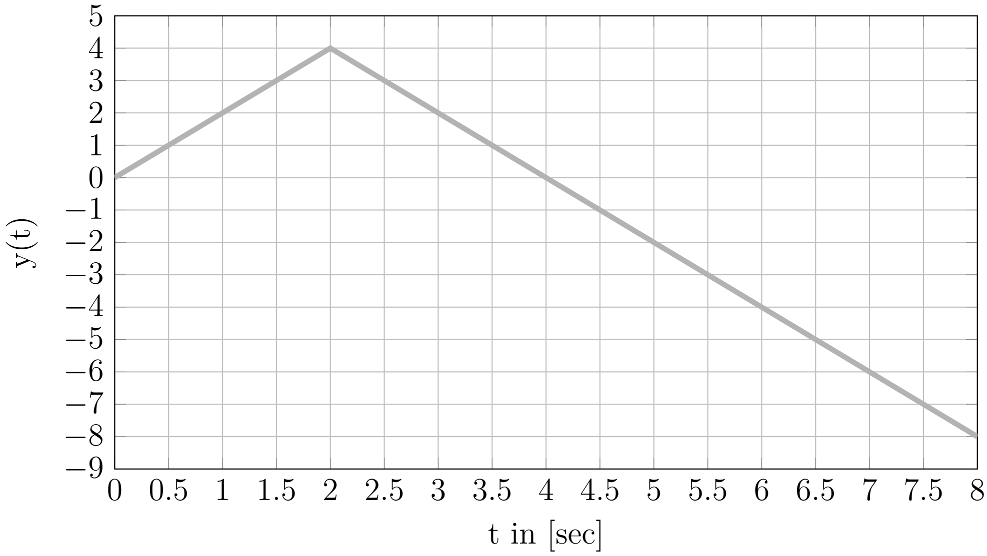


Fig. 4: Quantization with 2-bit A/D converter

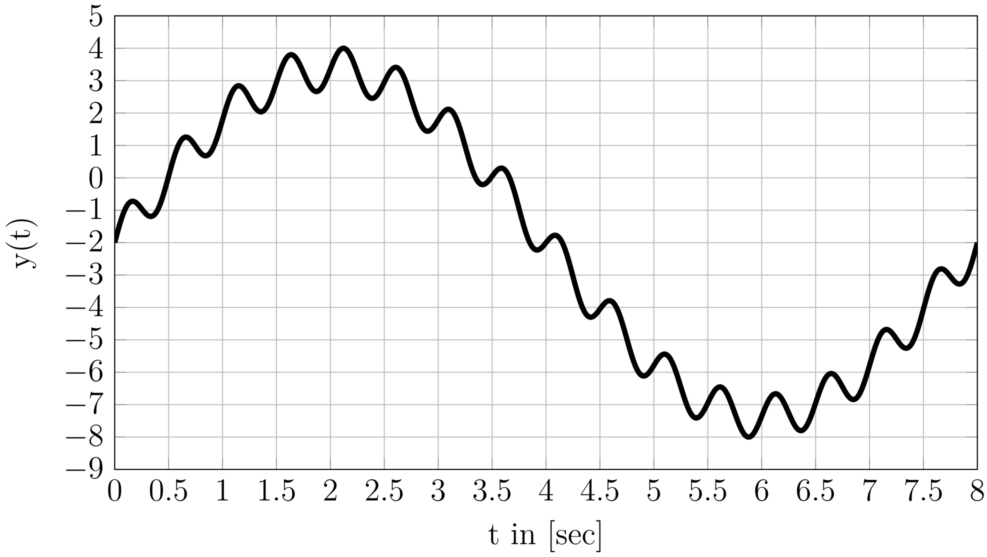


Fig. 5: Quantization with 2-bit A/D converter

Task 4: Probability Density

In Fig. 6 four different PDFs are shown.

a) What type of probability density distribution (PDF) are the signals in Fig. 7 presumably based upon?

b) Which probability density function from Fig. 6 possesses the biggest area from $u = -\infty$ to $u = \infty$?

c) Assign the probability density functions from Fig. 6 to a signal from Fig. 7 $(u_A(k), u_B(k), \ldots, u_H(k))$. Consider the scaling of the $u(k)$-axis in 7!

	PDF 1	PDF 2	PDF 3	PDF 4
Signal				

d) Describe your choices from c) in a short sentence:

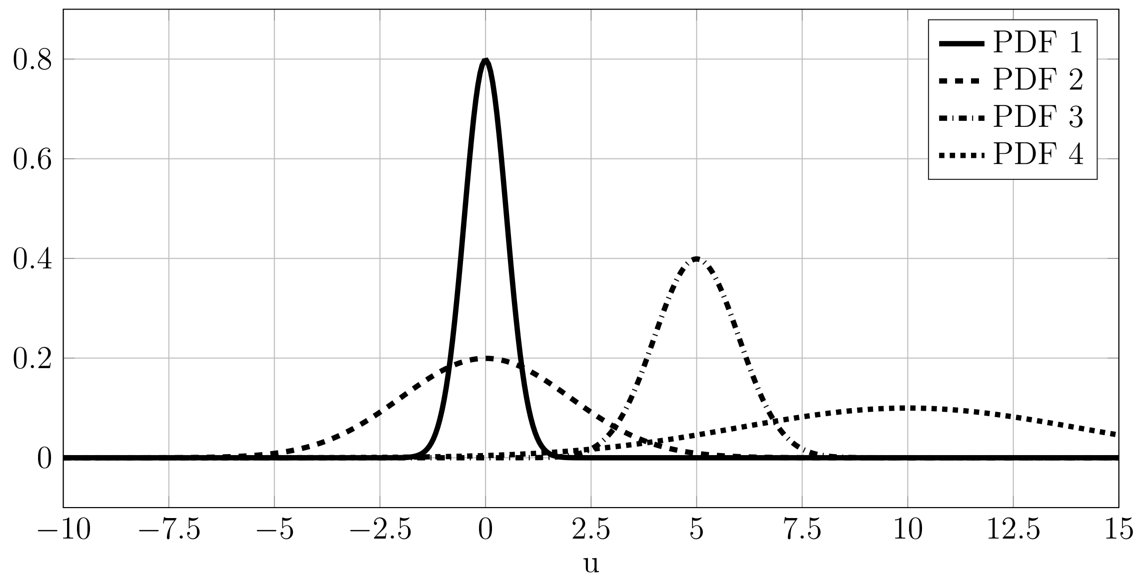


Fig. 6: Probability density functions

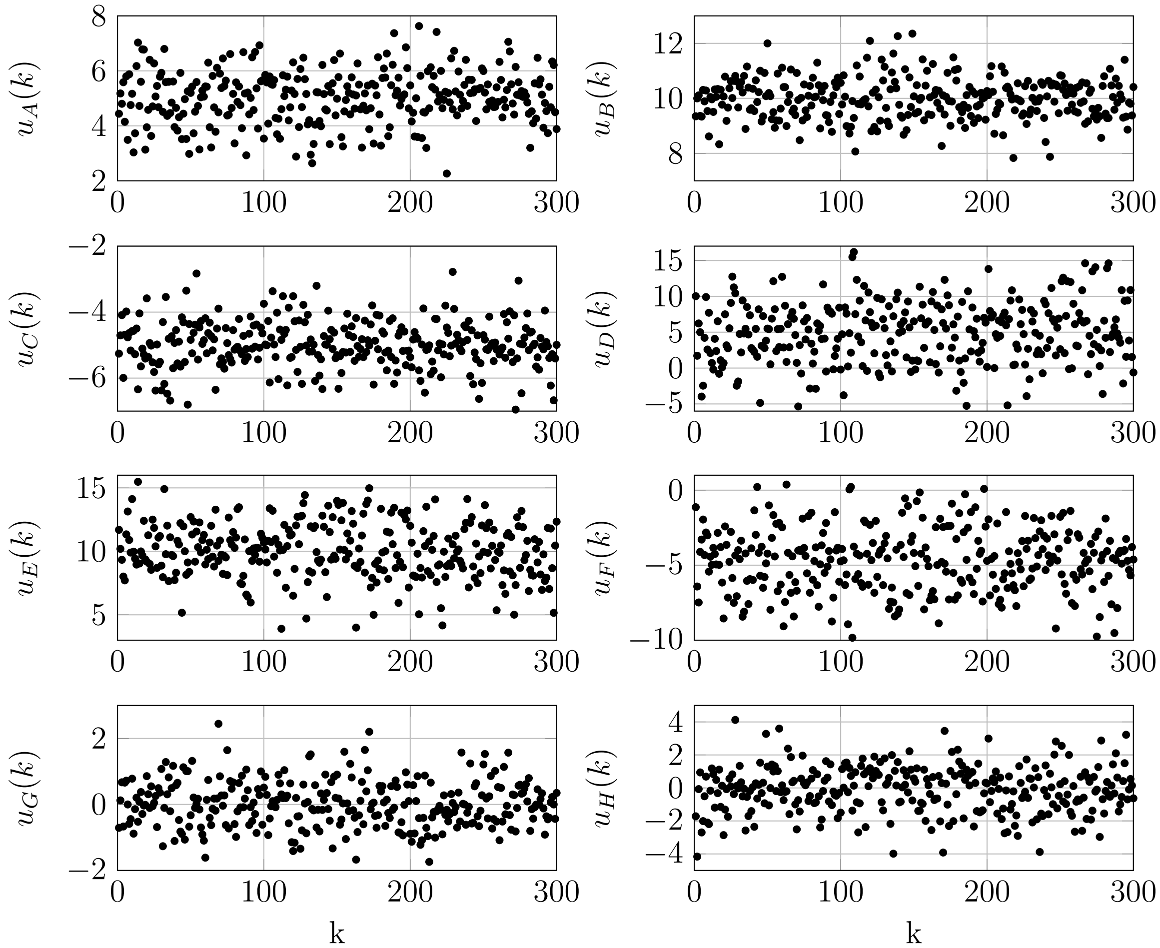


Fig. 7: Signals from PDFs

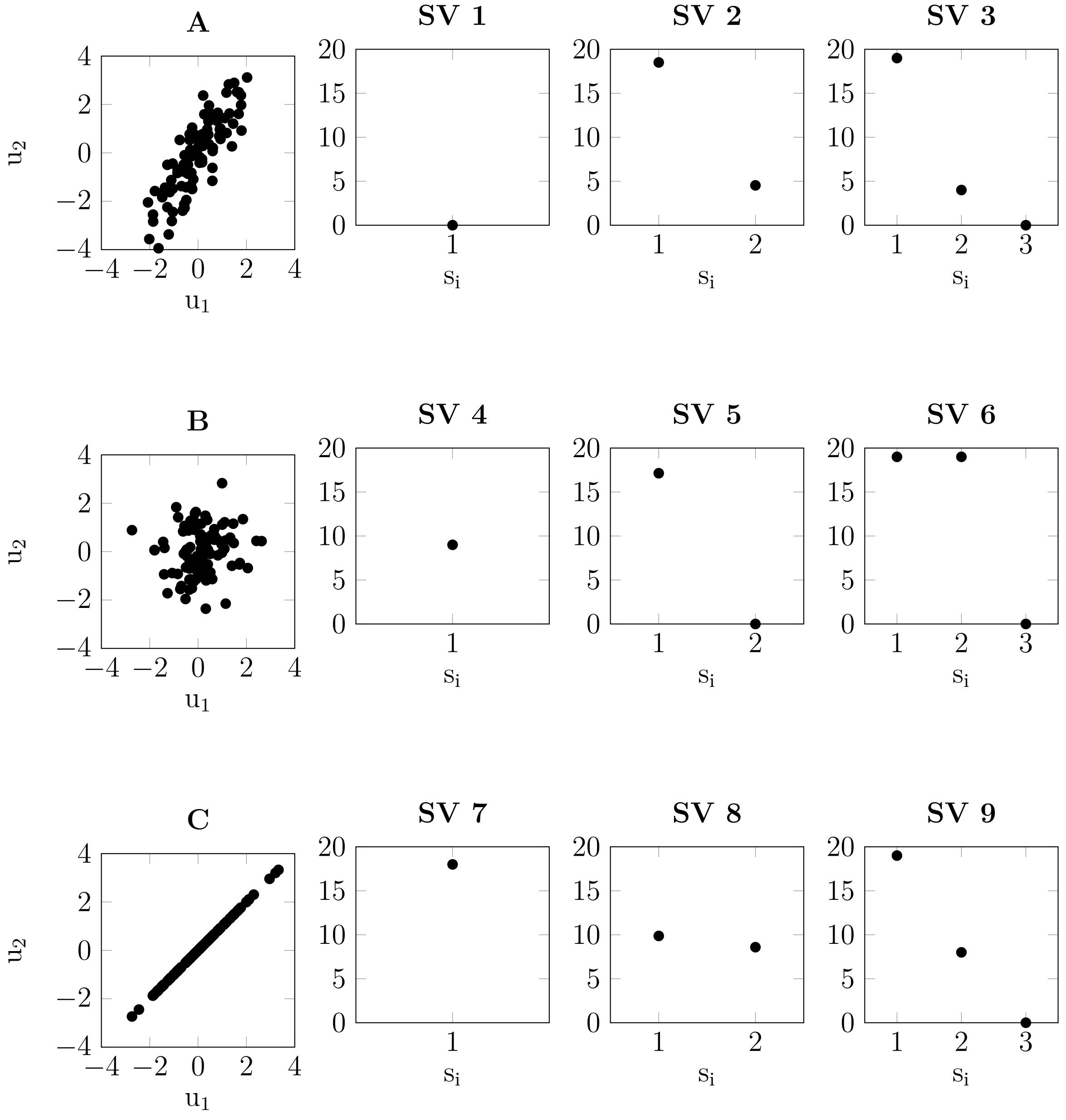
Task 5: PCA

From different inputs u_1 and u_2, the multivariate distributions A, B and C arise.

a) Assign the correct singular values to each data distribution, that result from a principal component analysis (PCA). For each data distribution only one singular value combination from SV1 up to SV9 is correct.

b) Give reasons for your choices (short explanation).

Data distribution	A	B	C
Singular value			



Task 6: Downsampling

The time-discrete signal $u(k)$ with sampling time T_0 is shown in Fig. 8. The signal $u(k)$ should be compressed by downsampling.

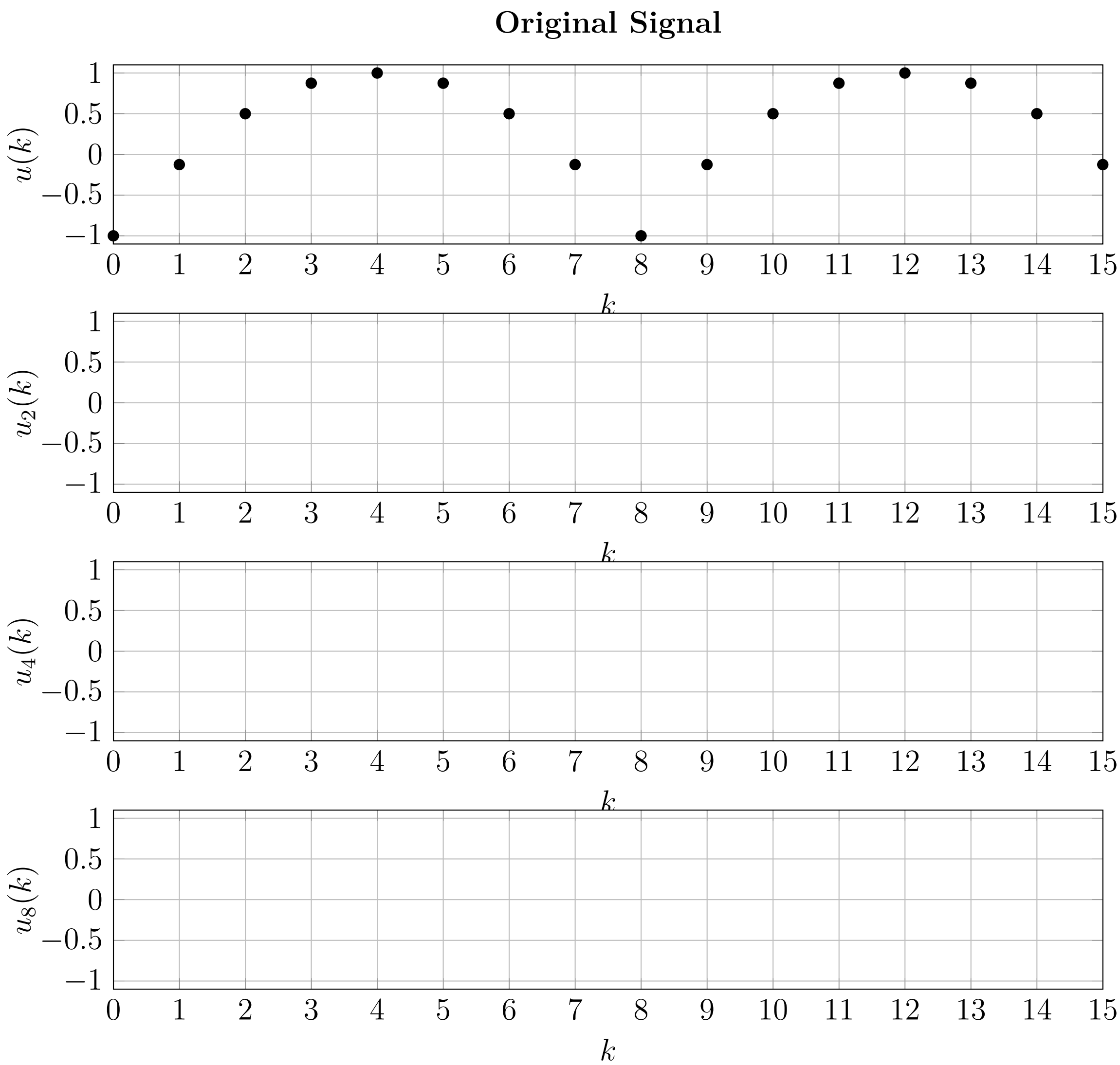


Fig. 8: Zeitdiskretes Signal

a) Compress the signal $u(k)$ by factors of 2, 4 and 8. In what way changes the sampling time T_0? Sketch the results ($u_2(k)$,$u_4(k)$ and $u_8(k)$) into Fig. 8. Hint: The first value is always retained. Think about the meaning of k for different sampling times T_0.

b) For which of the compressions do problems arise? How is the appearing phenomenon called?

c) How can the problem from subtask b) be prevented?

d) How would the signal look like, if the solution from subtask c) would be applied?

Task 7: Dynamic System

The following time-discrete system is given:

$$G(z) = \frac{Y(z)}{U(z)} = \frac{z}{z - 0.5} \tag{2}$$

a) How is a discrete impulse defined in the z-domain (equation)?

b) How does the signal $Y(z)$ of the given system $G(z)$ look like in the z-domain, if an impulse is applied?

c) Determine the final value of the impulse response for $G(z)$ with the help of the final value theorem.

d) Transform the signal $Y(k)$ from b) to the discrete-time domain and calculate $y(k)$ for $k = \{0, 1, \ldots, 5\}$ ($y(k) = 0$ für $k < 0$).

e) Specify the transfer function of a common FIR filter of order 4 in the z-domain?

f) Specify coefficients of the FIR filter from subtask e), if it should be used to approximate the transfer function (2) from Eq. 2.

Task 8: Filter

In Fig. 9 two different filters are shown.

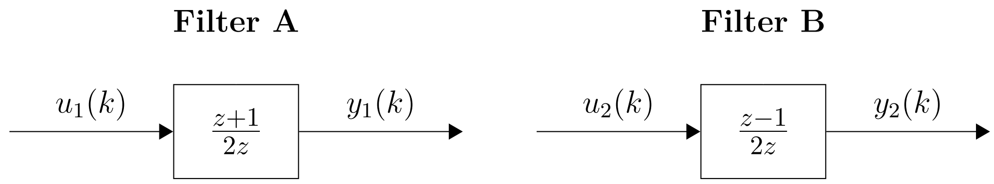
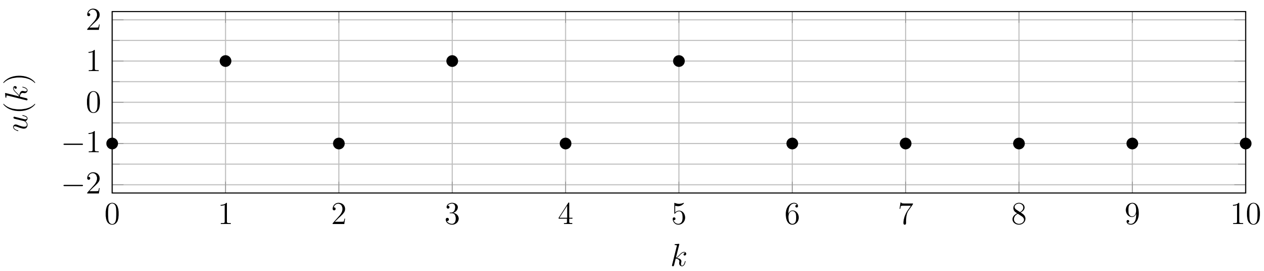


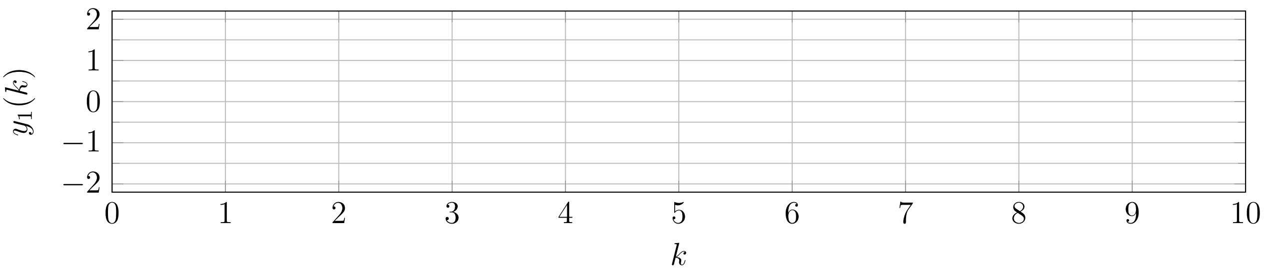
Fig. 9: Filter transfer functions

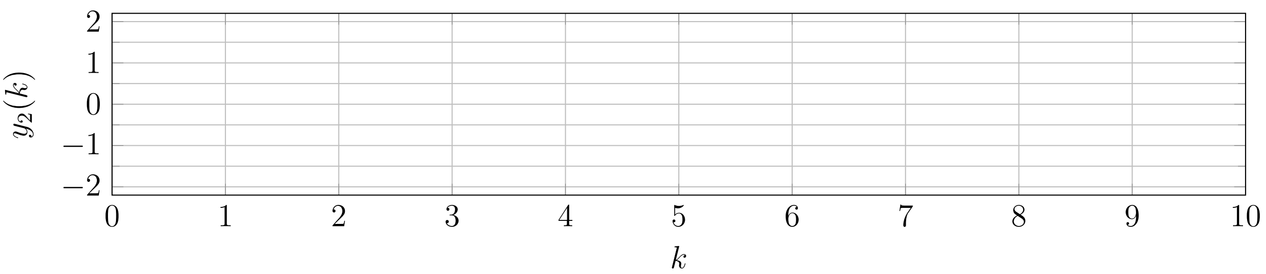
a) Calculate the amplitude response for each filter.

b) Assign the correct filter type to each of the two filters. Choose from the following types: High-pas, low-pass, band-pass or band-stop. Hint: Set $\omega = 0$ and $\omega = \frac{\pi}{T_0}$ in the solution from subtask 1).

c) The signal $u(k)$ shown below is applied to both filters ($u(k) = u_1(k) = u_2(k)$). Sketch the filter's responses $y_1(k)$ and $y_2(k)$ into the corresponding diagrams ($u(k) = 0$ for $k < 0$).







d) Both filters are now connected in parallel as shown in Fig. 10. The input signal $u(k)$ from subtask 3) is applied to this system. Sketch the system response $y_3(k)$ into the corresponding diagram. Give reasons for the resulting behavior based on the overall transfer function.

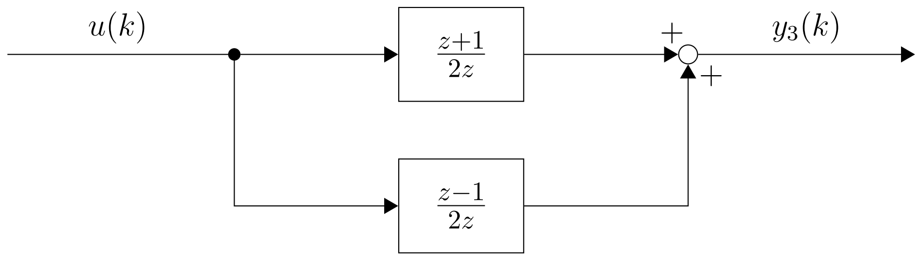
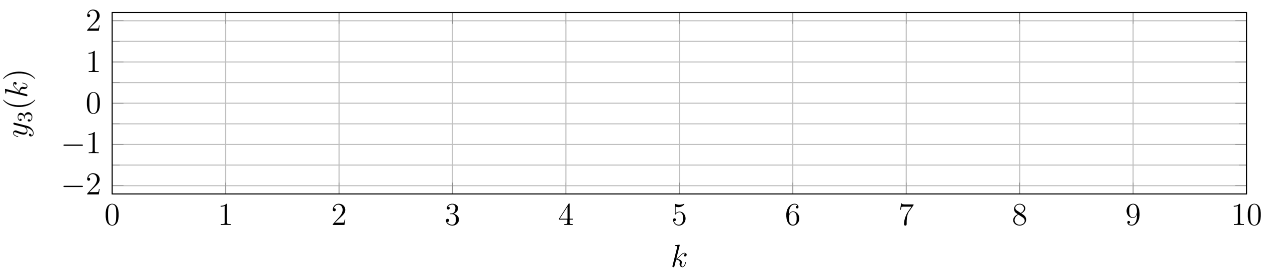


Fig. 10: Filters A and B connected in parallel



Task 9: Block Diagram

The following block diagram is given:

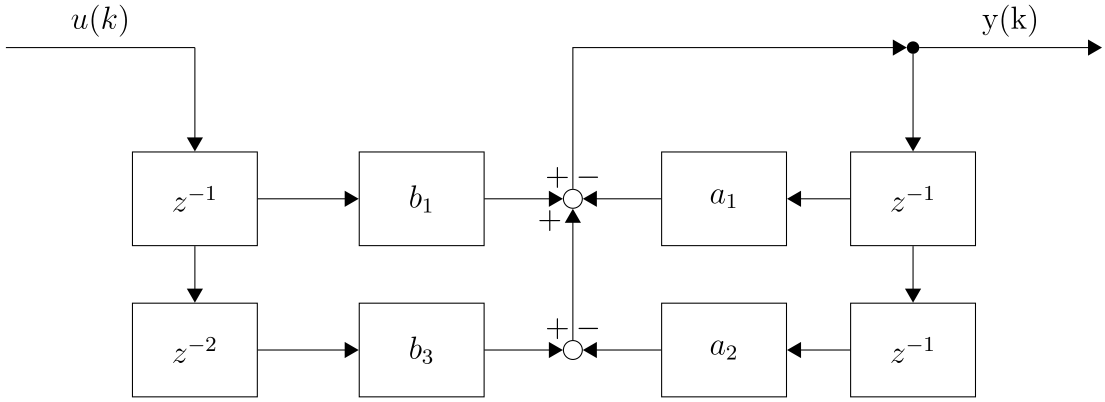


Fig. 11: Direct form I block diagram.

a) Draw an equivalent block diagram, where the number of delay-blocks is reduced. This block diagram is known as the direct form II block diagram (first $\frac{1}{A(z)}$, then $B(z)$).

b) Derive the difference equation for the block diagram given in Fig. 11.

c) Determine the transfer function $G(z)$ for the block diagram given in Fig. 11.

d) The following table lists several possible coefficient combinations for the block diagram given in Fig. 11. Assign the correct step response **A** to **H** (see Fig. 12 next page) to each coefficient combination. Explain your choices shortly. Hint: It is not necessary to calculate the poles for any of the given coefficient combinations in order to solve this subtask.

a_1	a_2	b_1	b_3	Step Response
-0.2	-0.2	-0.2	-0.4	
0.3	0.3	0	0.4	
0.5	0.5	1.2	-0.2	
0.5	-0.5	-0.5	0.5	

e) Assume to approximate the IIR system that corresponds to the block diagram shown in Fig. 11 by an FIR system. If step response in Fig. 12 **C** would be the step response $H_{IIR}(z)$ of the IIR system, what FIR system order do you need, if the absolute maximum error between the FIR step response $H_{FIR}(z)$ and H_{IIR} should be less than 0.05 at any time. Explain the choice of the FIR system order shortly.

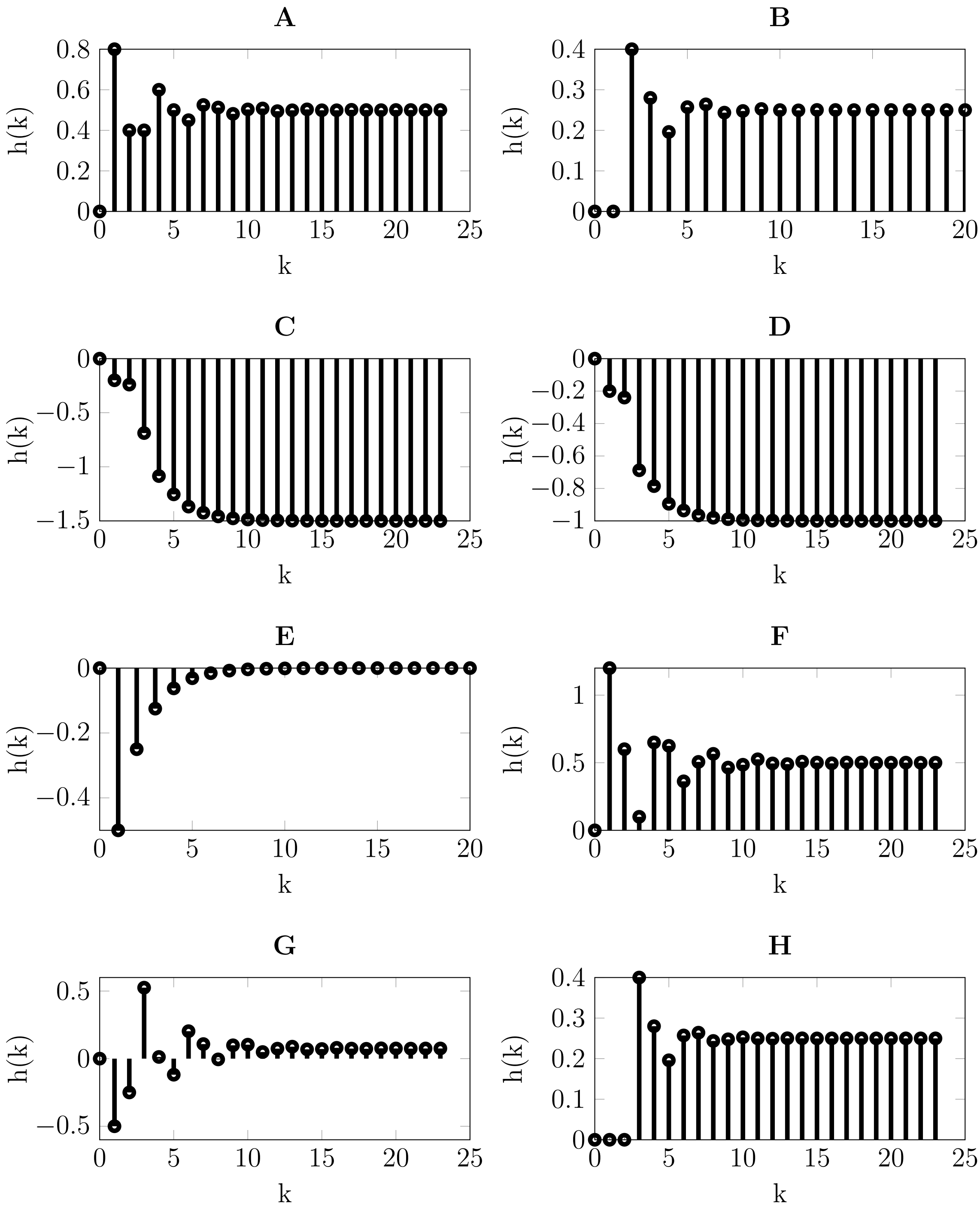


Fig. 12: Step responses **A** to **H**

Solutions:

Task 1: Comprehension Questions

a) Which statements are true for strain gauges?

- [x] Environmental influences can easily be compensated through a clever arrangement of the strain gauges.
- [x] Strain gauges utilize the resistance change caused by a change in length and a change of the cross section area of a conductor for the measurement.
- [ ] Strain gauges utilize the change of its capacity for the measurement.

b) How can speed be measured?

- [ ] Via the measurement of the acceleration and a subsequent differentiation.
- [x] With the help of the Doppler-effect.
- [ ] Via the measurement of the rotational speed and a subsequent division by the radius.

c) How large should the internal resistance of a voltage meter be?

- [x] Very large, in the ideal case infinite large.
- [ ] Very small, in the ideal case zero.
- [ ] It depends on the electrical circuit, what internal resistance is ideal.

d) Which statements are true for the measurement of temperatures?

- [ ] Thermocouples are more accurate than resistance thermometer.
- [x] Thermocouples are suitable for point-wise measurements.
- [x] Thermocouples have a smaller time constant than resistance thermometer.

e) The discrete Fourier transform is periodic ...

- [ ] ... only in time.
- [ ] ... only in frequency.
- [x] ... in time and frequency.

f) A temporal sequence of N measurements that is transformed with the DFT results in a number of ...

- [x] ... N discrete frequencies.
- [ ] ... $N/2$ discrete frequencies.
- [ ] ... 2^N discrete frequencies.

g) An increase in the number of measurements has the following effect on the frequency resolution:

- [x] It becomes finer.
- [ ] It becomes coarser.
- [ ] It remains unaffected.

h) In order to make a meaningful statement about the contained frequencies in a non-stationary signal, ...

- ■ the signal can be examined with a short-time DFT.
- □ the signal can not be examined with a short-time DFT.
- ■ the signal can be examined with a wavelet transform.

i) The median filter . . .

- ■ . . . is commonly used to eliminate outliers.
- ■ . . . is a nonlinear filter.
- □ . . . calculates the arithmetic average.

j) Assess the following statements regarding parametric methods.

- ■ Only a few number of parameters n are estimated using a relatively large number of data samples N $(n << N)$.
- □ An FIR system is a parametric approach.
- □ Their parameters have no direct physical meaning or interpretation.

k) According to the Shannon theorem . . .

- □ . . . nonlinear filters can be approximated by linear filters by using feedback-control.
- ■ . . . the highest significant component of the signal f_{max} must not be higher than the Nyquist frequency $f_n = \frac{f_0}{2}$.
- □ . . . the quantization error is reduced if the sampling time is increased.

l) Assess the following statements regarding measurement techniques.

- □ Voltage meters should have an internal resistance as small as possible.
- ■ The ideal operational amplifier has an infinite input resistance.
- □ The ideal operational amplifier has an infinite output resistance.

m) The direct measurement of speed can be done by . . .

- ■ . . . using the Doppler effect of acoustic or electromagnetic waves.
- ■ . . . using the Combination of 2 cameras and correlation analysis.
- □ . . . using the piezoelectric effect.

n) If you derive a noisy signal with respect to time t . . .

- □ . . . the gain of the transfer function changes.
- ■ . . . the noise is amplified.
- □ . . . the system becomes unstable.

$\sum$ 19

Task 2: Displacement Measurement

a) Determine the relationship between the inductance $L(x)$ and the displacement x for the simple inductance measurement (case 1).

$$L(x) = \frac{N^2 \mu_0 A}{x + s_0}$$

1

b) Determine the equation for the calculation the upper inductance $L_u(x)$ and the lower inductance $L_l(x)$ depending on the displacement x for the differential inductance measurement (case 2).

$$L_l(x) = \frac{N^2 \mu_0 A}{x + s_0}$$
$$L_u(x) = \frac{N^2 \mu_0 A}{s_0 - x}$$

1

c) Derive the equation that describes the bridge voltage U_d depending on the inductance $L(x)$ for the bridge circuit shown in Fig. 2. L is a constant inductance, U_0 is the input voltage and R are resistances. Hint: The complex impedance of an inductance is $Z_L = j\omega L$.

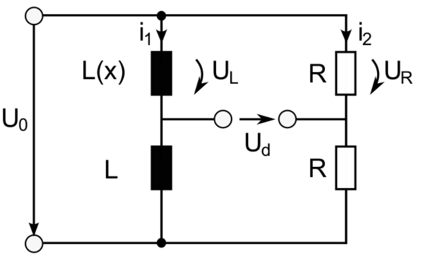


Fig. 13: Bridge circuit to measure the inductance.

$$\begin{aligned} U_d &= \frac{R}{2R} U_0 - \frac{Z_{L(x)}}{Z_{L(x)} + Z_L} U_0 \\ &= \frac{1}{2} U_0 - \frac{j\omega L(x)}{j\omega L(x) + j\omega L} U_0 \\ &= \frac{1}{2} U_0 - \frac{L(x)}{L(x) + L} U_0 \end{aligned}$$

3

d) Substitute $L(x)$ with the results from subtask a).

$$
\begin{aligned}
U_d &= \frac{1}{2}U_0 - \frac{\frac{N^2\mu_0 A}{x+s_0}}{\frac{N^2\mu_0 A}{x+s_0} + L}U_0 \\
&= \frac{1}{2}U_0 - \frac{N^2\mu_0 A}{N^2\mu_0 A + L(x+s_0)}U_0
\end{aligned}
$$

$\boxed{2}$

e) Derive the equation that describes the bridge voltage U_d depending on the **displacement** x in case of the differential inductance measurement. Sketch qualitatively the curve of the bridge voltage U_d over the displacement x for the differential inductance measurement.

We just have to replace L by $L(-x)$:

$$
\begin{aligned}
U_d &= \frac{1}{2}U_0 - \frac{N^2\mu_0 A}{N^2\mu_0 A + L_u(x)(x+s_0)}U_0 \\
&= \frac{1}{2}U_0 - \frac{N^2\mu_0 A}{N^2\mu_0 A + \frac{N^2\mu_0 A}{s_0-x}(x+s_0)}U_0 \\
&= \frac{1}{2}U_0 - \frac{N^2\mu_0 A(s_0-x)}{N^2\mu_0 A(s_0-x) + N^2\mu_0 A(x+s_0)}U_0 \\
&= \frac{1}{2}U_0 - \frac{N^2\mu_0 A(s_0-x)}{2s_0 N^2\mu_0 A}U_0 \\
&= \frac{1}{2}U_0 - \frac{s_0-x}{2s_0}U_0 \\
&= \frac{1}{2}U_0 - \frac{1}{2}U_0 + \frac{x}{2s_0}U_0 = \frac{x}{2s_0}U_0
\end{aligned}
$$

$\boxed{5}$

As a result of the differential inductance measurement, the relationship between the bridge voltage U_d and the displacement x is linear. Therefore any sketch is correct, if the linear dependency is recognizable, i.e. a straight line through the origin with positive slope. $\boxed{2}$

f) Compare the relationship between the bridge voltage U_d and the displacement x for both cases, the simple and the differential inductance measurement. What relationship would you prefer and why?

As can be seen by the results of the upper subtasks, the relationship between the bridge voltage U_d and the displacement x is nonlinear in case of the simple inductance and linear for the differential inductance measurement. The linear relationship is preferable, since the sensitivity of the system is the same for all displacements x. Furthermore the behavior is easy to understand and to handle. $\boxed{2}$

$\boxed{\sum 16}$

Task 3: Quantization

a) How many quantization levels exist for the 2-bit A/D converter?

$2^n = 2^2 = 4$ 1

b) In order to determine the intervals for the quantization, use the signal's minimum and maximum values as outer limits. Determine the intermediate boundaries between all quantization levels.

	Minimum continuous value	Maximum continuous value
Quantization level 0	-8	-5
Quantization level 1	-5	-2
Quantization level 2	-2	1
Quantization level 3	1	4

5

c) Sketch the quantized signal, if you **round off** all values to the corresponding quantization interval floor. 2

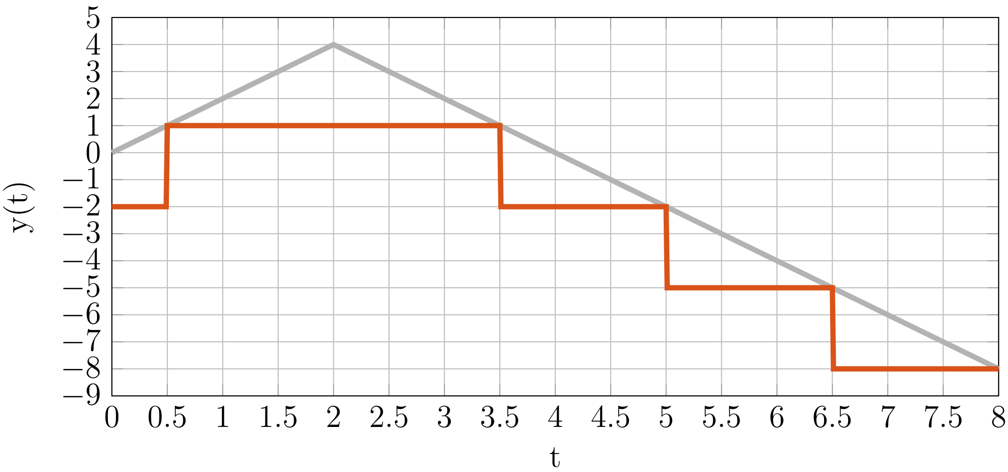


Fig. 14: Quantization with 2-bit A/D converter

d) Sketch the quantized signal, if you **round up** all values to the corresponding quantization interval ceiling. 2

e) See Fig. 16. 3

$\sum 13$

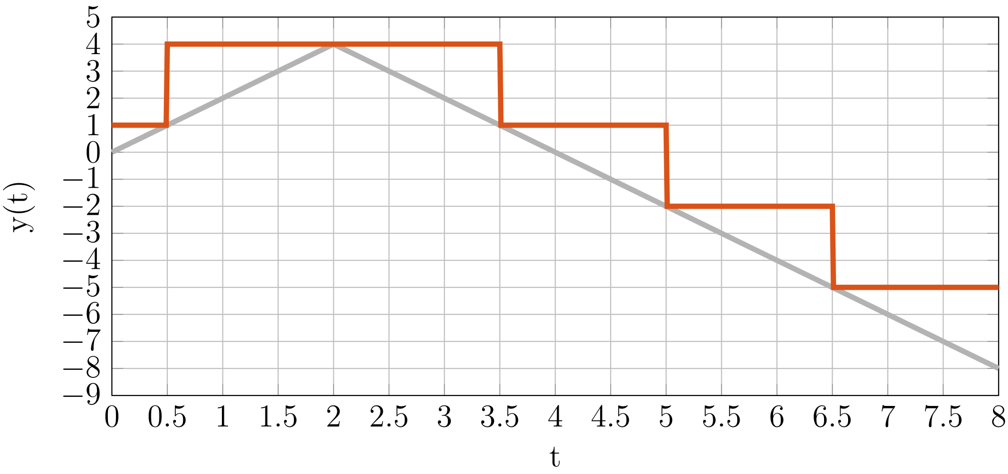


Fig. 15: Quantization with 2-bit A/D converter

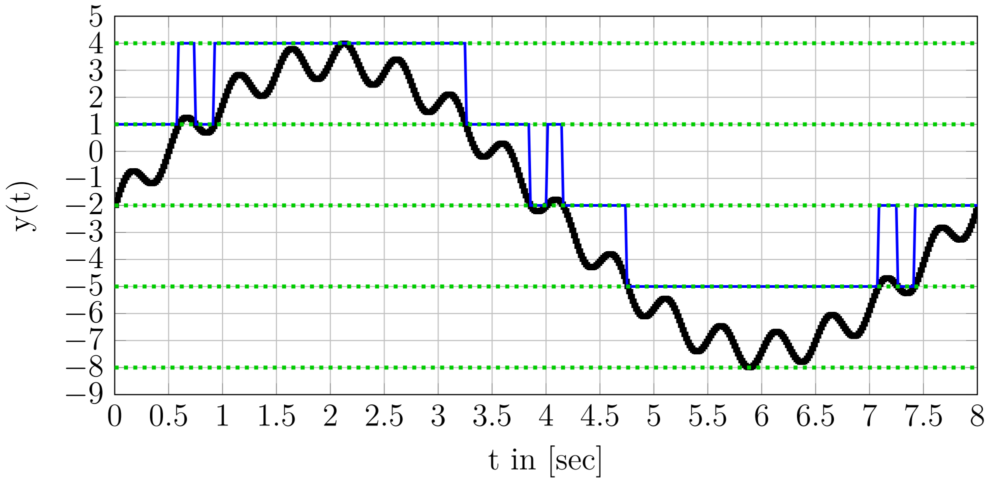


Fig. 16: Quantization with 2-bit A/D converter

Task 4: Wahrscheinlichkeitsdichte (8 Punkte)

a) Normal distribution. $\boxed{1}$

b) The integral of all PDFs is

$$\int_{-\infty}^{\infty} \mathrm{PDF}(u)\, du = 1 \tag{3}$$

$\boxed{1}$

c)

	PDF 1	PDF 2	PDF 3	PDF 4
Signal	$u_G(k)$	$u_H(k)$	$u_A(k)$	$u_E(k)$

$\boxed{4}$

d) Example for 2 different PDFs:

PDF 1 $\mu = 0 \Rightarrow u_G$ / u_H. At $u \approx 2$ the PDF is close to zero. Values for $u > 2$ or $u < 2$ are very unlikely. Only u_G remains.

PDF 2 $\mu = 0 \Rightarrow u_G$ / u_H. At $u \approx 5$ the PDF is close to zero. In the value range $2 < u < 5$ the probability is obviously above zero. The data is for u_H denser than for u_G. $\Rightarrow u_H$.

$\boxed{2}$

$\boxed{\sum 8}$

Task 5: PCA

a)

Data distribution	A	B	C
Singular values	SV 2	SV 8	SV 5

3

b) The shown data distributions are described by two inputs. It follows, that only two singular values exist for each data distribution, such that SV 1, SV 3, SV 4, SV 6, SV 7 and SV 9 can't be correct.

For data distribution B, knowing the value of u_1 contains almost no information about the corresponding value of u_2. Both inputs are equally important. The singular values represent the importance of each input. It follows, that SV8 belongs to data distribution 8.

In data distribution C a linear dependency between u_1 and u_2 exists. If one value of one input is known, the value of the other input can be determined exactly. Therefore one singular value has to be zero $\rightarrow$ SV5 is correct.

For data distribution A the range in which u_2 values might occur given a value for u_1 is small. This leads to a big difference between the two singular values, but no singular value is exactly zero $\rightarrow$ SV2. 3

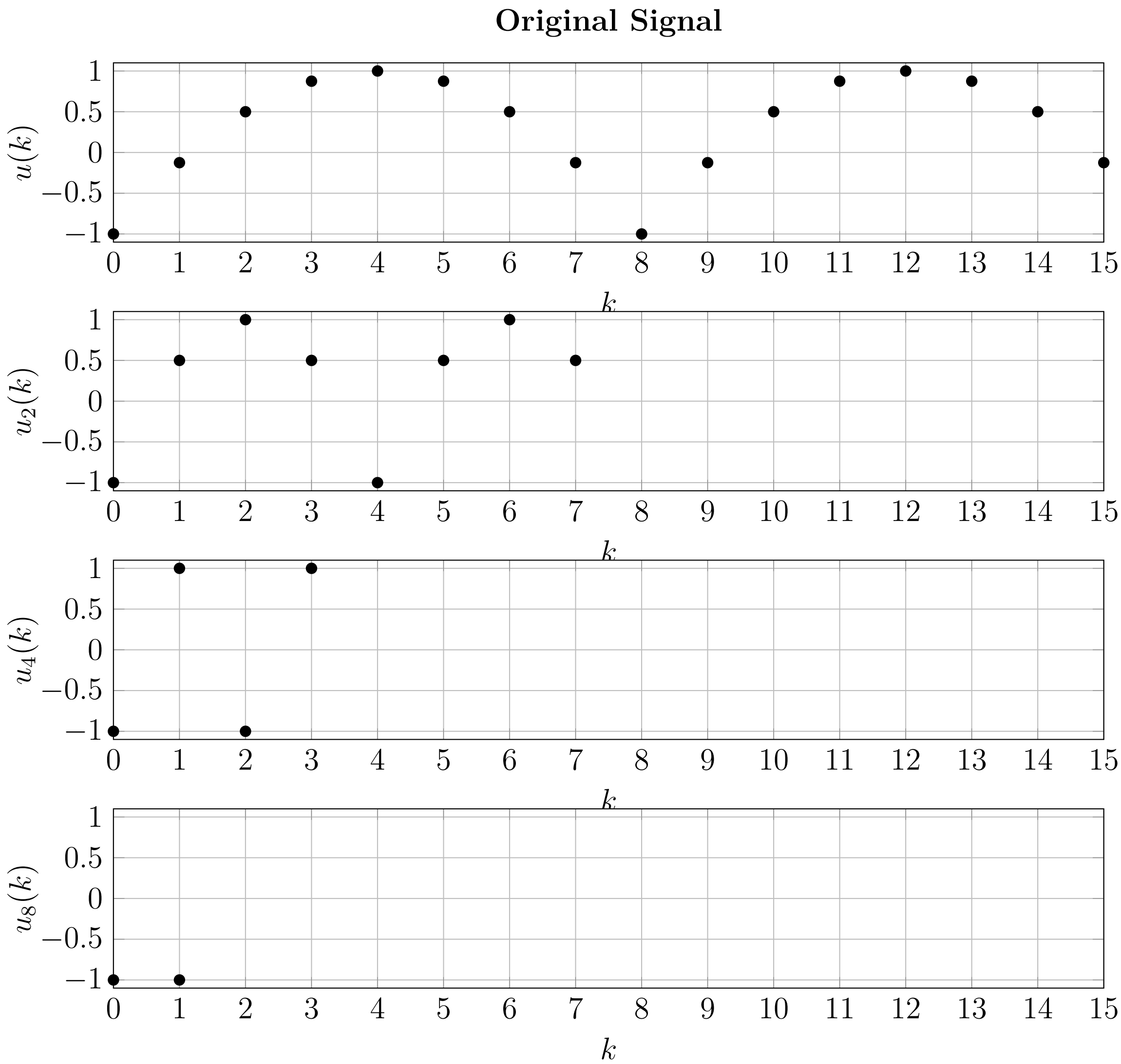
$\sum 6$

Task 6: Downsampling

a) The sampling times are:

$$T_{0,2} = 2 \cdot T_0 \quad ; \quad T_{0,4} = 4 \cdot T_0 \quad ; \quad T_{0,8} = 8 \cdot T_0 \tag{4}$$

The correct solutions for the compressed signals are shown below. 1



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b) For a compression factor of 8, the signal?s frequency is no longer correctly described. The sampling theorem is violated, aliasing occurs. 1

c) Because of the violation of the sampling theorem, a constant signal appears with mean value $\mu = 1$. In contrast, the original signal has a mean value of $\mu = 0$. To prevent the occurring aliasing a low-pass filter should be used before the downsampling (anti aliasing filter). 1

d) Ideally the result would be $u_8(k) = 0$. 1

$\sum 7$

Task 7: Dynamic System

a) An impulse in the z-domain is defined as follows:

$$U(z) = 1 \tag{5}$$

$\boxed{1}$

b) The impulse response in z corresponds to the multiplication of the system with an impulse:

$$Y(z) = G(z) \cdot U(z) \tag{6}$$
$$Y(z) = \frac{z}{z - 0.5} \cdot 1 \tag{7}$$
$$Y(z) = \frac{z}{z - 0.5} \tag{8}$$

$\boxed{1}$

c) The final value theorem in general is defined as:

$$y(k \to \infty) = \lim_{z \to 1} \left((z - 1) \cdot G(z) \cdot U(z)\right) \tag{9}$$
$$y(k \to \infty) = \lim_{z \to 1} \left((z - 1) \cdot Y(z)\right) \tag{10}$$

If follows for the given system:

$$y(k \to \infty) = \lim_{z \to 1} \left((z - 1) \cdot \frac{z}{z - 0.5}\right) \tag{11}$$
$$y(k \to \infty) = 0 \cdot \frac{z}{z - 0.5} \tag{12}$$
$$y(k \to \infty) = 0 \cdot \frac{1}{1 - 0.5} \tag{13}$$
$$\Rightarrow y(k \to \infty) = 0 \tag{14}$$

$\boxed{2}$

d) The transformation is as follows:

$$Y(z) = \frac{z}{z - 0.5} \tag{15}$$
$$Y(z) = \frac{1}{1 - 0.5 \cdot z^{-1}} \tag{16}$$
$$\Leftrightarrow \quad Y(z) - 0.5 \cdot z^{-1} \cdot Y(z) = 1 \tag{17}$$
$$\bullet\!\!-\!\!\!-\!\!\circ \tag{18}$$
$$y(k) = 0.5 y(k - 1) + \delta_k(k) \tag{19}$$

The first 6 discrete values of the signal are:

$\boxed{2}$

$$
\begin{aligned}
y(k=0) &= 0.5y(k=-1) + \delta_k(k=0) && (20)\\
&= 0.5 \cdot 0 + 1 = 1 && (21)\\
y(k=1) &= 0.5y(k=0) + \delta_k(k=1) && (22)\\
&= 0.5 \cdot 1 + 0 = 0.5 && (23)\\
y(k=2) &= 0.5y(k=1) + \delta_k(k=2) && (24)\\
&= 0.5 \cdot 0.5 + 0 = 0.25 && (25)\\
y(k=3) &= 0.5y(k=2) + \delta_k(k=3) && (26)\\
&= 0.5 \cdot 0.25 + 0 = 0.125 && (27)\\
y(k=4) &= 0.5y(k=3) + \delta_k(k=4) && (28)\\
&= 0.5 \cdot 0.125 + 0 = 0.0625 && (29)\\
y(k=5) &= 0.5y(k=4) + \delta_k(k=5) && (30)\\
&= 0.5 \cdot 0.0625 + 0 = 0.03125 && (31)
\end{aligned}
$$

$\boxed{2}$

e) In general the transfer function is defined as:

$$G(z) = b_0 + b_1 \cdot z^{-1} + b_2 \cdot z^{-2} + b_3 \cdot z^{-3} + b_4 \cdot z^{-4} \quad (32)$$

$\boxed{1}$

f) The impulse response values of the IIR system are utilized as coefficients for the FIR filter:

$$
\begin{aligned}
&b_i = y(k=i) && (33)\\
&b_0 = y(k=0) \quad ; \quad b_1 = y(k=1) \quad ; \quad b_2 = y(k=2) && (34)\\
&b_3 = y(k=3) \quad ; \quad b_4 = y(k=4) && (35)
\end{aligned}
$$

$\boxed{1}$

$\boxed{\sum 10}$

Task 8: Filter

a) Calculation of the amplitude response for **Filter A:**

$$\begin{aligned} G_1(z) &= \frac{1}{2}\left(1+z^{-1}\right) && (36)\\ G_1(i\omega) &= \frac{1}{2}\left(1+e^{-i\omega T_0}\right) && (37)\\ G_1(i\omega) &= \frac{1}{2}\left(1+\cos(\omega T_0) - i\sin\left(\omega T_0\right)\right) && (38)\\ \|G_1(i\omega)\| &= \sqrt{\left(\frac{1}{2}+\frac{1}{2}\cos\left(\omega T_0\right)\right)^2 + \frac{1}{4}\sin^2\left(\omega T_0\right)} && (39)\\ &= \sqrt{\frac{1}{2}+\frac{1}{2}\cos\left(\omega T_0\right)} && (40) \end{aligned}$$

for **Filter B:** $\boxed{3}$

$$\begin{aligned} G_2(z) &= \frac{1}{2}\left(1-z^{-1}\right) && (41)\\ G_2(i\omega) &= \frac{1}{2}\left(1-e^{-i\omega T_0}\right) && (42)\\ G_2(i\omega) &= \frac{1}{2}\left(1-\cos(\omega T_0) - i\sin\left(\omega T_0\right)\right) && (43)\\ \|G_2(i\omega)\| &= \sqrt{\left(\frac{1}{2}-\frac{1}{2}\cos\left(\omega T_0\right)\right)^2 + \frac{1}{4}\sin^2\left(\omega T_0\right)} && (44)\\ &= \sqrt{\frac{1}{2}-\frac{1}{2}\cos\left(\omega T_0\right)} && (45) \end{aligned}$$

$\boxed{3}$

b) The filters are of type:
Filter A: Low-pass, because

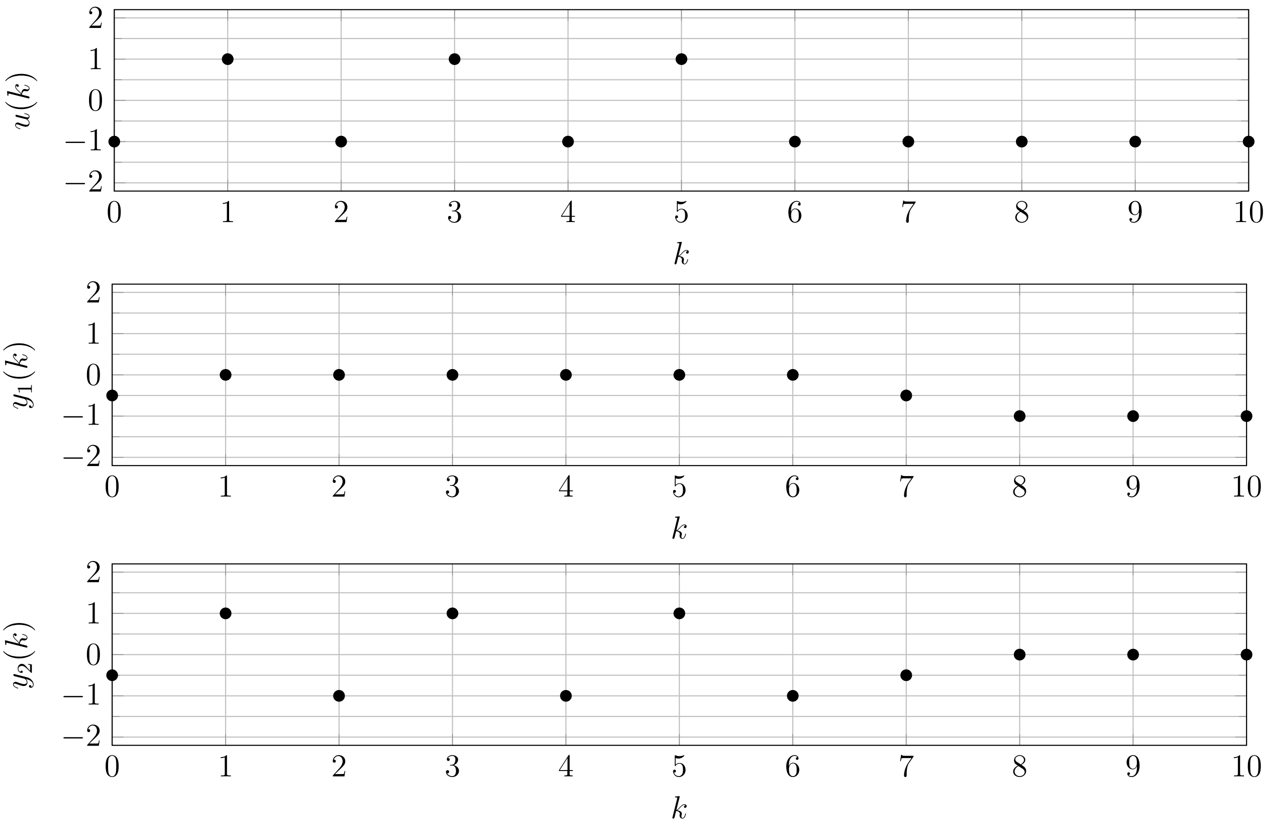
$$\|G_1(0)\| = 1 \qquad \left\|G_1\left(i\frac{\pi}{T_0}\right)\right\| = 0 \tag{46}$$

Filter B: High-pass, because

$$\|G_2(0)\| = 0 \qquad \left\|G_2\left(i\frac{\pi}{T_0}\right)\right\| = 1 \tag{47}$$

$\boxed{2}$

c) The correct signals are shown below.



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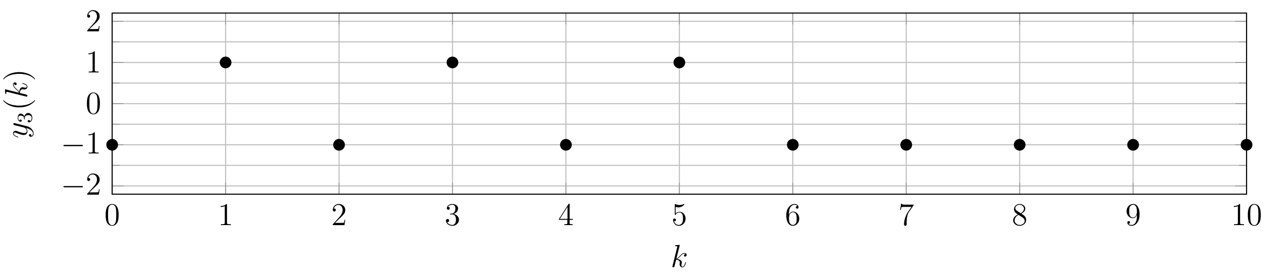
d) The sum of both transfer functions is 1. Therefore the output of the filters connected in parallel equals exactly the input signal $u(k)$.



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$\sum 14$

Task 9: Block Diagram

a) Draw an equivalent block diagram, where the number of delay-blocks is reduced. This block diagram is known as the direct form II block diagram (first $\frac{1}{A(z)}$, then $B(z)$).

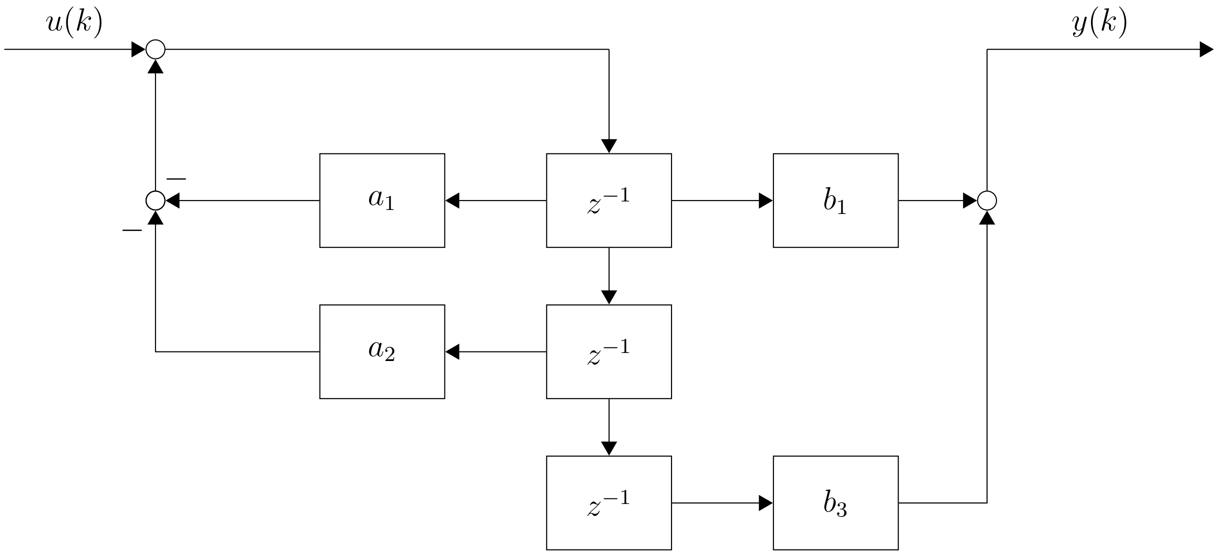

Fig. 17: Non-redundant direct form II block diagram.

5

b) Derive the difference equation for the block diagram given in Fig. 11.

$$y(k) + a_1 y(k-1) + a_2 y(k-2) = b_1 u(k-1) + b_3 u(k-3). \tag{48}$$

1

c) Determine the transfer function $G(z)$ for the block diagram given in Fig. 11.

$$G_(z) = \frac{b_1 z^{-1} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

1

d) Assign the correct step response .

a_1	a_2	b_1	b_3	Step Response
-0.2	-0.2	-0.2	-0.4	**D**
0.3	0.3	0	0.4	**H**
0.5	0.5	1.2	-0.2	**F**
0.5	-0.5	-0.5	0.5	**E**

16

D: The only step response, with gain 1 0.5

H: The only step response with correct delay (3 time steps) and correct first non-zero value (0.4) 0.5

F: Only step response with correct time delay (1 time step) and correct first non-zero value (1.2) 0.5

E: Only system with gain 0. 0.5

e) Assume to approximate the IIR system that corresponds to the block diagram shown in Fig. 11 by an FIR system. If step response in Fig. 12 **C** would be the step response $H_{IIR}(z)$ of the IIR system, what FIR system order do you need, if the absolute maximum error between the FIR step response $H_{FIR}(z)$ and H_{IIR} should be less than 0.05 at any time. Explain the choice of the FIR system order shortly.

The order should be at least 8. After 8 discrete time steps the final value of the step response is almost reached. 1 The FIR approximation of order n is exact until the n-th time step, after that the final value of the FIR system is exactly reached and the difference between $H_{FIR}(z)$ and $H_{IIR}(z)$ will stay constant. 1

$\sum 27$