Sensorics Exam

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26th of March 2019

Name:	
Mat.-No.:	
Grade:	

Task:	T1	T2	T3	T4	T5	T6	T7	T8	Sum
Scores:	18	19	14	16	17	13	11	12	120
Accomplished:									

Task 1: **Comprehension Questions (18 points)**

Mark the correct answers clearly.
Every question has one, two or three correct answers!
For every correctly marked answer you will get one point. If there is one correct answer marked and one incorrect answer marked, you will get no point for that subtask.

a) A system has minimal phase if ...
- ☐ ...it has stable poles, zeros inside the unit circle and no dead time.
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- ☐ ...it has stable poles, zeros outside the unit circle and a dead time.

b) A system in the z-domain is ...
- ☐ ...stable if all poles are outside the unit circle.
- ☐ ...stable if all poles are inside the unit circle.
- ☐ ...stable if it was stable in s-domain and the bilinear transformation was used.
- ☐ ...unstable if poles have no imaginary part.

c) Assess the following statements regarding measurements:
- ☐ The Doppler effect can be used to measure a distance and the relative velocity.
- ☐ Acceleration can <u>only</u> be obtained by taking the derivative of speed signals.
- ☐ The resistance in strain gauges changes proportionally to the change in length.
- ☐ Pressure measurement is typically based on the measurement of force.

d) Discretization ...
- ☐ ... in time is called sampling.
- ☐ ... in amplitude is called sampling.
- ☐ ... in time is called quantization.
- ☐ ... in amplitude is called quantization.

e) Assess following statements regarding digital measurement techniques.
- ☐ Digital electronics are sensitive with respect to environmental influences (e.g. temperature).
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- ☐ Shannons sampling theorem: The lowest sampling frequency has to be half of the highest frequency component of the signal.
- ☐ The aliasing effect enables a signal to be reconstructed perfectly.

f) Assess following statements regarding errors:
- ☐ The relative error of a measurement is the difference between the displayed/outputted value and the true value.
- ☐ The absolute error of a measurement is the difference between the displayed/outputted value and the true value.
- ☐ The quadratic error is not suitable as a criterion for optimization.
- ☐ The mean quadratic error is suitable as a criterion for the performance.

g) Assess the following statements regarding statistics:
- ☐ Any value of a continuous probability density function represents the probability of the variable.
- ☐ The mean value is an important parameter of the normal distribution.
- ☐ The Gaussian distribution is different from the normal distribution.
- ☐ A higher number of measurements usually decreases the variance of an estimation, compared to a very low number of measurements.

h) Assess following statements regarding sensor characteristics:
- ☐ An affine relationship is a linear relationship as well.
- ☐ A dead zero measurement allows to distinguish between a zero measurement and a disconnection or other wire breakage.
- ☐ The sensitivity of an instrument is determined by the slope of its sensor characteristics in the operating point.
- ☐ If the sensitivity is low, a change in the measured value hardly affects the output of the instrument.

i) These signals are <u>not</u> deterministic:

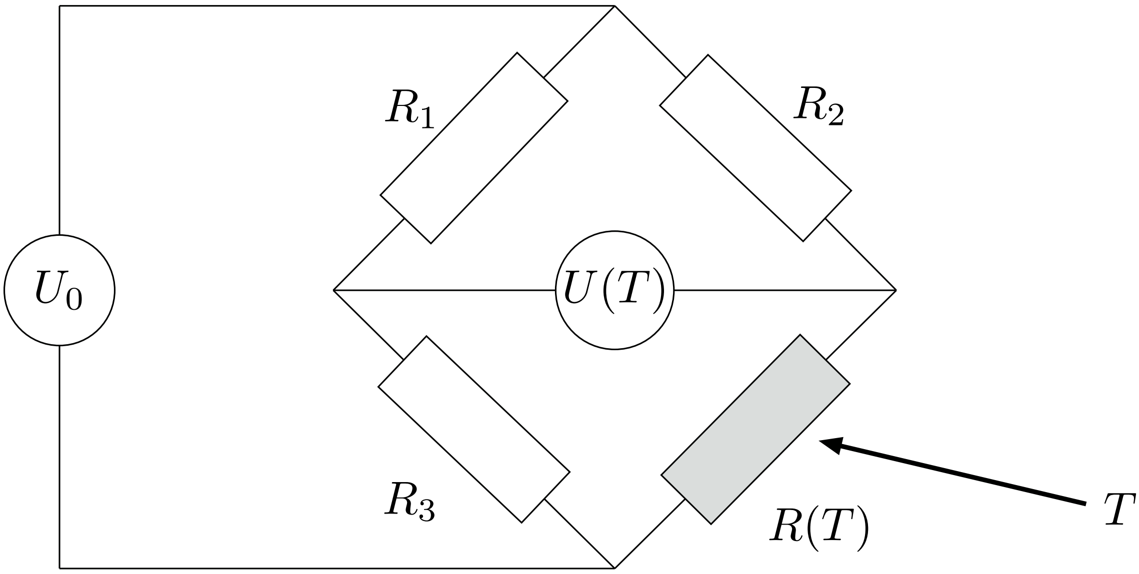
- [ ] Step
- [ ] Noise
- [ ] Sine
- [ ] Ramp

j) Assume $F(x)$ and $G(x)$ are linear functions. Which statement is true?

- [ ] $F(G(x)) = G(F(x))$
- [ ] $F(G(x)) \neq G(F(x))$
- [ ] $K \cdot F(x) = F(K \cdot x)$
- [ ] $c + K \cdot F(x) = F(c + Kx)$

Task 2: **Temperature Measurement (19 points)**

The temperature T can be measured with the following circuit. The relevant operating range is between $T = 0°C$ and $200°C$, whereas accuracy is more important in the lower end.



$R(T)$ is the temperature dependent resistance of a PRT-sensor. The characteristic of this sensor is given by

$$R(T) = R_{ref}[1 + a_1 T + a_2 T^2 + a_3 (T - 100)^3]. \tag{1}$$

The resistance of this sensor can be measured by the depicted bridge circuit. The relationship between $R(T)$ and $U(T)$ is described by

$$U(T) = U(R(T)) = U_0 \left(\frac{R(T)}{R_3 + R(T)} - \frac{R_2}{R_1 + R_2} \right). \tag{2}$$

Constants of the above equations are listed in the following table.

a_1	$3.9083 \cdot 10^{-3}\ °C^{-1}$
a_2	$-5.7750 \cdot 10^{-7}\ °C^{-2}$
a_3	$-4.1830 \cdot 10^{-12}\ °C^{-3}$
R_{ref}	100Ω
R_1	1000Ω
R_2	1000Ω
R_3	100Ω
V_0	$7.27V$

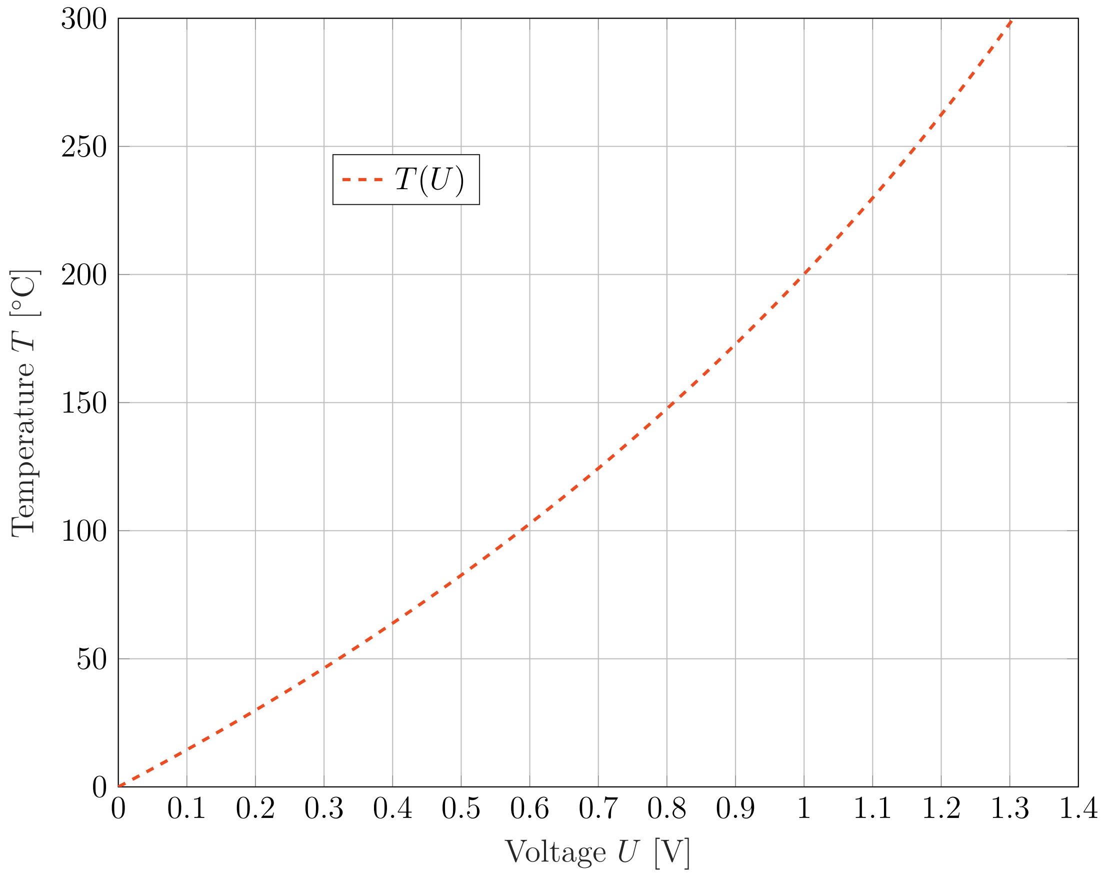
a) Linearize $R(T)$(around a reasonable operating point). The linearized function should be called $R_l(T)$. Explain your linearization or give a calculation.

b) Use equation (2) to determine the equation for $R(U(T))$.

c) The voltage U is measured now. Use your functions $R(U(T))$ and $R_l(T)$ to obtain $T_l(R(U))$. Calculate the corresponding temperatures for given voltages in the table.

U *in* V	$T_l(R(U))$ in° C	$T(U)$ in° C
0		0
0.3		46
0.6		103
0.9		172
1		200
1.2		262

d) With the calculated points draw a the graph of $T_l(R(U))$ into the following figure. How does the error develop with increasing temperature?



e) What is the maximum absolute error in the relevant range? At which Voltage does it occur?

f) What type of static error is introduced by doing an approximation?

Task 3: Mean Filter (14 points)

Given is following input/output relation in discrete time:

$$y(k) = \frac{1}{3}\left(u(k+2) + u(k+1) + u(k)\right) \tag{3}$$

a) Use the given relationship to calculate $G(z)$.

b) Draw the response $y(k)$ of the input $u(k)$ from Fig. 1 into the same graph.

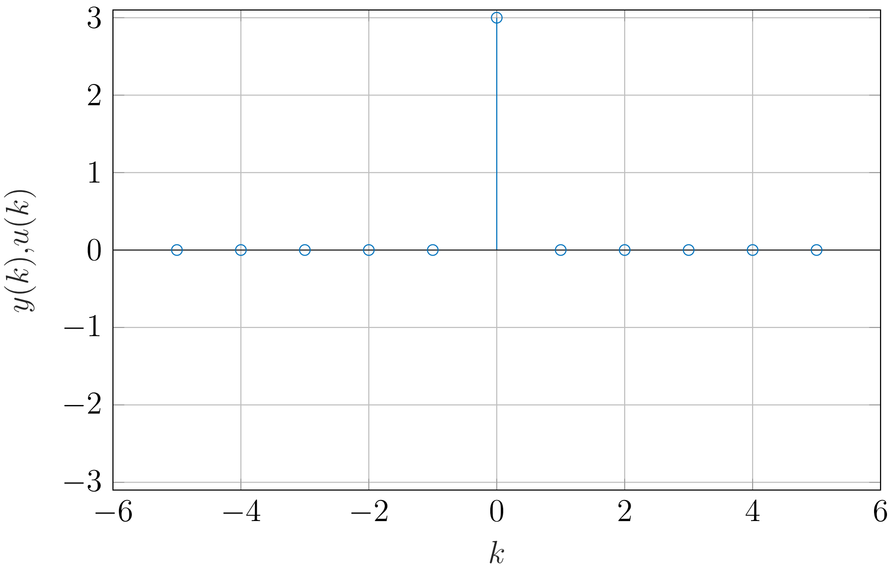


Figure 1: Input $u(k)$: (○)

c) Create the transfer function $G_d(z)$ by delaying $G(z)$ so that $G_d(z)$ has no phase shift. Give an explanation or calculation for your answer.
Hint: You can use following relationship: $e^{i\phi} = \cos(\phi) + i\sin(\phi)$

d) Does the transfer function $G_d(z)$ belong to a causal or non-causal system? Explain why.

e) Draw the response $y_d(k)$ (which corresponds to the transfer function $G_d(Z)$) of the input $u(k)$ from Fig. 2 into the same graph.

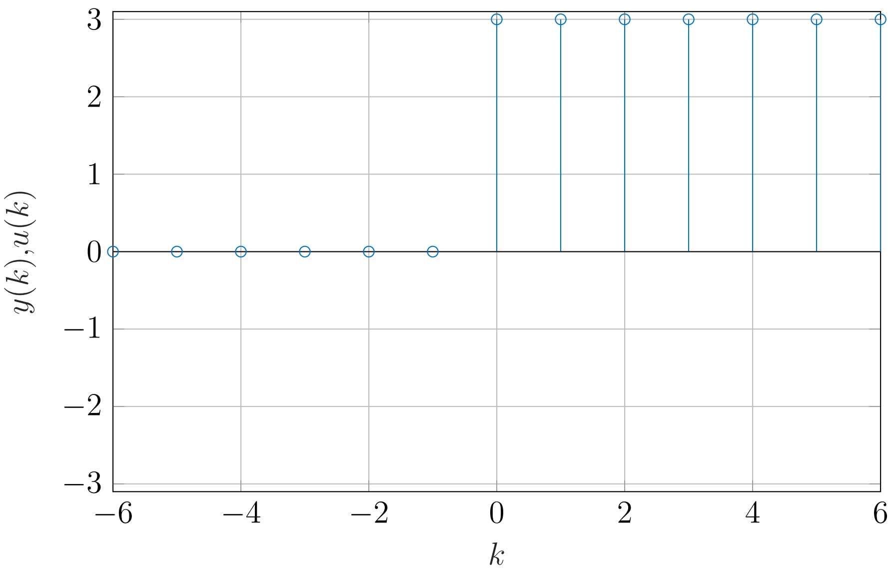


Figure 2: Input $u(k)$: (∘)

Task 4: Statistics

a) Draw the contour lines of the two-dimensional normal distributions for the given signals into Fig. 3.

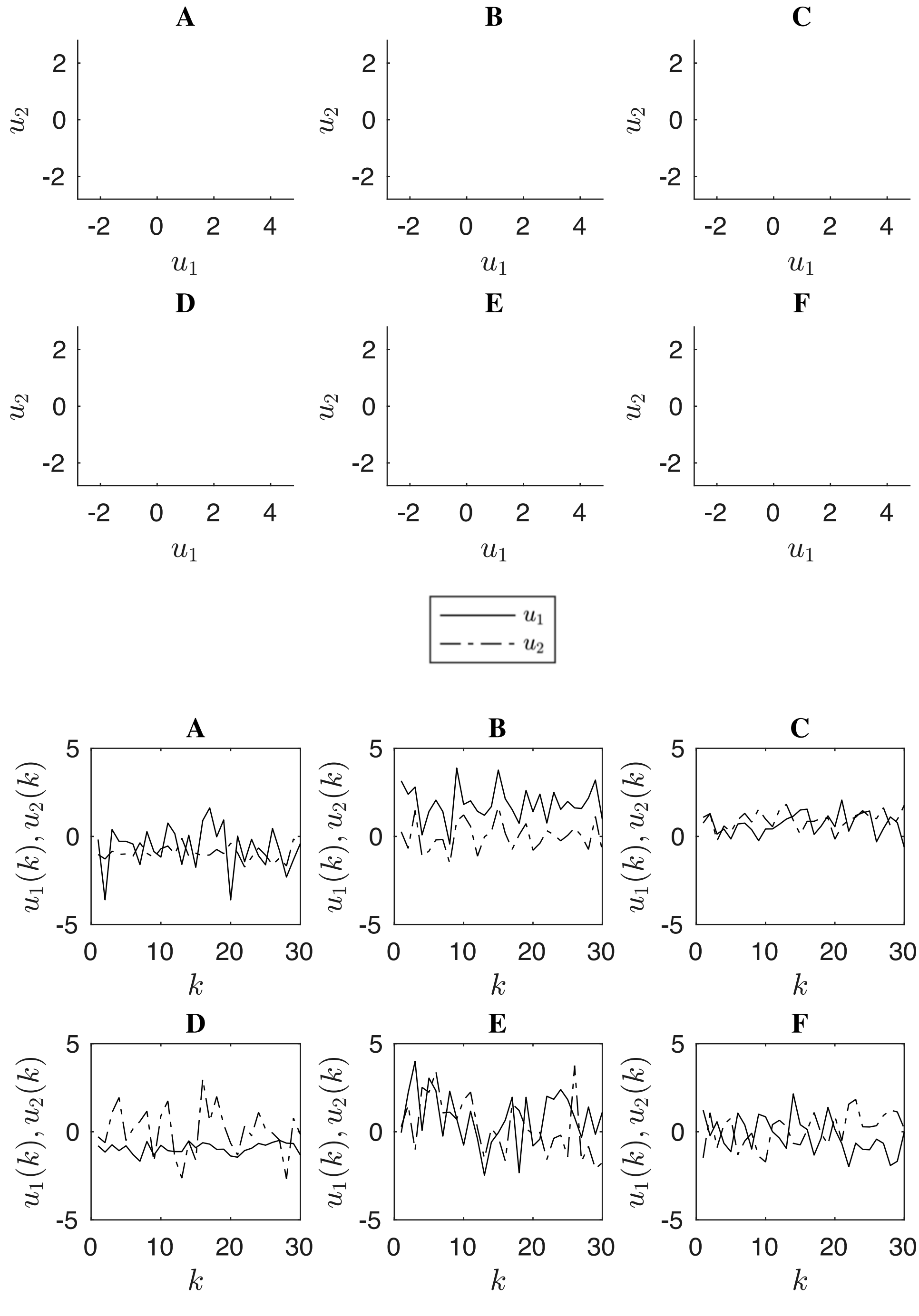


Figure 3: Signals u_1 and u_2 from different distributions.

b) A two-dimensional normal distribution has perfectly circular shaped contour lines. What properties does the distribution posses?

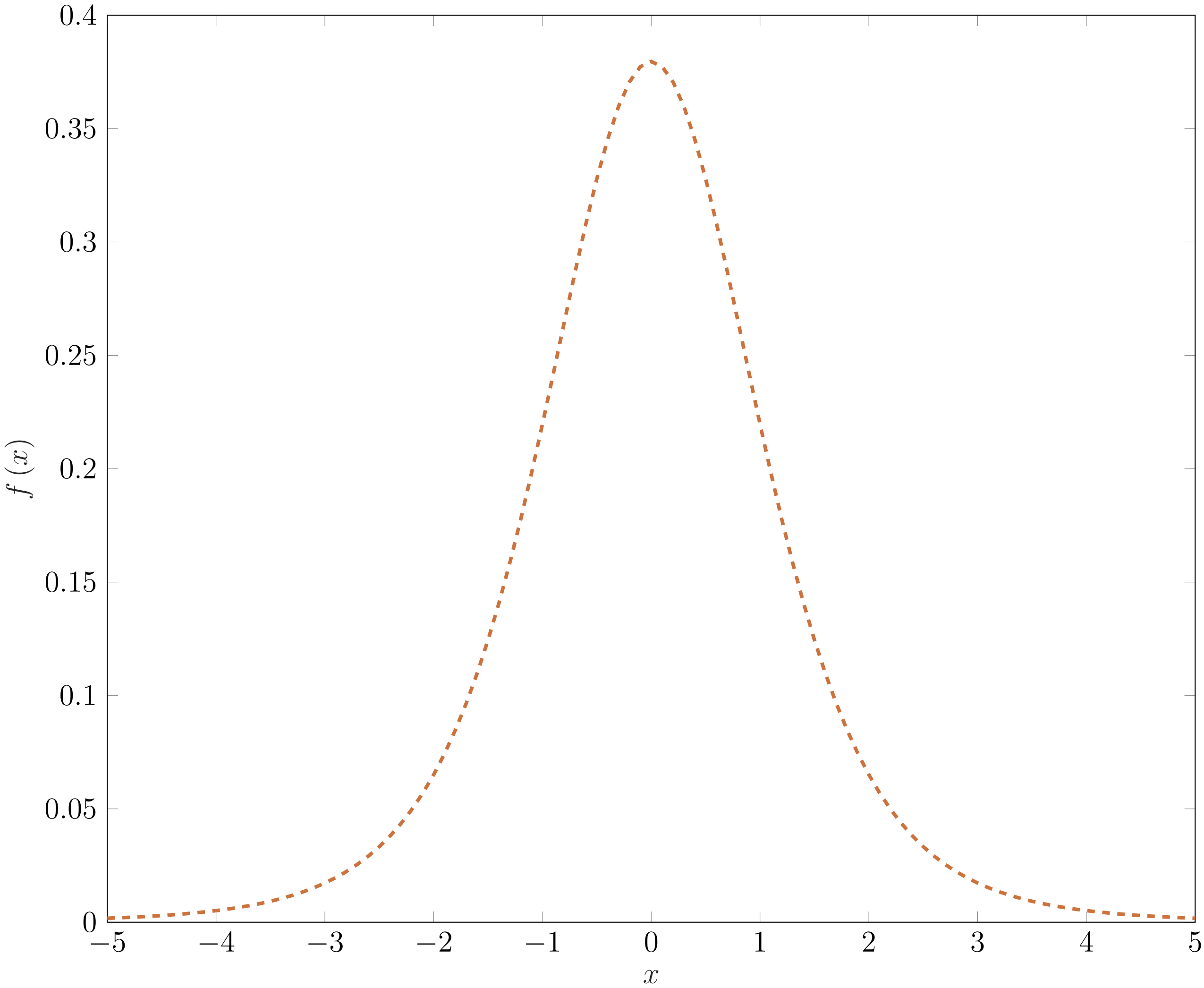


Figure 4: Student's t-distribution

c) This subtask (and the following ones) is about Student's distribution, also known as the t-distribution. Figure 4 shows a t-distribution for a medium amount of degrees of freedom.
Draw qualitatively a t-distribution for a higher number of degrees of freedom into Fig. 4. Name the line according to the task.

d) Draw qualitatively a t-distribution for a lower number of degrees of freedom into Fig. 4. Name the line according to the task.

e) How does the integral $\left(\int_{-\infty}^{\infty} f(x)\, dx\right)$ of a t-distribution change if the number of degrees of freedom is doubled?

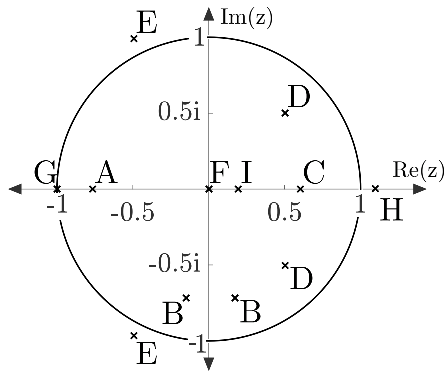
f) Under which circumstances does the t-distribution become equal to a normal distribution?

Task 5: Step Responses (17 points)

Given are the pole locations for nine different systems (A-I) in the pole zero map and six different step responses (1-6).

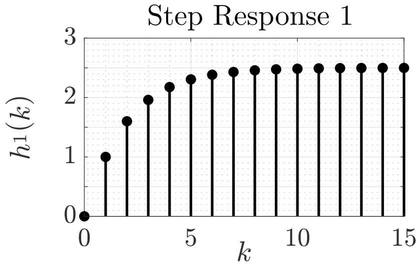
a) Match the pole locations to the step responses. Note down your answers in the given table. (Note: Three pole locations do not belong to any step response. All transfer functions have the structure $G(z) = \frac{1}{A(z)}$ with $A(z) = \prod_{i=1}^{n}(z - p_i)$.)

Poles

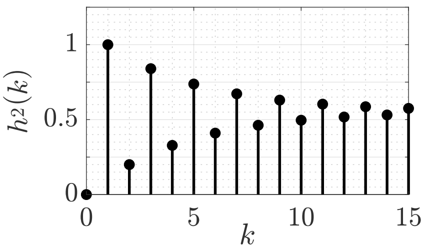


Step Response	Poles
1	
2	
3	
4	
5	
6	

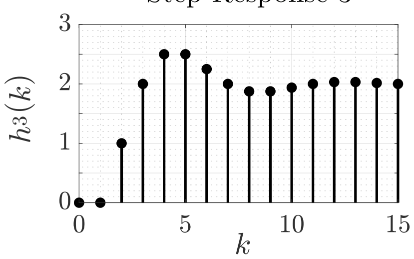
Step Response 1



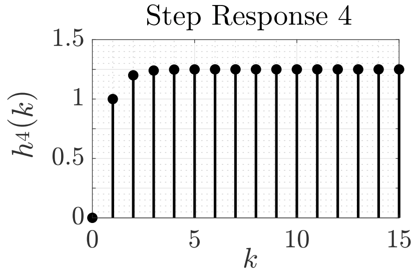
Step Response 2



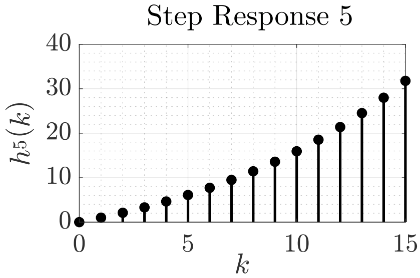
Step Response 3



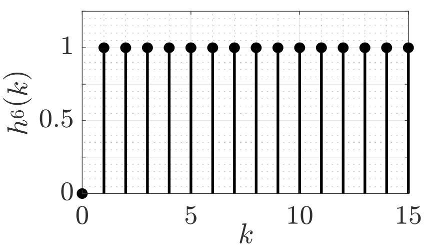
Step Response 4



Step Response 5



Step Response 6

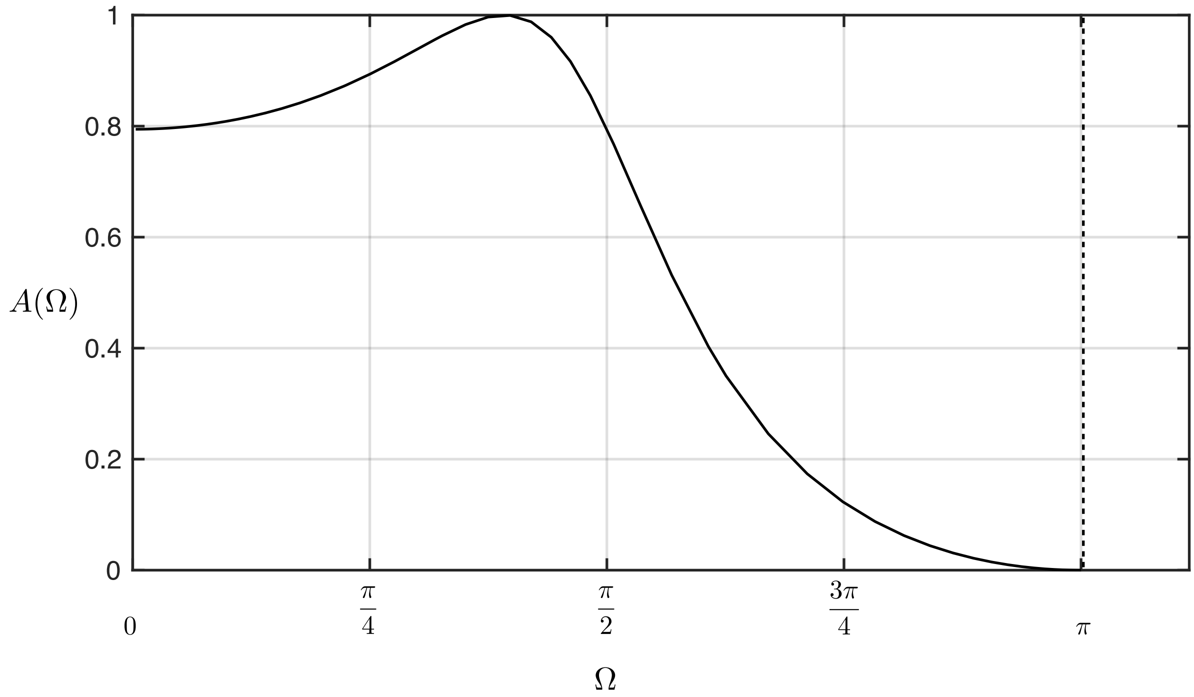


b) Assume that the pole of system C is at $p_c = 0.6$. Calculate the systems transfer function $G_C(z)$ and apply the final value theorem to the systems response with input signal $u(k) = 2 \cdot \sigma(k)$.

c) System $G_C(z)$ is now expanded to system $G_{CC}(z)$, which is $G_C(z)$ with $G_C(z)$ series-connected. What is the gain of $G_{CC}(z)$?

Task 6: Filter (13 points)

The amplitude response of a 2nd order filter is given. The normalized cut-off frequency is $\Omega_0 = \pi/2$.



a) Define the filter type (lowpass, highpass, bandpass, or bandstop).

b) What filter design was used (Butterworth, Cauer, Type I Chebyshev or Type II Chebyshev)?

c) Give a short explanation why $A(\Omega)$ is plotted only in the range $0 \leq \Omega \leq \pi$.

d) A step signal ($\sigma(k)$) is filtered using the filter above. What is the final value of the filtered step signal?

e) Sketch the remaining filters (2nd order each with cut-off frequency $\Omega_0 = \pi/2$) in the given diagram. Indicate which amplitude response corresponds to which filter design. Please make sure that the characteristic features of each filter is visible.

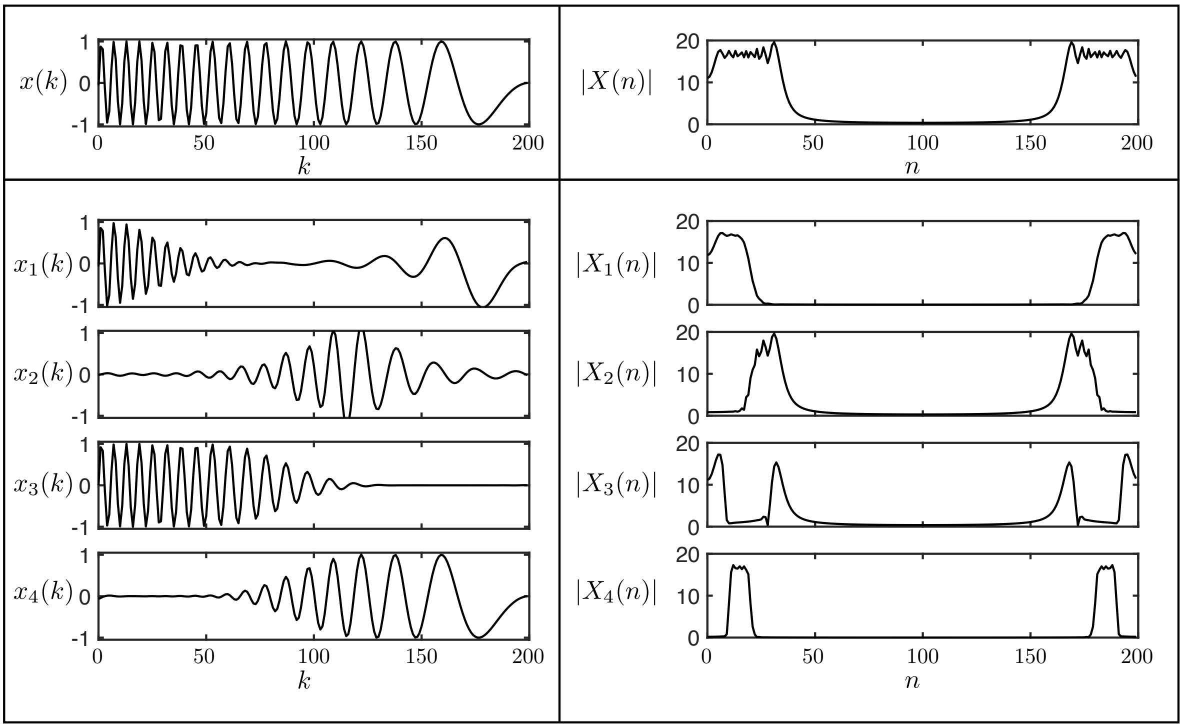
f) Draw a block diagram of a general IIR-filter of order 2. Use only blocks containing constants or z^{-1} values.

g) Give the difference-equation of the filter from sub-task f).

h) What defines the stability of the filter from f) / g) (short answer please)?

Task 7: DFT (11 points)

A sine-shaped signal $x(k)$ is given. The frequency of the signal changes linearly with the time. The absolute values of the corresponding DFT is also given by $|X(n)|$.



Now the signal $x(k)$ is filtered with different filters. The four resulting signals $x_1(k)$ - $x_4(k)$ are shown as well as the corresponding absolute values of the DFTs $|X_1(n)|, \ldots, |X_4(n)|$.

a) Assign to each signal $x_i(k)$ the corresponding absolute value of the DFT $|X_j(n)|$.

Signal $x_i(k)$	$x_1(k)$	$x_2(k)$	$x_3(k)$	$x_4(k)$
Corresponding $\lvert X_j(n)\rvert$				

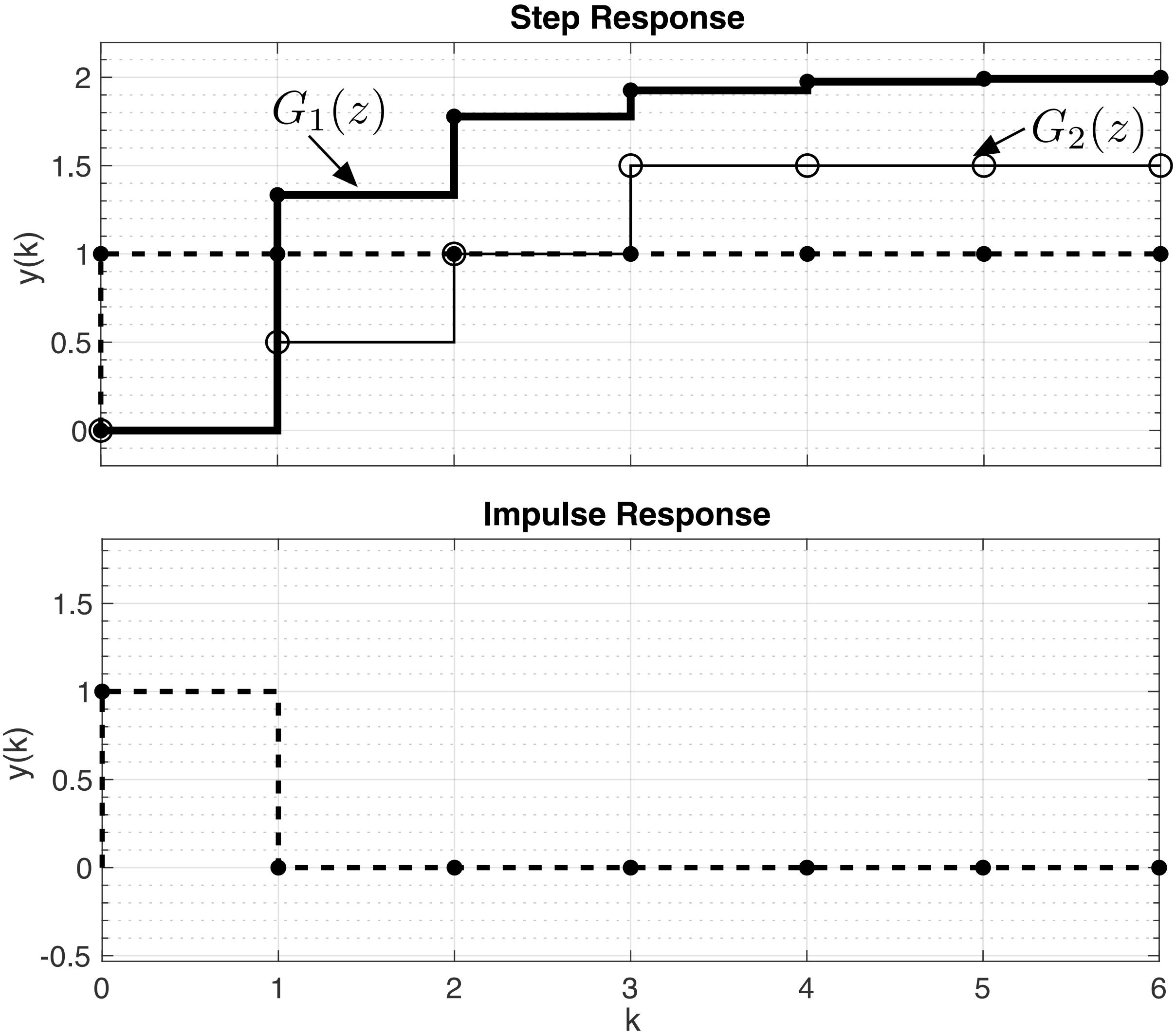
b) The frequency of the signal $x(k)$ changes over time. What can be done to analyze such a behavior? Mention at least one procedure and explain the approach in **a few** sentences.

c) Imagine four further DFTs $X_1(n), \ldots, X_4(n)$, all with length N. The values of the DFTs are equal to zero for $n = 0, 1, 2, \ldots, n_0 - 1, n_0 + 1, \ldots, N/2$. For n_0 the DFTs contain the following values:
$X_5(n_0) = 10, \quad X_6(n_0) = 10i, \quad X_7(n_0) = -10,$ and $X_8(n_0) = -10i\,.$
How do the corresponding signals $x_5(k)$, $x_6(k)$, $x_7(k)$, and $x_8(k)$ differ?
Hint: A calculation is not necessary!

d) Assume a signal with $2f_0$ is sampled with a sampling frequency f_0 at which normalized discrete frequency n occurs the highest peak?

Task 8: FIR- und IIR-Systems (12 points)

In the following diagram step responses for the transfer functions $G_1(z)$ und $G_2(z)$ are given.



a) Draw the impulse responses for $G_1(z)$ und $G_2(z)$ in the diagram above. The values can be rounded to the grid interval (0.1).

b) Explain, based on the drawn impulse responses, which transfer function has FIR and which has IIR behavior. If you couldn't draw the impulse responses in the task before, you can use the step responses for your explanation instead.

c) Explain (e.g. using gain, poles, zeros) for each of the given transfer functions $G_A(z), G_B(z), \dots G_F(z)$, if they represent $G_1(z)$, $G_2(z)$ or none of the two.

$$G_A(z) = \frac{2z^{-1}}{3 - z^{-1}}, \quad G_B(z) = \frac{(z^{-1} + z^{-2} + z^{-3})}{2}, \quad G_C(z) = \frac{2}{3 - z^{-1}}$$

$$G_D(z) = \frac{2z^{-1}}{1{,}5 - 0{,}5z^{-1}}, \quad G_E(z) = \frac{1 + z^{-1} + z^{-2}}{2} \quad , G_F(z) = \frac{4z^{-1}}{0{,}5 + 1{,}5z^{-1}},$$

Solutions:

Task 1: Comprehension Questions (18 Points)

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b) A system in the z-domain is …

- ☐ … stable if all poles are outside the unit circle.
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- ☐ … unstable if poles have no imaginary part.

c) Assess the following statements regarding measurements:

- ■ The Doppler effect can be used to measure a distance and the relative velocity.
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- ■ The resistance in strain gauges changes proportionally to the change in length.
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■ A higher number of measurements usually decreases the variance of an estimation, compared to a very low number of measurements.

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□ An affine relationship is a linear relationship as well.

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■ The sensitivity of an instrument is determined by the slope of its sensor characteristics in the operating point.

■ If the sensitivity is low, a change in the measured value hardly affects the output of the instrument.

i) These signals are <u>not</u> deterministic:

□ Step

■ Noise

□ Sine

□ Ramp

j) Assume $F(x)$ and $G(x)$ are linear functions. Which statement is true?

■ $F(G(x)) = G(F(x))$

□ $F(G(x)) \neq G(F(x))$

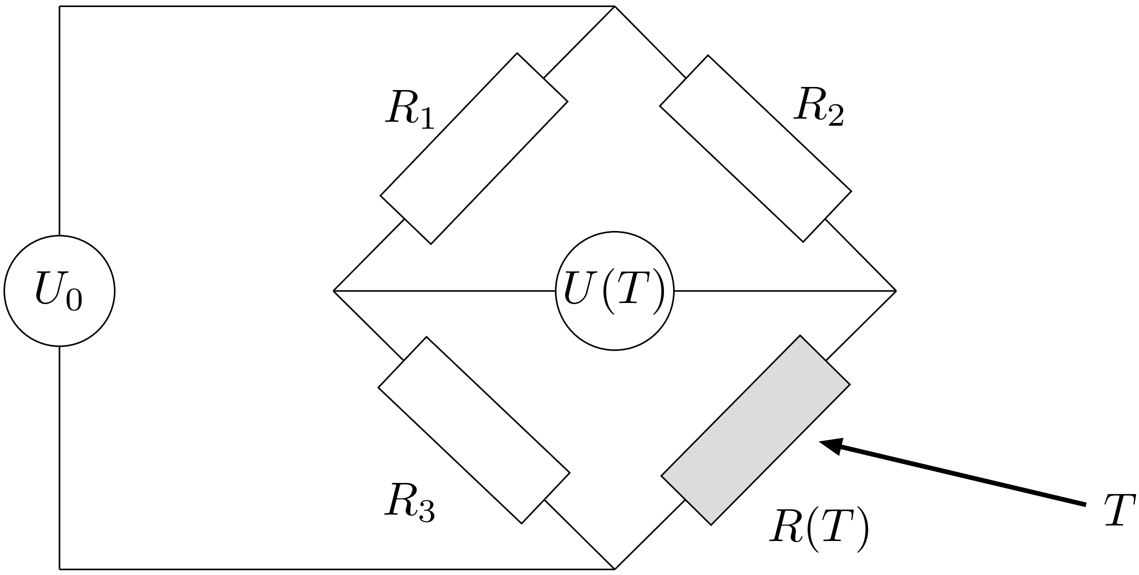
■ $K \cdot F(x) = F(K \cdot x)$

□ $c + K \cdot F(x) = F(c + Kx)$

$\boxed{\sum 18}$

Task 2: Temperature Measurement (19 points)

The temperature T can be measured with the following circuit. The relevant operating range is between $T = 0°C$ and $200°C$, whereas accuracy is more important in the lower end.



$R(T)$ is the temperature dependent resistance of a PRT-sensor. The characteristic of this sensor is given by

$$R(T) = R_{ref}[1 + a_1T + a_2T^2 + a_3(T - 100)^3]. \tag{4}$$

The resistance of this sensor can be measured by the depicted bridge circuit. The relationship between $R(T)$ and $U(T)$ is described by

$$U\left(T\right) = U\left(R\left(T\right)\right) = U_0\left(\frac{R\left(T\right)}{R_3 + R\left(T\right)} - \frac{R_2}{R_1 + R_2}\right). \tag{5}$$

Constants of the above equations are listed in the following table.

a_1	$3.9083 \cdot 10^{-3}\ °C^{-1}$
a_2	$-5.7750 \cdot 10^{-7}\ °C^{-2}$
a_3	$-4.1830 \cdot 10^{-12}\ °C^{-3}$
R_{ref}	100Ω
R_1	1000Ω
R_2	1000Ω
R_3	100Ω
V_0	$7.27V$

a) Linearize $R(T)$(around a reasonable operating point). The linearized function should be called $R_l(T)$. Explain your linearization or give a calculation.
Answer:
Possible solutions:
-Graphical estimation

-Taylor approximation
Hint: In general the Taylor series for a function $f(x)$ around an operating point x_0 is given by

$$Tf(x;x_0) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x-x_0)^n$$

Here, $n!$ denotes the factorial of n and $f^{(n)}(x_0)$ denotes the n-th derivative of $f(x)$ evaluated at x_0.

$$\frac{\partial R\left(\right)}{\partial T} = R_{ref}\left[a_1 + 2a_2 T + 3a_3 (T-100)^2\right]$$

For:

$$\begin{aligned} R_{Taylor}(T) &= R_{ref}\left[1 + a_1 T_0 + a_2 T_0^2 + a_3 (T_0-100)^3\right](T-T_0)^0 \\ &+ R_{ref}\left[a_1 + 2a_2 T_0 + 3a_3 (T_0-100)^2\right](T-T_0)^1 \end{aligned}$$

With T_0 being 0 °C:

$$\begin{aligned} R_{Taylor_0} &= R_{ref}\left[1 + a_3 (-100)^3\right] + R_{ref}\left[a_1 + 3a_3 (-100)^2\right] T \\ R_{Taylor_0} &= R_{ref}\left[1 + 4.183 \cdot 10^{-6}\right] + R_{ref}\left[3.9083 \cdot 10^{-3} - 1.25 \cdot 10^{-7}\right] T \\ R_{Taylor_0} &= R_{ref}\left[1 + a_1 T\right] \end{aligned}$$

-Cutting higher order terms with low impact

$$R_l(T) = R_{ref}[1 + a_1 T]$$

This is a valid approach in this case because the higher order terms do not have a big impact in the relevant range. $\boxed{3}$

b) Use equation (5) to determine the equation for $R\left(U\left(T\right)\right)$.
Answer:

$$U\left(R\left(T\right)\right) = U_0\left(\frac{R\left(T\right)}{R_3 + R\left(T\right)} - \frac{R_2}{R_1 + R_2}\right)$$
$$\frac{U\left(T\right)}{U_0} + \frac{R_2}{R_1 + R_2} = \frac{R\left(T\right)}{R_3 + R\left(T\right)}$$
$$\frac{U\left(T\right)}{U_0} + \frac{R_2}{R_1 + R_2} = \frac{1}{\frac{R_3}{R\left(T\right)} + 1}$$
$$\left(\frac{U\left(T\right)}{U_0} + \frac{R_2}{R_1 + R_2}\right)^{-1} = \frac{R_3}{R\left(T\right)} + 1$$
$$\frac{R\left(T\right)}{\frac{U\left(T\right)}{U_0} + \frac{R_2}{R_1 + R_2}} = R_3 + R\left(T\right)$$
$$\frac{R\left(T\right)}{\frac{U\left(T\right)}{U_0} + \frac{R_2}{R_1 + R_2}} - R\left(T\right) = R_3$$
$$\left(\frac{1}{\frac{U\left(T\right)}{U_0} + \frac{R_2}{R_1 + R_2}} + 1\right) R\left(T\right) = R_3$$
$$R\left(U\left(T\right)\right) = R_3\left(\frac{1}{\frac{U\left(T\right)}{U_0} + \frac{R_2}{R_1 + R_2}} - 1\right)^{-1}$$

$\boxed{5}$

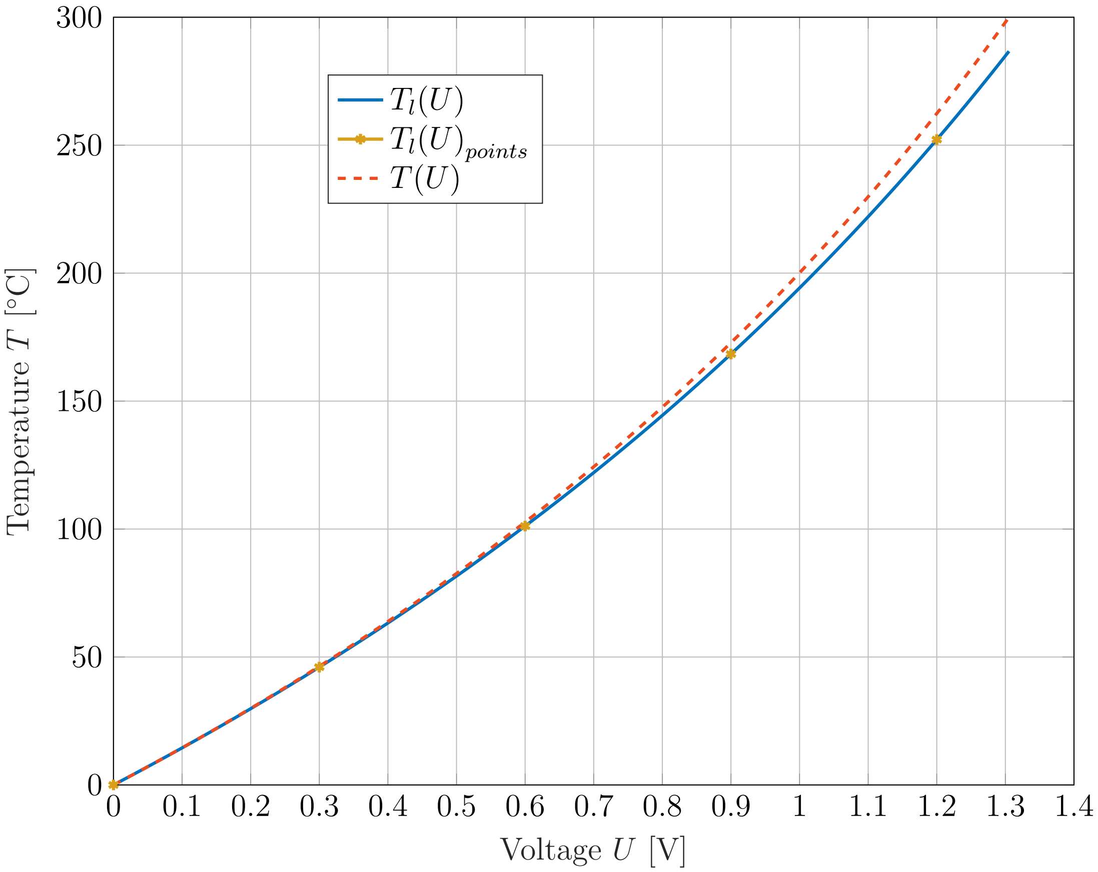
c) The voltage U is measured now. Use your functions $R\left(U\left(T\right)\right)$ and $R_l(T)$ to obtain $T_l\left(R\left(U\right)\right)$. Calculate the corresponding temperatures for given voltages in the table.
Answer:

$$From$$
$$R_l(T) = R_{ref}[1 + a_1 T]$$
$$to$$
$$T_l\left(U\right) = \frac{\frac{R\left(U\right)}{R_{ref}} - 1}{a_1}$$

U in V	$T(U)$ in °C	$T\left(U\right)$ in °C
0	0	0
0.3	46	46
0.6	101	103
0.9	168	172
1	194	200
1.2	252	262

$\boxed{3}$

d) With the calculated points draw a the graph of $T_l(U)$ into the following figure. How does the error develop with increasing temperature?
Answer:
Error increases with increasing nonlinearity of the original formula. It rises with rising temperature.



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e) What is the maximum absolute error in the relevant range? At which Voltage does it occur?
Answer:

$$e_{abs} = |y - y_{true}|$$
$$e_{abs} = |194°\text{C} - 200°\text{C}| = 6°\text{C}$$

At 1 V.

3

f) What type of static error is introduced by doing an approximation?
Answer:
A systematic error.

1

$\sum 19$

Task 3: Mean Filter

Given is following input/output relation in discrete time:

$$y\left(k\right) = \frac{1}{3}\left(u\left(k+2\right) + u\left(k+1\right) + u\left(k\right)\right) \tag{6}$$

a) Use the given relationship to calculate $G\left(z\right)$.
Answer:

$$Y\left(z\right) = \frac{1}{3}\left(U\left(z\right)z^0 + U\left(z\right)z^1 + U\left(z\right)z^2\right) \tag{7}$$

$$G\left(z\right) = \frac{1}{5}\left(z^2 + z^1 + z^0\right) \tag{8}$$

2

b) Draw the response $y(k)$ of the input $u(k)$ from Fig. 5 into the same graph.
Answer:

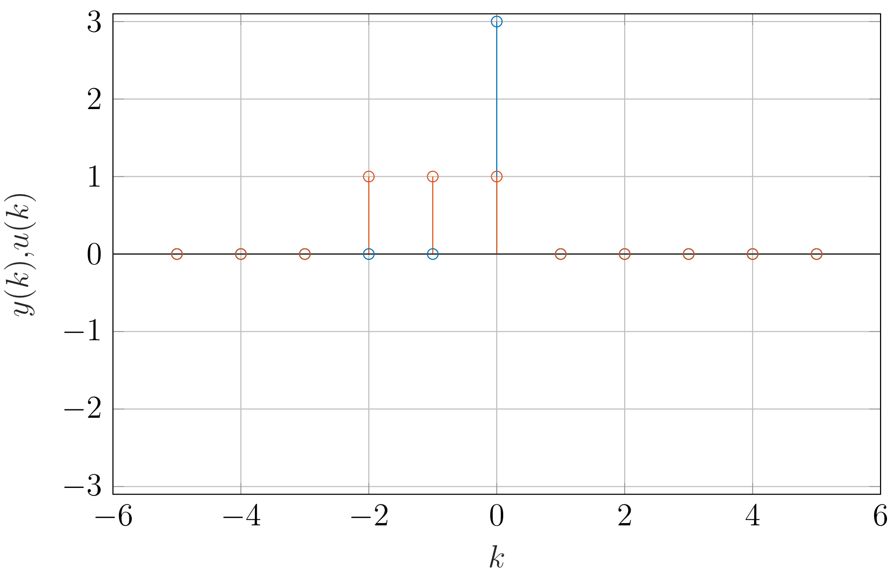


Figure 5: Input $u(k)$: (○), response $y(k)$: (○)

3

c) Create the transfer function $G_d(z)$ by delaying $G(z)$ so that $G_d(z)$ has no phase shift. Give an explanation or calculation for your answer.
Hint: You can use following relationship: $e^{i\phi} = \cos(\phi) + i\sin(\phi)$
Answer:
No phase shift means Imaginary parts cancel each other.
With a sufficient explanation eq. (9) is enough to acquire full score in this subtask.

$$G_d(z) = G(z)z^{-1} = \frac{1}{3}\left(z^{-1} + 1 + z^{1}\right) \tag{9}$$

$$G_d(i\omega) = \frac{1}{3}\left(e^{-1i\omega T_0} + e^{0i\omega T_0} + e^{1i\omega T_0}\right) \tag{10}$$

$$= \frac{1}{3}\left(\cos(\omega T_0) + i\sin(\omega T_0) + 1 + \cos(-\omega T_0) + i\sin(-\omega T_0)\right) \tag{11}$$

$$G_d(i\omega) = \frac{1}{3}\left(2\cos(\omega T_0) + 1\right) \tag{12}$$

4

d) Does the transfer function $G_d(z)$ belong to a causal or non-causal system? Explain why..
Answer:
Non-causal because it relies on input values from the future $u(k+1)$. 2

e) Draw the response $y_d(k)$ (which corresponds to the transfer function $G_d(Z)$) of the input $u(k)$ from Fig. 6 into the same graph.
Answer:

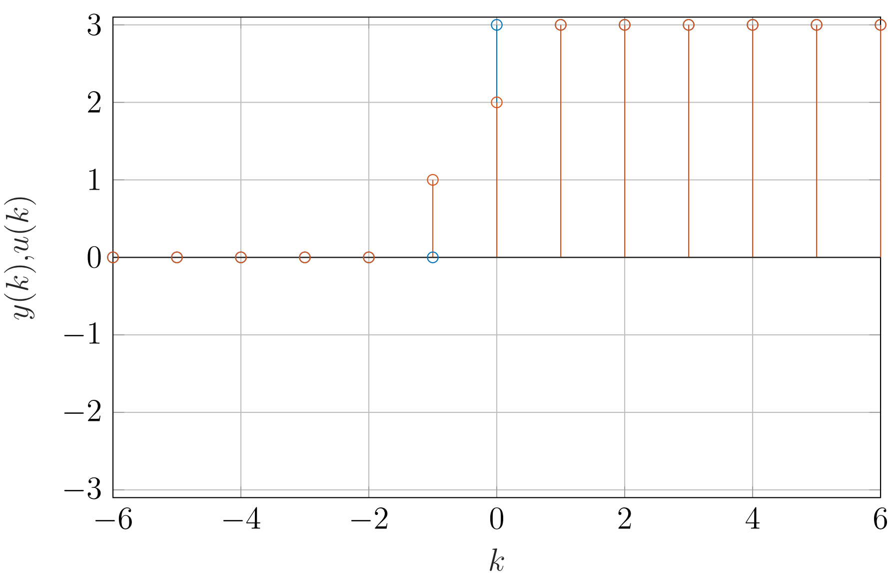


Figure 6: Input $u(k)$: (○), response $y(k)$: (○)

3

$\sum$ 14

Task 4: **Statistics**

a) Draw the two-dimensional normal distributions for the given signals into Fig. 7.

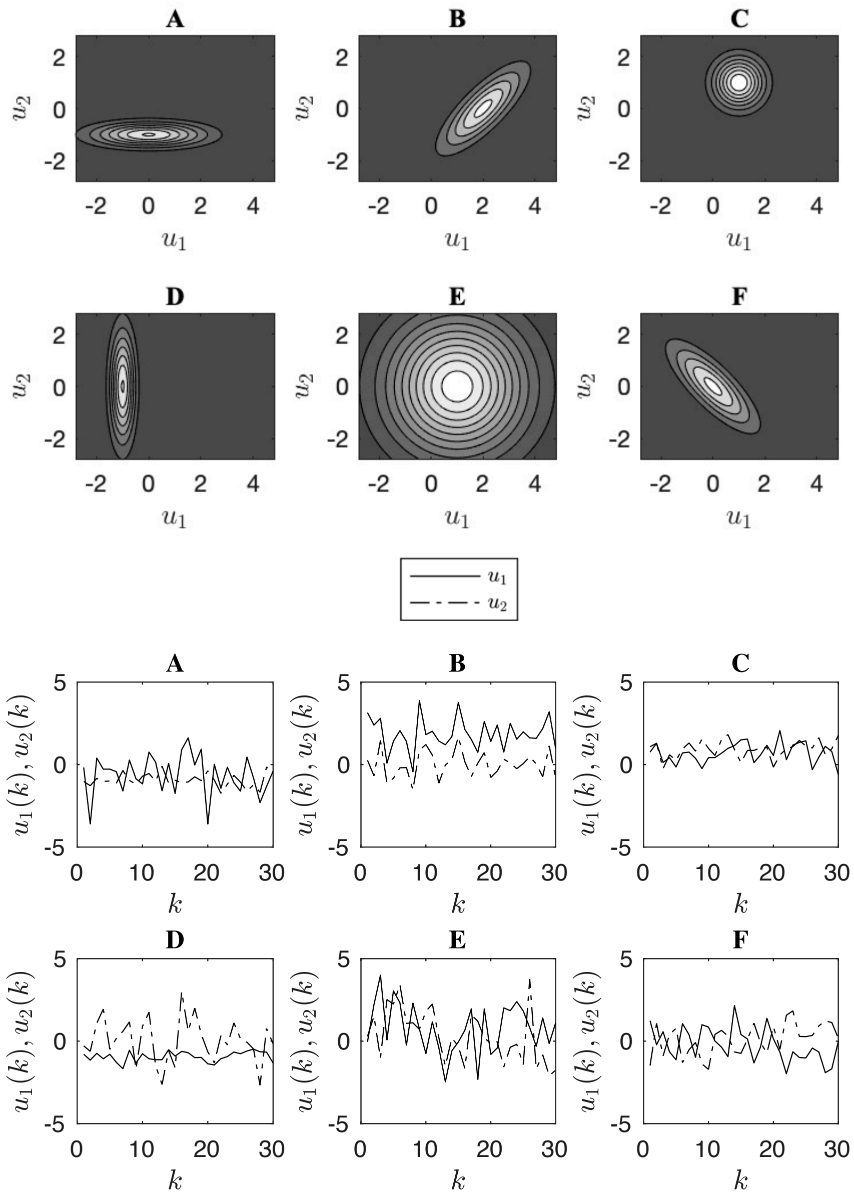


Figure 7: Correct pdf countour plots

6

b) A two-dimensional normal distribution has perfectly circular shaped contour lines. What properties does the distribution posses?
Answer:
The probabilities are independent of one another, thus the signals are uncorrelated.
$\rho_{u_1,u_2} = 0$

2

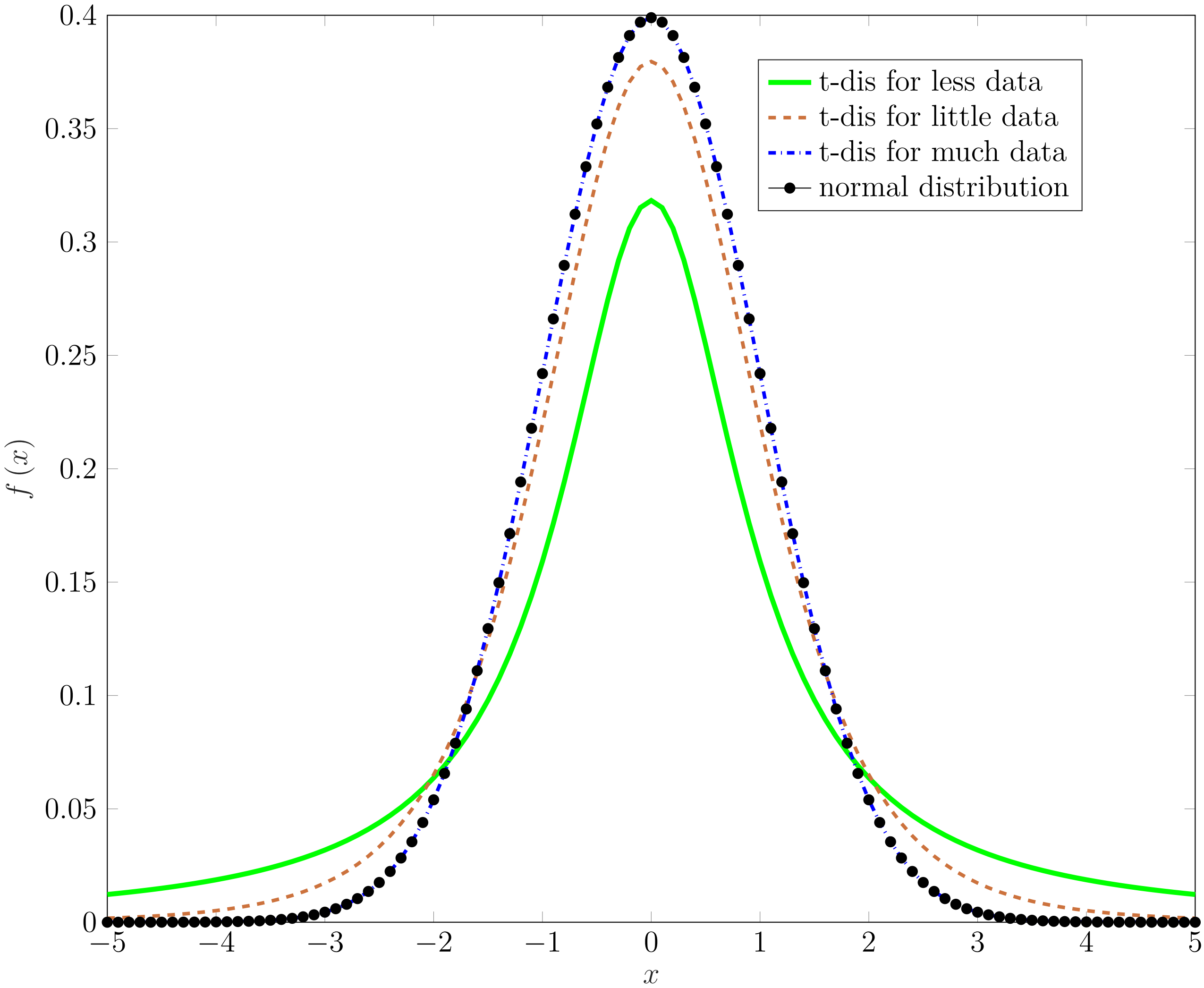


Figure 8: Student's- and normal distribution

c) This subtask (and the following ones) is about the Student's t-distribution, also known as the t-distribution. Figure 8 shows a t-distribution for medium amount of data.
Draw qualitatively a t-distribution for a higher number of degrees of freedom into Fig. 8. Name the line according to the task.
Answer: Same /almost the same as normal distribution. Higher mid, smaller tails. Dashed blue line. 2

d) Draw qualitatively a t-distribution for a lower number of degrees of freedom into Fig. 8. Name the line according to the task.
Answer: Flat middle, fat tails. Green line 2

e) How does the integral $\left(\int_{-\infty}^{\infty} f(x)\,dx\right)$ of a t-distribution change if the number of degrees of freedom is doubled?
Answer: It does not change. 2

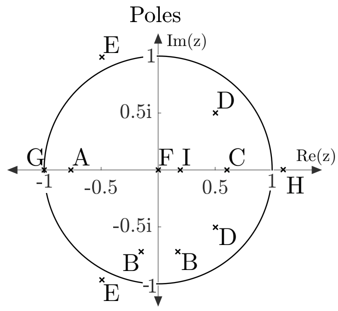
f) Under which circumstances does the t-distribution become equal to a normal distribution?
Answer: Infinitely high number of measurements/ degrees of freedom. 2

$\sum 16$

Task 5: Step Responses (17 points)

Given are the pole locations for nine different systems (A-I) in the pole zero map and six different step responses (1-6).

a) Match the pole locations to the step responses. Note down your answers in the given table. (Note: Three pole locations do not belong to any step response. All transfer functions have the structure $G(z) = \frac{1}{A(z)}$ with $A(z) = \prod_{i=1}^{n}(z - p_i)$.)



Step Response	Poles
1	C
2	A
3	D
4	I
5	H
6	F

2

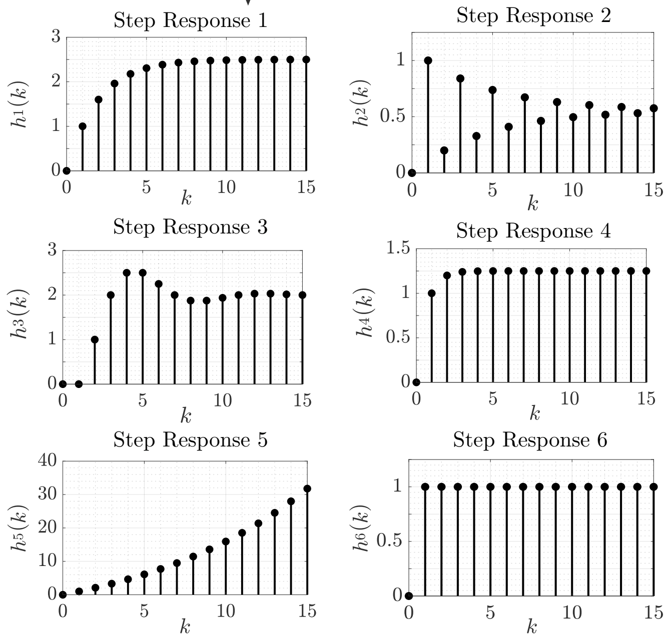
2

2

2

2

2



b) Assume that the pole of system C is at $p_c = 0.6$. Calculate the systems transfer function $G_C(z)$ and apply the final value theorem to the systems response with input signal $u(k) = 2 \cdot \sigma(k)$.

$$G_C(z) = \frac{1}{z - 0.6} = \frac{z^{-1}}{1 - 0.6z^{-1}} \qquad U(z) = \mathcal{Z}\{u(k)\} = 2 \cdot \frac{z}{z-1}$$

$$\lim_{z\to 1}(z-1)Y(z) = \lim_{z\to 1}(z-1)G_C(z)U(z) = \lim_{z\to 1}\cancel{(z-1)} \cdot \frac{1}{z-0.6}\frac{2z}{\cancel{z-1}} = 5$$

3

c) System $G_C(z)$ is now expanded to system $G_{CC}(z)$, which is $G_C(z)$ with $G_C(z)$ series-connected. What is the gain of $G_{CC}(z)$?

$$G_{CC}(z) = G_C(z) \cdot G_C(z) = \frac{1}{(z-0.6)^2}$$

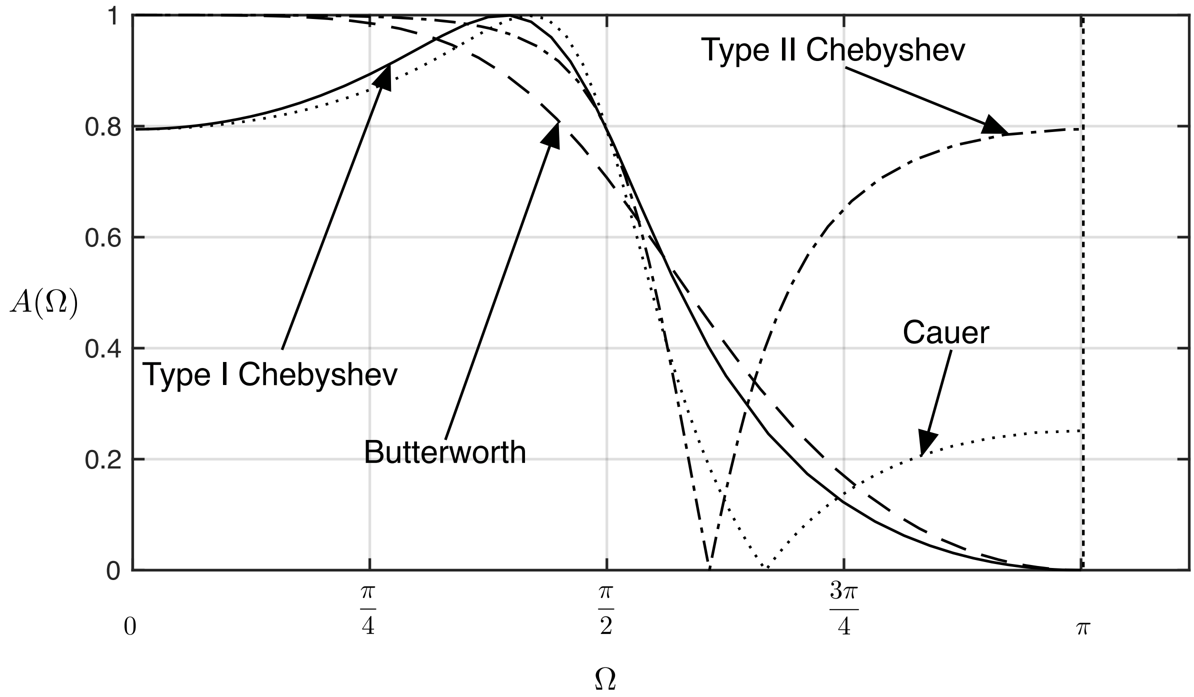
$$\lim_{z\to 1}(z-1)Y(z) = \lim_{z\to 1}(z-1)G_{CC}(z)U(z) = \lim_{z\to 1}\cancel{(z-1)} \cdot \frac{1}{(z-0.6)^2}\frac{z}{\cancel{z-1}} = 6.25$$

2

$\sum$ 12

Task 6: Filter, filter, filter, ... (13 points)

The amplitude response of a 2nd order filter is given. The normalized cut-off frequency is $\Omega_0 = \pi/2$.



a) Define the filter type (lowpass, highpass, bandpass, or bandstop).
Just one cut-off frequency ⇒ lowpass or highpass.
low frequency pass the filter ⇒ lowpass. 1

b) What filter design was used (Butterworth, Cauer, Type I Chebyshev or Type II Chebyshev)?
answer: see figure. 1

c) Give a short explanation why $A(\Omega)$ is plotted in the range $0 \leq \Omega \leq \pi$.
$0 \leq \Omega$: A negative frequency is the same as a positive frequency with phase shift.
$\Omega \leq \pi$: For signals with frequency greater than π aliasing may occur. 2

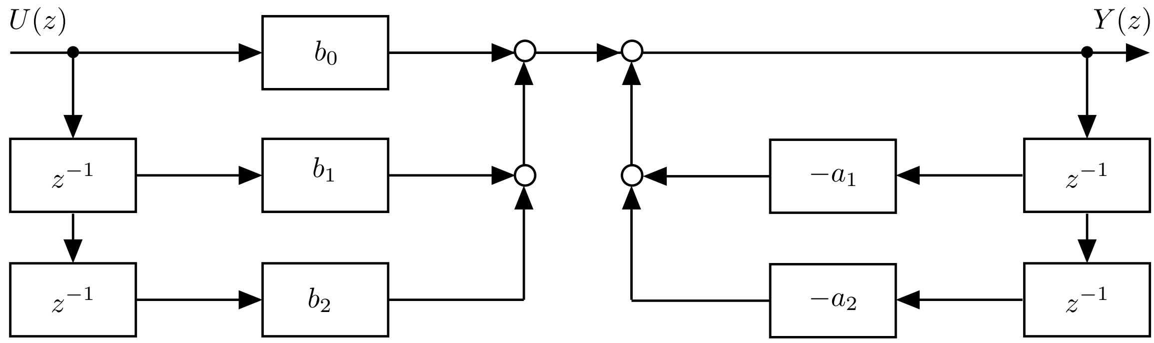
d) A step signal ($\sigma(k)$) is filtered using the filter above. What is the final value of the filtered step signal?
The amplification factor for $\Omega \to 0$ is ≈ 0.8. Therfore the final value of the filtered step signal is 0.8 (gain of the filter). 2

e) Sketch the remaining filters (2nd order each with cut-off frequency $\Omega_0 = \pi/2$) in the given diagram. Indicate which amplitude response corresponds to which filter design. Please make sure that the characteristic features of each filter is visible.
Characteristic features:
Type II Chebyshev ⇒ Ripple in stopband, transition steepness similar to Type I Chebyshev. 1

Cauer $\Rightarrow$ Ripple in passband and stopband, transition steepness higher compared to Chebyshev. $\boxed{1}$

Butterworth$\Rightarrow$ Monotonous amplitude response transition steepness lower compared to Chebyshev. $\boxed{1}$

f) Draw a block diagram of a general IIR-filter of order 2. Use only blocks containing constants or z^{-1} values.



$\boxed{2}$

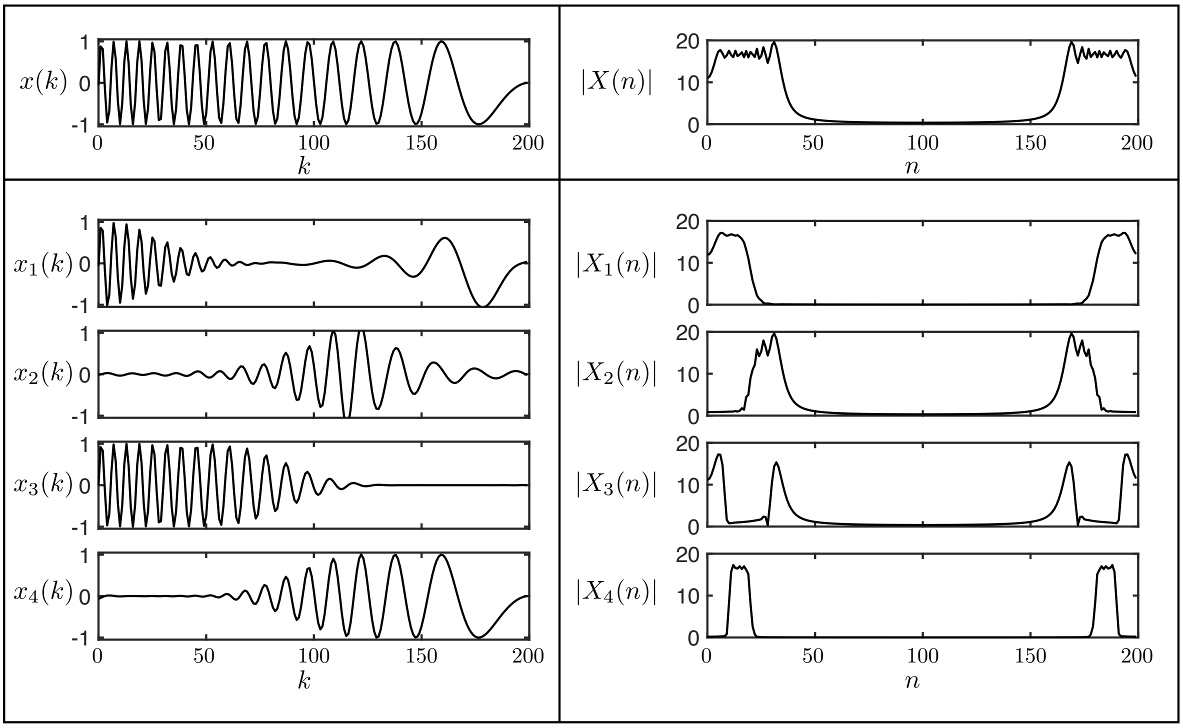
g) Give the difference-equation of the filter from sub-task f).
$y(k) = b_0u(k) + b_1u(k-1) + b_2u(k-2) - a_1y(k-1) - a_2y(k-2)$ $\boxed{1}$

h) What defines the stability of the filter from f) / g) (short answer please)?
The stability is defined by the poles of the transfer function. The poles are determined by the a-coefficients. $\boxed{1}$

$\boxed{\sum 13}$

Task 7: DFT (11 points)

A sine-shaped signal $x(k)$ is given. The frequency of the signal changes linearly with the time. The absolute values of the corresponding DFT is also given ($|X(n)|$).



Now the Signal $x(k)$ is filtered with different filters. The resulting four signals $x_1(k)$ - $x_4(k)$ are shown as well as the corresponding DFTs.

a) Assign to each signal $x_i(k)$ the corresponding absolute value of the DFT $|X_j(n)|$.

Signal	$x_1(k)$	$x_2(k)$	$x_3(k)$	$x_4(k)$
Zugehöriges $\|X_j(n)\|$	$\|X_3(n)\|$	$\|X_4(n)\|$	$\|X_2(n)\|$	$\|X_1(n)\|$

4

b) The signal $x(k)$ changes the frequency over time. What can be done to analyze such a behavior? Mention at least one procedure and explain the approach in **a few** sentences.

Short-Time Discrete Fourier Transform (STDFT):
Windowed DFT: Width of the window determines the time resolution and also the frequency resolution. The width is a parameter defined by the user. It should be guided by the expected rate of change in the spectrum.
The DFT does not only depend on the frequency for n but also on a second variable: the time shift of the window. It indicates the time around which the DFT is valid. 2

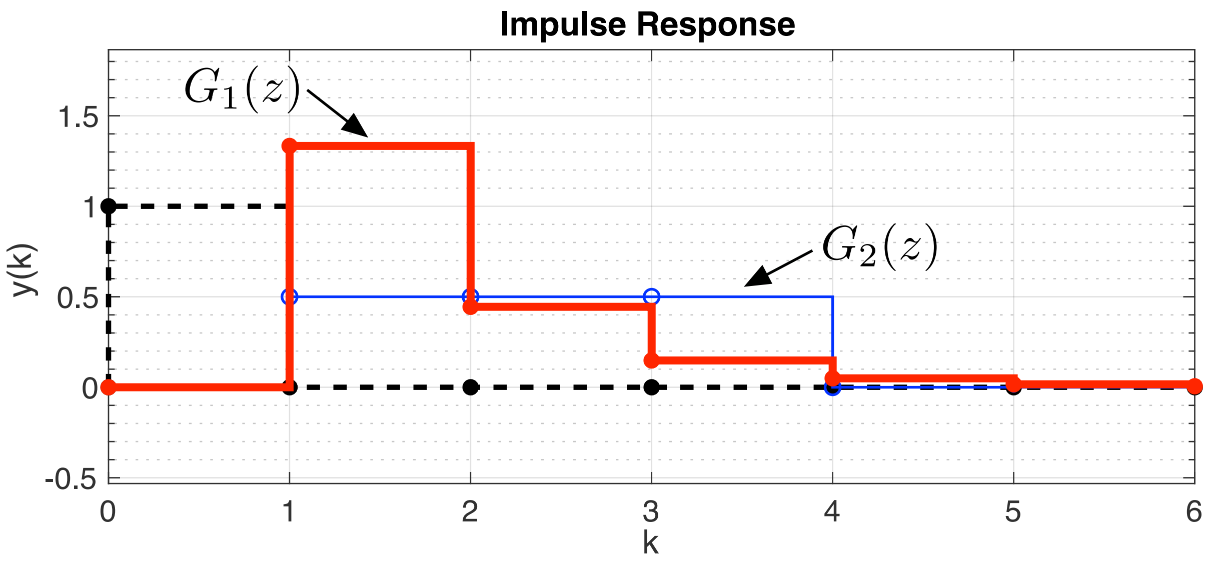
c) Now the DFTs $X_5(n)$, $X_6(n)$, $X_7(n)$, and $X_8(n)$ are analyzed (all with length N). The absolute values of the DFTs are equal to zero for $n = 0, 1, 2, \ldots, n_0-1, n_0+1, \ldots, N/2$. For n_0 the DFTs contain the following values:
$X_5(n_0) = 10, \quad X_6(n_0) = 10i, \quad X_7(n_0) = -10,$ and $X_8(n_0) = -10i\,.$
How do the corresponding signals $x_5(k)$, $x_6(k)$, $x_7(k)$, and $x_8(k)$ differ? Hint: A

calculation is not necessary!
All the DFTS have just one peak at n_0 so the corresponding signal is a sine shaped signal. The $X_i(n)$ differ only in the phase, thus the signal differ only in a phase shift too:
$x_5(k) \to \varphi_0 + 0^o$, $x_6(k) \to \varphi_0 + 90^o$, $x_7(k) \to \varphi_0 + 180^o$, and $x_8(k) \to \varphi_0 + 270^o$. The signals $x_5(k)$ and $x_7(k)$, as well as $x_6(k)$ and $x_8(k)$ are out-of-phase: $x_5(k) = -x_7(k)$ and $x_6(k) = -x_8(k)$. 3

d) Assume a signal with $2f_0$ is sampled with a sampling frequency f_0 at which normalized frequency occurs the highest peak?
The signal frequency violates the Shannons theorem. The signal is sampled at the exact same value. Thus, the sampled signal is constant. A constant signal has a (normalized) frequency of zero (1/sec). The DFT has therefore the highest peak at $n = 0$. 2

$\sum 11$

Task 8: FIR- und IIR-systems (12 points)



a) Impulse responses of $G_1(z)$ and $G_2(z)$ see diagram above. Values are obtained by calculating the difference to the previous sample value. $\boxed{4}$

b) Which system has FIR and which has IIR behavior:

- The impulse response of $G_1(z)$ approaches zero asymptotically (or the step response asymptotically approaches a constant value), which implies IIR behavior.
- The impulse response of $G_2(z)$ reaches exactly zero after 4 steps (or the step response remains exactly constant), which implies FIR behaviour. $\boxed{2}$

c) Explanation, which of the given transfer functions represent $G_1(z)$, $G_2(z)$ or none of the two:

- $G_A(z)$: Gain equals 1, $G_1(z)$ and $G_2(z)$ have gain 2 or 1.5 respectively $\Rightarrow$ None!
- $G_B(z)$: Gain equals 1.5, has no poles (FIR) and is strictly proper $\Rightarrow G_2(z)$
- $G_C(z)$: Gain equals 1, $G_1(z)$ and $G_2(z)$ have gain 2 or 1,5 respectively $\Rightarrow$ None!
- $G_D(z)$: Gain equals 2, has a stable pole (IIR) and is strictly proper $\Rightarrow G_1(z)$
- $G_E(z)$: Gain equals 1.5, is not strictly proper (would respond directly, not with delay) $\Rightarrow$ None!
- $G_F(z)$: Gain equals 2, but has an unstable pole at -3 $\Rightarrow$ None!

$\boxed{6}$

$\boxed{\sum 12}$