Sensorics Exam

Prof. Dr.-Ing. O. Nelles
Institute of Mechanics and Control Engineering - Mechatronics
University of Siegen

27th of March 2018

Name:	
Mat.-No.:	
Grade:	

Task:	T1	T2	T3	T4	T5	T6	T7	T8	Sum
Scores:	9	21	20	11	15	15	15	14	120
Accomplished:									

Duration of examination: 2 hours

You are allowed to use a calculator and four pages of notes

Task 1: Comprehension Questions (9 Points)

Mark the correct answers clearly. **Every question has one or two correct answers!** For every correctly marked answer you will get one point. If there is one correct answer marked and one incorrect answer marked, you will get no point for that subtask.

a) How can you change the range of a moving coil mechanism meter?

- ☐ If used as a voltmeter: connecting resistances in series.
- ☐ If used as an amperemeter: connecting resistances in parallel.
- ☐ If used as a voltmeter: connecting resistances in parallel.

b) Supervised learning...

- ☐ ...does not require the desired output values.
- ☐ ...is often used for data-preprocessing.
- ☐ ...requires the desired output values.

c) The so-called leakage effect in the DFT can be reduced by ...

- ☐ ... addition of a window in the frequency domain.
- ☐ ... convolution with a window in the frequency domain.
- ☐ ... multiplication of a window in the frequency domain.

d) What are characteristics of an idealized operational amplifier?

- ☐ Output resistance = 0.
- ☐ Output resistance = ∞.
- ☐ Gain = 0.

e) A temporal sequence of N measurements that is transformed with the DFT results in a number of ...

- ☐ ... $\frac{N}{2}$ discrete frequencies.
- ☐ ... N discrete frequencies.
- ☐ ... 2^N discrete frequencies.

f) The central limit theorem of statistics says that ...

- ☐ ... the sum of several random variables follows approximately a normal distribution.
- ☐ ... the product of several random variables follows approximately a normal distribution.
- ☐ ... the sum of several random variables follows approximately an unique distribution.

g) A median filter...

- ☐ ...is always causal.
- ☐ ...can be used to remove outliers.
- ☐ ...has a step-like output as a response to a step input.

Task 2: Temperature Measurement (21 Points)

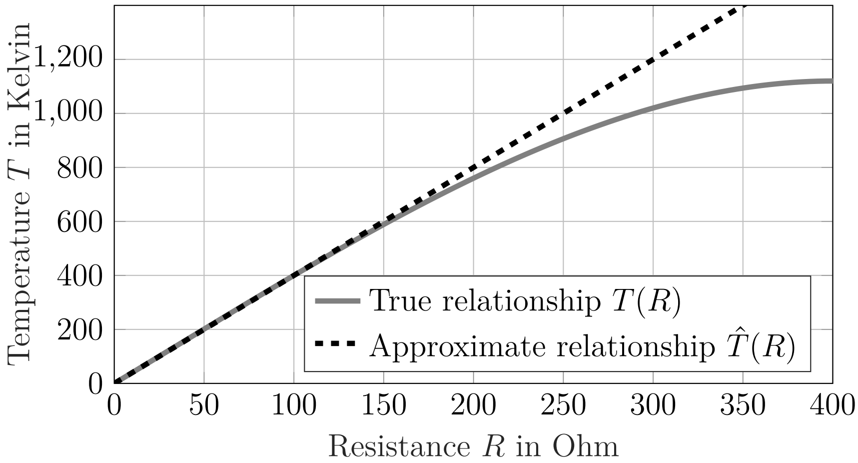


Fig. 1: Relationship of the temperature and the resistance of a PTC sensor.

The true relationship between the temperature T and the resistance R of a positive-temperature-coefficient (PTC) sensor follows the equation:

$$T(R) = 4R + 0.001R^2 - 10^{-5}R^3\,. \tag{1}$$

One possible simplification is to neglect the quadratic and the cubic term, since they have very small coefficients. This approximate relationship $\hat{T}(R)$ is shown as the dashed curve in Fig. 1 together with the true relationship (solid curve).

a) What type of error occurs through the usage of the approximate relationship $\hat{T}(R)$?

b) Determine up to which resistance the relative error between the approximate relationship $\hat{T}(R)$ and the true relationship $T(R)$ is less or equal to an absolute value of 10 %.

c) At which temperature (only the temperature range shown in Fig. 1) does the sensor (true relationship) have the highest sensitivity? Explain your answer briefly.
Hint: No calculation needed.

d) Use the Taylor series expansion to linearize the true relationship $T(R)$ for a operating point R_0. In general the Taylor series for a function $f(x)$ around an operating point x_0 is given by

$$Tf(x; x_0) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}\,(x - x_0)^n \tag{2}$$

Here, $n!$ denotes the factorial of n and $f^{(n)}(x_0)$ denotes the n-th derivative of $f(x)$ evaluated at x_0. How are the linearization through the Taylor series expansion and the one shown in Fig. 1 ($\hat{T}(R)$) related?

e) Sketch into Fig. 1 a good linear approximation, such that the mean error is minimized.

Task 3: Cross-Correlation (20 Points)

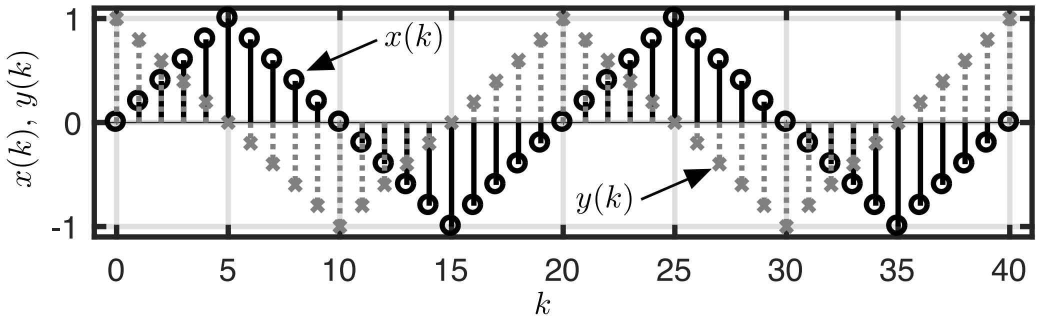


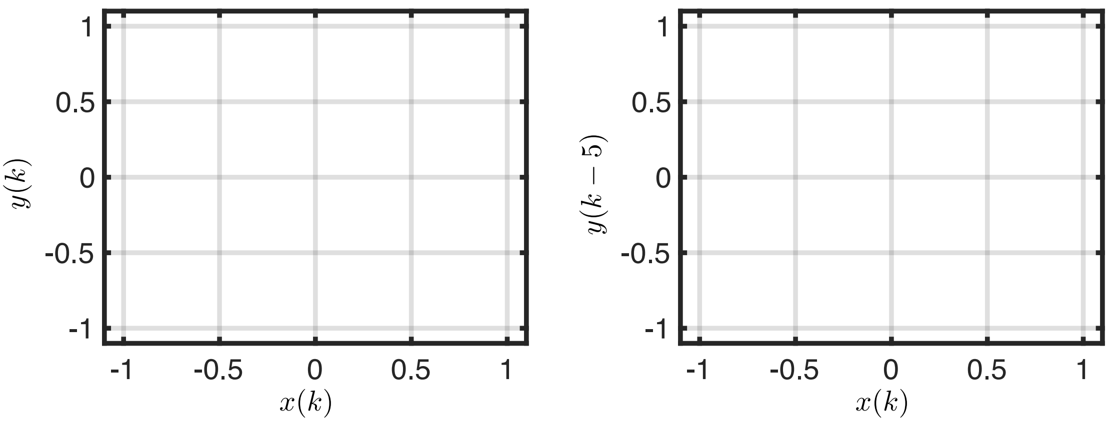
Fig. 2: Two full cycles of the periodic signals $x(k)$ and $y(k)$.

The cross-correlation for the two shown signals $x(k)$ and $y(k)$, both consisting of $N = 41$ samples, is calculated for a time shift κ as follows:

$$r_{xy}(\kappa) = \begin{cases} \frac{1}{N} \sum\limits_{k=0}^{N-1-\kappa} x(k) \cdot y(k+\kappa), & \text{if } \kappa \geq 0 \\ \frac{1}{N} \sum\limits_{k=-\kappa}^{N-1} x(k) \cdot y(k+\kappa), & \text{otherwise} \end{cases} \tag{3}$$

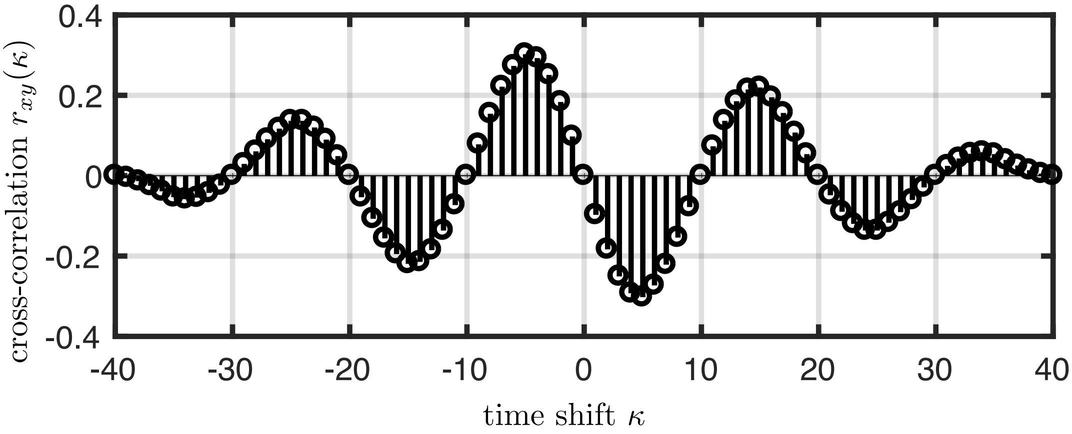
a) Which of the cross-correlation functions shown in Fig. 3 belongs to the signals $x(k)$ and $y(k)$ shown in Fig. 2?

b) Sketch the time shifted signal $y(k+\kappa)$ versus the signal $x(k)$ in the graphs below for $\kappa = 0$ and $\kappa = -5$.

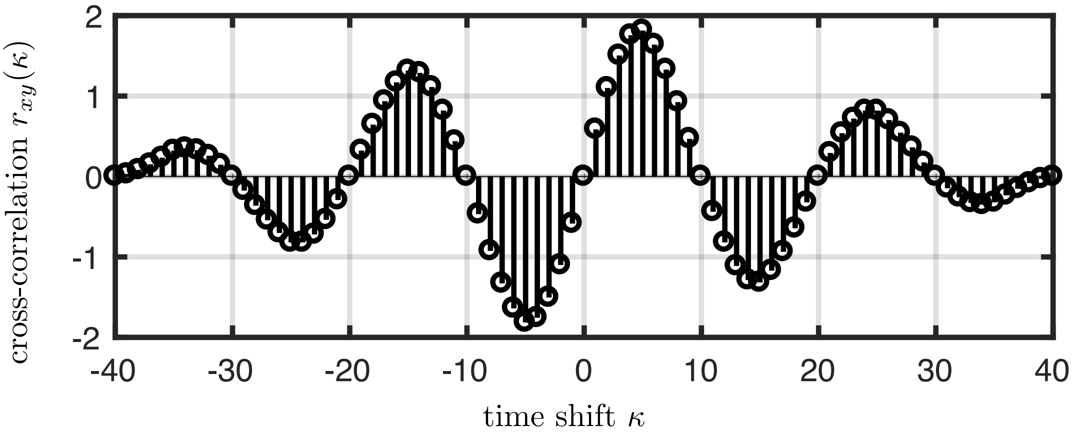


c) Now assume that the sampling frequency is doubled. How would the time shift change at which the cross-correlation reaches its maximum?

$a)$



$b)$



$c)$

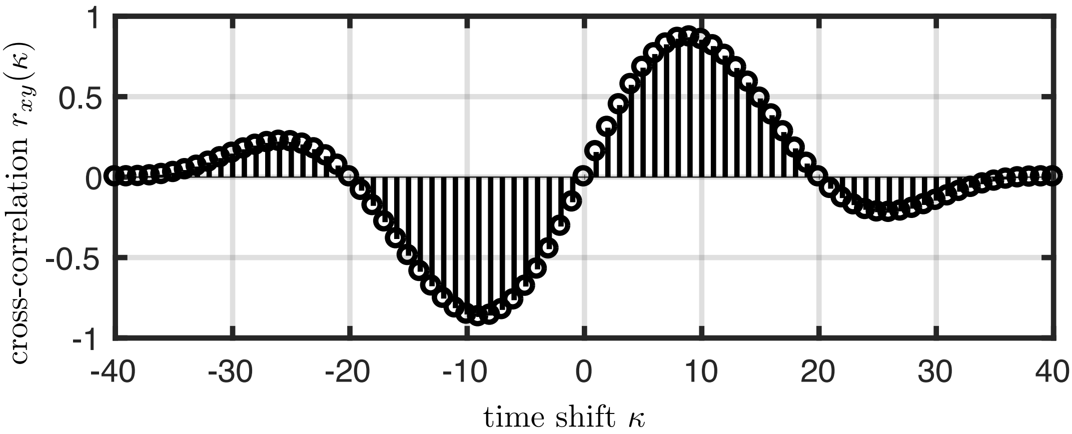


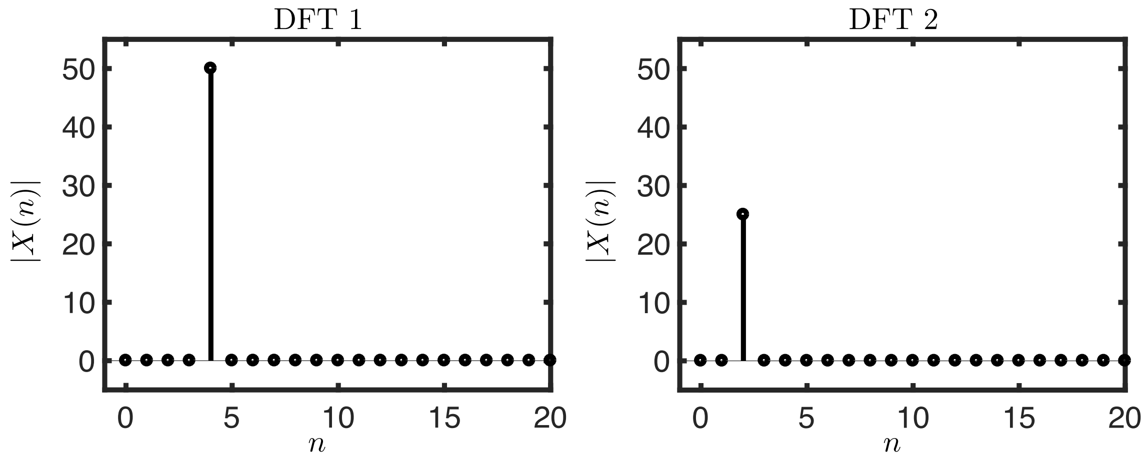
Fig. 3: Kreuzkorrelationsfunktionen

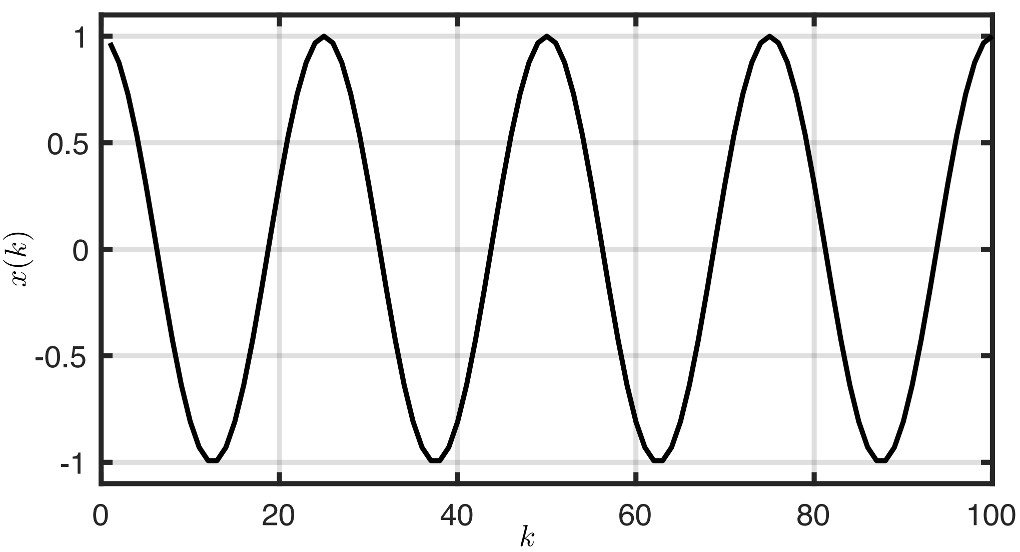
Task 4: Analysis of Signals (11 Points)

a) The following Figure shows two different DFTs based on two different signals containing $N = 100$ samples each. The signal $x(k)$ belonging to DFT 1 is shown in the following Figure.

Sketch a signal which corresponds to DFT 2 in the diagram of $x(k)$.

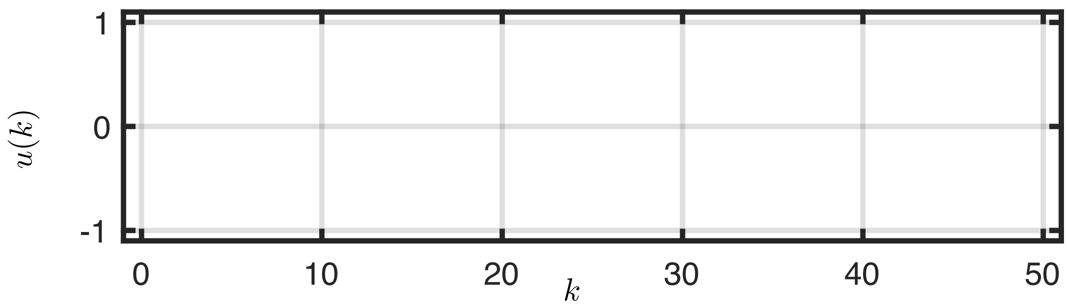
Note that the DFTs show only the first 20 values. For $20 < n \leq 49$ the absolute value of the DFT is $|X(n)| = 0$.



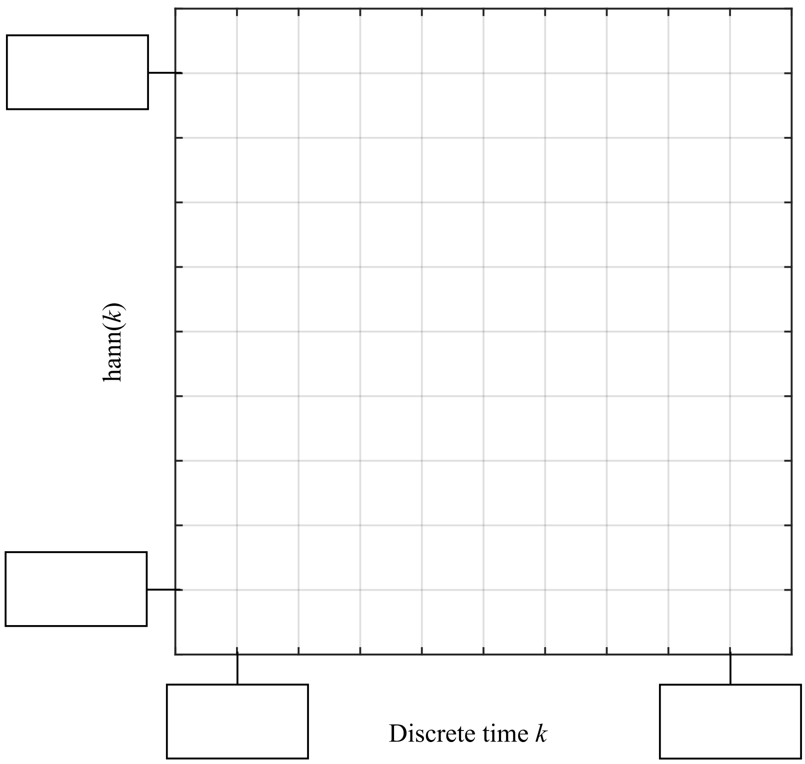


b) Sketch in the following diagram a cosine-shaped signal where the leakage effect appears significantly. Besides this, the signal must fulfill the following properties:

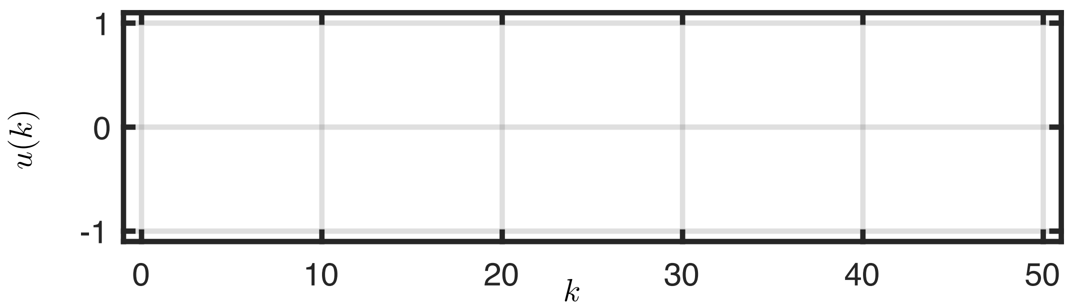
- $k = 0, 1, 2, \ldots 50$
- only one frequency,
- a maximum of 3 oscillations, and
- amplitude equal to one.



c) To minimize the leakage effect, a Hann-window should be applied to the signal from b). Sketch the Hann window in the following diagram and complete the description on the axes.



d) Now, apply the Hann-window from c) to the signal from b). Sketch the resulting signal in the following diagram.



Task 5: Static and Dynamic Behavior of Sensors (15 Points)

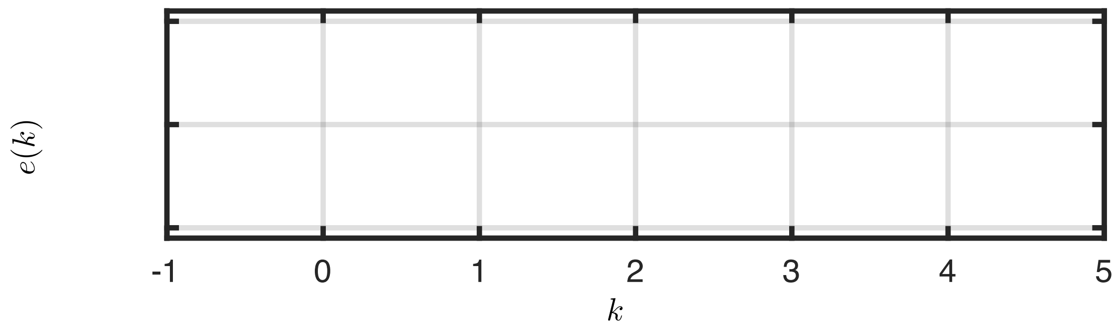
The behavior of a temperature sensor can be described by a linear IIR-filter:

$$T_m(z) = \frac{0.5\,z}{z - 0.5} T_{real}(z)$$

where $T_m(z)$ is the displayed temperature at the instrument and $T_{real}(z)$ is the true temperature.

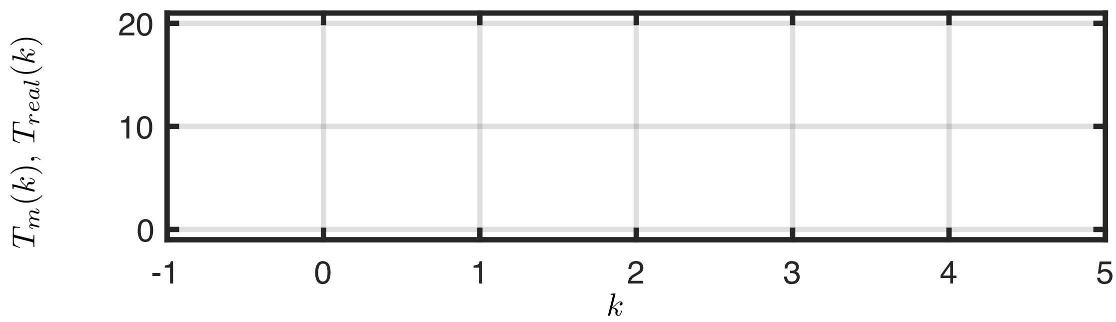
a) Determine the transfer function $G_1(z)$, with input $T_{real}(z)$ and the occurring errors $E(z)$ between the measured temperature $T_m(z)$ and the real temperature $T_{real}(z)$ as output.

b) Assume that the real and the measured temperature is 0°C at $k = 0$. At time instance $k = 1$ the real temperature changes to 20°C and stays constant for all $k > 1$. Calculate the response of the error and sketch signal of the error for $k = 0, 1, 2, 3, 4$ in the following diagram and write the correct values on the vertical axis.



c) How is the error called that arises due to this non-ideal behavior of the sensor and what can be done to avoid this error?

d) Now assume, that due to wear the gain of the sensor has changed to 1/2. Determine the transfer function $G_2(z) = T_m(z)/T_{real}(z)$ with this gain-error. Sketch the step response of $G_2(z)$ **qualitatively** if the true temperature T_{real} steps from 0°C ($k < 1$) to 20°C ($k \geq 1$). Draw the step of $T_{real}(k)$ in the same diagram. Note: Here is no need for an accurate calculation of the signal values.



e) How is the error called that arises due to the wrong sensor gain and what can be done to compensate this error?

Task 6: Discrete-Time Systems / Amplitude Response (15 points)

Let the following transfer function be given in the z-domain

$$G(z) = \frac{ez - 1}{e - z} \tag{4}$$

with e denoting Euler's number.

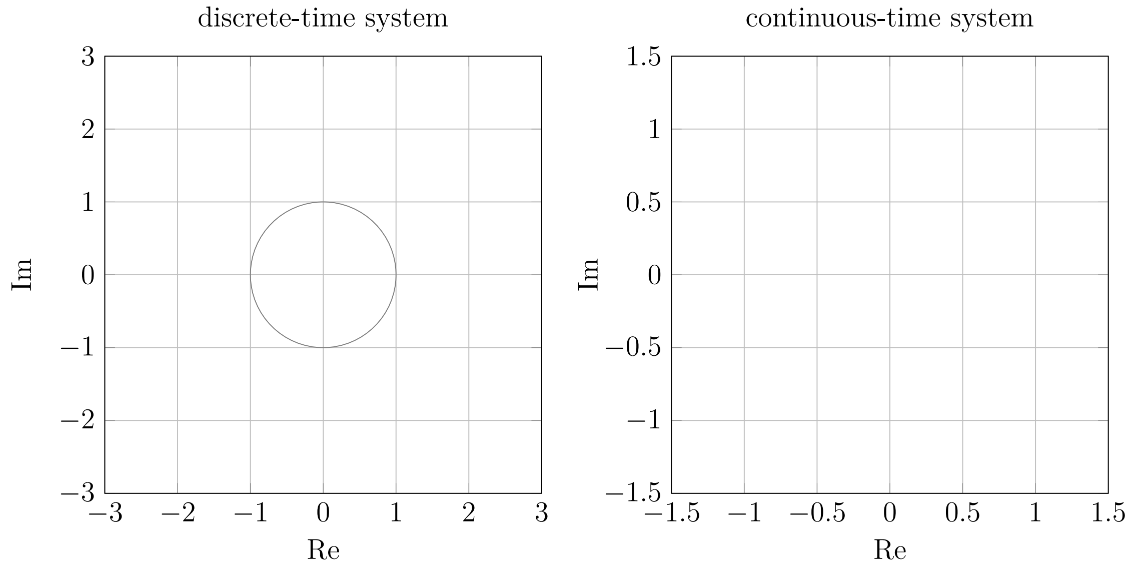
a) Calculate the pole and the zero of the transfer function. Draw them in the following diagram.

b) Calculate the amplitude response of the system, if the sampling time is given by $T_0 = 1$.

c) How is a system with this amplitude response called?

d) Determine the continuous-time system which has the same poles and zeros in the s-domain.

e) Draw the poles and zeros of the continuous system in the corresponding diagram. What is the difference for pole and zero locations between continuous time and discrete-time system?



Task 7: Combined System (15 points)

Two different systems are given. The transfer function of system (1) is

$$G_1(z) = \frac{2}{z} + \frac{0.8}{z^2} \ ,$$

while system (2) is described by the difference equation

$$y(k+1) + 0.6y(k) = 2u(k) + 0.8u(k-1) \ .$$

a) Carry out a z-transform of the difference equation of system (2) and form the transfer function in z. Shift in time, so that only powers ≤ 0 of z remain.

If you were not able to perform task (a) completely, use the following alternative transfer functions:

$$G_1'(z) = 3z^{-1} + 0.5z^{-3}$$

$$G_2'(z) = \frac{3z^{-1} + 0.5z^{-3}}{2 + 0.7z^{-1}}$$

b) How can systems (1) and (2) be characterized? Mark with „yes“/„no“.

	causal	non-causal	IIR	FIR
system (1)				
system (2)				

c) Draw block diagrams of systems (1) and (2). Herefore, use only z^{-1}-blocks and blocks for the coefficients.

d) Which system's impulse response is equal to 0, if $k \geq 4$?

e) System (3) is defined by $Y = G_1U_1 + G_2U_2$. Complete the following block diagram correctly.
Hint: Watch for similarities of the transfer functions.

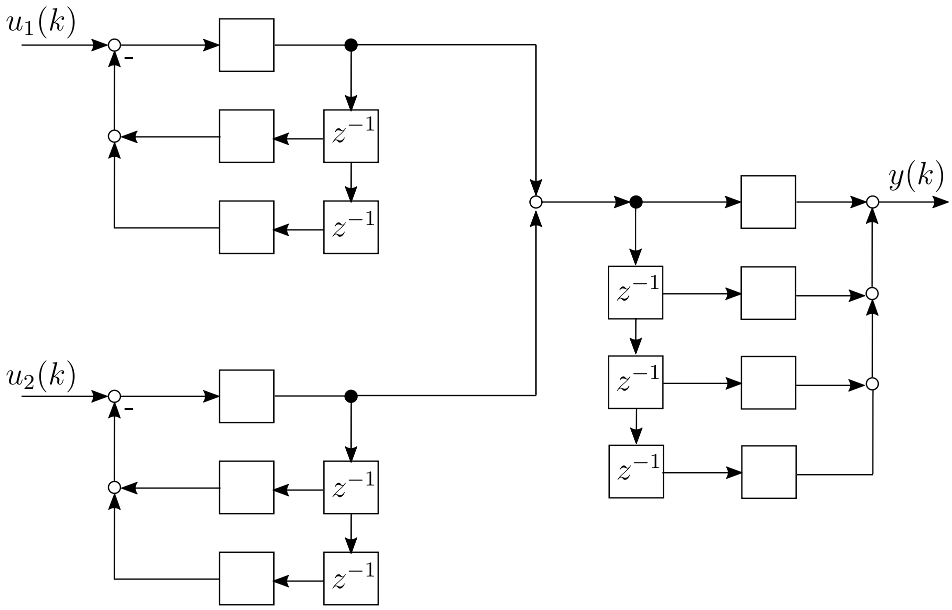
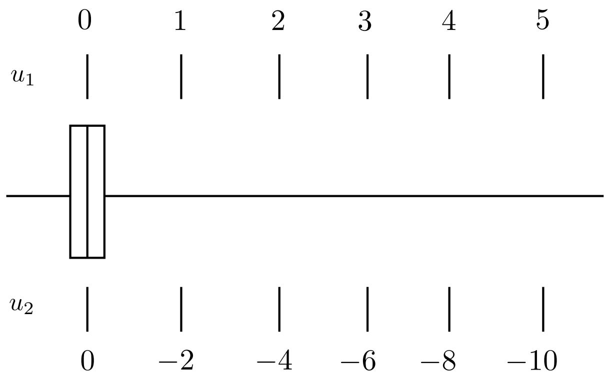


Fig. 4: Block diagram of system (3)

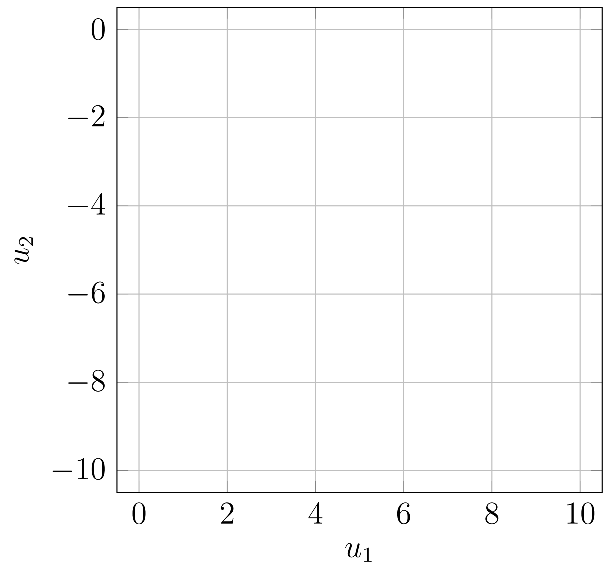
Task 8: Singular Value Decomposition (14 Points)

The inputs of a plant u_1 and u_2 can be changed with the depicted slide control. At the shown position $u_1 = u_2 = 0$ holds.



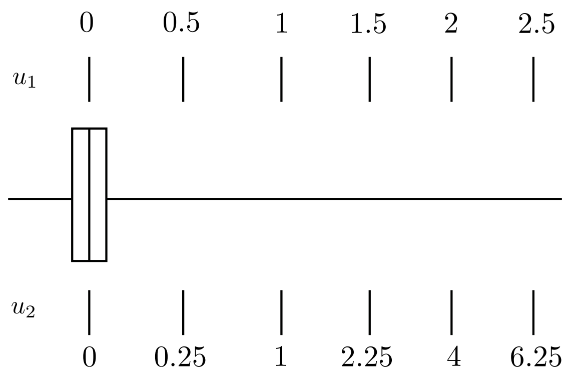
a) Plot the possible input combinations for $u_1 = 1, 2, 3, 4, 5$ in the diagram.

task a)/b)



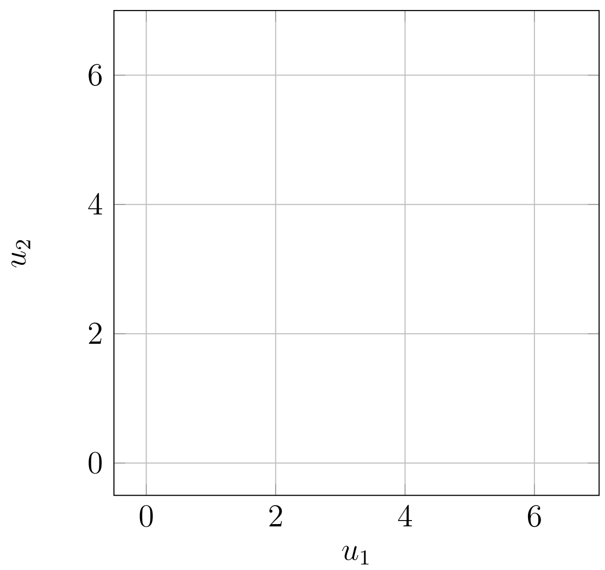
b) Perform a PCA geometrically. Now draw the principal axes and mark the principal axis with the singular value 0.

c) At another plant the inputs u_1 and u_2 can be modified with the slide control shown below.



Plot the possible input combinations for $u_1 = 0, 0.5, 1, 1.5, 2, 2.5$ in the diagram.

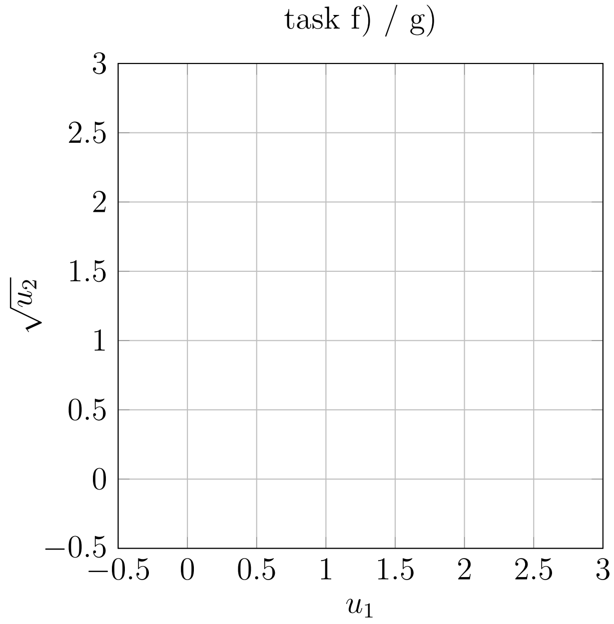
task c)/d)



d) Perform a PCA geometrically. Now draw the principal axes qualitatively.

e) Why is none of the singular vectors 0 although due to the slide controler the input dimension is 1.

f) Now plot in the diagram shown below u_1 on the one axis and $\sqrt{u_2}$ on the other axis.



g) Perform a PCA geometrically. Now draw the principal axes. Why is now one of the singular values 0 again?

Solutions:

Task 1: Comprehension Questions

a) How can you change the range of a moving coil mechanism meter?

- [x] If used as a voltmeter: connecting resistances in series.
- [x] If used as an amperemeter: connecting resistances in parallel.
- [ ] If used as a voltmeter: connecting resistances in parallel.

b) Supervised learning...

- [ ] ...does not require the desired output values.
- [ ] ...is often used for data-preprocessing.
- [x] ...requires the desired output values.

c) The so called leakage effect in the DFT can be reduced by ...

- [ ] ... addition of a window in the frequency domain.
- [x] ... convolution with a window in the frequency domain.
- [ ] ... multiplication of a window in the frequency domain.

d) What are characteristics of an idealized operational amplifier?

- [x] Output resistance = 0.
- [ ] Output resistance = ∞.
- [ ] Gain = 0.

e) A temporal sequence of N measurements that is transformed with the DFT results in a number of ...

- [ ] ... $\frac{N}{2}$ discrete frequencies.
- [x] ... N discrete frequencies.
- [ ] ... 2^N discrete frequencies.

f) The central limit theorem of statistics says that ...

- [x] ... the sum of several random variables follows approximately a normal distribution.
- [ ] ... the product of several random variables follows approximately a normal distribution.
- [ ] ... the sum of several random variables follows approximately an unique distribution.

g) A median filter...

- [ ] ...is always causal.
- [x] ...can be used to remove outliers.
- [x] ...has a step-like output as a response to a step input.

$\boxed{\sum 9}$

Task 2: Temperature Measurement

a) What type of error occurs through the usage of the approximate relationship $\hat{T}(R)$?

A systematic, static error. 2

b) Determine up to which resistance the relative absolute error between the approximate relationship $\hat{T}(R)$ and the true relationship $T(R)$ is less or equal to 10 %.

The relative absolute error is defined as:

$$e_{rel}(R) = \frac{|\hat{T}(R) - T(R)|}{T(R)}\,. \tag{5}$$

Because the true temperature is always above its approximation, we can write (5) as:

$$e_{rel}(R) = \frac{T(R) - \hat{T}(R)}{T(R)}\,.$$ 1

This relative absolute error should be less or equal to 0.1:

$$\begin{aligned}
e_{rel}(R) &\leq 0.1 \\
\frac{\hat{T}(R) - T(R)}{T(R)} &\leq 0.1 \\
\frac{4R - (4R + 0.001R^2 - 10^{-5}R^3)}{4R + 0.001R^2 - 10^{-5}R^3} &\leq 0.1 \\
\frac{R \cdot (-0.001R + 10^{-5}R^2)}{R \cdot (4 + 0.001R - 10^{-5}R^2)} &\leq 0.1 \\
-0.001R + 10^{-5}R^2 &\leq 0.4 + 10^{-4}R - 10^{-6}R^2 \\
0 &\leq -1.1 \cdot 10^{-5}R^2 + 1.1 \cdot 10^{-3}R + 0.4
\end{aligned}$$ 6

Solving the quadratic equation leads to two results from which only the positive one makes sense in this context $\rightarrow R_{10\%} \approx 247.139\,\Omega$. The solution $R = 0\,\Omega$ is not useful as well. Therefore the relative absolute error is smaller than 10 % as long as the resistance $R < 247.139\,\Omega$. 3

c) At which temperature (only the temperature range shown in Fig. 1) does the sensor (true relationship) have the highest sensitivity? Explain your answer briefly.

The highest sensitivity occurs at the range with the smallest gradient. Here, the highest sensitivity occurs at the highest shown resistances, approximately at 1120 K. 2

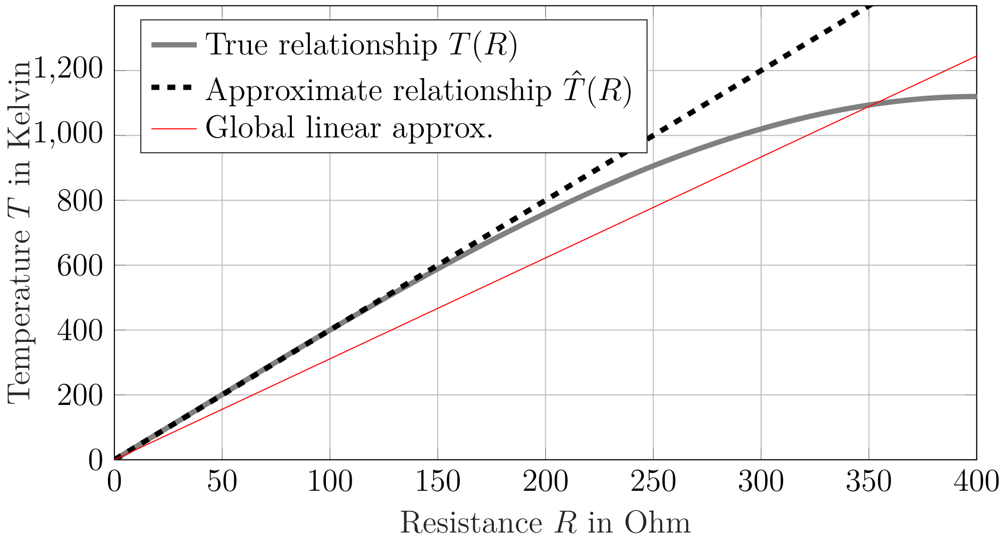
d) Use the Taylor series expansion to linearize the true relationship $T(R)$ for a operating point R_0. How are the linearization through the Taylor series expansion and the one shown in Fig. 1 ($\hat{T}(R)$) related?

For the linearization all terms with a higher order than one are neglected:

$$\begin{aligned}
Tf(R; R_0) &\approx \frac{T^{(0)}(R_0)}{0!}(R - R_0)^0 + \frac{T^{(1)}(R_0)}{1!}(R - R_0)^1 \\
&\approx \underbrace{4 \cdot R_0 + 0.001 \cdot R_0^2 - 10^{-5} \cdot R_0^3}_{T(R_0)} + \underbrace{(4 + 0.001 \cdot R_0 - 10^{-5} \cdot R_0^2)}_{T^{(1)}(R_0)}(R - R_0)
\end{aligned} \tag{6}$$ 4

Connection to $\hat{T}(R)$: If the operating point is chosen to be $R_0 = 0$, $\hat{T}(R)$ is obtained. 1

e) Sketch into Fig. 1 a good linear approximation, such that the mean error is minimized.



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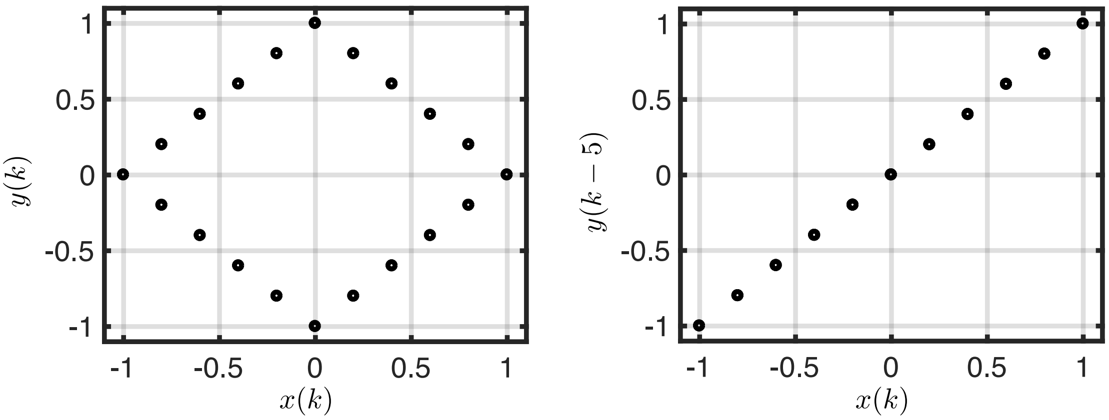
$\sum$ 21

Task 3: Cross-Correlation

a) Which of the cross-correlation functions shown in Fig. 3 belongs to the signals $x(k)$ and $y(k)$ shown in Fig. 2?

Cross-correlation a) belongs to the signals $x(k)$ and $y(k)$ shown in Fig. 2, because for a time shift of $\kappa = -5$ the two signals lie upon each other. Therefore the maximum cross-correlation value has to be reached for that time shift. $\boxed{6}$

b) Sketch the time shifted signal $y(k+\kappa)$ versus the signal $x(k)$ in the graphs below for $\kappa = 0$ and $\kappa = -5$.



$\boxed{12}$

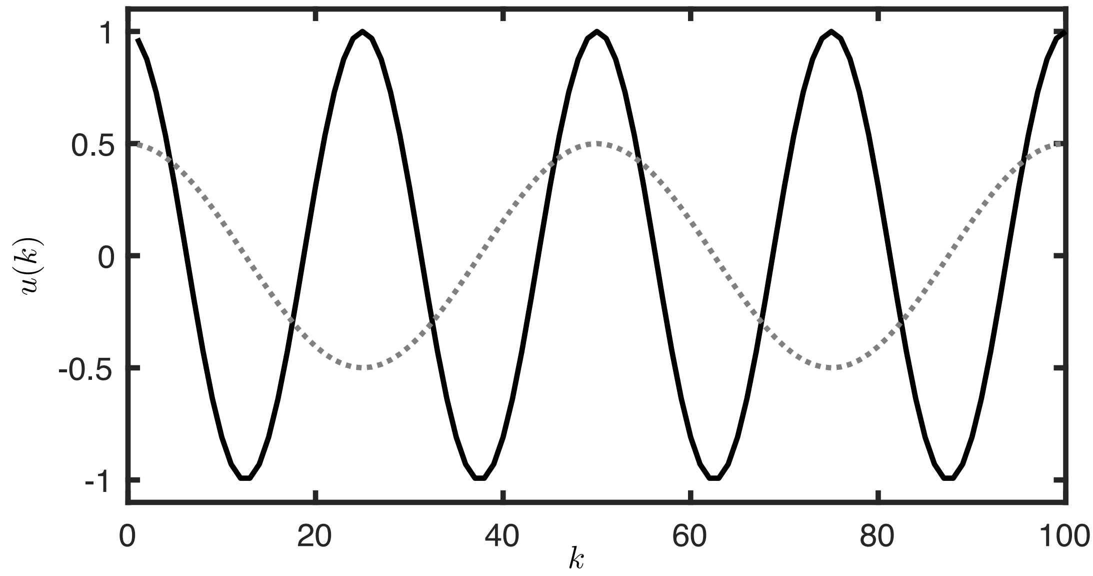
c) Now assume that the sampling frequency is doubled. How would the time shift change at which the cross-correlation reaches its maximum?

The time shift at which the maximum cross-correlation reaches its maximum value will be doubled $\kappa_{new} = 2\kappa_{old}$, because the number of samples is increased by a factor of two. $\boxed{2}$

$\boxed{\sum 20}$

<u>Task 4:</u> Analysis of Signals (11 Points)

a) Sketch a signal which corresponds to DFT 2 in the diagram of $x(k)$. There is a peak at $n = 4$ in DFT 1. Thus, the signal has an integer number of oscillations. In DFT 2 is a single peak at $n = 2$. This indicates that the frequency of the signal is the half of the frequency of signal $x(k)$. Furthermore, the peak has the half of the height compared to the peak in DFT 1. Thus, the signal corresponding to DFT 2 has the half amplitude compared to signal $x(k)$.



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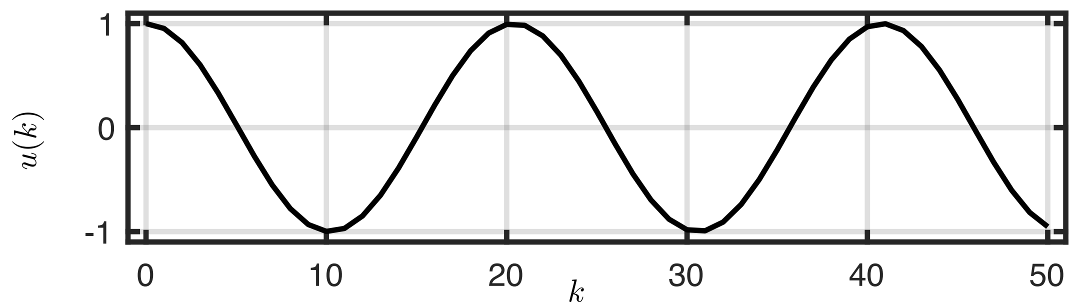
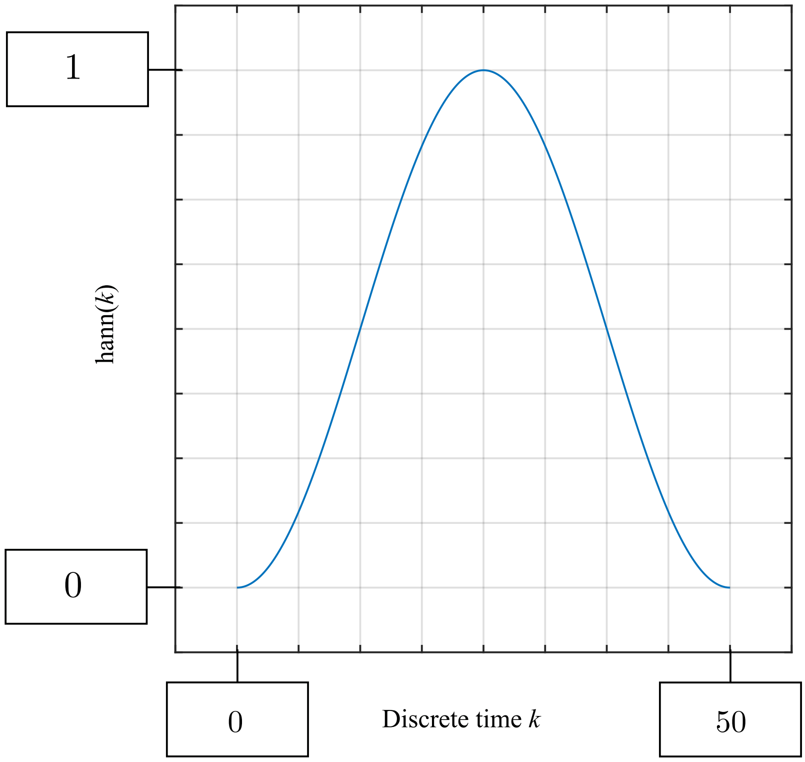
b) Sketch in the following diagram a cosine-shaped signal where the leakage effect appear significantly. Besides this, the signal must fulfill the following properties:



2

c) To minimize the leakage effect, a Hann-window should be applied to the signal from b). Sketch the Hann window in the following diagram and complete the description on the axes.



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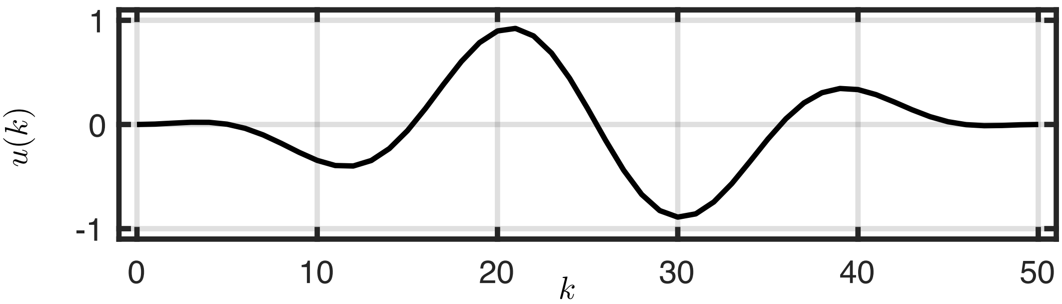
d) Now, apply the Hann-window from c) to the signal from b). Sketch the resulting signal in the following diagram.



2

Task 5: Static and Dynamic Behavior of Sensors (15 Points)

a) Determine the transfer function $G_1(z)$, with input $T_{real}(z)$ and the occurring errors $E(z)$ between the measured temperature $T_m(z)$ and the real temperature $T_{real}(z)$ as output.

$$E(z) = T_{real}(z) - T_m(z) \tag{7}$$

$$E(z) = T_{real}(z) - \frac{0.5z}{z-0.5} T_{real}(z) \tag{8}$$

$$E(z) = T_{real}(z) \left(1 - \frac{0.5z}{z-0.5}\right) \tag{9}$$

$$\Rightarrow \quad G_1(z) = 1 - \frac{0.5z}{z-0.5} \tag{10}$$

$$G_1(z) = \frac{0.5z - 0.5}{z-0.5} \tag{11}$$

$\boxed{1}$

b) Assume that the real and the measured temperature is 0°C at $k = 0$. At time instance $k = 1$ the real temperature changes to 20°C and stays constant for all $k > 1$. Calculate the response of the error and sketch signal of the error for $k = 0, 1, 2, 3, 4$ in the following diagram and write the correct values on the vertical axis.

$$E(z) = \frac{0.5z - 0.5}{z-0.5} T_{real}(z) \tag{12}$$

$$E(z)(z-0.5) = T_{real}(z)(0.5z - 0.5) \tag{13}$$

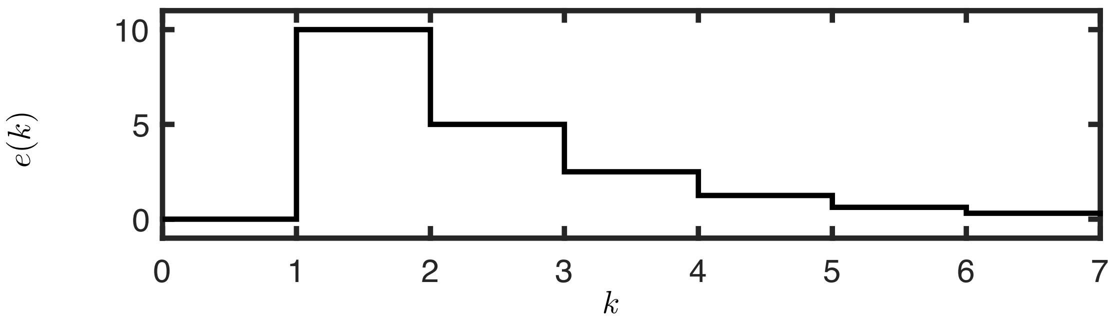
$$E(z)z - E(z)0.5 = T_{real}(z)0.5z - T_{real}(z)0.5 \tag{14}$$

$$\overset{\bullet}{\underset{\circ}{|}} \tag{15}$$

$$e(k+1) - 0.5e(k) = 0.5T_{real}(k+1) - 0.5T_{real}(k) \tag{16}$$

$$\Rightarrow \quad e(k) = 0.5e(k-1) + 0.5T_{real}(k) - 0.5T_{real}(k-1) \tag{17}$$

$k=0$	$T_{real}(0) = 0$	$e(0) = 0.5e(-1) + 0.5T_{real}(0) - 0.5T_{real}(-1)$	
		$e(0) = 0.5 \cdot 0 + 0.5 \cdot 0 - 0.5 \cdot 0$	$e(0) = 0$
$k=1$	$T_{real}(1) = 20$	$e(1) = 0.5e(0) + 0.5T_{real}(1) - 0.5T_{real}(0)$	
		$e(1) = 0.5 \cdot 0 + 0.5 \cdot 20 - 0.5 \cdot 0$	$e(1) = 10$
$k=2$	$T_{real}(2) = 20$	$e(2) = 0.5e(1) + 0.5T_{real}(2) - 0.5T_{real}(1)$	
		$e(1) = 0.5 \cdot 10 + 0.5 \cdot 20 - 0.5 \cdot 20$	$e(2) = 5$
$k=3$	$T_{real}(3) = 20$	$e(3) = 0.5e(2) + 0.5T_{real}(3) - 0.5T_{real}(2)$	
		$e(1) = 0.5 \cdot 5 + 0.5 \cdot 20 - 0.5 \cdot 20$	$e(3) = 2.5$
$k=4$	$T_{real}(4) = 20$	$e(4) = 0.5e(3) + 0.5T_{real}(4) - 0.5T_{real}(3)$	
		$e(4) = 0.5 \cdot 2.5 + 0.5 \cdot 20 - 0.5 \cdot 20$	$e(4) = 1.25$



6

c) How is the error called that arises due to this non-ideal behavior of the sensor and what can be done to avoid this error?

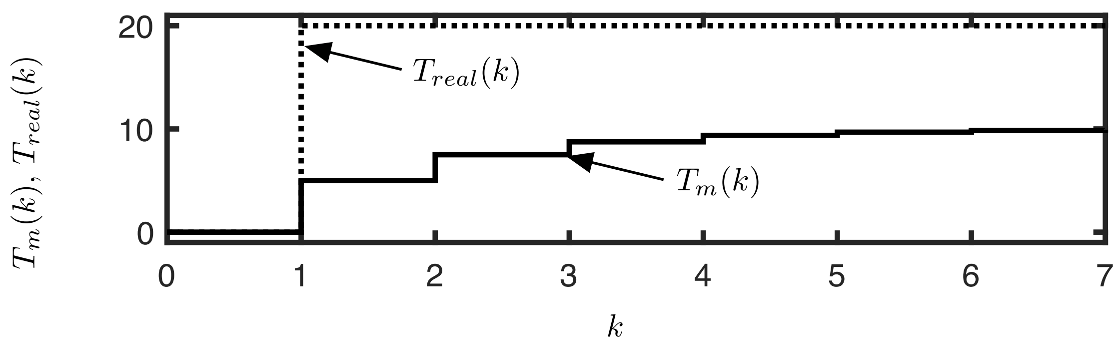
Dynamic error. 1

Can be avoided if one waits long enough. 1

d) Now assume, that due to wear the gain of the sensor has changed to 1/2. Determine the transfer function $G_2 = T_m(z)/T_{real}(z)$ with this gain-error. Sketch the step response of a sensor with this gain-error **qualitatively** if the step in the true temperature goes T_{real} from 0°C ($k < 1$) to 20°C ($k \geq 1$). Draw the step of T_{real} in the same diagram. Note: Here is no accurate calculation of the signal values needed, just a rough sketch of the resulting signal.

$$G_2(z) = \frac{0.25z}{z - 0.5} \tag{18}$$

1



3

e) How is the error called that arises due to the wrong sensor gain and what can be done to compensate this error?

Static error. 1

A correction factor is needed to compensate this error. 1

$\sum$ 15

Task 6: Discrete-Time System/ Amplitude Response (15 Points)

a) The zero is $n = e$, the pole $p = \frac{1}{e}$. 2

b) For the amplitude response it holds that

$$
\begin{aligned}
|G(i\omega)| &= \frac{|e^{i\omega} - e|}{|1 - e \cdot e^{i\omega}|} && (19)\\
&= \frac{|\sin(\omega) - e + i\cos(\omega)|}{|1 - e\sin(\omega) - ie\cos(\omega)|} && (20)\\
&= \frac{\sqrt{(\sin(\omega) - e)^2 + \cos^2(\omega)}}{\sqrt{(1 - e\sin(\omega))^2 + e^2\cos^2(\omega)}} && (21)\\
&= \frac{\sqrt{\sin^2(\omega) - 2e\sin(\omega) + e^2 + \cos^2(\omega)}}{\sqrt{1 - 2e\sin^2(\omega) + e^2\sin(\omega) + e^2\cos^2(\omega)}} && (22)\\
&= \frac{\sqrt{1 - 2e\sin(\omega) + e^2}}{\sqrt{1 - 2e\sin(\omega) + e^2}} && (23)\\
&= 1 && (24)
\end{aligned}
$$

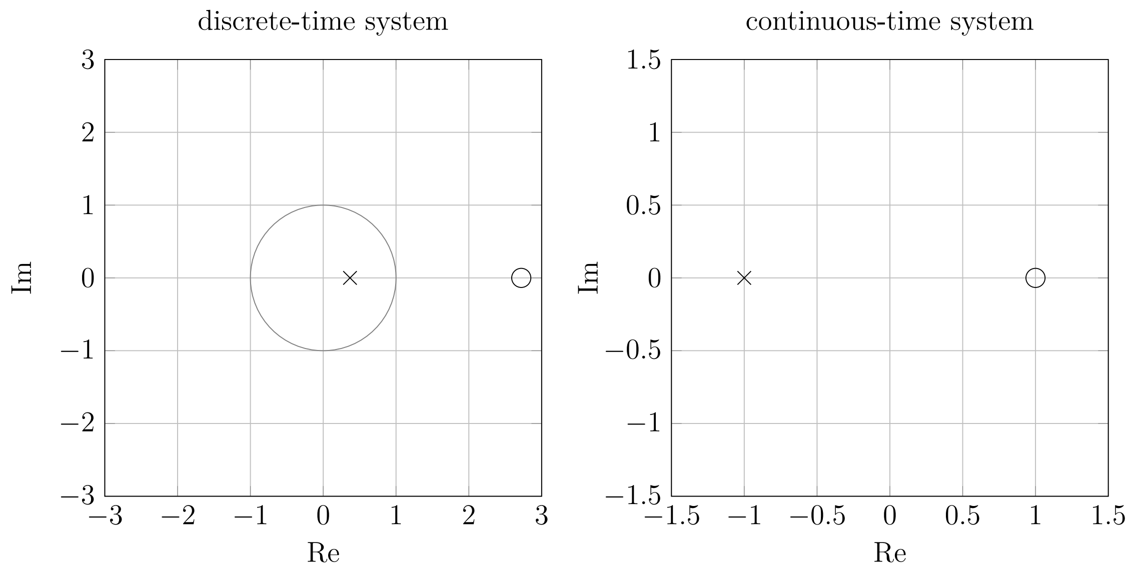
5

c) allpass 1

d) It holds that $z = e^{iT_s s}$ and thus $s = \frac{1}{T_s}\ln(z)$. Therefore it holds for the zero that $n_c = \ln(e) = 1$ and for the pole $p_c = \ln\left(\frac{1}{e}\right) = -1$. The transfer function thus is

$$G(s) = \frac{1 - s}{1 + s}. \tag{25}$$

4



e) For the continuous system the poles are mirrored at the imaginary axis. For the discrete system it holds that the zero is $\frac{1}{p}$. 3

$\sum 15$

Task 7: Combined System (15 points)

Two different systems are given. The transfer function of system (1) is

$$G_1(z) = \frac{2}{z} + \frac{0.8}{z^2} \ ,$$

while system (2) is described by the difference equation

$$y(k+1) + 0.6y(k) = 2u(k) + 0.8u(k-1) \ .$$

a) Carry out a z-transform for the difference equation of system (2) and form the transfer function in z. Shift in time, so that only powers ≤ 0 of z remain.

$$\begin{aligned} y(k+1) + 0.6y(k) &= 2u(k) + 0.8u(k-1) \\ y(k) + 0.6y(k-1) &= 2u(k-1) + 0.4u(k-2) \\ Y(z) + 0.6z^{-1}Y(z) &= 2U(z)z^{-1} + 0.4U(z)z^{-2} \\ Y(z) \cdot (1 + 0.6z^{-1}) &= U(z) \cdot (2z^{-1} + 0.4z^{-2}) \\ \Rightarrow G(z) = \frac{Y(z)}{U(z)} &= \frac{2z^{-1} + 0.8z^{-2}}{1 + 0.6z^{-1}} \end{aligned}$$

$\boxed{3}$

b) How can systems (1) and (2) be characterized? Mark with „yes“/„no“.

	kausal	akausal	IIR	FIR
System (I)	yes	no	no	yes
System (II)	yes	no	yes	no

$\boxed{2}$

c) Draw block diagrams of systems (1) and (2). Herefore, use only z^{-1}-blocks and blocks for the coefficients.

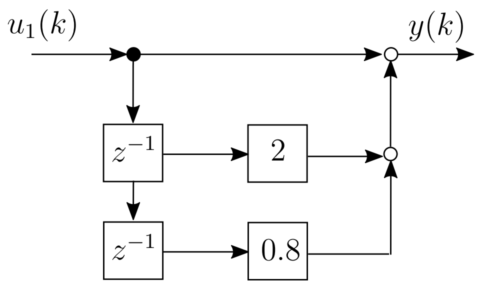


Fig. 5: FIR-System (System (1))

$\boxed{2}$

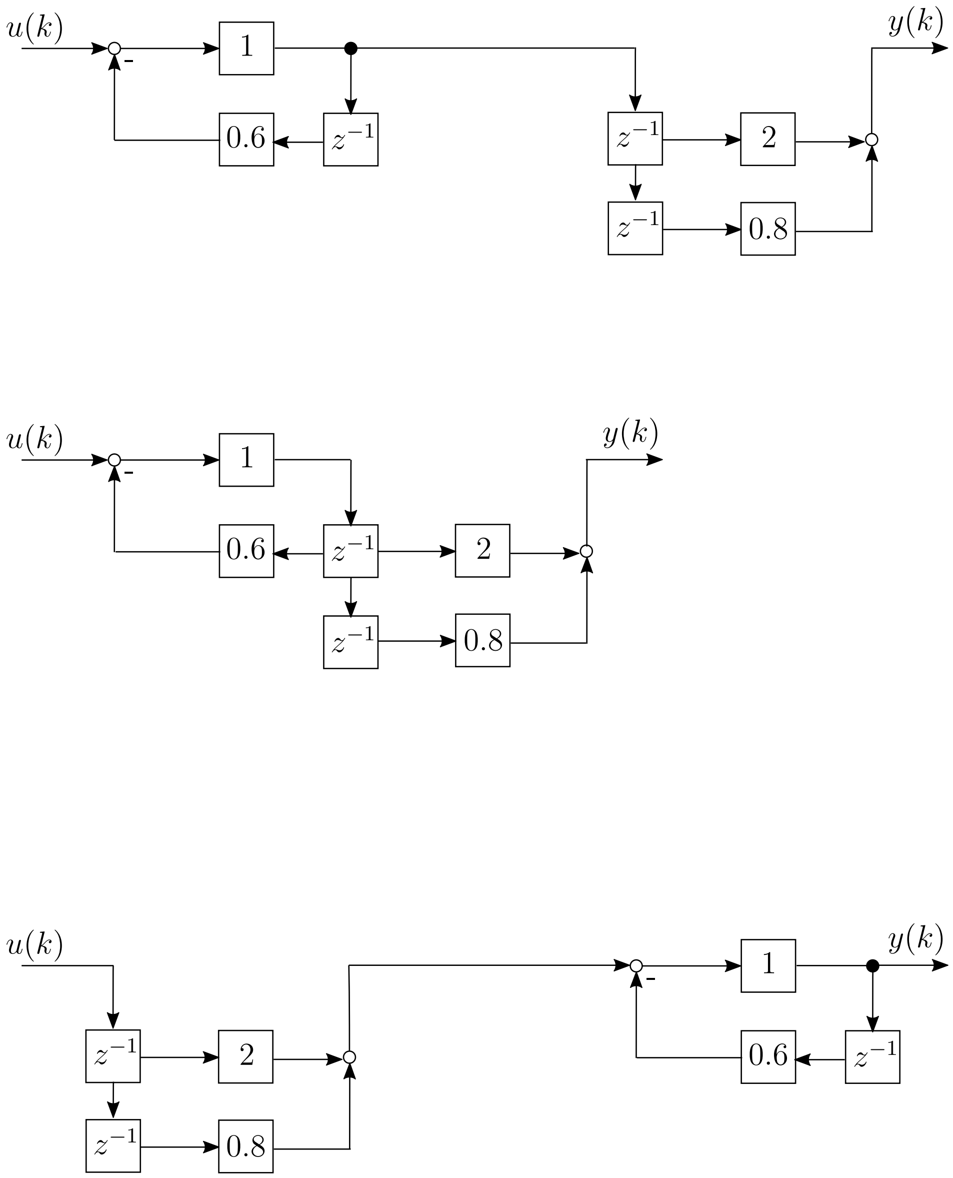


Fig. 6: IIR-system (System (2)), different variations

4

d) The impulse response of which system is equal to 0, if $k \geq 4$?

The impulse response of the FIR-system. (System (1)) 1

e) System (3) is defined by $Y = G_1 U_1 + G_2 U_2$. Complete the following block diagram correctly.

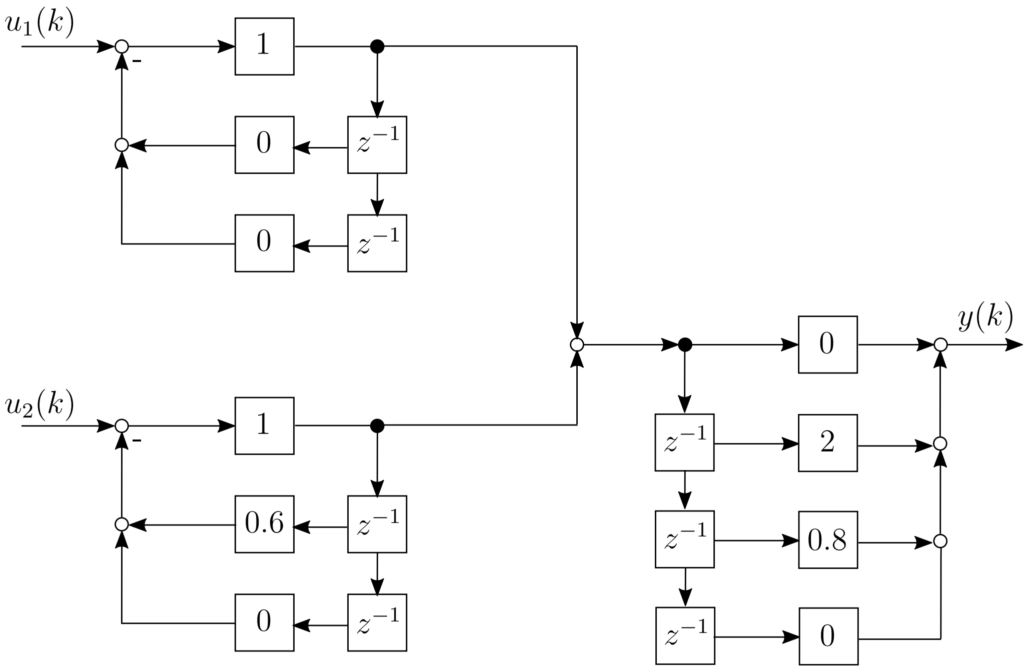
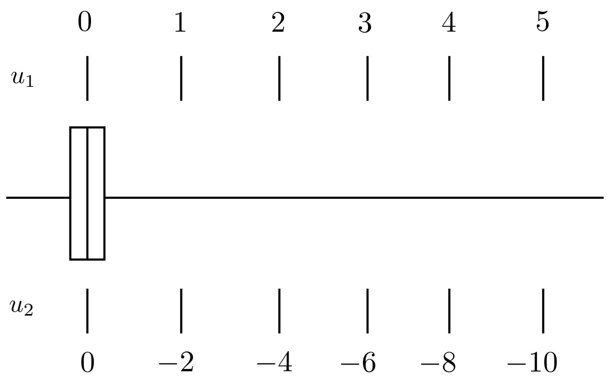


Fig. 7: Block diagram of system (3)

3

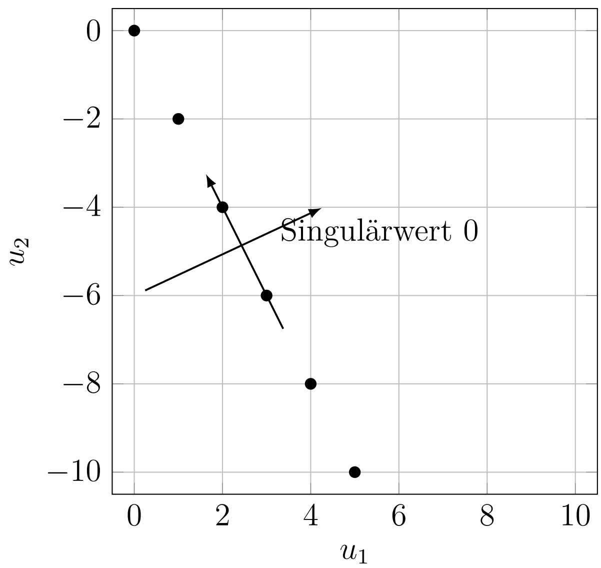
$\sum 20$

Task 8: Singular value decomposition

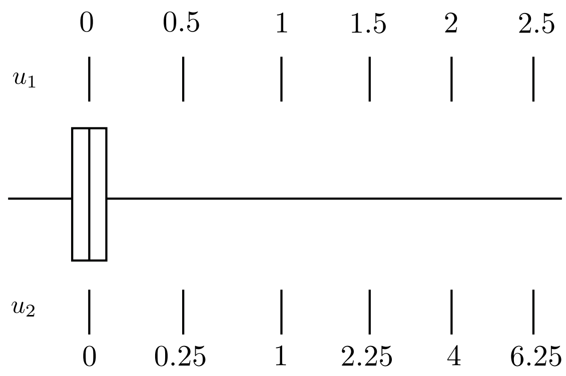


a) see diagram 1

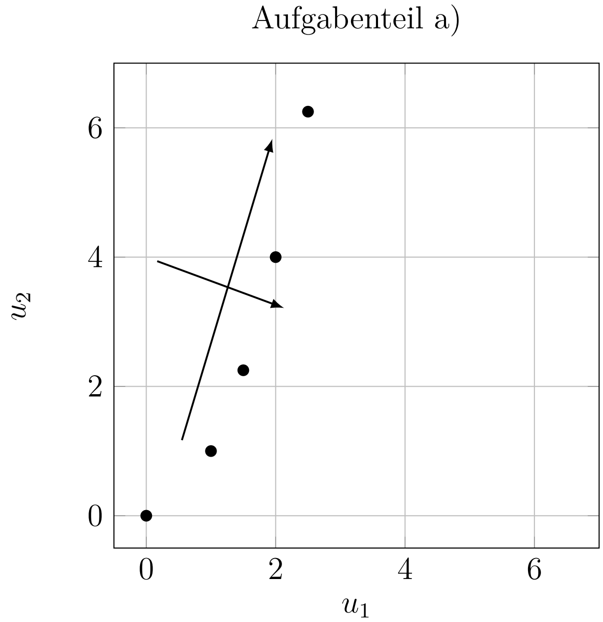
Aufgabenteil a)



b) see diagram 3



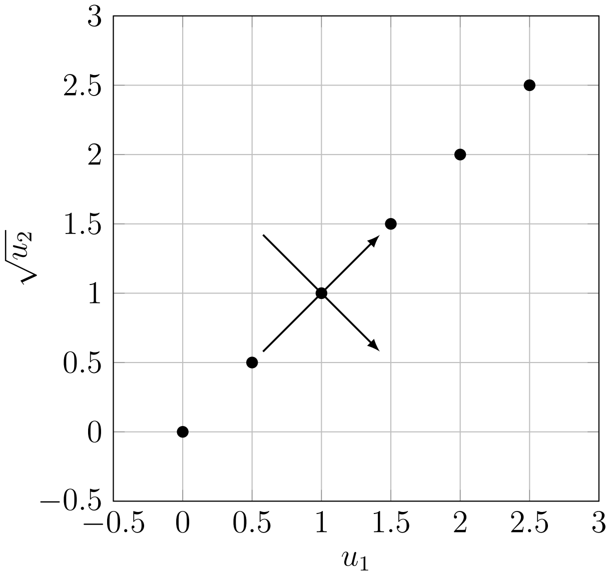
c) see diagramm 1



d) see diagram 1

e) The relation between $u_2 = u_1^2$ ist nolinear. Thus there is a variance orthogonal to the first singular vector. 3 1

Aufgabenteil a)



f) see diagramm 1

g) Because due to the nonlinear transformation$u_3^* = \sqrt{u_2}$ a linear relationship between u_1 und u_3^* holds. 3

$\sum$ 14