

Sensorics Exam

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Name:	Task:	T1	T2	T3	T4	T5	T6	Sum
Mat.-No.:	Scores:	9	24	19	19	27	22	120
Grade:	Accomplished:							

Task 1: Comprehension Questions

Mark the correct answers clearly.

Every question has one or two correct answers!

For every correctly marked answer you will get one point. If there is one correct answer marked and one incorrect answer marked, you will get no point for that subtask.

- a) If you differentiate a noisy signal with respect to time t ...
- ... the system becomes unstable.
 - ... the gain of the transfer function changes.
 - ... the noise is amplified.
- b) Assess the following statements regarding measurement techniques.
- The doppler effect is used to measure the effective force inside a fluid.
 - PTC resistance thermometers are more accurate than thermocouples.
 - The piezo-electric effect can be used to measure a static displacement.
- c) Non-parametric methods ...
- ... are more flexible than parametric methods.
 - ... can only be used for time-continuous systems.
 - ... are more robust with respect to noise.
- d) Assess the following statements regarding adaptive filters.
- An adaptive filter is a time-invariant, static system.
 - An adaptive filter changes its parameters based on the error of the filter.
 - An adaptive filter is always stable.
- e) The parametric frequency analysis ...
- ... leads to a continuous amplitude spectrum
 - ... is more insensitive with respect to noise than non-parametric methods.
 - ... leads to a discrete amplitude spectrum.

f) An FIR-filter ...

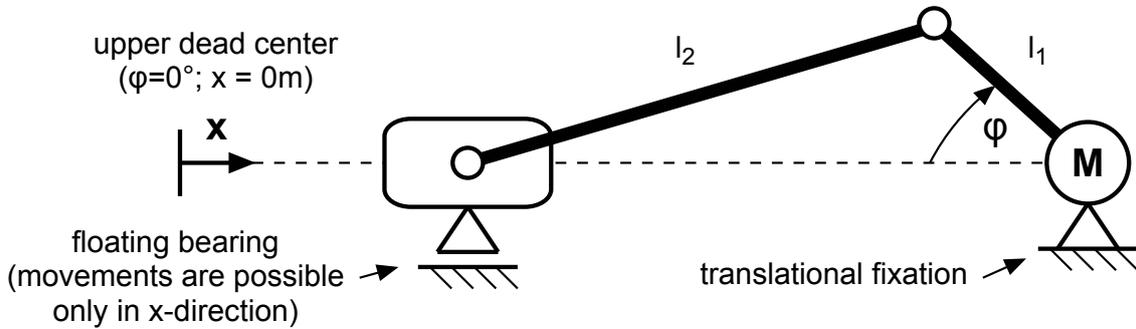
- ... always has a linear phase.
- ... is always stable.
- ... is better suited for adaption than IIR-filters.

g) Shannon's sampling theorem says, that ...

- ... the sampling frequency must be at least the double of the highest significant frequency component of the signal.
- ... aliasing only appears sampling a noisy signal.
- ... an amplitude spectrum only ranges from 0 to $\frac{\omega_0}{2}$ to avoid redundancy.

Task 2: Slider-Crank Mechanism

The picture below shows a slider-crank mechanism, that is driven by an electric motor **M** (on top of the translational fixation). l_1 and l_2 denote the length of the two cranks. The x -position of the slider is measured from the upper dead center.



The position x , the velocity v and the acceleration a of the slider should be measured.

- Name two possible measurement methods, that can be used to measure the position x of the slider.
- Name two measurement principles, that can be used to measure speed in general.
- If a Piezoelectric sensor is applied to the slider, would this type of sensor be suited to measure the slider's acceleration? What drawback do Piezoelectric sensors have (Hint: What about a rotational speed near zero $\dot{\varphi} \approx 0$)? Sketch the position of the sensor into the picture above, where you would apply this sensor.
- Now it is assumed to have only one sensor measuring the angle φ . The angular velocity as well as the angular acceleration should be determined with the help of derivatives of the measured angle φ . What could be problematic with this approach in the real world? Hint: What happens when the angle measurement is disturbed? How does the frequency of the disturbance influence the speed and acceleration?
- The position of the slider can be approximated with the following equation:

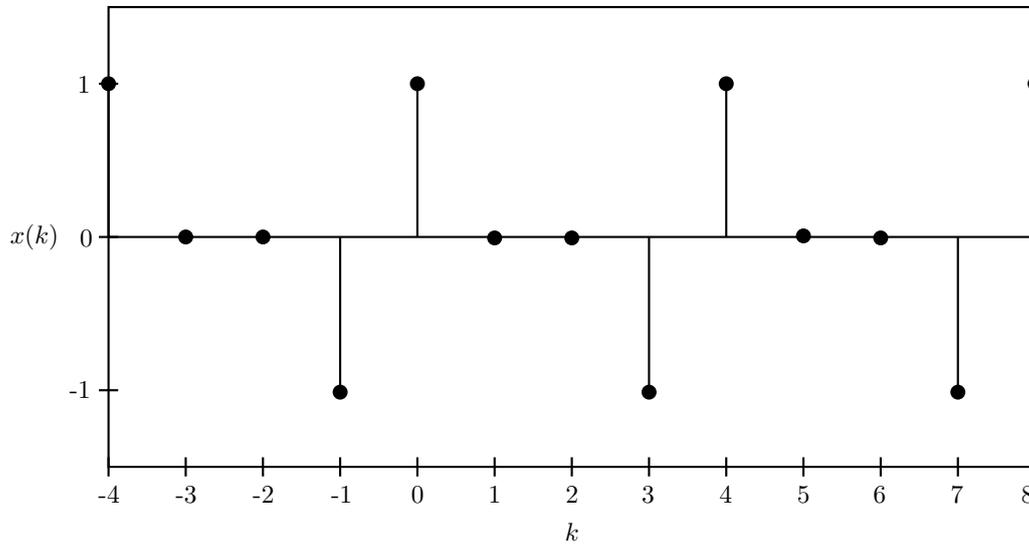
$$x(\varphi) = l_1 \left(1 - \cos(\varphi) + \frac{\lambda}{4} - \frac{\lambda}{4} \cos(2\varphi) \right),$$

where $\lambda = \frac{l_1}{l_2}$ denotes the fraction between the two crank lengths. Derive the equations to calculate the translational speed and acceleration of the slider.

- The maximum rotational speed is $n_{max} = 3000\text{rpm}$ (rounds per minute). The translational speed maximum is reached at the angle $\varphi = 73.7^\circ$ for $l_1 = 1\text{m}$ and $l_2 = 3\text{m}$. Calculate the maximum translational speed v_{max} for the given values.
- Determine the sampling frequency f_0 for the angle measurement φ such that the maximum error of the translational position x is less or equal 0.1m at any time. The maximum rotational speed is still $n_{max} = 3000\text{rpm}$.

Task 3: Diskrete Fourier-Transformation

The following periodic sequence $x(k)$ is given.

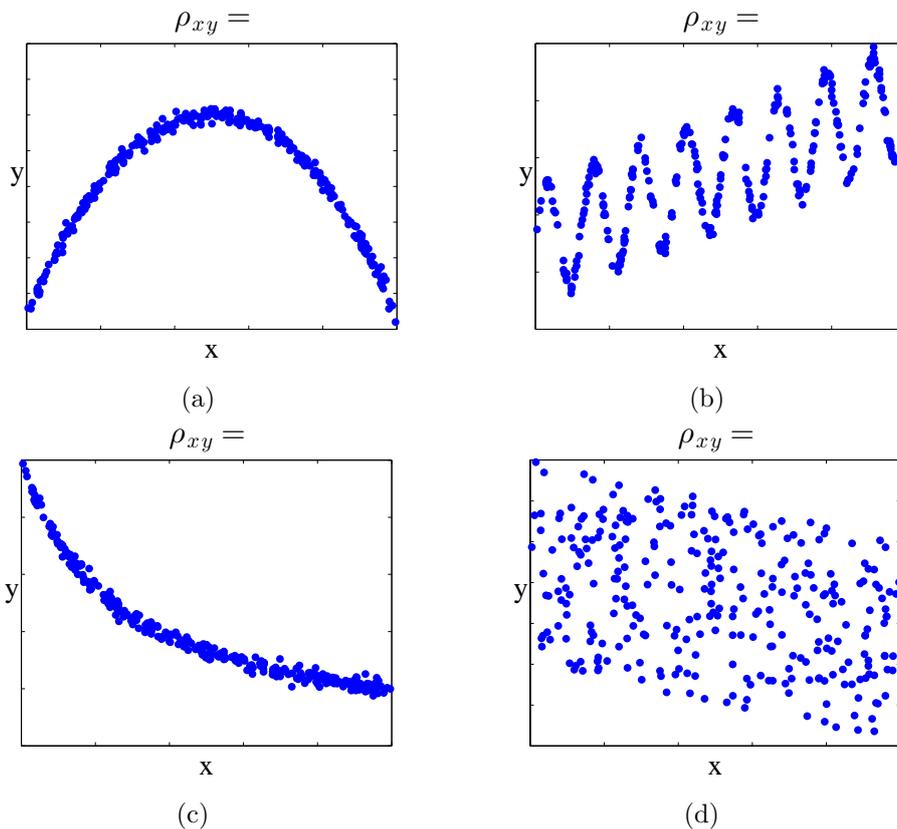


- a) After how many values N does the series repeat?
- b) First derive the equation system, that has to be solved to achieve the discrete Fourier-Transform $X(n)$, and than express the equation system with matrix-vector notation $\underline{X} = \underline{F} \underline{x}$.
 Use in both cases the abbreviation $W_N = e^{-i\frac{2\pi}{N}}$.
 For the equation system use a general signal $x(k)$ and for the matrix-vector notation use a general signal vector \underline{x} .
- c) Now use the given signal to solve the equation system for $X(n)$, depending on the not yet specified values of W_N .
- d) Calculate all required powers of W_N and plot them in the complex plane.
- e) Use the calculated powers of W_N to calculate the discrete amplitude spectrum $|X(n)|$.

Task 4: Correlation, Probability Densities and Confidence Intervals

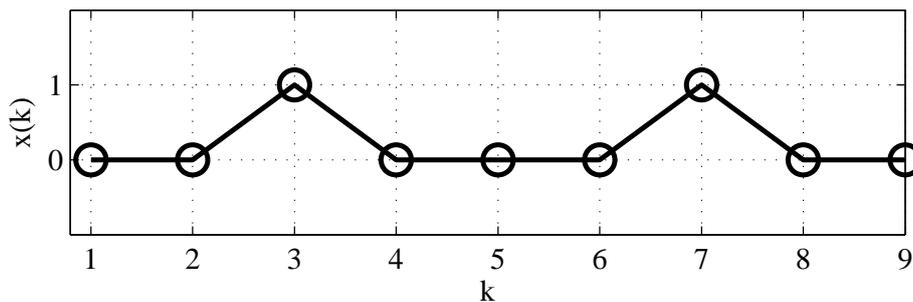
All subtasks can be solved independently.

- a) Assign the following correlation coefficients to one of the figures below. Every correlation coefficient can only be assigned once. Here are the correlation coefficients: -0.9, -0.3, 0, 0.7.



- b) Calculate the autocorrelation function values for the signal $u(k)$ below. Use the following equation for the calculation of time shifts $\kappa = 0 \dots 8$:

$$r_{xx}(\kappa) = \frac{1}{N} \sum_{k=1}^{N-|\kappa|} x(k)x(k + \kappa) . \tag{1}$$



State if equation 1 is biased or unbiased.

c) Probability Densities

1) Sketch **two** normal distributions into **one** coordinate system, that have the following properties:

- Mean $\mu_1 = 0$, Variance $\sigma_1^2 = 1$.
- Mean $\mu_2 = 0$, Variance $\sigma_2^2 = 4$.

Assign each variance to the corresponding (sketched) normal distribution.

2) Sketch the probability density, where all values between 0 and 2 are equally likely (uniform distribution). Take care of the coordinate axis' scaling.

Make sure, that all coordinate axes are labeled.

d) Confidence Intervals

1) The temperature of a fluid in a chemical manufacturing process should be measured. It can be assumed, that the overall error follows a normal distribution. The accuracy properties of the measurement instrument are well known. The standard deviation of the disturbance is determined to be $\sigma_T = 0.2$. Five measurements are carried out and the following values are obtained:

$$T[^\circ C] = [65.1 \quad 64.8 \quad 64.9 \quad 65.3 \quad 65.15]$$

Calculate the range in which the true temperature will be for an accepted error probability of 0.27%. Therefore use the following table.

Interval	Probability
$\mu_x - 1\sigma_x < x < \mu_x + 1\sigma_x$	68.27%
$\mu_x - 2\sigma_x < x < \mu_x + 2\sigma_x$	95.45%
$\mu_x - 3\sigma_x < x < \mu_x + 3\sigma_x$	99.73%
$\mu_x - 4\sigma_x < x < \mu_x + 4\sigma_x$	99.99%

- 2) Now it is assumed, that the standard deviation σ_T is not known, but can be estimated from the given values above. Calculate the estimated standard deviation s_T with the help of the five given temperature values.
- 3) Given s_T from subtask d2), will there anything change regarding the confidence interval calculation for the true temperature value compared with the one from subtask d1)?

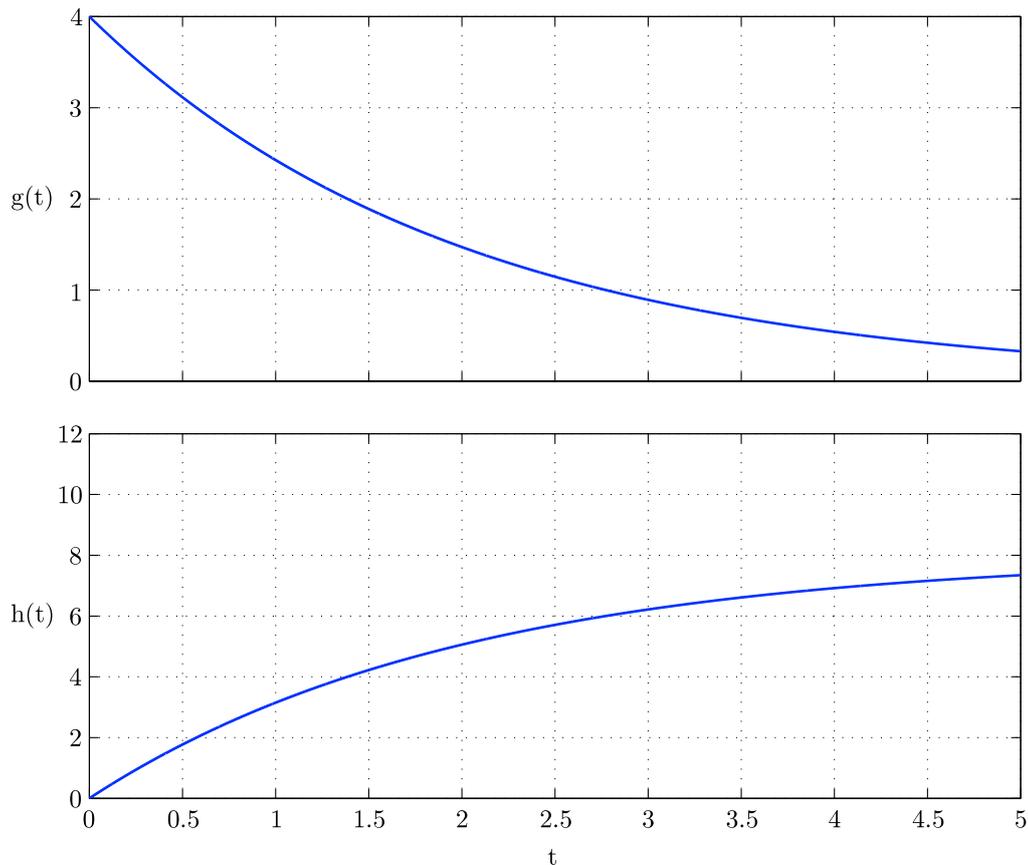
Task 5: Time discrete system

The following time continuous step response of a PT_1 -system is given:

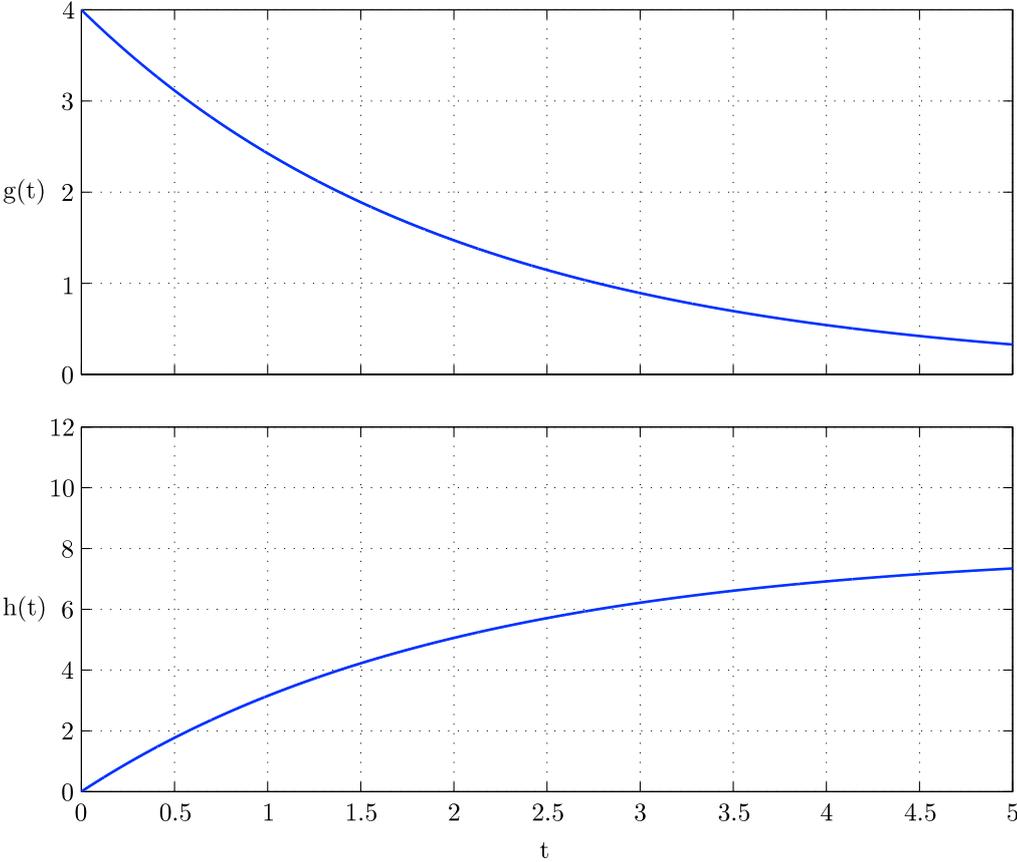
$$h(t) = K \cdot \left(1 - e^{-\frac{t}{T}}\right),$$

with gain $K = 8$ and the time constant $T = 2$ sec.

- Determine the impulse response (weighting function) $g(t)$ of the given PT_1 system.
- Determine the transfer function $G_I(z)$ via **impulse response invariance**. Use the sampling time $T_0 = 0.5$ sec.
- Determine the transfer function $G_S(z)$ via **step response invariance**. Use the sampling time $T_0 = 0.5$ sec.
- To compare the two Systems $G_I(z)$ and $G_S(z)$ calculate the gain, the properness and the stability.
- Plot the first $k = 0, 1, 2, 3$ samples of the impulse and step response for the **System $G_I(z)$** .



- Plot the first $k = 0, 1, 2, 3$ samples of the impulse and step response for the **System $G_S(z)$** .



Task 6: Filter

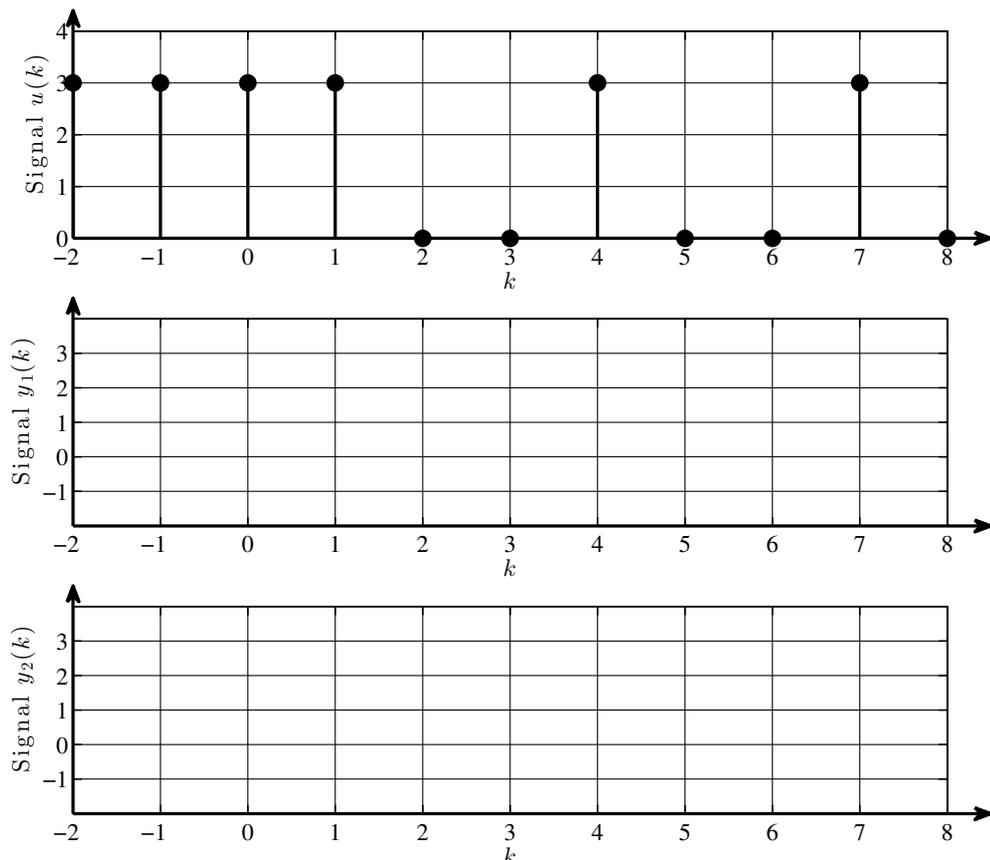
The following transfer function is given in the z -domain:

$$G_1(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}} .$$

- a) Determine the difference equation for the given filter.
- b) Sketch the block diagram of the given filter.
- c) In the figure below, you can see the time signal $u(k)$ that has to be filtered. For times k lower than -2 and greater than 8 the signal $u(k)$ is equal to 0. Two different filters should be used. The first filter is the one given above with the coefficients $b_0 = \frac{1}{3}$, $b_1 = -\frac{1}{3}$ and $a_1 = 0$ (filtered signal $y_1(k)$). The second filter G_2 is described by the following difference equation and leads to the filtered signal $y_2(k)$:

$$y_2(k) = \frac{1}{3} (u(k+1) + u(k) + u(k-1)) .$$

Calculate and sketch the values of the filtered signals $y_1(k)$ and $y_2(k)$.



- d) Specify the type of filter for both filters (high-pass or low-pass).
- e) Make a statement on causality for each filter.
- f) Is G_1 a FIR- or a IIR-Filter, given the values for the coefficients from subtask c)? State also if G_2 is an FIR- or IIR-Filter.

Solutions:

Task 1: Comprehension Questions

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- ... always has a linear phase.
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 - ... aliasing only appears sampling a noisy signal.
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Task 2: Slider-Crank Mechanism

- a) Name two possible measurement methods, that can be used to measure the position x of the slider.

Here a few possible measurement methods:

- 1) Inductive measurement method
- 2) Capacitive measurement method
- 3) Optical angle measurement and transformation via the given equation
- 4) Optical displacement measurement of the slider
- 5) etc.

2

- b) Name two measurement principles, that can be used to measure speed in general.

Possibilities for speed measurement:

- 1) Measure time interval Δt and distance Δx , then calculate the velocity $v = \frac{\Delta x}{\Delta t}$.
- 2) Measurement of rotational speed ω and conversion into the translational speed with $v = \omega r$.
- 3) Direct measurement of speed by the use of the Doppler effect (of acoustic or electromagnetic waves).

2

- c) If a Piezoelectric sensor is applied to the slider, would this type of sensor be suited to measure the slider's acceleration? What drawback do Piezoelectric sensors have (Hint: What about rotational speed near zero $\dot{\varphi} \approx 0$)?

In this case a piezoelectric sensor is suited for acceleration measurement. Only for a rotational speed of $\dot{\varphi} \approx 0$ the acceleration might not be accurate, because it would be a quasi-stationary measurement. Piezoelectric sensors are only suited for dynamic measurements, because of their transient behavior. (Sensor Position: see sketch...)

3

- d) What could be problematic with this approach in the real world?

Derivatives enhance the noise! Especially for the acceleration where the signal is derived two times with respect to time, the signal might become very noisy. It has to be checked if the achieved accuracy is sufficient for the task at hand.

5

- e) Derive the equations to calculate the translational speed and acceleration of the slider.

$$\dot{x}(\varphi) = v(\varphi) = l_1 \dot{\varphi} \left(\sin(\varphi) + \frac{\lambda}{2} \sin(2\varphi) \right)$$

$$\ddot{x}(\varphi) = a(\varphi) = l_1 \dot{\varphi}^2 (\cos(\varphi) + \lambda \cos(2\varphi)) + l_1 \ddot{\varphi} \left(\sin(\varphi) + \frac{\lambda}{2} \sin(2\varphi) \right) ,$$

6

f) Calculate the maximum translational speed v_{max} for the given values.

$$\begin{aligned}
 v(\varphi) &= l_1 \dot{\varphi} \left(\sin(\varphi) + \frac{\lambda}{2} \sin(2\varphi) \right) \\
 &= \frac{1\text{m} \cdot 3000 \cdot \frac{2\pi}{\text{per round}}}{60 \frac{\text{sec}}{\text{min}} \cdot \text{min}} \left(\sin(73.7^\circ \cdot \frac{\pi}{180^\circ}) + \frac{1}{6} \sin(2 \cdot 73.7^\circ \cdot \frac{\pi}{180^\circ}) \right) \\
 &= 329.7 \frac{\text{m}}{\text{sec}} = v_{max}
 \end{aligned}$$

2

g) Determine the sampling frequency f_0 for the angle measurement φ such that...

To guarantee one measured value at least every $\Delta x = 0.1\text{m}$, we determine how long it takes to cover that distance at maximum translational speed v_{max} :

$$\begin{aligned}
 \Delta t &= \frac{\Delta x}{v_{max}} \\
 &= \frac{0.1\text{m}}{329.7 \frac{\text{m}}{\text{sec}}} \\
 &= 0.000303\text{sec} .
 \end{aligned}$$

With the help of this time period, the sampling frequency becomes:

$$\begin{aligned}
 f_0 &= \frac{1}{\Delta t} \\
 &= 3297.07\text{Hz} .
 \end{aligned}$$

4

Σ 24

Task 3: Discrete Fourier-Transformation

In general: $\text{DFT}\{x(k)\} = X(n) = \sum_{k=0}^{N-1} x(k) \cdot W_N^{nk}$ with $W_N = e^{-i\frac{2\pi}{N}}$.

- a) The periodic, time discrete signal is $N = 4$ long. □ 1
- b) Therefore the equation for the discrete Fourier transformation is:

$$\text{DFT}\{x(k)\} = X(n) = \sum_{k=0}^3 x(k) \cdot W_4^{nk} .$$

The equation system for achieving the Fourier transform $X(n)$ is:

$$\begin{aligned} X(0) &= W_4^0 \cdot x(0) + W_4^0 \cdot x(1) + W_4^0 \cdot x(2) + W_4^0 \cdot x(3) \\ X(1) &= W_4^0 \cdot x(0) + W_4^1 \cdot x(1) + W_4^2 \cdot x(2) + W_4^3 \cdot x(3) \\ X(2) &= W_4^0 \cdot x(0) + W_4^2 \cdot x(1) + W_4^4 \cdot x(2) + W_4^6 \cdot x(3) \\ X(3) &= W_4^0 \cdot x(0) + W_4^3 \cdot x(1) + W_4^6 \cdot x(2) + W_4^9 \cdot x(3) . \end{aligned} \quad \square 4$$

The same in matrix-vector notation:

$$\underbrace{\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix}}_{\underline{X}} = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}}_F \cdot \underbrace{\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}}_{\underline{x}} . \quad (2) \quad \square 2$$

- c) The given signal with periodic time $N = 4$ is:

$$\underline{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} .$$

Used in equation 2 it leads to \underline{X} :

$$\underline{X} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 - 1 \\ 1 - W_4^3 \\ 1 - W_4^6 \\ 1 - W_4^9 \end{bmatrix} . \quad \square 2$$

d) The only powers of W_N to calculate are W_4^3 , W_4^6 und W_4^9 :

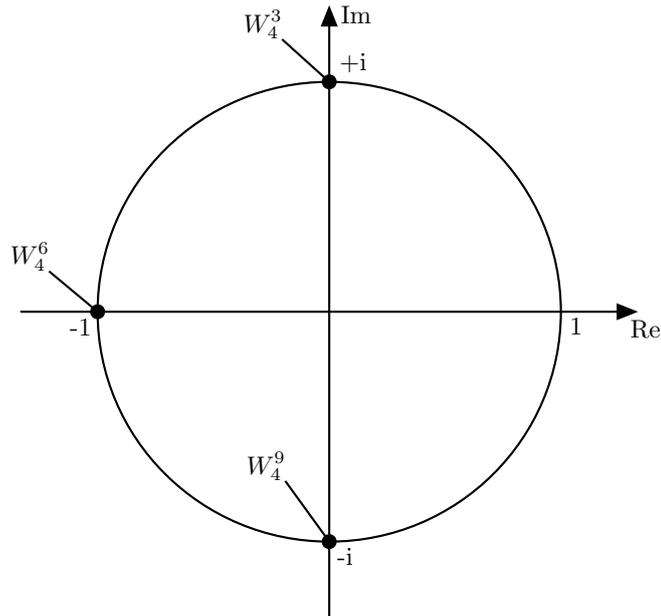
$$W_4^3 = e^{-i\frac{2\pi}{4}3} = \cos(-\frac{3}{2}\pi) + i \cdot \sin(-\frac{3}{2}\pi) = i$$

$$W_4^6 = e^{-i\frac{2\pi}{4}6} = \cos(-\frac{6}{2}\pi) + i \cdot \sin(-\frac{6}{2}\pi) = -1$$

$$W_4^9 = e^{-i\frac{2\pi}{4}9} = \cos(-\frac{9}{2}\pi) + i \cdot \sin(-\frac{9}{2}\pi) = -i .$$

3

Plot of them in the complex plane.



5

e) The discrete amplitude spectrum $|\underline{X}|$ is:

$$|\underline{X}| = \left\| \begin{bmatrix} 1 - 1 \\ 1 - i \\ 1 - (-1) \\ 1 - (-i) \end{bmatrix} \right\| = \left\| \begin{bmatrix} 0 \\ 1 - i \\ 2 \\ 1 + i \end{bmatrix} \right\| = \begin{bmatrix} 0 \\ \sqrt{2} \\ 2 \\ \sqrt{2} \end{bmatrix} .$$

2

$\sum 19$

Task 4: Correlation, Probability Densities and Confidence Intervals

a) Assign the following correlation coefficients to one of the figures below. Every correlation coefficient can only be assigned once.

- (a): 0
- (b): 0.7
- (c): -0.9
- (d): -0.3

4

b) Calculate the autocorrelation function values for the signal $u(k)$.

The autocorrelation function values differ from zero only for two time shifts: For $\kappa = 0$ and $\kappa = 4$.

$$r_{xx}(\kappa = 0) = \frac{2}{9}$$

$$r_{xx}(\kappa = 4) = \frac{1}{9} .$$

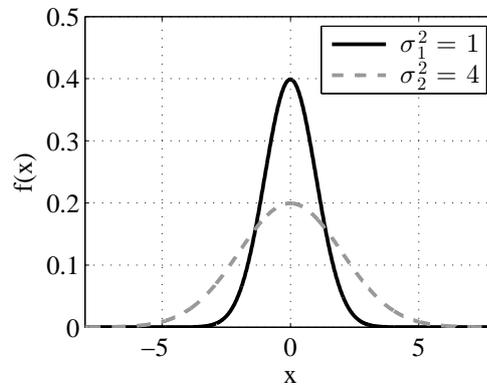
The estimation with the given equation is biased.

3

c) Probability Densities

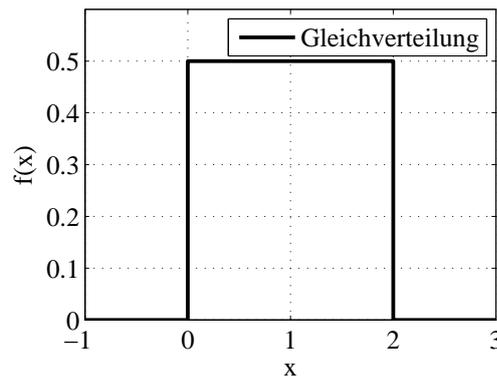
1

1) Normal distributions



3

2) Uniform distribution



2

d) Confidence Intervals

- 1) Calculate the range in which the true temperature will be for an accepted error probability of 0.27%.

Sample mean: $\bar{T} = \frac{1}{5} \sum_{i=1}^5 T(i) = \frac{325.1}{5} = 65.02^\circ C$.

Standard deviation of the sample mean: $\sigma_{\bar{T}} = \frac{\sigma_T}{\sqrt{5}} = \frac{0.2}{2.24} = 0.09$.

From the accepted error probability it follows a requested confidence level of 99.73%. This corresponds to $c = 3$ for the range calculation. The range is:

$$\begin{aligned} \bar{T} - c \cdot \sigma_{\bar{T}} < T < \bar{T} + c \cdot \sigma_{\bar{T}} \\ 64.78 < T < 65.32 \end{aligned}$$

3

- 2) Estimate the instrument's standard deviation of the temperature disturbance with the help of the measured values from above.

$$\begin{aligned} s_T &= \sqrt{\frac{1}{4} \sum_{i=1}^5 [T(i) - \bar{T}]^2} \\ &= 0.2 \end{aligned}$$

1

- 3) Given s_T from subtask d2), will there anything change regarding the confidence interval calculation for the true temperature value compared with the one from subtask d1)?

Even though the estimated and the true standard deviation are exactly equal, the resulting interval will be different. In the case, where the estimated standard deviation is used to calculate the range, the factor c has to be determined from a t-student distribution.

2

Σ 19

Task 5: Time discrete system

a) The impulse response is the derivative of the step response:

$$g(t) = \frac{d}{dt}h(t) = \frac{d}{dt} \left(K \cdot \left(1 - e^{-\frac{t}{T}} \right) \right) = K \cdot \left(-\frac{1}{T} \right) \cdot -e^{-\frac{t}{T}} = \frac{K}{T} \cdot e^{-\frac{t}{T}} .$$

Considering the given values leads to:

$$g(t) = \frac{8}{2} \cdot e^{-\frac{t}{2\text{sec}}} = 4 \cdot e^{-\frac{t}{2\text{sec}}} .$$

b) The first thing to do is to discretize the impulse response with the sampling time $T_0 = 0.5$ sec:

$$g(k) = g(t = T_0 \cdot k) = \frac{K}{T} \cdot e^{-\frac{T_0 \cdot k}{T}} = \frac{8}{2} \cdot e^{-\frac{0.5\text{sec} \cdot k}{2\text{sec}}} = 4 \cdot e^{-0.25k} .$$

Used in the general equation of the z-transformation results in $G(z)$:

$$\begin{aligned} G_I(z) &= \sum_{k=0}^{\infty} \left(g(k) \cdot z^{-k} \right) = \sum_{k=0}^{\infty} \left(4 \cdot e^{-0.25k} \cdot z^{-k} \right) = 4 \cdot \sum_{k=0}^{\infty} \left(e^{-0.25k} \cdot z^{-k} \right) \\ &= 4 \cdot \sum_{k=0}^{\infty} \left(e^{-0.25} \cdot z^{-1} \right)^k . \end{aligned}$$

Using the equation of a geometric series $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$ leads to:

$$G_I(z) = 4 \cdot \sum_{k=0}^{\infty} \left(e^{-0.25} \cdot z^{-1} \right)^k = \frac{4}{1 - e^{-0.25} \cdot z^{-1}} .$$

c) Similar to c) the first step is the discretization of the time continuous step response with the sampling time $T_0 = 0.5$ sec:

$$h(k) = h(t = T_0 \cdot k) = K \cdot \left(1 - e^{-\frac{T_0 \cdot k}{T}} \right) = 8 \cdot \left(1 - e^{-\frac{0.5\text{sec} \cdot k}{2\text{sec}}} \right) = 8 \cdot \left(1 - e^{-0.25 \cdot k} \right) .$$

Used in the general equation of the z-transformation results in $H(z)$:

$$\begin{aligned} H_S(z) &= \sum_{k=0}^{\infty} \left(h(k) \cdot z^{-k} \right) = \sum_{k=0}^{\infty} \left(8 \cdot \left(1 - e^{-0.25 \cdot k} \right) \cdot z^{-k} \right) \\ &= 8 \cdot \left[\sum_{k=0}^{\infty} \left(z^{-k} \right) - \sum_{k=0}^{\infty} \left(e^{-0.25 \cdot k} \cdot z^{-k} \right) \right] \\ &= 8 \cdot \left[\sum_{k=0}^{\infty} \left(z^{-1} \right)^k - \sum_{k=0}^{\infty} \left(e^{-0.25} \cdot z^{-1} \right)^k \right] . \end{aligned}$$

Using the equation of a geometric series $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$ leads to:

$$\begin{aligned} \sum_{k=0}^{\infty} \left(z^{-1} \right)^k &= \frac{1}{1 - z^{-1}} \\ \sum_{k=0}^{\infty} \left(e^{-0.25} \cdot z^{-1} \right)^k &= \frac{1}{1 - e^{-0.25} \cdot z^{-1}} \\ \Rightarrow H_S(z) &= 8 \cdot \left[\frac{1}{1 - z^{-1}} - \frac{1}{1 - e^{-0.25} \cdot z^{-1}} \right] . \end{aligned}$$

Next step is converting the transfer function of the step response $H_S(z)$ into the transfer function $G_S(z)$.

$$\begin{aligned}
 H_S(z) &= G_S(z) \cdot \underbrace{\frac{1}{1-z^{-1}}}_{\text{step } \sigma(k)} \\
 G_S(z) &= H_S(z) \cdot (1-z^{-1}) \\
 G_S(z) &= 8 \cdot \left(\frac{1}{1-z^{-1}} - \frac{1}{1-e^{-0.25} \cdot z^{-1}} \right) \cdot (1-z^{-1}) \\
 G_S(z) &= 8 \cdot \frac{1 - e^{-0.25} \cdot z^{-1}}{1 - e^{-0.25} \cdot z^{-1}} \\
 G_S(z) &= 8 \cdot \frac{(1 - e^{-0.25}) \cdot z^{-1}}{1 - e^{-0.25} \cdot z^{-1}} .
 \end{aligned}$$

d) Comparison of the two discrete systems:

1) different gains : $h(k \rightarrow \infty) = G(z = 1)$

$$\begin{aligned}
 G_I(z = 1) &= \frac{4}{1 - e^{-0.25}} = 18.19 \\
 G_S(z = 1) &= 8 \cdot \frac{1 - e^{-0.25}}{1 - e^{-0.25}} = 8 .
 \end{aligned}$$

2) properness :

$$\begin{aligned}
 G_I(z) &= \frac{4}{1 - e^{-0.25} \cdot z^{-1}} \\
 \Leftrightarrow Y(z) - e^{-0.25} \cdot z^{-1} \cdot Y(z) &= 4 \cdot U(z) \\
 &\quad \updownarrow \\
 y(k) - e^{-0.25} \cdot y(k-1) &= 4 \cdot u(k) .
 \end{aligned}$$

\Rightarrow The system is proper. 1

$$\begin{aligned}
 G_S(z) &= 8 \cdot \frac{(1 - e^{-0.25}) \cdot z^{-1}}{1 - e^{-0.25} \cdot z^{-1}} \\
 \Leftrightarrow Y(z) - e^{-0.25} \cdot z^{-1} \cdot Y(z) &= 8 \cdot (1 - e^{-0.25}) \cdot z^{-1} \cdot U(z) \\
 &\quad \updownarrow \\
 y(k) - e^{-0.25} \cdot y(k-1) &= 8 \cdot (1 - e^{-0.25}) \cdot u(k-1) .
 \end{aligned}$$

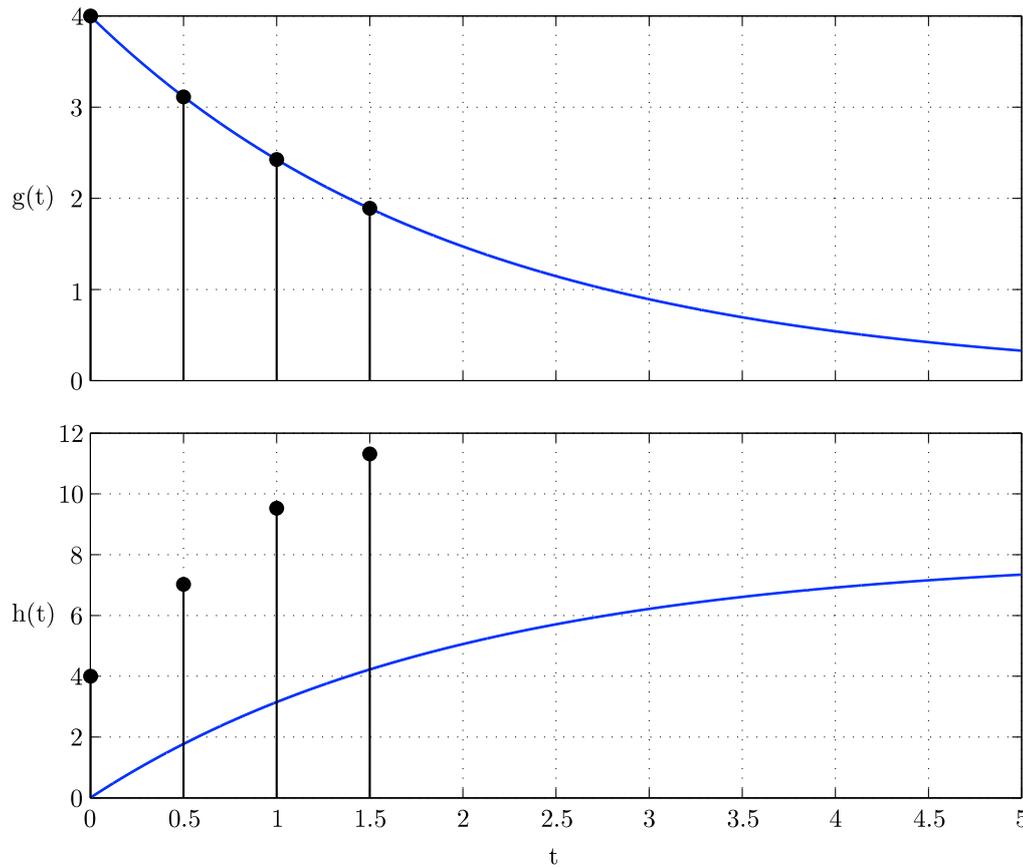
\Rightarrow The system is strictly proper. 1

3) stability:

$$\begin{aligned}
 G_I(z) &= \frac{4}{1 - e^{-0.25} \cdot z^{-1}} = \frac{4 \cdot z}{z - e^{-0.25}} \\
 G_S(z) &= 8 \cdot \frac{(1 - e^{-0.25}) \cdot z^{-1}}{1 - e^{-0.25} \cdot z^{-1}} = \frac{8 \cdot (1 - e^{-0.25})}{z - e^{-0.25}} .
 \end{aligned}$$

\Rightarrow both systems have the same pole.

e) The impulse and step response for the system $G_I(z)$ is:



2

2

The impulse response of the system $G_I(z)$ has the same values on the discrete points as the continuous impulse response of the system. To get the value of the step response it is necessary to calculate the first points $k=0,1,2,3$ of the difference equation of the system $G_I(z)$

$$G_i(z) = \frac{4}{1 - e^{-0.25} \cdot z^{-1}}$$

$$\circ$$

$$y(k) = 4 \cdot u(k) + e^{-0.25} \cdot y(k-1) .$$

with $u(k) = \sigma(k)$ and $y(-1) = 0$:

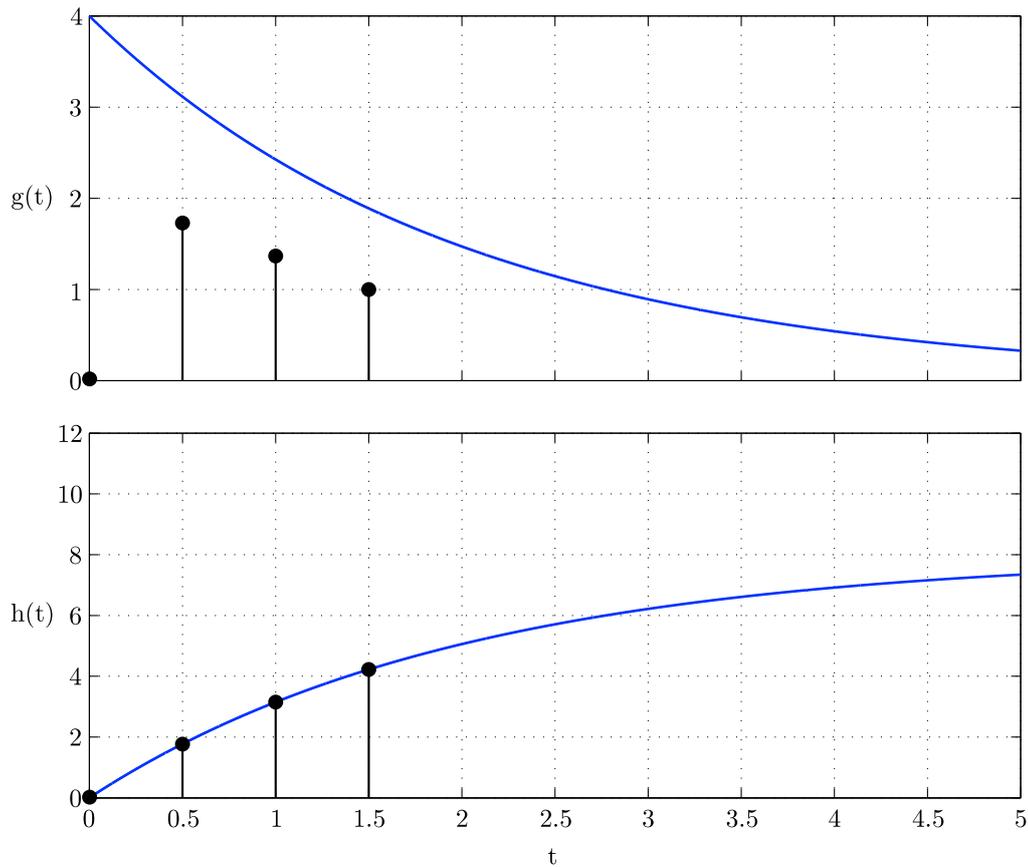
$$h(0) = 4 \cdot 1 + e^{-0.25} \cdot 0 = 4$$

$$h(1) = 4 \cdot 1 + e^{-0.25} \cdot 4 = 4 \cdot (1 + e^{-0.25}) = 7.1152$$

$$h(2) = 4 + e^{-0.25} \cdot 4 \cdot (1 + e^{-0.25}) = 4 \cdot (1 + e^{-0.25} + e^{-0.25 \cdot 2}) = 9.5413$$

$$\begin{aligned} h(3) &= 4 + e^{-0.25} \cdot 4 \cdot (1 + e^{-0.25} + e^{-0.25 \cdot 2}) \\ &= 4 \cdot (1 + e^{-0.25} + e^{-0.25 \cdot 2} + e^{-0.25 \cdot 3}) = 11.4308 . \end{aligned}$$

f) The impulse and step response for the system $G_S(z)$ is:



2

2

The step response of the system $G_S(z)$ has the same values on the discrete points as the continuous step response of the system. To get the value of the impulse response it is necessary to calculate the first points $k=0,1,2,3$ of the difference equation of the system $G_S(z)$

$$G_i(z) = 8 \cdot \frac{(1 - e^{-0.25}) \cdot z^{-1}}{1 - e^{-0.25} \cdot z^{-1}}$$

↓

$$y(k) = 8 \cdot (1 - e^{-0.25}) \cdot u(k - 1) + e^{-0.25} \cdot y(k - 1) .$$

with $u(k) = \delta_K(k)$ and $u(k - 1) = y(k - 1) = 0$:

$$g(0) = 8 \cdot (1 - e^{-0.25}) \cdot 0 + e^{-0.25} \cdot 0 = 0$$

$$g(1) = 8 \cdot (1 - e^{-0.25}) \cdot 1 + e^{-0.25} \cdot 0 = 8 \cdot (1 - e^{-0.25}) = 1.7696$$

$$g(2) = 8 \cdot (1 - e^{-0.25}) \cdot 0 + e^{-0.25} \cdot 8 \cdot (1 - e^{-0.25}) = e^{-0.25} \cdot 8 \cdot (1 - e^{-0.25}) = 1.3782$$

$$g(3) = 0 + e^{-0.25} \cdot e^{-0.25} \cdot 8 \cdot (1 - e^{-0.25}) = e^{-0.25 \cdot 2} \cdot 8 \cdot (1 - e^{-0.25}) = 1.0733 .$$

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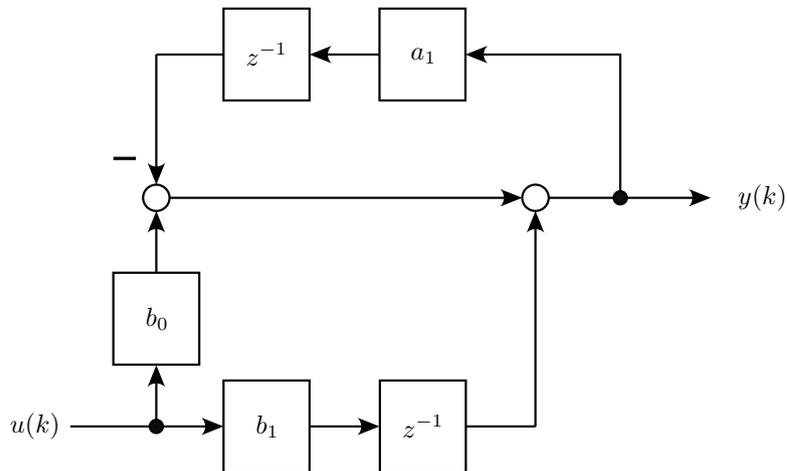
Task 6: Filter

a) Determine the difference equation for the given filter.

$$y_1(k) = b_0u(k) + b_1u(k - 1) - a_1y(k - 1)$$

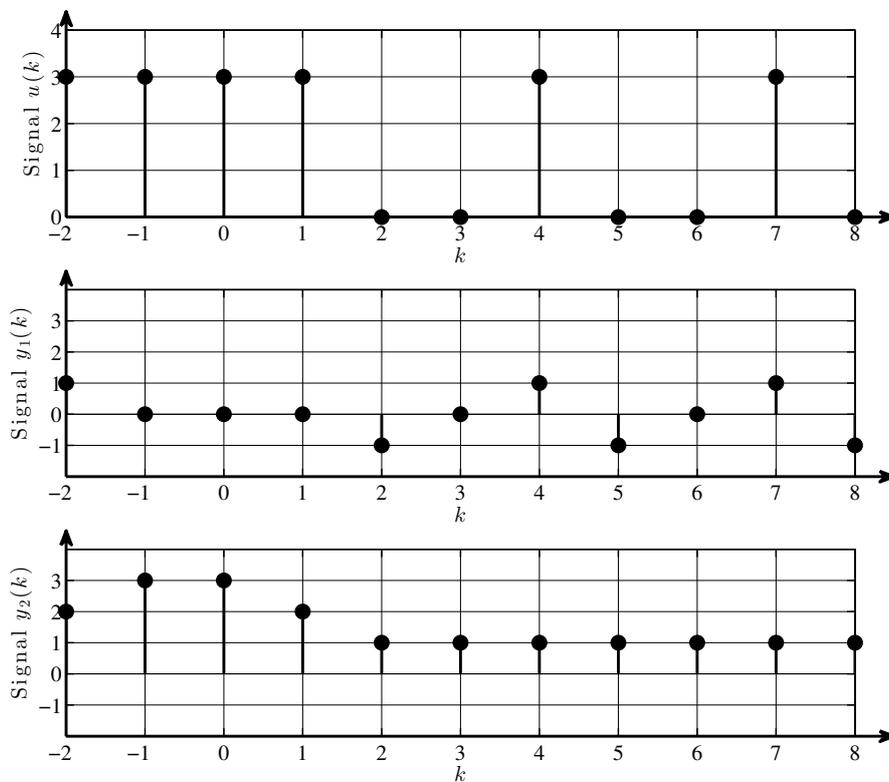
1

b) Sketch the block diagram of the given filter.



5

c) Calculate and sketch the values of the filtered signals $y_1(k)$ and $y_2(k)$.



10

d) Specify the type of filter for both filters (high-pass or low-pass?).

G_1 : high-pass; G_2 : low-pass.

2

e) Make a statement on causality for each filter.

G_1 : causal filter; G_2 : acausal filter.

2

f) Is G_1 a FIR- or a IIR-Filter, given the values for the coefficients from subtask c)?
State also if G_2 is an FIR- or IIR-Filter.

G_1 : FIR-Filter; G_2 : FIR-Filter.

2

$\sum 22$