Sensorics Exam

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23rd of July 2019

Name:	
Mat.-No.:	
Grade:	

Task:	T1	T2	T3	T4	T5	T6	T7	Sum
Scores:	18	16	18	12	23	15	18	120
Accomplished:								

Task 1: Comprehension Questions (18 points)

Mark the correct answers clearly.
Every question has one, two or three correct answers!
For every correctly marked answer you will get one point. For every wrong answer a point will be subtracted, but a question will never be rated with negative points.

a) A system $G(z)$ in the z-domain is always ...
- ☐ ... stable if all poles are inside the unit circle.
- ☐ ... stable if all poles have a negative imaginary part.
- ☐ ... stable if it is calculated from a stable G(s) by using bilinear transformation.
- ☐ ... unstable if poles have no imaginary part.

b) A non-parametric method typically ...
- ☐ ... needs a **large** number of parameters to model a large number of data samples.
- ☐ ... needs a **small** number of parameters to model a large number of data samples.
- ☐ ... has (e.g. physically) interpretable parameters.
- ☐ ... is not the result of structural considerations, but mainly motivated by model accuracy.

c) Which statements concerning parametric and non-parametric models are true?
- ☐ Impulse response models (FIR) are parametric.
- ☐ IIR, AR and ARMA models are parametric.
- ☐ Transfer function models are parametric.
- ☐ The Discrete Fourier Transformation (DFT) is non-parametric.

d) Which statements regarding *Parametric Frequency Analysis* are true?
- ☐ The method is less influenced by noise compared to DFT.
- ☐ The peaks of the spectrum are not distorted by leakage or the picket fence effect.
- ☐ The frequency is discretized (similar to DFT).
- ☐ The calculation of the parameters requires nonlinear optimization.

e) Assess following statements regarding errors:
- ☐ The quantization error is half as large if a 16 bit A/D converter is used instead of an 8 bit converter.
- ☐ The absolute error of a measurement is the difference between the measured value and the true value, divided by the true value.
- ☐ Stochastic errors can be reduced by averaging over several measurements.
- ☐ Random errors are typically reduced with $1/N$, where N is the number of measurements.

f) What is the meaning of the terms *dead zero* and *life zero*?

- ☐ Live zero is a measurement technique which allows detection of a broken wire.
- ☐ Live zero means a measurement device is broken and has to be replaced.
- ☐ If a measurement of a signal $\neq 0$ has a value $= 0$, it is called a dead zero.
- ☐ If a measurement of a signal $= 0$ has a value $= 0$, it is called a dead zero.

.

g) Explain apparent, active and reactive power.

- ☐ Apparent power is the power an electrical device (e.g. a motor) actually delivers.
- ☐ Active and reactive power depend on the phase shift φ between voltage and current.
- ☐ Active power is the power lost due to phase shifting and should usually be avoided.
- ☐ Reactive power is calculated from apparent power P_S as follows: $P_B = P_S \cdot \cos(\varphi)$.

h) Operational amplifiers ...

- ☐ ... are passive components (need no external energy source).
- ☐ ... are amplifiers with a very low gain.
- ☐ ... have a very high **output** resistance.
- ☐ ... have a very high **input** resistance.

i) What is a principal component analysis (PCA)?

- ☐ PCA is a supervised learning method.
- ☐ PCA is often used for data preprocessing.
- ☐ PCA is used for feature **selection**, meaning some of the inputs are completely discarded from further use.
- ☐ PCA is used for feature **extraction**, meaning all original inputs may still be necessary to calculate a lower number of features for the next processing step.

j) Explain positive (PTC) and negative temperature coefficient (NTC) resistance thermometers.

- ☐ For PTC thermometers no wire calibration is necessary due to the very low resistance of the temperature sensor.
- ☐ NTC thermometers are semiconductor elements, PTC thermometers use a sensor made out of metal.
- ☐ NTC thermometers have strongly nonlinear characteristics.
- ☐ NTC thermometers are by far more accurate then PTC thermometers.

Task 2: Statistics (16 points)

Figure 1 shows contour plots of two-dimensional Gaussian probability density functions $p(u_1, u_2)$. Each distribution is characterized by its mean $(\mu_1\ \mu_2)$ and its covariance matrix $(\underline{\Sigma})$.

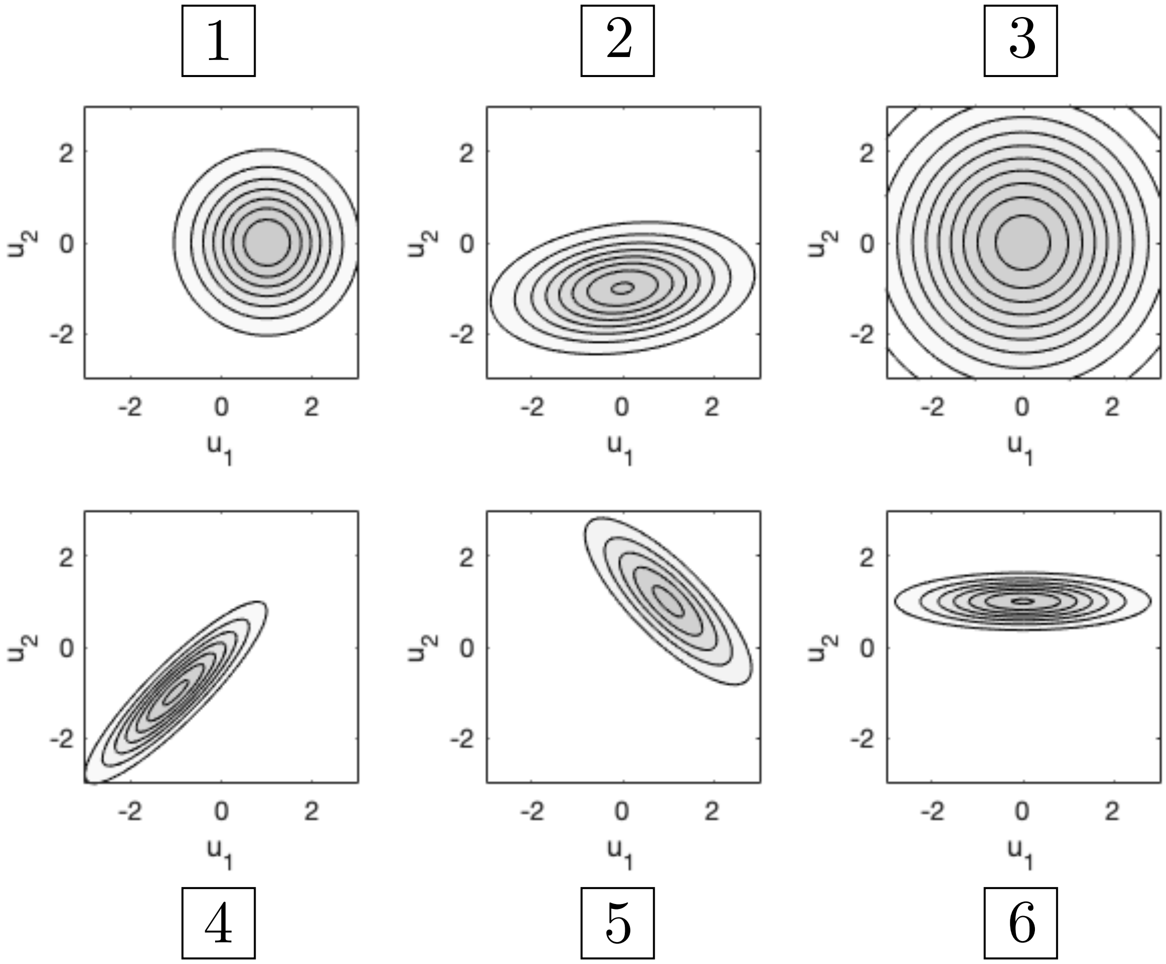


Figure 1: Contour plots of two-dimensional Gaussian probability density functions

a) Estimate the mean $(\mu_1\ \mu_2)$ of $p(u_1, u_2)$ for every density (1-6) and write them in the table below.

b) Match every density from Fig. 1 to the correct covariance matrix $\underline{\Sigma}$ of the following matrices $\underline{A}$ to $\underline{L}$.

$$\underline{A} = \begin{bmatrix} 1 & -0.4 \\ 0 & -1 \end{bmatrix} \qquad \underline{B} = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix} \qquad \underline{C} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\underline{D} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} \qquad \underline{E} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \qquad \underline{F} = \begin{bmatrix} 1 & -0.8 \\ -0.8 & 1 \end{bmatrix}$$

$$\underline{G} = \begin{bmatrix} 2 & 0.2 \\ 0.2 & 0.5 \end{bmatrix} \qquad \underline{H} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \underline{I} = \begin{bmatrix} 1 & 0.2 \\ 0.2 & -0.5 \end{bmatrix}$$

$$\underline{J} = \begin{bmatrix} 2 & 0 \\ 0 & 0.1 \end{bmatrix} \qquad \underline{K} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \underline{L} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Plot	μ_1	μ_2	$\underline{\Sigma}$
1			
2			
3			
4			
5			
6			

c) Draw the graph of a Student's t-distribution for a medium number of degrees of freedom (label the axis accordingly).

d) Draw the graph of a Student's t-distribution for a lower number of degrees of freedom into the same figure. Use a dashed line for this graph in order to distinguish both graphs.

e) How do the integrals from -∞ to ∞ of these t-distributions differ?

Task 3: Nonlinearity (18 points)

The input signal

$$u(k) = 1 + A\sin(k\frac{\pi}{5})$$

is fed into a nonlinear function

$$f\left(u\left(k\right)\right) = u^2\left(k\right) = y\left(k\right). \qquad | \qquad \underline{\mathit{Hint}}: \sin^2\left(x\right) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

The block diagram is shown in Fig. 2.

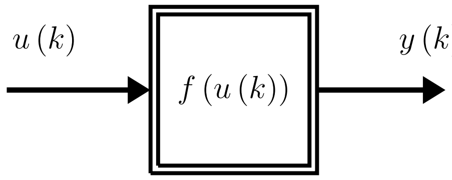


Figure 2: I/O-diagram of nonlinear function f

a) Imagine you carry out a DFT of an arbitrary signal of length N in your computer. What is the difference between $|X(n)|$ and $|X(N-n-1)|$ if n is an integer in the range $0 \leq n \leq \frac{N}{2}$?

b) Calculate $y(k)$ if $A = 1$ and plot it into Fig. 3.

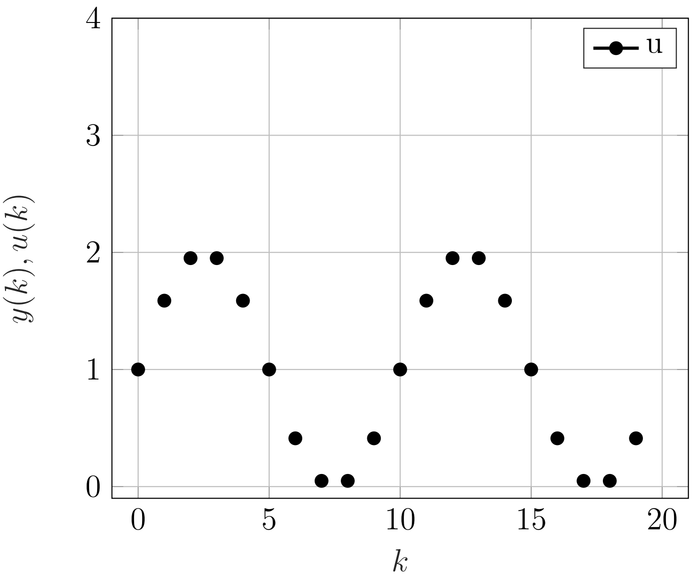


Figure 3: In- and outputs for $A = 1$

c) Draw qualitatively the absolute value of the DFT of the signal $u(k) = 1 + A\sin\left(k\frac{\pi}{5}\right)$ for $A = 1$. Label the axes according to the task. Take care that relations can be seen.

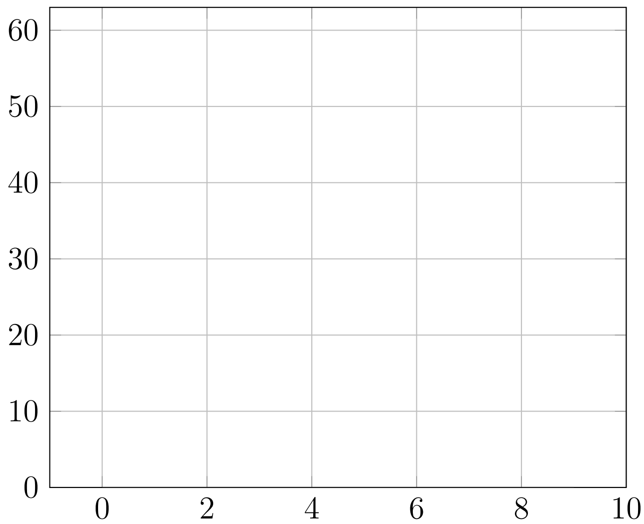


Figure 4: Frequency spectrum from the signal $u(k)$ with $A = 1$

d) Draw qualitatively the absolute value of the DFT of the signal $y(k) = u^2(k)$ with $A = 1$. Label the axes according to the task. Take care that relations can be seen. The relation of the offset can be neglected in this regard.

Figure 5: Frequency spectrum from the signal $y(k)$ with $A = 1$

e) Draw qualitatively the absolute value of the DFT of the signal $y(k) = u^2(k)$ with $A = 2$. Label the axes according to the task. Take care that relations can be seen. The relation of the offset can be neglected in this regard. What is different compared to the case for $A = 1$?

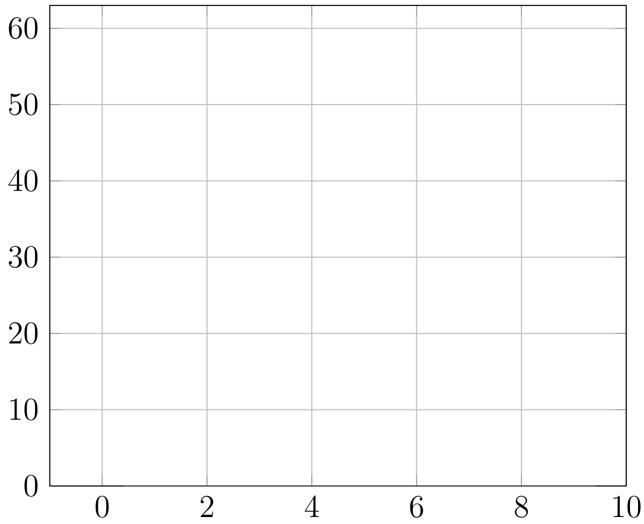
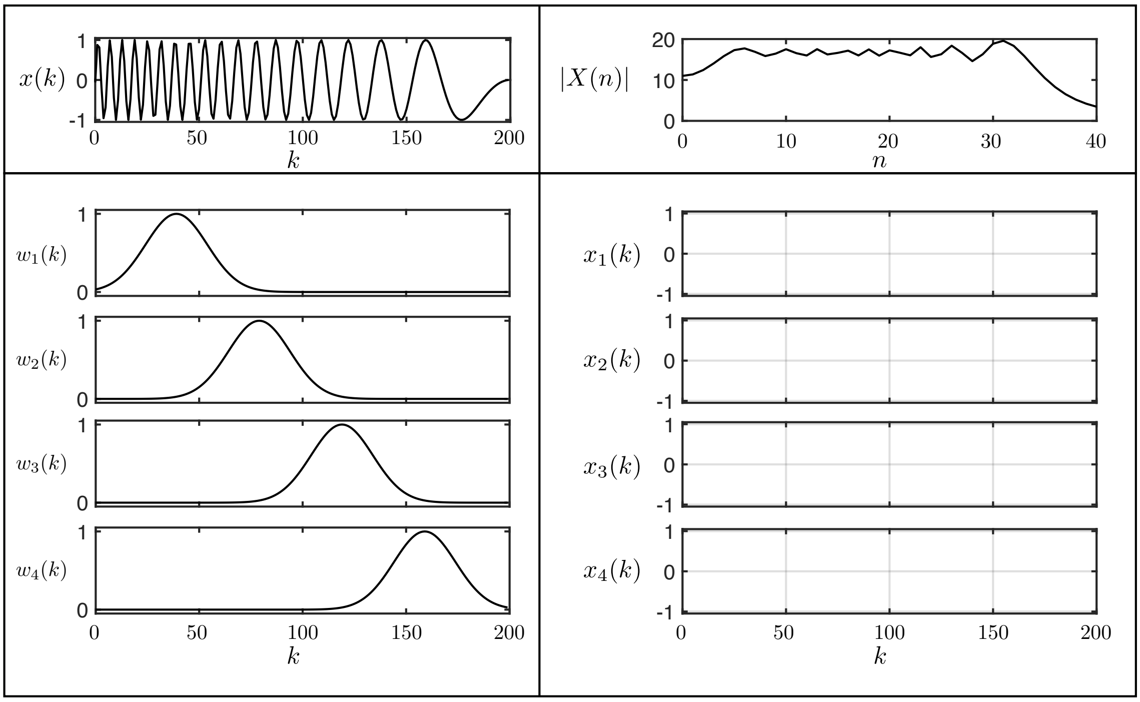


Figure 6: Frequency spectrum from the signal $y(k)$ with $A = 2$

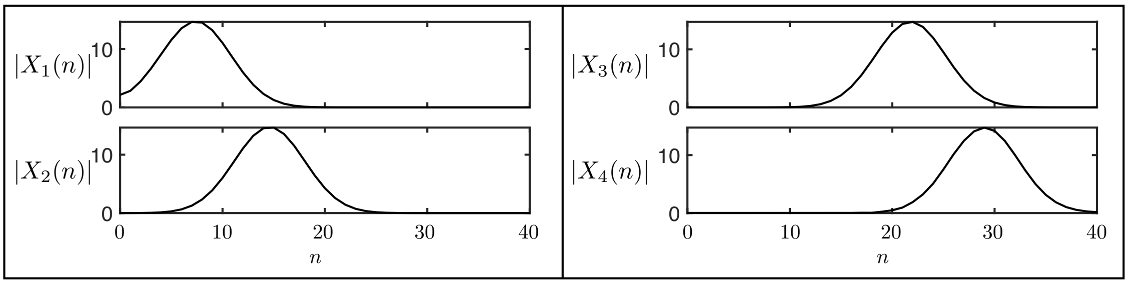
Task 4: DFT (12 points)

A sine-shaped signal $x(k)$ is given. The frequency of the signal changes linearly with the time. The first 40 absolute values of the corresponding DFT are also given by $|X(n)|$ as well as four different window functions $(w_1(k), \ldots, w_4(k))$.

a) Apply each window function $(w_1(k), \ldots, w_4(k))$ to the signal $x(k)$ and sketch the resulting signals $x_1(k), \ldots, x_4(k)$.



b) Assign to each signal $x_i(k)$ the corresponding absolute value of the DFT $|X_j(n)|$.



Signal $x_i(k)$	$x_1(k)$	$x_2(k)$	$x_3(k)$	$x_4(k)$
Corresponding $\lvert X_j(n)\rvert$				

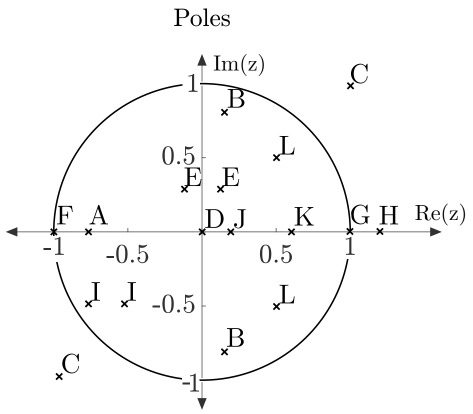
c) What is the gathered information by using different time shifted window functions? How is this procedure called?

d) Assume a sinusoidal signal with $2f_0$ is sampled with a sampling frequency f_0 at which normalized discrete frequency n occurs the highest peak?

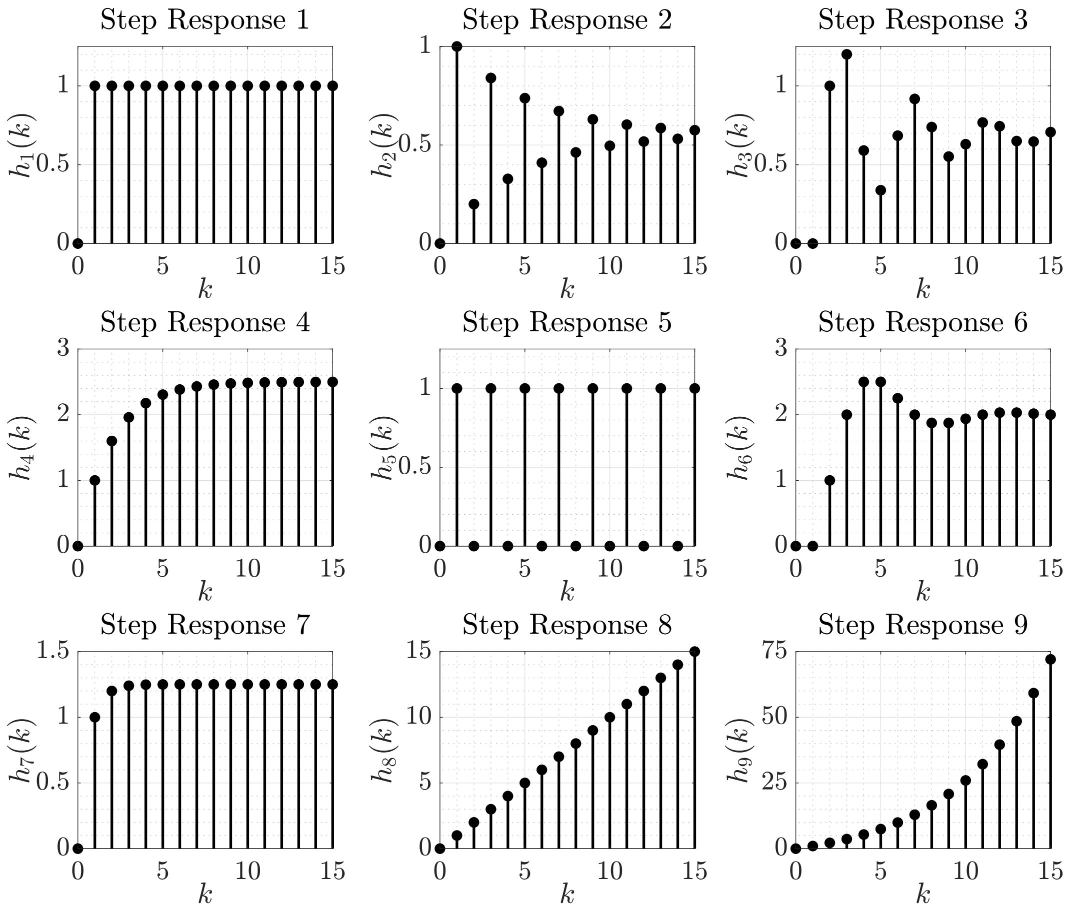
Task 5: Step Responses (23 points)

Given are the pole locations for 12 different systems (A-L) in the pole zero map and 9 different step responses (1-9).

a) Match the pole locations to the step responses. Note down your answers in the given table. (Note: Three pole locations do not belong to any step response. All transfer functions have the structure $G(z) = \frac{1}{A(z)}$ with $A(z) = \prod_{i=1}^{n}(z - p_i)$.)



Step Response	Poles
1	
2	
3	
4	
5	
6	
7	
8	
9	



b) Assume that the pole of system J is at $p_J = 0.2$. Calculate the systems transfer function $G_J(z)$ and apply the final value theorem to the systems response with input signal $u(k) = 2 \cdot \sigma(k)$.

c) System $G_J(z)$ is now expanded to system $G_{JJ}(z)$, which is $G_J(z)$ with $G_J(z)$ series-connected. What is the gain of $G_{JJ}(z)$?

<u>Task 6:</u> Time-Discrete Systems (15 Points)

A system with output $y(k) = b_0 \cdot u(k) - a_1 \cdot y(k-1)$ is given with $a_1 = -1$ and $b_0 = 0.3$.

a) Calculate the output of the system with the given input signal for $k = 0, 1, ..., 5$ and sketch the corresponding signal in Fig. 15. Use $y(k) = 0$ for all $k < 0$ as the initial condition.

b) What is the global behavior of the system $(P, PI, PD, PT_1, I, D, \ldots)$?

c) Sketch the block diagram of the given system.

d) Determine the poles of the given system.

e) Calculate the final value of the impulse response.

f) Which of the following statements is correct?

- ☐ The system is stable.
- ☐ The system is unstable.
- ☐ The system is marginally stable.

g) How can you modify the given system to meet the two non-checked statements in task f)?

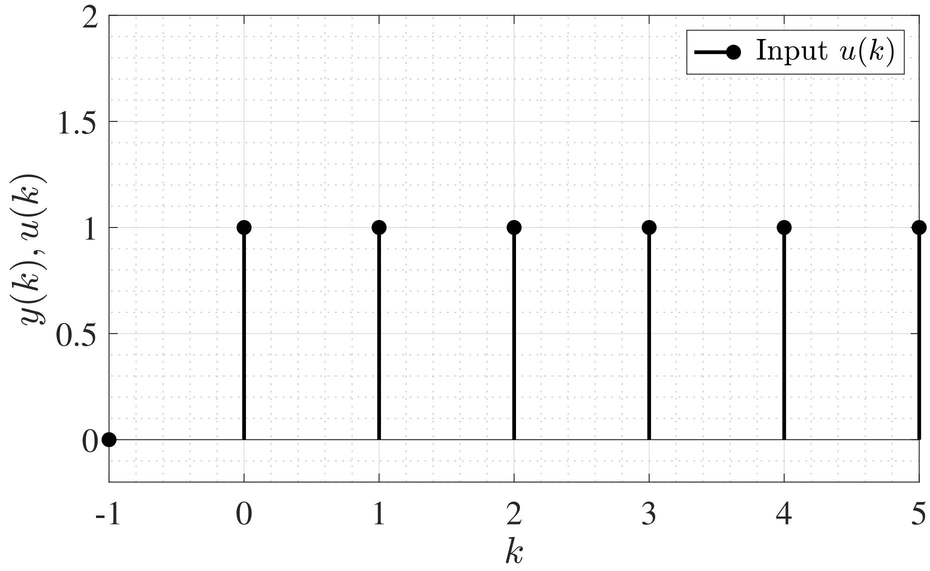


Figure 7: Input signal plotted over the discrete time.

Task 7: Comb Filter (18 Points)

The linear filter

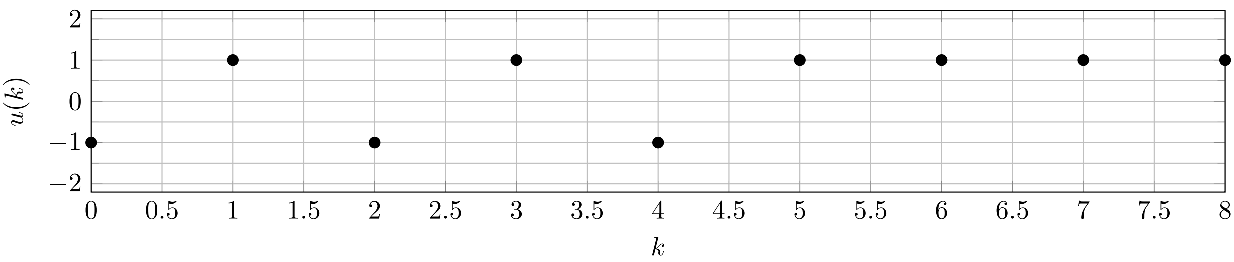
$$G(z) = \frac{Y(z)}{U(z)} = \frac{z^m + a}{z^m}$$

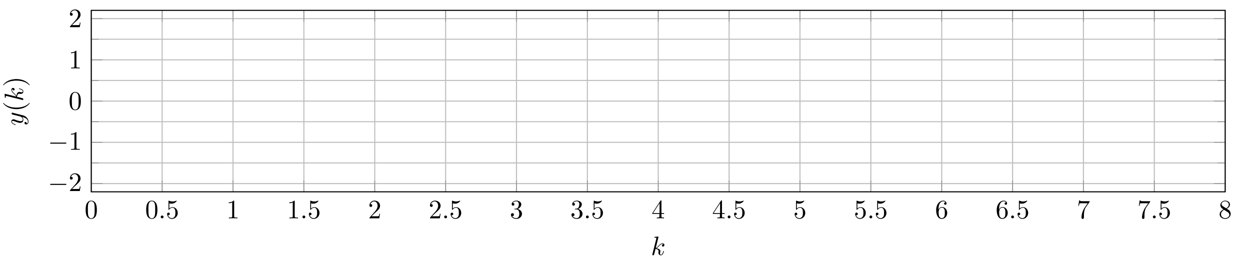
is given.

a) Is this an FIR or an IIR filter? Give a reason (1 sentence).

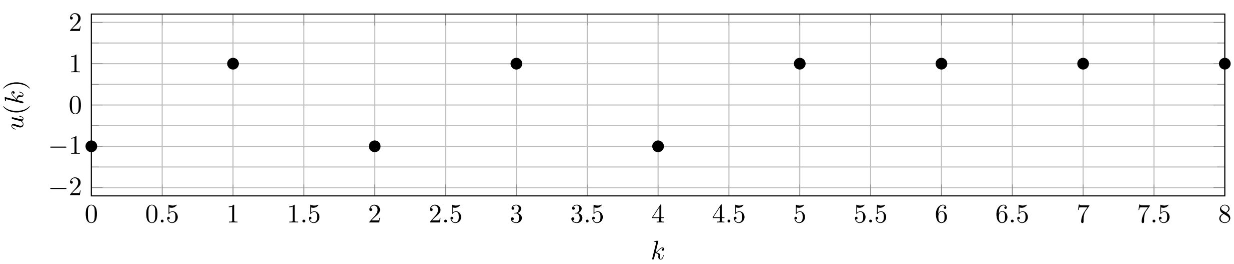
b) Write down the corresponding difference equation in the discrete time domain.

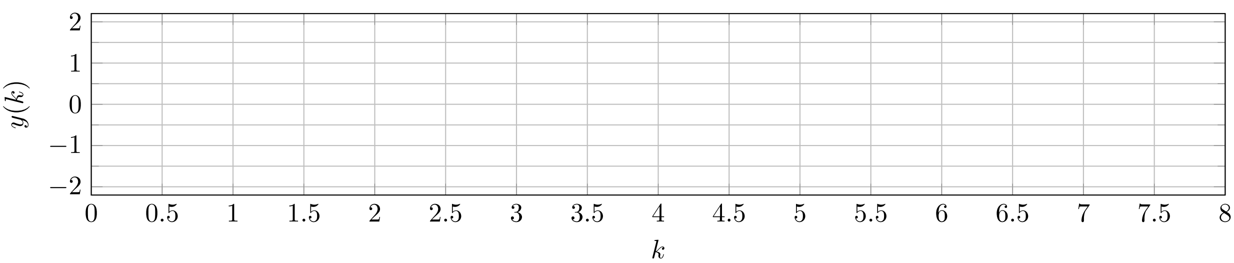
c) Let $a = \mathbf{-1}$ and $m = 2$. Draw the filter response for the following signal. Use $u(k) = 0$ for all $k < 0$ as the initial condition.





d) Now use $a = \mathbf{1}$ and $m = 2$. Draw the filter response for the following input signal. Use $u(k) = 0$ for all $k < 0$ as the initial condition.





Now let $a = -1$ and let m be an arbitrary positive integer.

e) Write down the expression for the frequency response of the filter and use T_0 as the sampling time. Do not calculate the absolute value yet.

f) What is the value of the frequency response at $\omega = 0$. Calculate the damping at this frequency in dB.

g) Decide whether this filter has a low-pass or a high-pass characteristic. Give a reason (1 sentence).

Solutions:

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- [x] ... needs a **large** number of parameters to model a large number of data samples.
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e) Assess following statements regarding errors:

- [ ] The quantization error is half as large if a 16 bit A/D converter is used instead of an 8 bit converter.
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f) What is the meaning of the terms *dead zero* and *life zero*?

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g) Explain apparent, active and reactive power.

- [ ] Apparent power is the power an electrical device (e.g. a motor) actually delivers.
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- [ ] Active power is the power lost due to phase shifting and should usually be avoided.
- [ ] Reactive power is calculated from apparent power P_S as follows: $P_B = P_S \cdot \cos(\varphi)$.

h) Operational amplifiers …

- [ ] … are passive components (need no external energy source).
- [ ] … are amplifiers with a very low gain.
- [ ] … have a very high **output** resistance.
- [x] … have a very high **input** resistance.

i) What is a principal component analysis (PCA)?

- [ ] PCA is a supervised learning method.
- [x] PCA is often used for data preprocessing.
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j) Explain positive (PTC) and negative temperature coefficient (NTC) resistance thermometers.

- [ ] For PTC thermometers no wire calibration is necessary due to the very low resistance of the temperature sensor.
- [x] NTC thermometers are semiconductor elements, PTC thermometers use a sensor made out of metal.
- [x] NTC thermometers have strongly nonlinear characteristics.
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Task 2: Statistics (16 points)

Figure 8 shows contour plots of two-dimensional Gaussian probability density functions $p(u_1, u_2)$. Each distribution is characterized by its mean $(\mu_1 \ \mu_2)$ and its covariance matrix $(\underline{\Sigma})$.

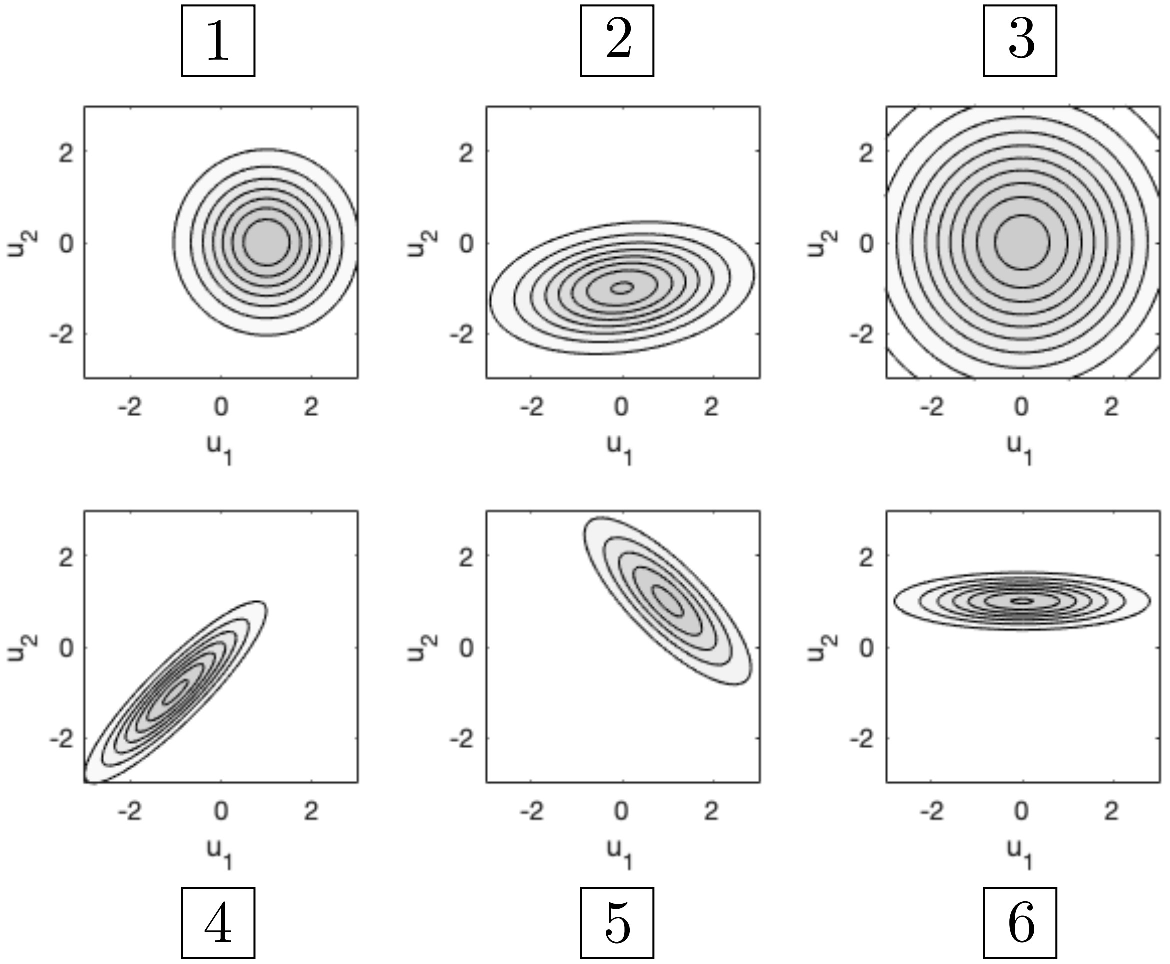


Figure 8: Contour plots of two-dimensional Gaussian probability density functions

a) Estimate the mean $(\mu_1 \ \mu_2)$ of $p(u_1, u_2)$ for every density (1-6) and write them in the table below. 3

b) Match every density from Fig. 8 to the correct covariance matrix $\underline{\Sigma}$ of the following matrices $\underline{A}$ to $\underline{L}$. 6

$$\underline{A} = \begin{bmatrix} 1 & -0.4 \\ 0 & -1 \end{bmatrix} \qquad \underline{B} = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix} \qquad \underline{C} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\underline{D} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} \qquad \underline{E} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \qquad \underline{F} = \begin{bmatrix} 1 & -0.8 \\ -0.8 & 1 \end{bmatrix}$$

$$\underline{G} = \begin{bmatrix} 2 & 0.2 \\ 0.2 & 0.5 \end{bmatrix} \qquad \underline{H} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \underline{I} = \begin{bmatrix} 1 & 0.2 \\ 0.2 & -0.5 \end{bmatrix}$$

$$\underline{J} = \begin{bmatrix} 2 & 0 \\ 0 & 0.1 \end{bmatrix} \qquad \underline{K} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \underline{L} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Answer:

Plot	μ_1	μ_2	Σ
1	1	0	$\underline{K}$
2	0	-1	$\underline{G}$
3	0	0	$\underline{E}$
4	-1	-1	$\underline{B}$
5	1	1	$\underline{F}$
6	0	1	$\underline{J}$

c) Draw the graph of a Student's t-distribution for a medium number of degrees of freedom (label the axis accordingly). 2

d) Draw the graph of a Student's t-distribution for a lower number of degrees of freedom into the same figure. Use a dashed line for this graph in order to distinguish both graphs. 4

Answer:

Bigger tails and smaller probability at mean for less data (degrees of freedom)

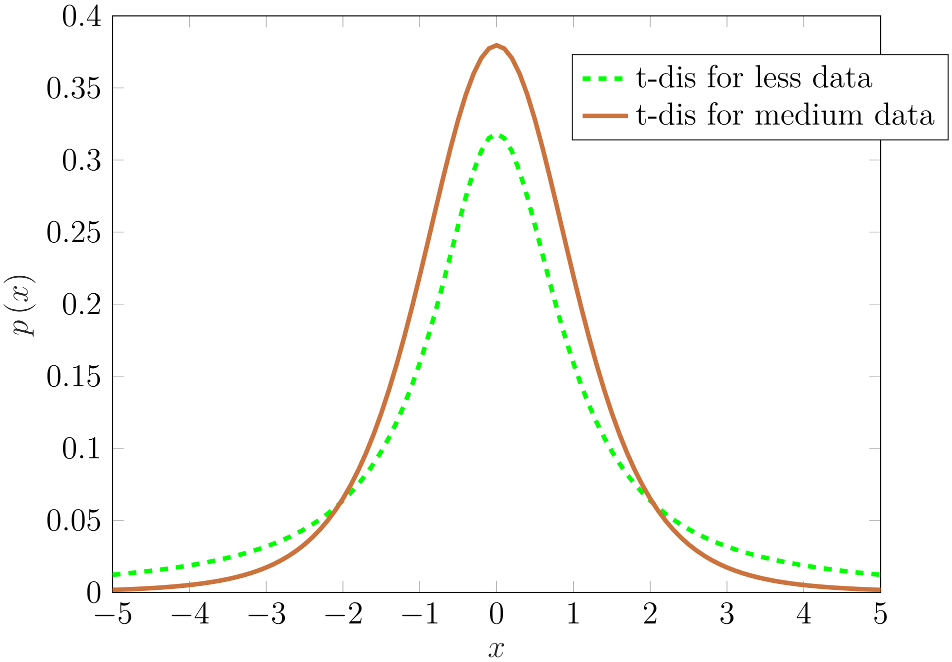


Figure 9: Student's- t-distribution

e) How do the integrals from -∞ to ∞ of these t-distributions differ?
Answer:
They do not differ, both are 1. $\boxed{1}$

$\boxed{\sum 16}$

Task 3: Nonlinearity (18 points)

The input signal

$$u(k) = 1 + A\sin(k\frac{\pi}{5})$$

is fed into a nonlinear function

$$f\left(u\left(k\right)\right) = u^2\left(k\right) = y\left(k\right). \qquad | \qquad \underline{\textit{Hint}}\text{: } \sin^2\left(x\right) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

The block diagram is shown in Fig. 10.

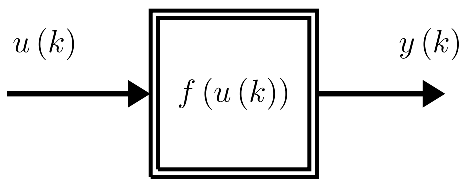


Figure 10: I/O-diagram of nonlinear function f

a) Imagine you carry out a DFT of an arbitrary signal of length N in your computer. What is the difference between $|X(n)|$ and $|X(N-n-1)|$ if n is an integer in the range $0 \leq n \leq \frac{N}{2}$?
Answer:
There is no difference as the spectrum is mirrored at $n = \frac{N-1}{2}$. 2

b) Calculate $y(k)$ if $A = 1$ and plot it into Fig. 11.

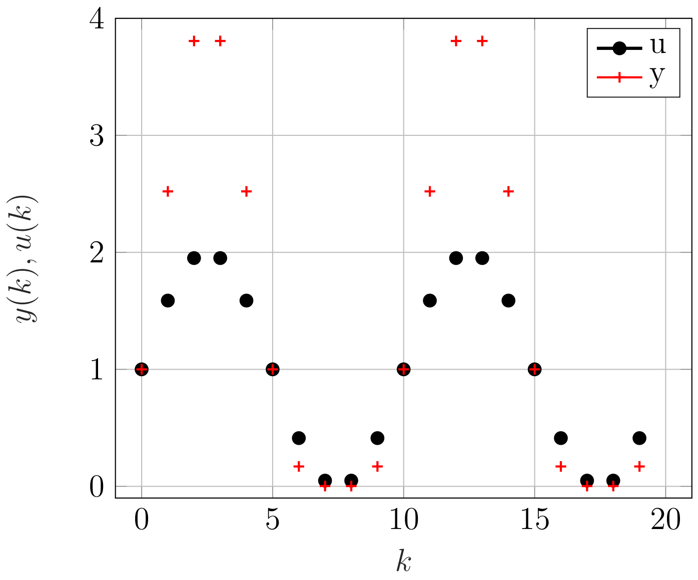


Figure 11: In- and outputs for $A = 1$

Answer:

$$y\left(k\right)=\left(1+\sin\left(k\frac{\pi}{5}\right)\right)^2$$
$$y\left(k\right)=1+2\sin\left(k\frac{\pi}{5}\right)+\sin^2\left(k\frac{\pi}{5}\right)$$
$$y\left(k\right)=1+2\sin\left(k\frac{\pi}{5}\right)+\frac{1-\cos(2k\frac{\pi}{5})}{2}$$

3

c) Draw qualitatively the absolute value of the DFT of the signal $u(k)=1+A\sin\left(k\frac{\pi}{5}\right)$ for $A=1$. Label the axes according to the task. Take care that relations can be seen.
Answer:
One peak for the offset, one peak for the sine at ω_1. See Fig. 12. 4

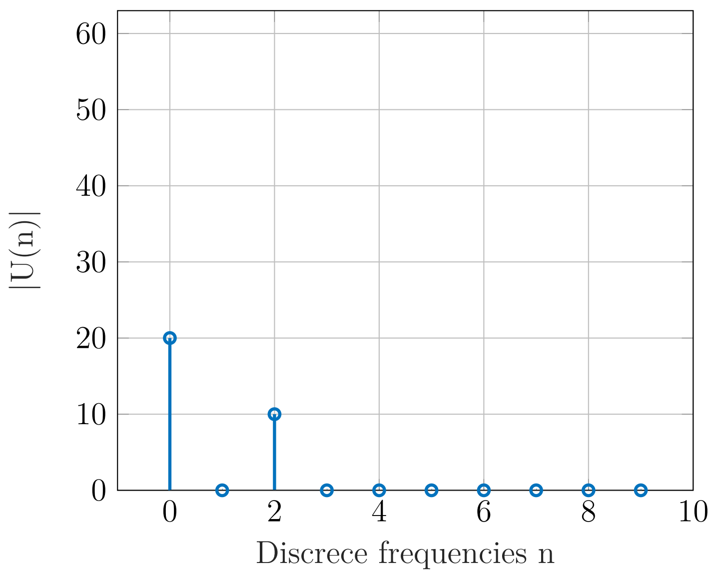


Figure 12: Frequency spectrum from the signal $u(k)$ with $A=1$

d) Draw qualitatively the absolute value of the DFT of the signal $y(k)=u^2(k)$ with $A=1$. Label the axes according to the task. Take care that relations can be seen. The relation of the offset can be neglected in this regard.
Answer:
One peak for the offset, one peak for the sine at ω_1 (double magnitude), one peak at $2\omega_1$ 0.25 times the peak at ω_1. See Fig. 12. 4

e) Draw qualitatively the absolute value of the DFT of the signal $y(k)=u^2(k)$ with $A=2$. Label the axes according to the task. Take care that relations can be seen. The relation of the offset can be neglected in this regard. What is different compared to the case for $A=1$?
Answer:
One peak for the offset, one peak for the sine at ω_1 (double magnitude, compared to the task before), one peak at $2\omega_1$, 0.5 times the peak at ω_1. See Fig. 13.

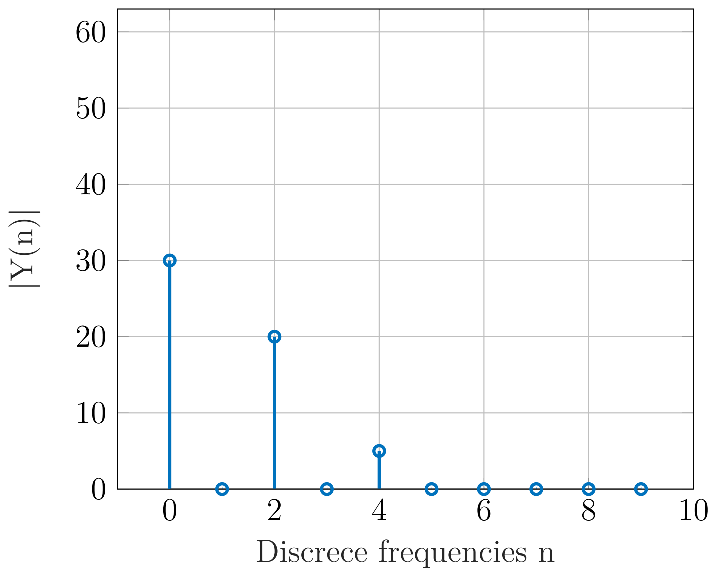


Figure 13: Frequency spectrum from the signal $y(k)$ with $A = 1$

Peak at $2\omega_1$ is now larger compared to the peak at ω_1 with $A = 2$ than it was for $A = 1$. $\boxed{5}$

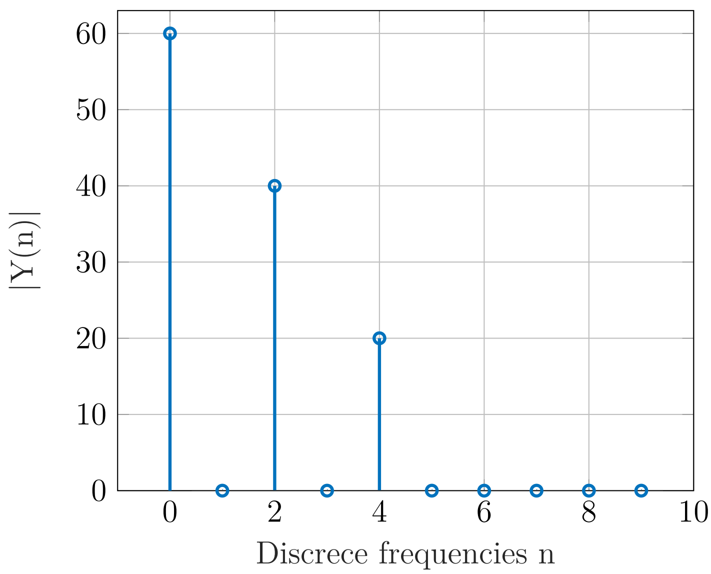


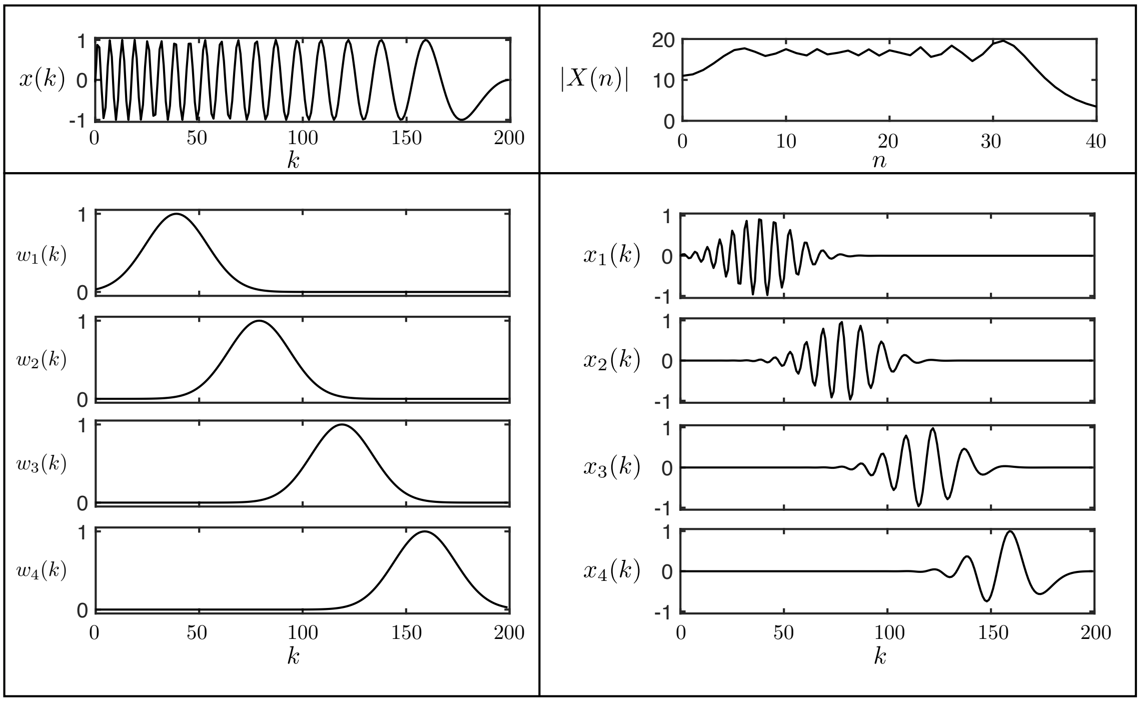
Figure 14: Frequency spectrum from the signal $y(k)$ with $A = 2$

$\boxed{\sum 18}$

<u>Task 4:</u> DFT (12 points)

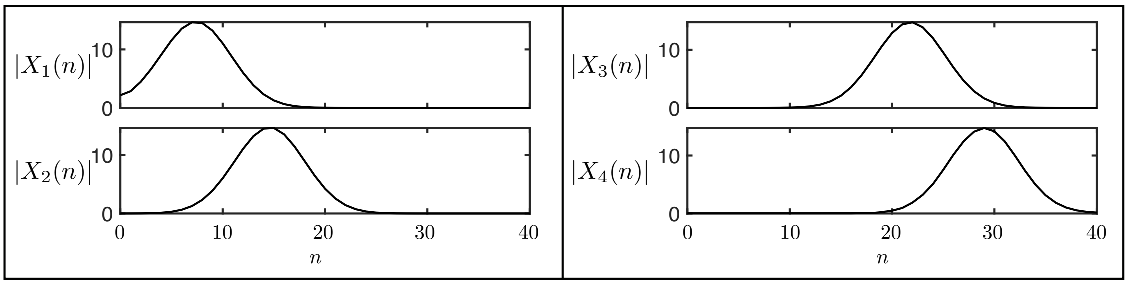
A sine-shaped signal $x(k)$ is given. The frequency of the signal changes linearly with the time. The first 40 absolute values of the corresponding DFT are also given by $|X(n)|$ as well as four different window functions $(w_1(k), \ldots, w_4(k))$.

a) Apply each window function $(w_1(k), \ldots, w_4(k))$ to the signal $x(k)$ and <u>sketch</u> the resulting signals $x_1(k), \ldots, x_4(k)$.



4

b) Assign to each signal $x_i(k)$ the corresponding absolute value of the DFT $|X_j(n)|$.



Signal	$x_1(k)$	$x_2(k)$	$x_3(k)$	$x_4(k)$
Corresponding $\|X_j(n)\|$	$\|X_4(n)\|$	$\|X_3(n)\|$	$\|X_2(n)\|$	$\|X_1(n)\|$

4

c) What is the gathered information by using different time shifted window functions? How is this procedure called?
Windowed DFT: The DFT does not only depend on the frequency for n but also on a second variable: the time shift of the window. It indicates the time around which the DFT is valid.
This procedure is called Short-Time Discrete Fourier Transform (STDFT). 2

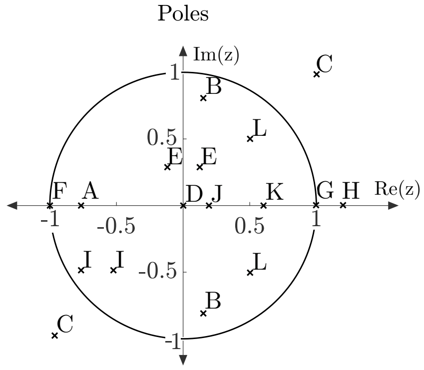
d) Assume a signal with $2f_0$ is sampled with a sampling frequency f_0 at which normalized frequency occurs the highest peak?
The signal frequency violates the Shannons theorem. The signal is sampled at the exact same value. Thus, the sampled signal is constant. A constant signal has a (normalized) frequency of zero (1/sec). The DFT has therefore the highest peak at $n = 0$. $\boxed{2}$

$\boxed{\sum 12}$

Task 5: Step Responses (23 points)

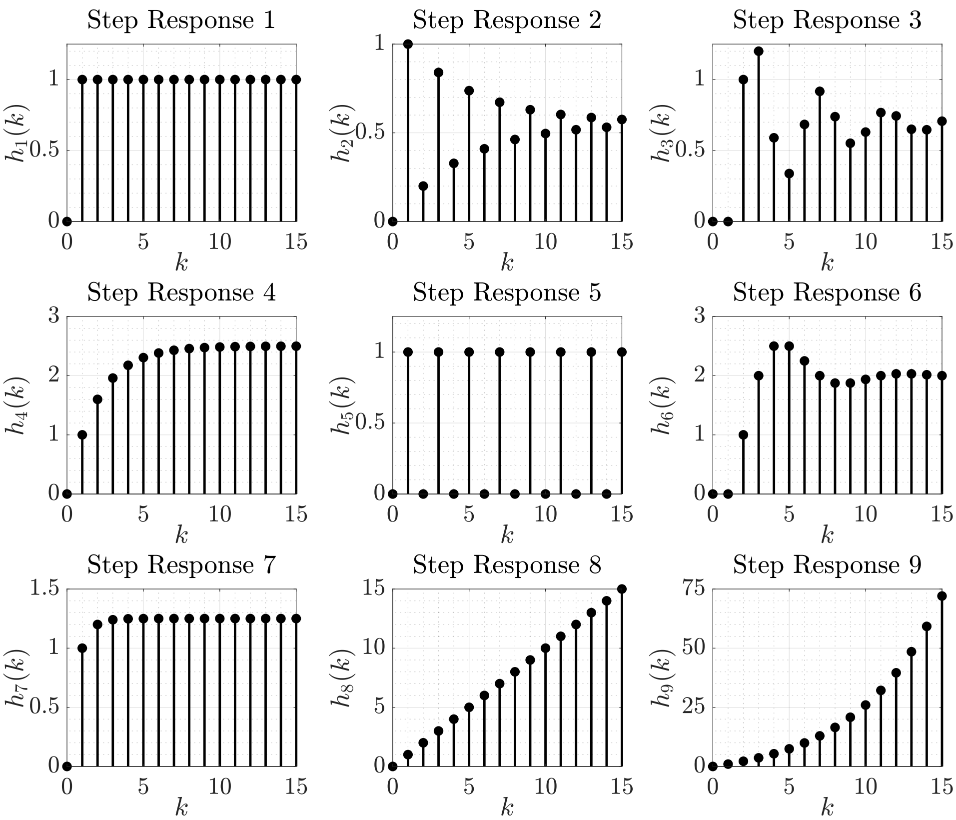
Given are the pole locations for 12 different systems (A-L) in the pole zero map and 9 different step responses (1-9).

a) Match the pole locations to the step responses. Note down your answers in the given table. (Note: Three pole locations do not belong to any step response. All transfer functions have the structure $G(z) = \frac{1}{A(z)}$ with $A(z) = \prod_{i=1}^{n}(z - p_i)$.)



Step Response	Poles
1	D
2	A
3	B
4	K
5	F
6	L
7	J
8	G
9	H

2 2 2 2 2 2 2 2 2 2



b) Assume that the pole of system J is at $p_J = 0.2$. Calculate the systems transfer function $G_J(z)$ and apply the final value theorem to the systems response with input signal $u(k) = 2 \cdot \sigma(k)$.

$$G_J(z) = \frac{1}{z - 0.2} = \frac{z^{-1}}{1 - 0.2z^{-1}} \qquad U(z) = \mathcal{Z}\{u(k)\} = 2 \cdot \frac{z}{z-1}$$

$$\lim_{z \to 1}(z-1)Y(z) = \lim_{z \to 1}(z-1)G_J(z)U(z) = \lim_{z \to 1}\cancel{(z-1)} \cdot \frac{1}{z-0.2}\frac{2z}{\cancel{z-1}} = 2.5$$

3

c) System $G_J(z)$ is now expanded to system $G_{JJ}(z)$, which is $G_J(z)$ with $G_J(z)$ series-connected. What is the gain of $G_{JJ}(z)$?

$$G_{JJ}(z) = G_J(z) \cdot G_J(z) = \frac{1}{(z-0.2)^2}$$

$$\lim_{z\to 1}(z-1)Y(z) = \lim_{z\to 1}(z-1)G_{JJ}(z)U(z) = \lim_{z\to 1}\cancel{(z-1)} \cdot \frac{1}{(z-0.2)^2}\frac{z}{\cancel{z-1}}$$

$$\frac{1}{\left(\frac{4}{5}\right)^2} = \frac{25}{16}\,(= 1.5625)$$

$\boxed{2}$

$\boxed{\sum 23}$

Task 6: Zeitdiskrete Systeme (15 Punkte)

A system with output $y(k) = b_0 \cdot u(k) - a_1 \cdot y(k-1)$ is given with $a_1 = -1$ and $b_0 = 0.3$.

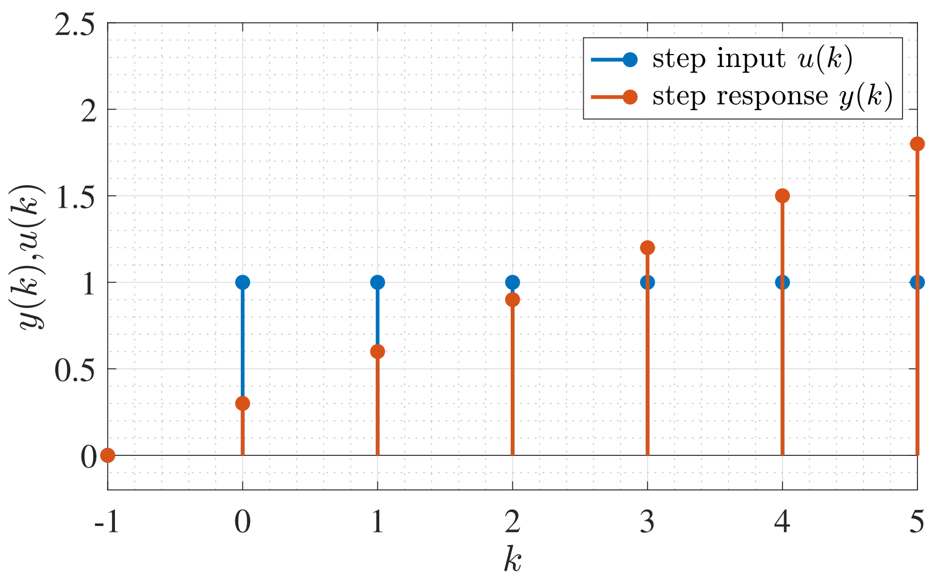
a) Calculate the output of the system with the given input signal for $k = 0, 1, ..., 5$ and sketch the corresponding signal in Fig. 15.

The given input signal is

$$u(k) = \begin{cases} 1, & \text{für } k \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

Calculation of the values of $y(k)$ for $k = 0, 1, ..., 5$: 3

$$y(k=0) = b_0 \cdot u(0) - a_1 \cdot y(0-1) = 0.3 \cdot 1 - (-1) \cdot 0 = 0.3$$
$$y(k=1) = 0.3 \cdot 1 - (-1) \cdot 0.3 = 0.6$$
$$y(k=2) = 0.3 \cdot 1 - (-1) \cdot 0.6 = 0.9$$
$$y(k=3) = 0.3 \cdot 1 - (-1) \cdot 0.9 = 1.2$$
$$y(k=4) = 0.3 \cdot 1 - (-1) \cdot 1.2 = 1.5$$
$$y(k=5) = 0.3 \cdot 1 - (-1) \cdot 1.5 = 1.8$$



1

b) What is the global behavior of the system $(P, PI, PD, PT_1, I, D, \ldots)$?

The system is an integrator (global I-behavior). 1

c) Sketch the block diagram of the given system. 2

d) Determine the poles of the given system.

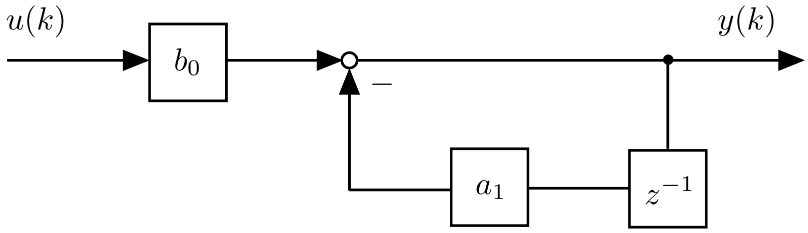


Figure 15: Block diagram of the system.

$$
\begin{aligned}
y(k) + a_1 \cdot y(k-1) &= b_0 \cdot u(k) \\
Y(z) + a_1 \cdot Y(z) \cdot z^{-1} &= b_0 \cdot U(z) \\
G(z) &= \frac{b_0}{1 + a_1 \cdot z^{-1}} \\
p &= 1
\end{aligned}
$$

3

e) Calculate the final value of the impulse response.

$$
\begin{aligned}
Y(z) &= G(z) \cdot U(z) \\
y(k \to \infty) &= \lim_{z \to 1} G(z) \cdot U(z) = (z-1) \cdot \frac{b_0}{1 + a_1 \cdot z^{-1}} \cdot \frac{z}{z} = \frac{b_0 \cdot z}{1} = b_0 = 0.3
\end{aligned}
$$

2

f) Which of the following statements is correct?

- ☐ The system is stable.
- ☐ The system is unstable.
- ■ The system is marginally stable. 1

g) How can you modify the given system to meet the two non-checked statements in task f)?

For an unstable system, $|a_1| > 1$. For a stable system, $|a_1| < 1$. 2

Task 7: Comb Filter

a) The filter is an FIR filter since there are only poles at 0. 1

b) The transfer function can be written as

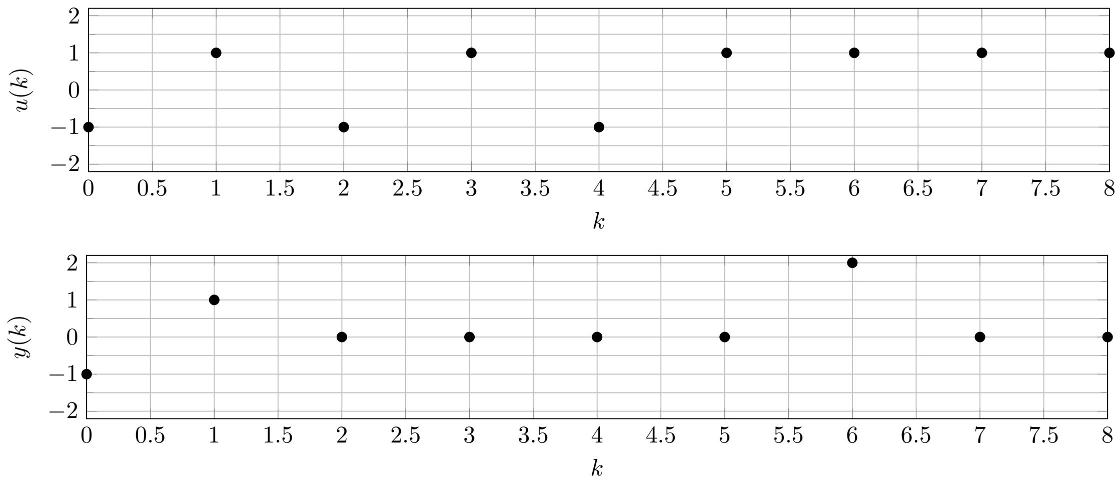
$$G(z) = \frac{Y(z)}{U(z)} \frac{z^m + a}{z^m} = 1 + az^{-m}$$

Transformation in the discrete time domain results in

$$y(k) = u(k) + au(k - m).$$

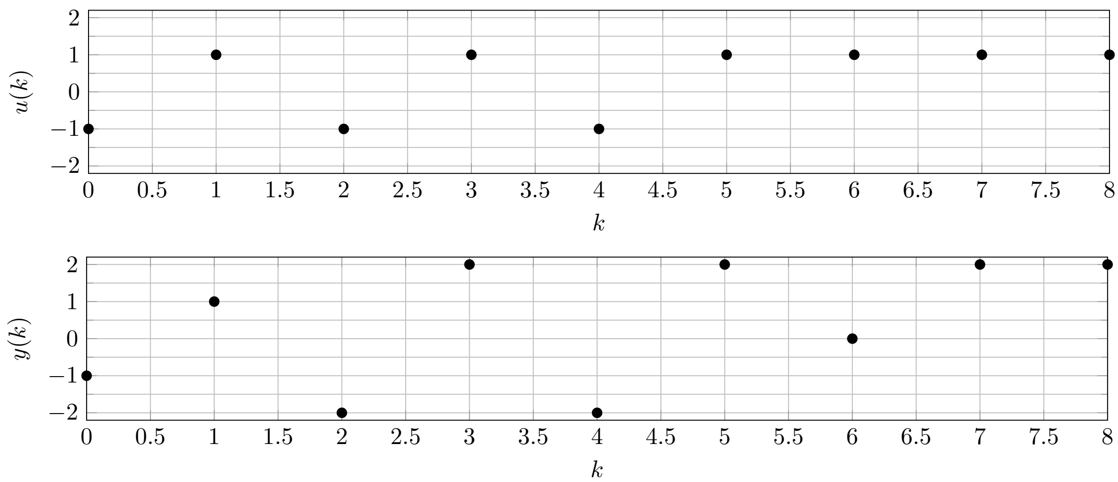
2

c) The difference equation is $y(k) = u(k) - u(k-2)$.



4

d) The difference equation is $y(k) = u(k) + u(k-2)$.



4

e) The frequency response is

$$G(i\omega) = 1 - \left(e^{-i\omega T_0}\right)^m = 1 - (\cos(-\omega T_0) - i\sin(-\omega T_0))^m$$

$\boxed{3}$

f) At $\omega = 0$

$$G(0) = 1 - (\cos(0) - i\sin(0))^m = 1 - 1^m = 0$$

The damping is $-\infty$ dB. $\boxed{3}$

g) The filter has a high-pass characteristic since for $\omega = 0$ the frequency response is 0.

$\boxed{1}$

$\boxed{\sum 18}$