Sensorics Exam

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14th September 2012

Name:	Task:	T1	T2	T3	T4	T5	T6	Sum
Mat.-No.:	Scores:	30	24	16	6	24	20	120
Grade:	Accomplished:							

Task 1: Comprehension Questions

Mark the correct answers clearly.
Every question has one or two correct answers!
For every correctly marked answer you will get one point. If there is one correct answer marked and one incorrect answer marked, you will get no point for that subtask.

a) To process a measurement signal in the computer, the signal...
- ☐ ...has to be discretized.
- ☐ ...has to be transformed into the frequency domain.
- ☐ ...has to be quantized.

b) Which statements are true for strain gauges?
- ☐ Environmental influences can easily be compensated through a clever arrangement of the strain gauges.
- ☐ Strain gauges utilize the resistance change caused by a change in length and a change of the cross section area of a metallic conductor for the measurement.
- ☐ Strain gauges utilize the change of its capacity for the measurement.

c) How can speed be measured?
- ☐ Via the measurement of the acceleration and a subsequent differentiation.
- ☐ With the help of the Doppler-effect.
- ☐ Via the measurement of the rotational speed and a subsequent division by the radius.

d) How large should the internal resistance of a voltage meter be?
- ☐ Very large, in the ideal case infinite large.
- ☐ Very small, in the ideal case zero.
- ☐ It depends on the electrical circuit, what internal resistance is ideal.

e) When measuring low frequencies...
- [ ] ...it is recommended to count the number of cycles.
- [ ] ...the gate time is usually defined by the signal, that should be measured. While the gate is open an artificially generated frequency produces pulses that are counted and utilized to calculate the low frequency.
- [ ] ...the gate time is usually defined by an artificially generated frequency. While the gate is open, the frequency that should be measured is used to produce pulses that are counted to calculate the signal's frequency.

f) Which statements are true for the measurement of temperatures?
- [ ] Thermocouples are more accurate than resistance thermometer.
- [ ] Thermocouples are suitable for point-wise measurements.
- [ ] Thermocouples have a smaller time constant than resistance thermometer.

g) Which of the following statements about measurement errors are correct?
- [ ] Systematic errors can be reduced by calculating the mean of multiple measurements.
- [ ] Reason and kind of the error action are known for systematic erros.
- [ ] Reason and kind of the error action are known for random erros.

h) A non-causal filter...
- [ ] ...requires only current and previous input values.
- [ ] ...requires future input values.
- [ ] ...requires future output values.

i) A median filter...
- [ ] ...is always causal.
- [ ] ...can be used to remove outliers.
- [ ] ...has a step output as a response to a step input.

j) Principal Component Analysis (PCA)...
- [ ] ...is a linear axis transformation.
- [ ] ...is a non-linear axis transformation.
- [ ] ...is a tool for dimensionality reduction.

k) Principal Component Analysis (PCA)...
- [ ] ...can be calculated with a filter.
- [ ] ...can be calculated with a singular value decomposition.
- [ ] ...can be calculated by solving an eigenvalue problem.

l) Clustering...
- [ ] ...is a method for data post-processing.
- [ ] ...is a method for data pre-processing.
- [ ] ...belongs to the class of supervised learning algorithms.

m) Unsupervised Learning...
- [ ] ...requires the desired output values.
- [ ] ...does not require the desired output values.
- [ ] ...is often used for data pre-processing.

n) What can be achieved with the discrete Fourier transform (DFT)?
- [ ] The transformation of a signal from the discrete frequency domain into the discrete time domain.
- [ ] The transformation of a discrete signal from the time domain to the discrete frequency domain.
- [ ] The transformation of a discrete signal from the time domain to the continuous range of frequencies.

o) The discrete Fourier transform is periodic ...
- [ ] ... only in time.
- [ ] ... only in frequency.
- [ ] ... in time and frequency.

p) A temporal sequence of N measurements that is transformed with the DFT results in a number of ...
- [ ] ... N discrete frequencies.
- [ ] ... $N/2$ discrete frequencies.
- [ ] ... 2^N discrete frequencies.

q) An increase in the number of measurements has the following effect on the frequency resolution:
- [ ] It becomes finer.
- [ ] It becomes coarser.
- [ ] It remains unaffected.

r) The so-called leakage effect can be reduced by ...
- [ ] convolution with a window in the time domain.
- [ ] subtraction of a window in the time domain.
- [ ] multiplication with a window in the time domain.

s) Stationary signals ...
- [ ] are time-variant.
- [ ] can be considered as non-stationary, when viewed over a short period.
- [ ] are time-invariant.

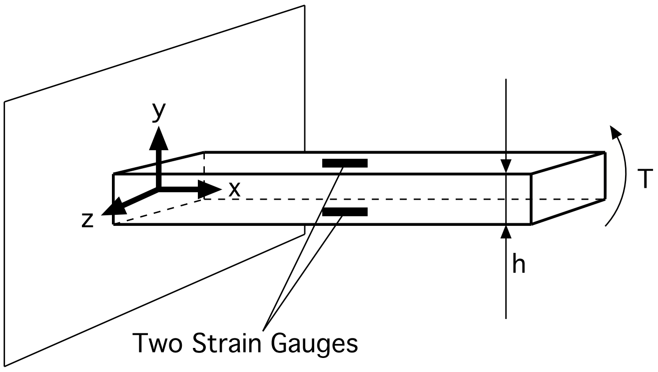
t) In order to make a meaningful statement about the contained frequencies in a non-stationary signal, ...

- ☐ the signal can be examined with a short-time DFT.
- ☐ the signal can not be examined with a short-time DFT.
- ☐ the signal can be examined with a wavelet transform.

u) Parametric frequency analysis ...

- ☐ leads to a continuous amplitude spectrum.
- ☐ leads to a discrete amplitude spectrum.
- ☐ is more robust with respect to measurement noise as a non-parametric method.

Task 2: Measurement of Torque



In the picture above you can see a beam, that is loaded with a torque T on one end. The other end is fixed to a wall and two strain gauges are applied to the beam to measure the torque. The following equations are given:

$$|\rho| = \frac{EI}{|T|} \quad ; \quad \Delta R = R_0 K \epsilon \quad ; \quad \epsilon = \frac{y}{|\rho|}$$

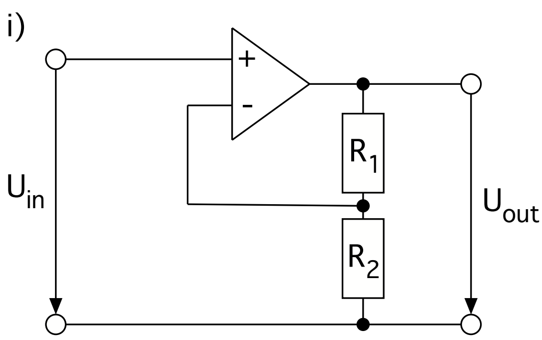
ρ: Curvature of the deflected beam; E: Young's modulus; I: Second moment of area; T: Torque (constant over the beam length); ΔR: The change in resistance of one strain gauge; R_0: The resistance of the strain gauge without any applied torque; K: Sensitivity of the strain gauges; ϵ: Deformation of the strain gauge, h: Beam height; y: Coordinate of the beam height, that starts exactly at the half of the beam height

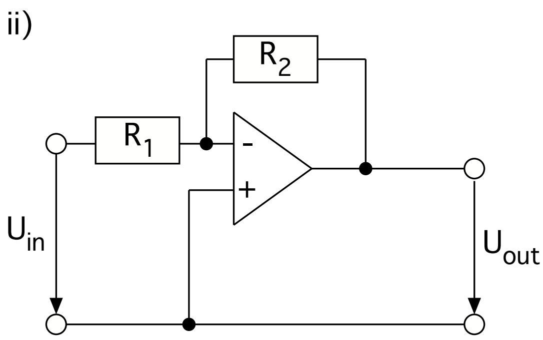
a) Evaluate the equation that describes the change of the resistances ΔR depending on the applied torque T. What relationship does the change of the resistance of the upper and the lower strain gauge have?

b) Sketch a bridge circuit that can be used to transform the change in resistance ΔR into a voltage U_d and derive the corresponding equation (Just use ΔR for the change of resistance - NOT the equation from a!).

c) In what way can the sensitivity of the torque measurement be increased through a change in the bridge circuit?

d) The measured voltage should be amplified with the help of an operational amplifier (OpAmp) circuit. Which of the following two circuits amplifies the input voltage U_{in} (assume ideal OpAmps)?



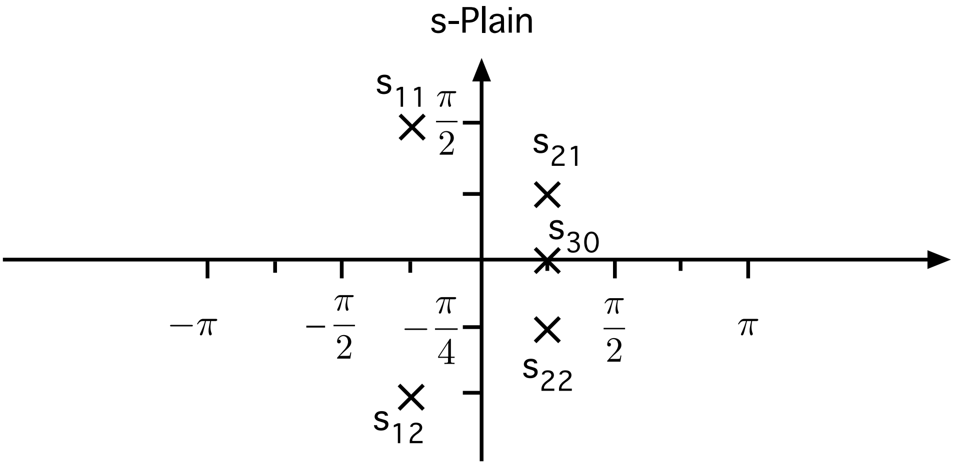


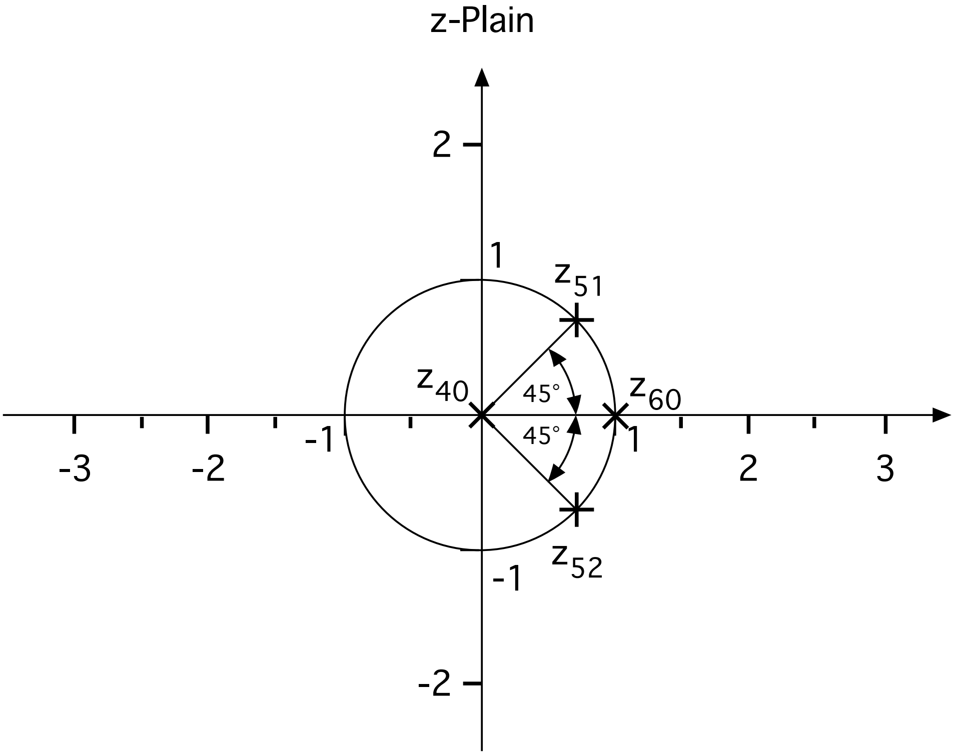
e) What relationship of R_1 and R_2 has to be fullfilled to achieve an amplification of the input voltage by a factor of two?

Task 3: Relation Between s-Plane and z-Plane

a) Specify the equation that transforms any value in the z-domain to the s-domain and specify the equation that transforms any value from the s-domain (back to) the z-domain.

b) In the following figure are six poles, three in the z-domain and three in the s-domain. Perform the transformation of all points in the z-domain to the s-domain and from the s-domain to the z-domain and sketch them in the corresponding plane. Denote related points by the same indices, i.e. the point s_{ij} in the s-domain corresponds to the point z_{ij} in the z-domain. To perform the transformation assume a sampling time $T_0 = 1$ second.





Task 4: Aliasing

The following periodic signal is given: $y(t) = \sin(2\pi \cdot 7\text{Hz} \cdot t)$.

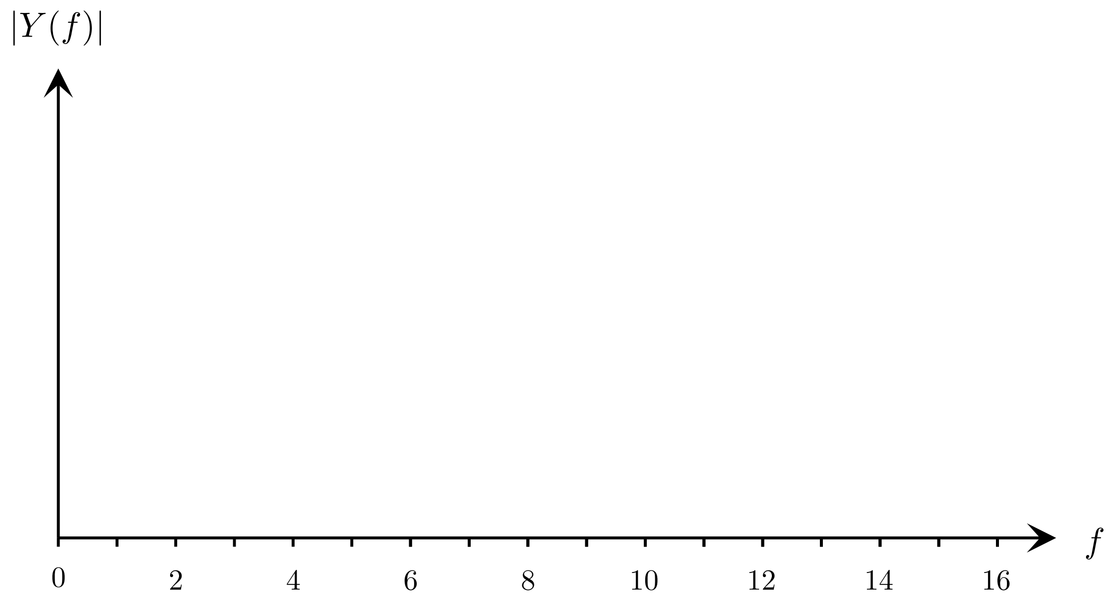
a) Sketch the amplitude spectrum for the given signal in the diagram below.

b) The signal is measured with the sampling frequency $f_0 = 6\text{Hz}$. Explain, why aliasing will occur in this case.

c) Highlight the Shannon frequency in the diagram below.

d) Sketch the shadow spectra that result from the sampling theorem.

e) What signal frequency can wrongly be guessed from the sampled signal?



Task 5: Time-Discrete Systems

Below, the block diagram of a first order system is given. First of all, assume that all coefficients are non-zero values, i.e. $a_1, b_0, b_1 \neq 0$.

a) Evaluate the underlying difference equation of the system shown in the block diagram.

b) Is the given system an IIR- or a FIR-system? Explain your answer.

c) Explain, if the system has a direct throughput from the input to the output. What is the crucial prerequisite for that?

d) Determine the corresponding transfer function $G(z)$ of the system in the z-domain.

e) Calculate the impulse response $g(k)$ (initial condition: $y(k < 0) = 0$).

f) Calculate the z-transformed step response $H(z)$.

g) What is the value of the step response in the time domain $h(k)$ for $k \to \infty$? What is the initial value of the step response for $k = 0$?

h) Now assume that the coefficients have the following values: $b_0 = 0$, $b_1 = 0.5$ and $a_1 = -0.95$. In this case, what is the universal equation for the step response $h(k)$?

i) Assume that the coefficient a_1 has a positive algebraic sign, i.e. $a_1 = +0.95$. What effect could be observed on the impulse response $g(k)$? Does the algebraic sign of the pole a_1 have any effect on the stability of the system?

Task 6: Filters

In the figure below, you can see the time signal $u(k)$ that has to be filtered. Two possible filters are given as follows:

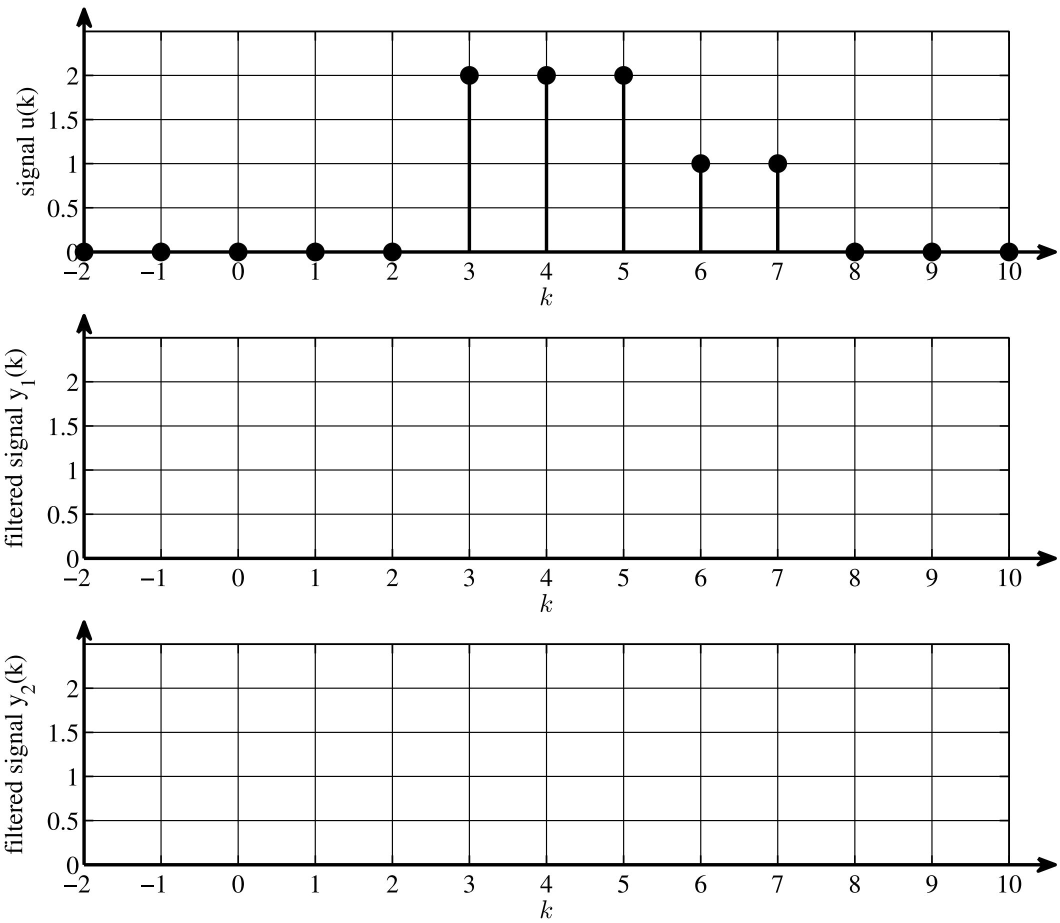
$$G_1(z) = \frac{1}{3}(z^{-1} + 1 + z)$$
$$G_2(z) = \frac{1}{3}(1 + z^{-1} + z^{-2})$$

a) Explain, if the filters $G_1(z)$ and $G_2(z)$ are low-pass or high-pass filters. Furthermore, specify if the filters are causal or non-causal.

b) What kind of phase shift will result from filtering with each filter (zero phase, linear phase, affine phase)? Determine the phase shifts $\varphi_1(\omega)$ and $\varphi_2(\omega)$.

c) Calculate and sketch both, the signal $y_1(k)$ which is filtered with the first filter $G_1(z)$ and the signal $y_2(k)$ which is filtered with the second filter $G_2(z)$. Use the given diagrams for that purpose. Additionally, write down the resulting output sequences in the following form: $y(k) = \{y(-2), y(-1), y(0), \ldots, y(10)\}$.

d) One of the two filters has to be applied for an application with real-time requirements. There is no possibility to apply any buffer or to store any data in the real-time environment, respectively. Which of the two filters is suitable in this case? Explain your answer.



Lösungen:

Task 1: Verständnisfragen

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□ ...belongs to the class of supervised learning algorithms.

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$\boxed{\sum 30}$

<u>Task 2:</u> Measurement of Torque

In the picture above you can see a beam, that is loaded with a torque T on one end. The other end is fixed to a wall and two strain gauges are applied to the beam to measure the torque. The following equations are given:

$$|\rho| = \frac{EI}{|T|} \quad ; \quad \Delta R = R_0 K \epsilon \quad ; \quad \epsilon = \frac{y}{|\rho|}$$

ρ: Curvature of the deflected beam; E: Young's modulus; I: Second moment of area; T: Torque (constant over the beam length); ΔR: The change in resistance of one strain gauge; R_0: The resistance of the strain gauge without any applied torque; K: Sensitivity of the strain gauges; ϵ: Deformation of the strain gauge, h: Beam height; y: Coordinate of the beam height, that starts exactly at the half of the beam height

a)

$$\Delta R = R_0 K \epsilon \tag{1}$$

$$|\rho| = \frac{EI}{|T|} \tag{2}$$

$$\epsilon = \frac{y}{|\rho|} \tag{3}$$

(2) in (3):

$$\epsilon = \frac{y|T|}{EI} \tag{4}$$

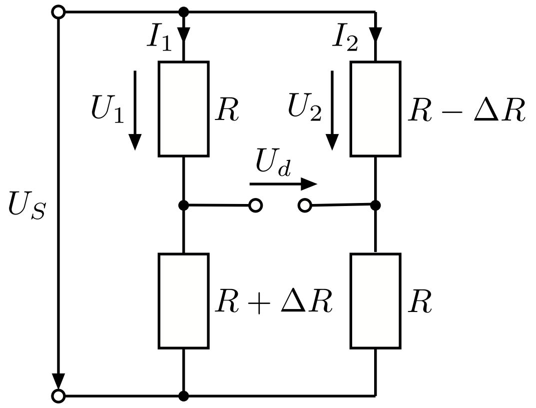
(4) in (1):

$$\Delta R = \frac{R_0 K y |T|}{EI} \tag{5}$$

$\boxed{5}$

Upper strain gauge ($y = +\frac{h}{2}$): $\Delta R_U = \frac{R_0 K h |T|}{2EI} \hat{=} \Delta R$
Lpper strain gauge ($y = -\frac{h}{2}$): $\Delta R_L = -\frac{R_0 K h |T|}{2EI} \hat{=} -\Delta R$

Relationship: $\Delta R_U = -\Delta R_L$ $\boxed{2}$



b) $\boxed{6}$

$$U_1 - U_2 + U_d = 0 \tag{6}$$
$$U_1 = RI_1 \tag{7}$$
$$U_2 = (R - \Delta R)I_2 \tag{8}$$
$$I_1 = \frac{U_S}{2R + \Delta R} \tag{9}$$
$$I_2 = \frac{U_S}{2R - \Delta R} \tag{10}$$

(9) in (7) and (10) in (8) :

$$U_1 = \frac{RU_S}{2R + \Delta R} \tag{11}$$
$$U_2 = \frac{(R - \Delta R)U_S}{2R - \Delta R} \tag{12}$$

(11) and (12) in (6):
(...)
$U_d = -\frac{\Delta R^2}{4R^2 - \Delta R^2} U_S$ 4

c) If two additional strain gauges are applied to the beam, they can be used to realize a full bridge circuit, which leads to an increase of the sensitivity by nearly a factor of two. 2

d) The first one amplifies the voltage.

$$U_{in} = \frac{R_2}{R_1 + R_2} U_{out}$$
$$\Leftrightarrow U_{out} = (\frac{R_1}{R_2} + 1)U_{in}$$

4

e) $R_1 = R_2$ (see equation above). 1

$\sum 24$

Task 3: Relation Between s-Plane and z-Plane

a) $z = e^{sT_0}$; $s = \frac{1}{T_0} ln(z)$ 2

b) With $T_0 = 1$: $z = e^s$; $s = ln(z)$

$z_{11} = e^{-\frac{\pi}{4}+i\frac{\pi}{2}} = e^{-\frac{\pi}{4}} \cdot e^{i\frac{\pi}{2}} = e^{-\frac{\pi}{4}}[cos(\frac{\pi}{2}) + isin(\frac{\pi}{2})] \approx 0 + i0.46$ 1

$z_{12} \approx 0 - i0.46$ 1

$z_{21} \approx 1.55 + i1.55$ 1

$z_{22} \approx 1.55 - i1.55$ 1

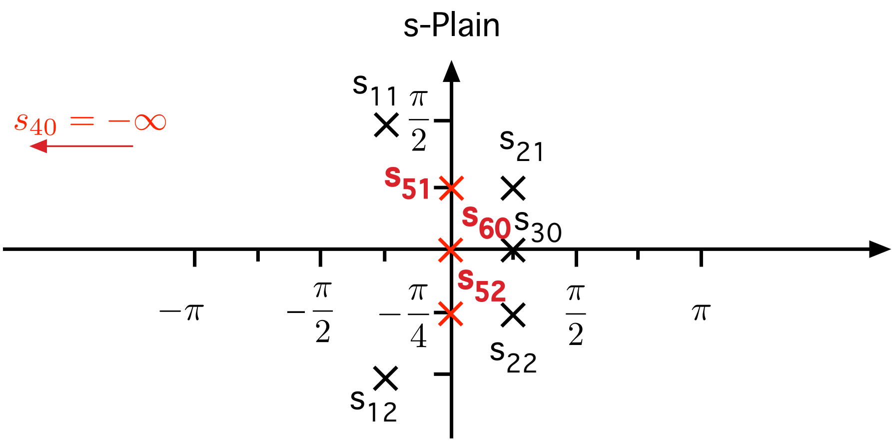
$z_{30} \approx 2.19$ 2

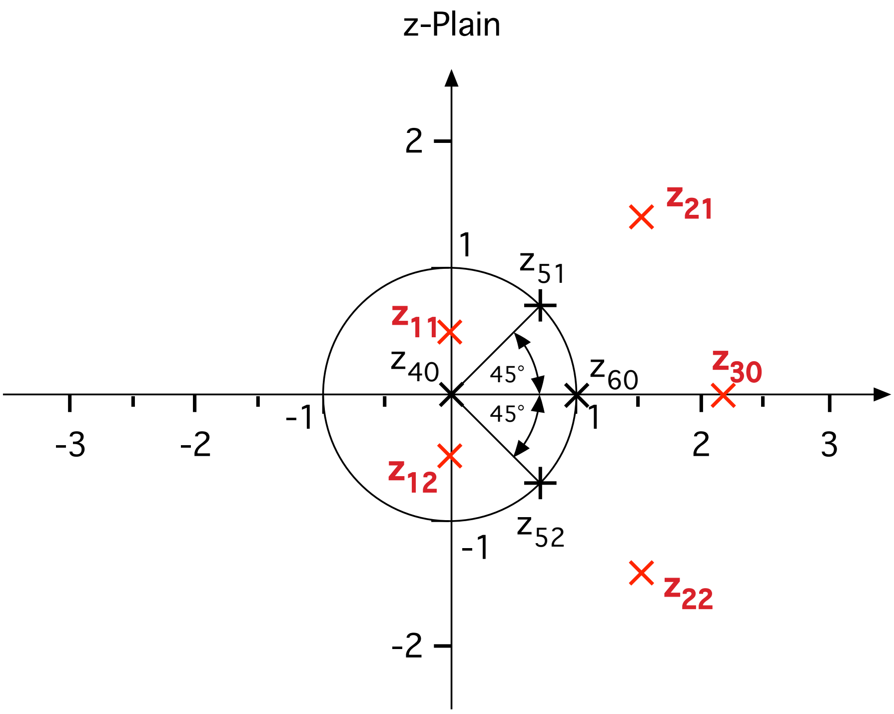
$s_{40} = ln(0) = -\infty$ 4

$s_{51} = ln(e^{i45^\circ}) = ln(e^{i\frac{\pi}{4}}) = 0 + i\frac{\pi}{4}$ 1

$s_{52} = 0 - i\frac{\pi}{4}$ 1

$s_{60} = ln(1) = 0$ 2





$\sum 16$

Task 4: Aliasing

Ein Signal $y(t) = \sin(2\pi \cdot 7\text{Hz} \cdot t)$ soll mit einer Abtastfrequenz von $f_0 = 6\text{Hz}$ gemessen werden.

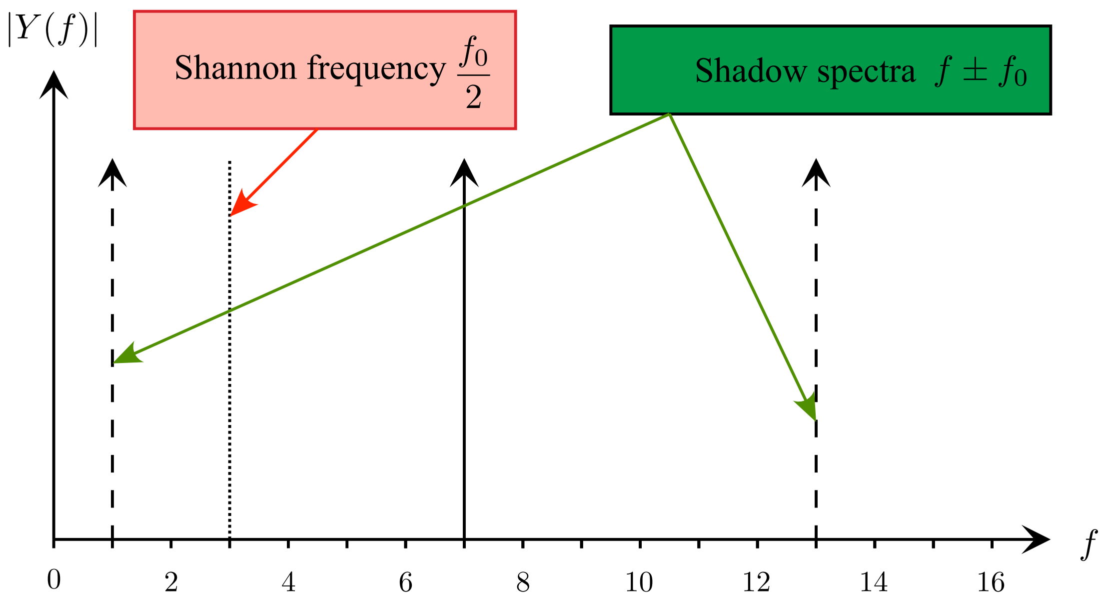
a) See figure below. 1

b) Aliasing occurs, if the highest frequency contained in the signal is greater than or equal the half of the sampling frequency. Here the highest frequency, that is contained in the signal is 7Hz, which is greater than the half of the sampling frequency $\frac{f_0}{2} = 3\text{Hz}$. 1

c) See figure below. 1

d) See figure below. 1

e) Here the sampled signal would have a frequency of 1Hz. 2



$\sum 6$

Task 5: Zeitdiskretes System

a) $y(k) = b_0 u(k) + b_1 u(k-1) - a_1 y(k-1)$ $\boxed{3}$

b) The given system has an infinite impulse response (IIR-system). The output at the present step k depends on former output values and therefore has to be calculated recursively. $\boxed{1}$

c) The system has a direct throughput from the input to the output. The crucial prerequisite is, that $b_0 \neq 0$. The input influences the output without any delay. $\boxed{1}$

d) The transfer function $G(z)$ is:

$$G(z) = \frac{Y(z)}{U(z)} = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}}$$

$\boxed{2}$

e) The impulse response $g(k)$ follows the equation:

$$g(k) = (-a_1)^{k-1}\,(b_1 - a_1\,b_0) = (-a_1)^{k-1}\,b_1 + (-a_1)^k\,b_0$$

$\boxed{6}$

f) To calculate the step response $H(z)$, the transfer function $G(z)$ has to be multiplied by the z-transform of the unit step:

$$H(z) = G(z)\frac{1}{1 - z^{-1}} = \frac{b_0 + b_1\,z^{-1}}{(1 + a_1\,z^{-1})(1 - z^{-1})}$$

$\boxed{2}$

g) Gain of the system:

$$h(k \to \infty) = \lim_{z \to 1}(z-1)H(z) = \lim_{z \to 1} G(z) = \frac{b_0 + b_1}{1 + a_1}\,.$$

Initial value ($k = 0$):

$$h(k \to 0) = \lim_{z \to \infty} H(z) = b_0\,.$$

$\boxed{2}$

h) With the given coefficients the step response looks as follows:

$$h(k) = \begin{cases} 0 & \text{for } k = 0 \\ 0.5 \cdot \sum_{i=1}^{k} (0.95)^{k-1} = 10 \cdot (1 - 0.95^k) & \text{for } k > 0 \end{cases}$$

$\boxed{6}$

i) If a_1 has a positive algebraic sign, the impulse response would alternate. The stability properties of the system would not be affected. $\boxed{1}$

$\boxed{\sum 24}$

Task 6: Filter

a) $G_1(z)$ and $G_2(z)$ are low-pass filters. $G_1(z)$ is acausal, $G_2(z)$ is causal. $\boxed{4}$

b)

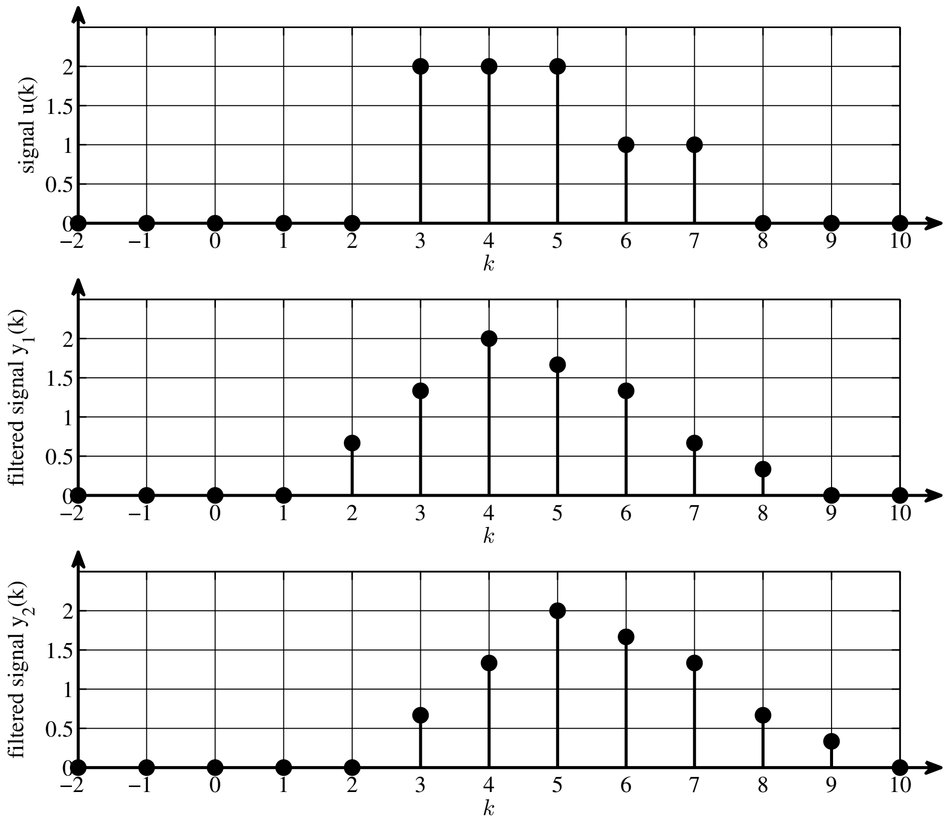
$$\begin{aligned} G_1(z) &= \frac{1}{3}(z^{-1} + 1 + z) = \frac{1}{3}(e^{-sT_0} + 1 + e^{sT_0}) \\ &= \frac{1}{3}(e^{-i\omega T_0} + 1 + e^{i\omega T_0}) = \frac{1}{3} \cdot (1 + 2\cos(\omega T_0)) \\ \varphi_1 &= tan^{-1}\left(\frac{0}{\frac{1}{3}(e^{-i\omega T_0} + 1 + e^{i\omega T_0})}\right) = 0 \rightarrow \text{zero phase} \end{aligned}$$

$$\begin{aligned} G_2(z) &= \frac{1}{3}(1 + z^{-1} + z^{-2}) = \frac{1}{3}(z^{-1} + 1 + z)z^{-1} \\ &= \frac{1}{3} \cdot (1 + 2\cos(\omega T_0)) \cdot \underbrace{e^{-i\omega T_0}}_{\text{Phase } e^{i\varphi}} \\ \varphi_2 &= -\omega T_0 \rightarrow \text{linear phase} \end{aligned}$$

$\boxed{4}$

c) See figure below. $\boxed{10}$

d) For an real-time application without the possibility to buffer any data, the causal filter is the only reasonable choice, because no future values are required. $\boxed{2}$



$\boxed{\sum 20}$