Sensorics Exam

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9th of August 2017

Name:	
Mat.-No.:	
Grade:	

Task:	T1	T2	T3	T4	T5	T6	T7	T8	Sum
Scores:	8	18	20	16	11	18	15	14	120
Accomplished:									

Duration of examination: 2 hours

You are allowed to use a calculator and four pages of notes

Task 1: Comprehension Questions (8 Points)

Mark the correct answers clearly. **Every question has one or two correct answers!** For every correctly marked answer you will get one point. If there is one correct answer marked and one incorrect answer marked, you will get no point for that subtask.

a) How large should the internal resistance of a voltage meter be?
- ☐ Very large, in the ideal case infinite large.
- ☐ Very small, in the ideal case zero.
- ☐ It depends on the electrical circuit, what internal resistance is ideal.

b) A non-causal filter...
- ☐ ...requires only current and previous input values.
- ☐ ...requires future input values.
- ☐ ...requires future output values.

c) Operational amplifier circuits ...
- ☐ ... can be used to realize a PID controller.
- ☐ ... need an external energy source.
- ☐ ... do not need an external energy source.

d) Bridge circuits ...
- ☐ ... measure current and voltage in order to determine purely ohmic impedances.
- ☐ ... can only be used to measure purely ohmic impedances.
- ☐ ... can be used to determine the value of impedances.

e) If you differentiate a noisy signal with respect to time t ...
- ☐ ... this can be realized in the Laplace domain by a multiplication with $1/s$.
- ☐ ... the noise is amplified.
- ☐ ... the system becomes unstable.

f) Assess the following statements regarding the Student's t-distribution.
- ☐ Its confidence interval $1 - \alpha$ is wider than the confidence interval of the corresponding normal distribution.
- ☐ If the size N of the regarded data set is big enough ($N \to \infty$), it converges to the cauchy distribution.
- ☐ It is used if the real standard deviation has to be estimated from data.

Task 2: Temperature Measurement (18 Points)

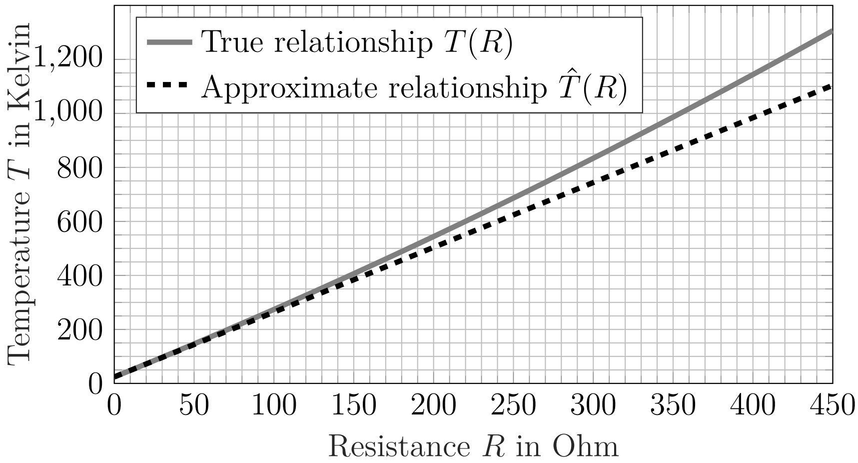


Fig. 1: Relationship of the Temperature and the resistance of a PTC sensor.

The true relationship between the Temperature T and the resistance R of a positive-temperature-coefficient (PTC) sensor follows the equation:

$$T(R) = 24 + 2.4R + 0.001R^2\,. \tag{1}$$

One possible simplification is to neglect the quadratic term, since it has a very small coefficient. This approximate relationship $\hat{T}(R)$ is shown as the dashed curve in Fig. 1 together with the true relationship (solid curve).

a) What type of error occurs through the usage of the approximate relationship $\hat{T}(R)$?

b) Determine up to which resistance the relative absolute error between the approximate relationship $\hat{T}(R)$ and the true relationship $T(R)$ is less or equal to 5 %.

c) What temperature would be measured if a wire from the PTC sensor to the measuring instrument is broken? Explain your answer briefly.

d) Use the Taylor series expansion to linearize the true relationship $T(R)$ for a operating point R_0. In general the Taylor series for a function $f(x)$ around an operating point x_0 is given by

$$Tf(x; x_0) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}\,(x - x_0)^n \tag{2}$$

Here, $n!$ denotes the factorial of n and $f^{(n)}(x_0)$ denotes the n-th derivative of $f(x)$ evaluated at x_0. How are the linearization through the Taylor series expansion and the one shown in Fig. 1 ($\hat{T}(R)$) related?

e) Sketch into Fig. 1 a good linear approximation, such that the mean error is minimized.

Task 3: Cross-Correlation (20 Points)

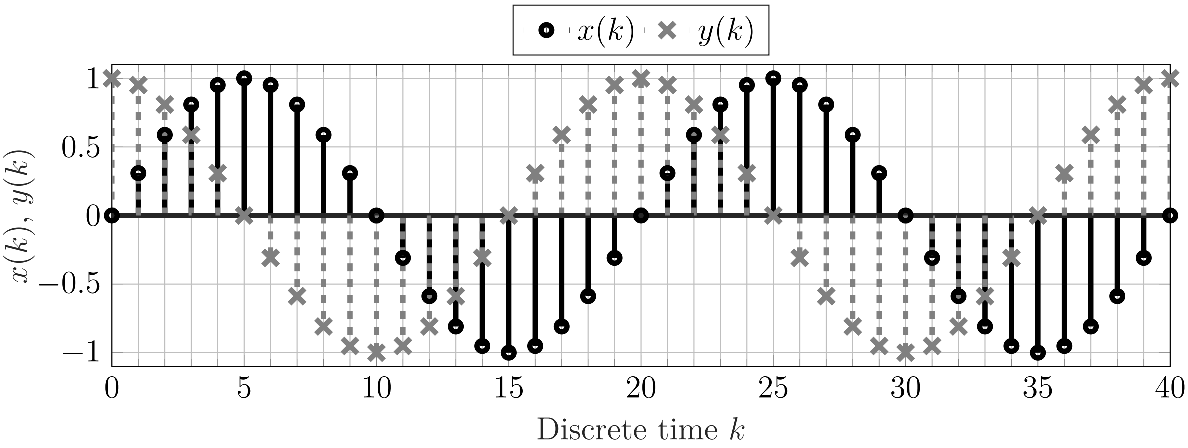


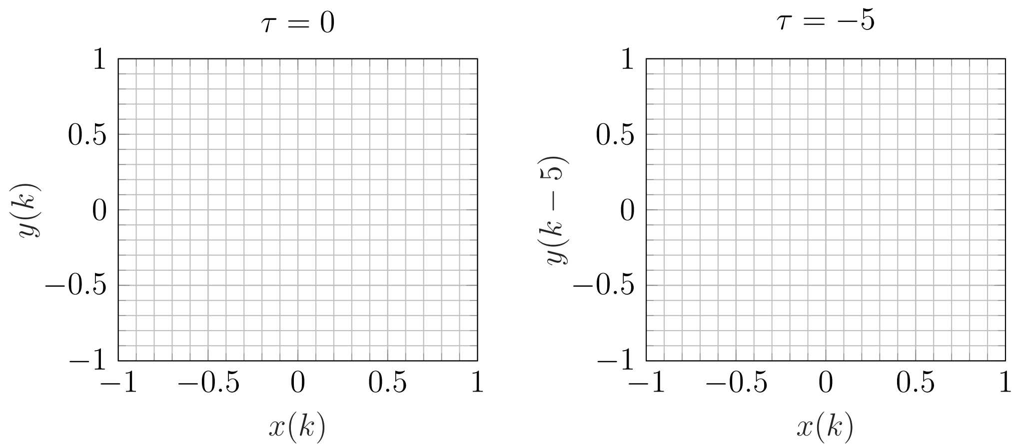
Fig. 2: Two full cycles of the periodic signals $x(k)$ and $y(k)$.

The cross-correlation for the two shown signals $x(k)$ and $y(k)$, both consisting of $N = 41$ samples, is calculated for a time shift τ as follows:

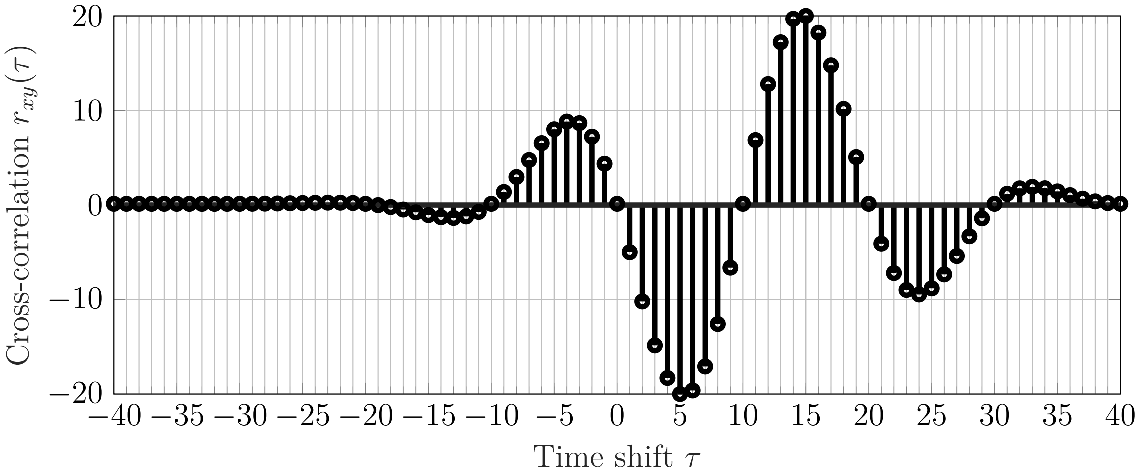
$$r_{xy}(\tau) = \begin{cases} \frac{1}{N} \sum\limits_{k=0}^{N-1-\tau} x(k) \cdot y(k+\tau), & \text{if } \tau \geq 0 \\ \frac{1}{N} \sum\limits_{k=-\tau}^{N-1} x(k) \cdot y(k+\tau), & \text{otherwise} \end{cases} \tag{3}$$

a) Which of the cross-correlation functions shown in Fig. 3 belongs to the signals $x(k)$ and $y(k)$ shown in Fig. 2?

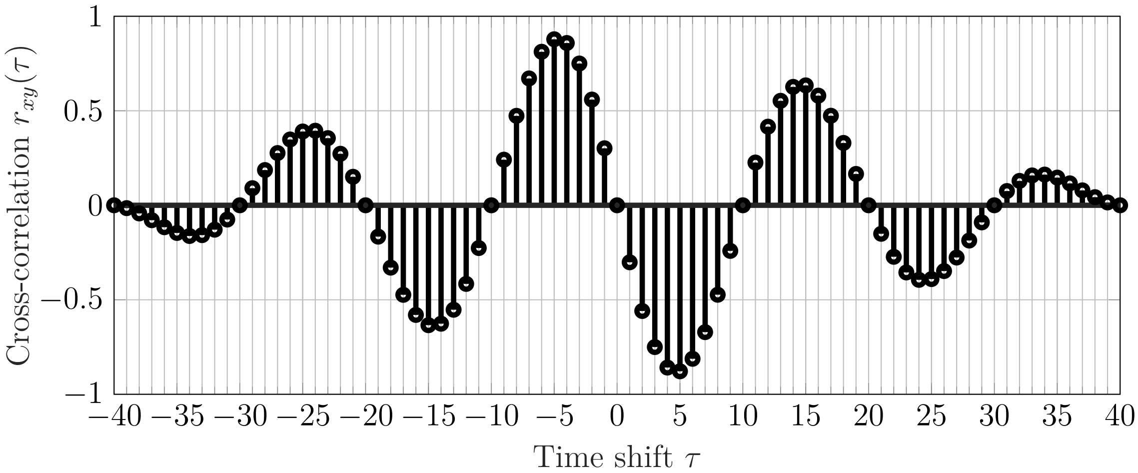
b) Sketch the time shifted signal $y(k - \tau)$ versus the signal $x(k)$ in the graphs below for $\tau = 0$ and $\tau = -5$.



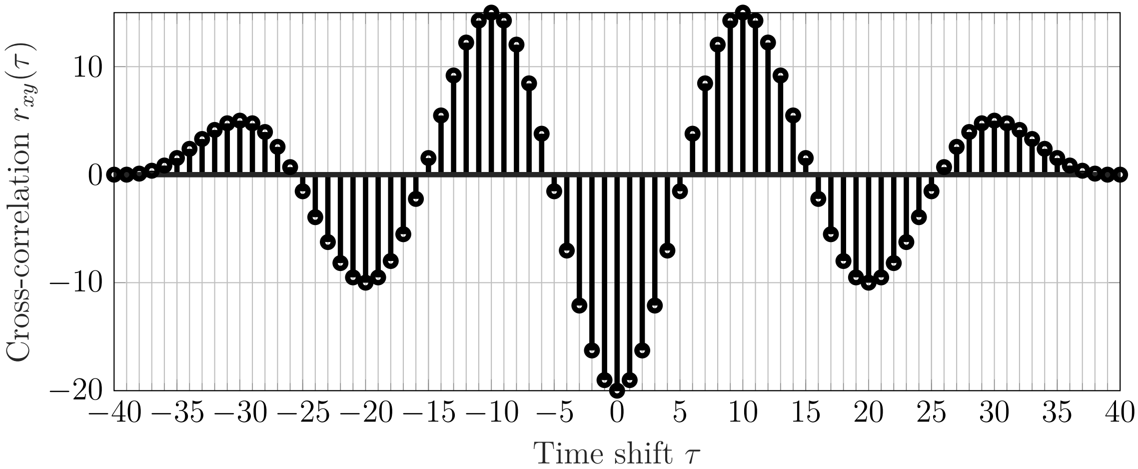
c) Now assume that the sampling frequency is doubled. How would the time shift change at which the cross-correlation reaches its maximum?



(a)



(b)

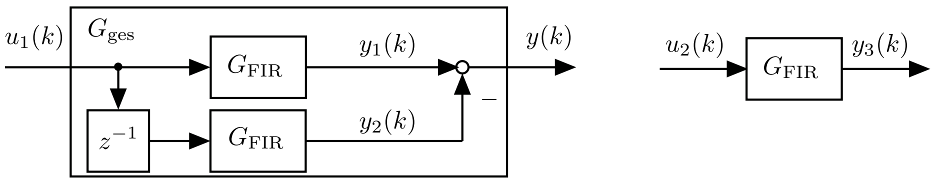


(c)

Fig. 3: Which of these correlation functions corresponds to the signals shown in Fig. 2?

Task 4: **FIR System (16 Points)**

a) How is a general FIR system with order m defined (time domain)?



b) The FIR coefficients are set to $b_0 = 4$, $b_1 = 2$, $b_2 = 1$ and $b_3 = 1$ ($b_i = 0$ for all $i > 3$). Calculate the step response for the outputs $y_1(k)$ and $y_2(k)$ for $k = 0, 1, \ldots, 4$.

c) Determine the transfer function $G_{\text{ges}}(z)$ in the z-domain. Use the given coefficients!

d) Calculate the step response of the system $G_{\text{ges}}(z)$ for $k = 0, 1, \ldots, 4$.

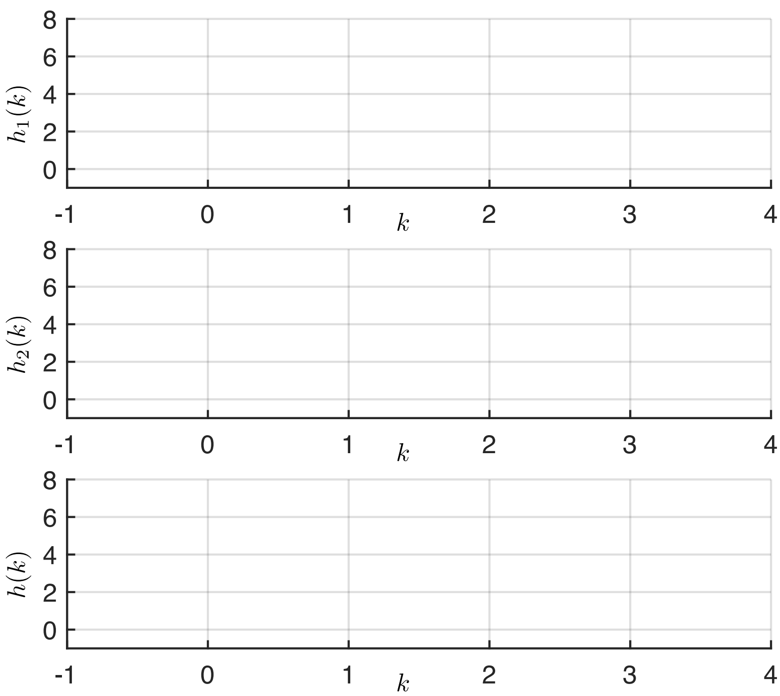
e) Sketch the step responses for the outputs $y_1(k)$, $y_2(k)$ and $y(k)$. Use the give diagrams.

f) What input signal $u_2(k)$ has to be chosen, if the output $y_3(k)$ should be identical with the step response of $G_{\text{ges}}(z)$ ($y_3(k) = y(k)$ with $u_1(k) = \sigma(k)$)?

g) Determine the relationship between the input signals $u_1(k)$ and $u_2(k)$, when the output signals $y_3(k)$ and $y(k)$ are identical. Determine the relationship $\frac{U_1(z)}{U_2(z)}$ in the z-domain.

h) How is the transfer function $\frac{U_1(z)}{U_2(z)}$ called?

i) Transform the relationship $\frac{U_1(z)}{U_2(z)}$ in the time-domain.



Task 5: DFT und Windowing (11 Points)

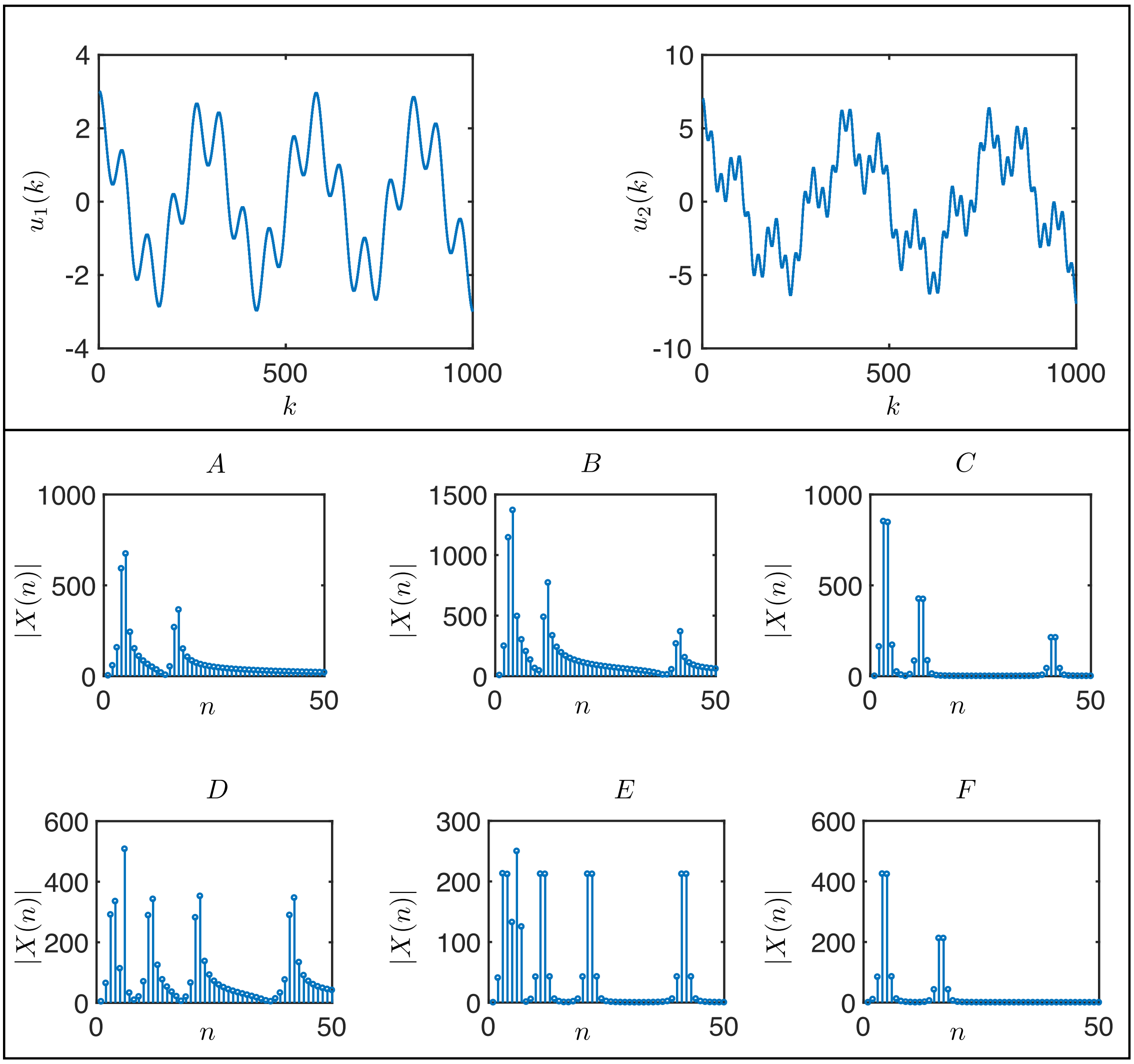
Given are the signals $u_1(k)$ and $u_2(k)$ with identical length $N = 1000$. All coefficients c_i are positive:

$$u_1(k) = c_1 \cos(\omega_1 k T_0) + c_2 \cos(\omega_2 k T_0)$$
$$u_2(k) = c_3 \cos(\omega_3 k T_0) + c_4 \cos(\omega_4 k T_0) + c_5 \cos(\omega_5 k T_0)\,.$$

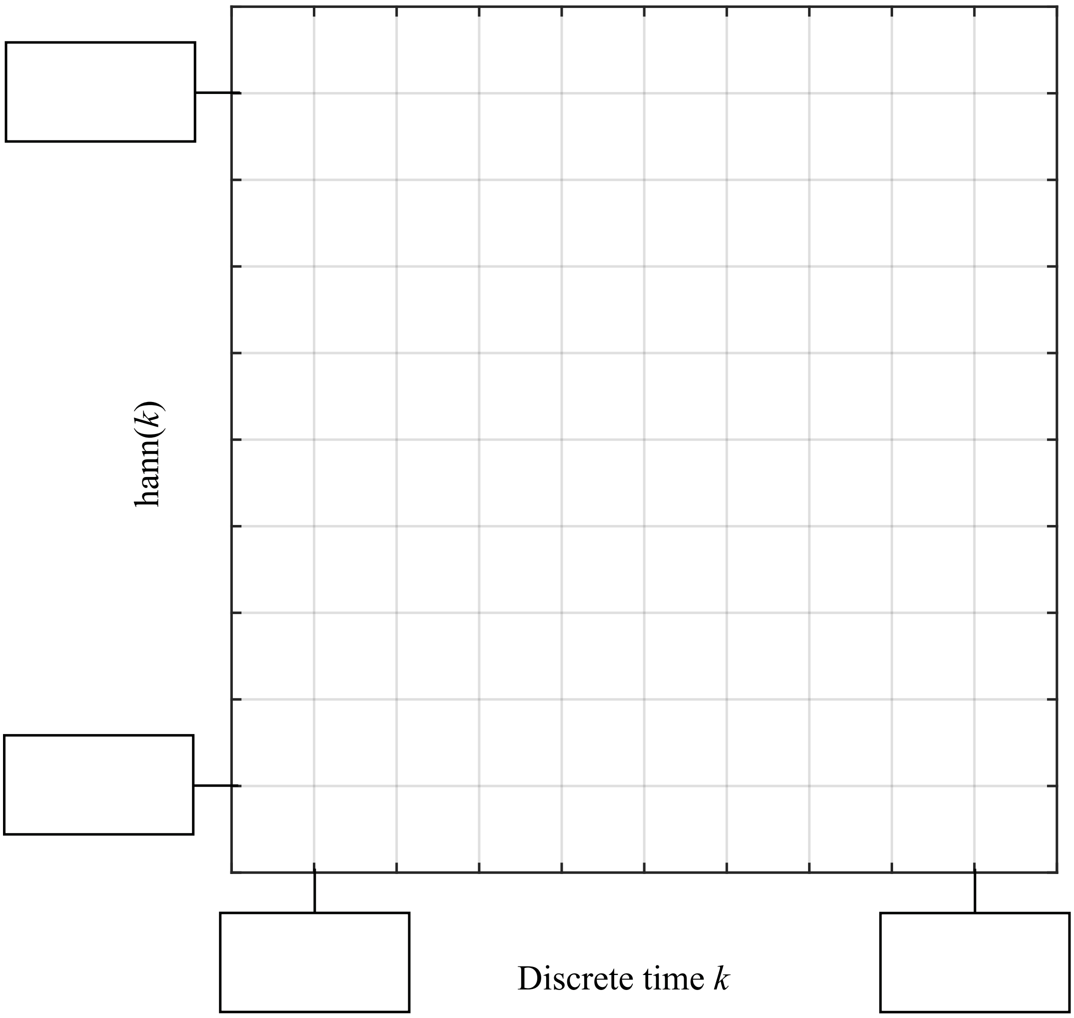
a) The signals $u_1(k)$ and $u_2(k)$ are fourier transformed. Which problem may arise? Describe the problem and the reason why it exists.

b) Assign $u_1(k)$ and $u_2(k)$ to one DFT respectively.
<u>Note:</u> Only the first 50 values of $|X(n)|$ are shown.

	$u_1(k)$	$u_2(k)$
DFT		



c) Sketch the hann window for the signals $u_1(k)$ and $u_2(k)$ in the given diagram. Fill the boxes with the minimum and maximum values of the axes.



d) The signals $u_1(k)$ and $u_2(k)$ are multiplied with a hann window. The signals $u_{1,h}(k)$ and $u_{2,h}(k)$ are the result. Assign $u_{1,h}(k)$ and $u_{2,h}(k)$ to one DFT respectively.

	$u_{1,h}(k)$	$u_{2,h}(k)$
DFT		

e) How do the amplitudes of the spectrum change if a hann window is applied? Explain why this effect occurs.

Task 6: Static and Dynamic Behavior of Sensors (18 Points)

The transient behavior of a temperature sensor is known to follow the equation:

$$T_m(t,\tau) = T_{real}(1 - e^{-t/\tau}) \text{ with time constant } \tau = 1\,\text{second}, \quad (4)$$

where $T_m(t,\tau)$ is the displayed temperature at the instrument and T_{real} is the true temperature. The continuous time t is reset to zero as soon as there is a change in the true temperature T_{real}.

a) Sketch the time course of the measured temperature $T_m(t,\tau)$ **qualitatively**, if a step in the true temperature T_{real} from 10°C to 20°C. Assume that the steady state of T_{real} = 10°C is already reached, before the step occurs. Take care to assign correct axis labels and sketch also the step of the true temperature into that diagram.

b) How is the error called that arises due to the transient behavior of the sensor and what can be done to avoid this error?

c) Due to external influences the time constant τ changes to $\tau_{new} = 2\tau$. Calculate at which time the maximum error between $T_m(t,\tau)$ and $T_m(t,\tau_{new})$ occurs.

d) Now assume, that due to wear the gain of the sensor has changed. Sketch the step response of a sensor with a gain-error **qualitatively** if the step in the true temperature goes T_{real} from 10°C to 20°C. Again, assume that the steady state of T_{real} = 10°C is already reached, before the step occurs. Take care to assign correct axis labels and sketch also the step of the true temperature into that diagram.

e) How is the error called that arises due to the wrong sensor gain and what can be done to compensate this error?

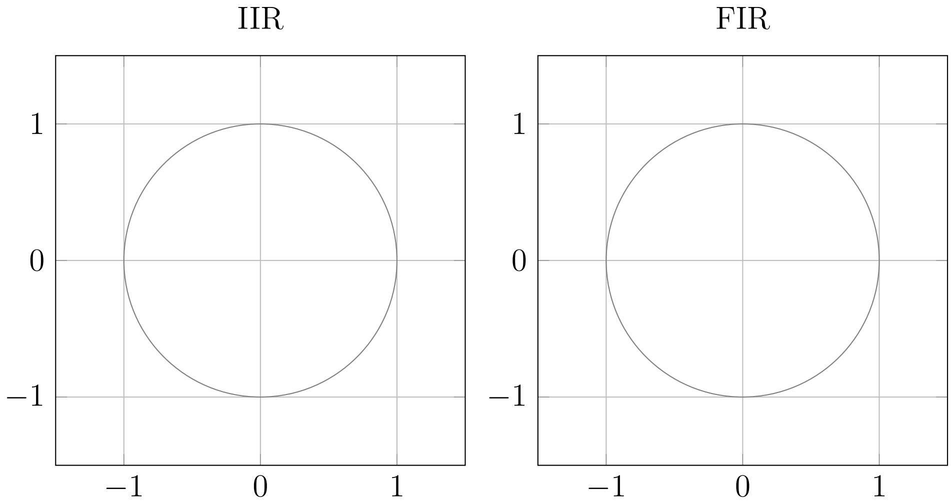
Task 7: FIR and IIR (15 Points)

An IIR system shall be approximated by an FIR system.

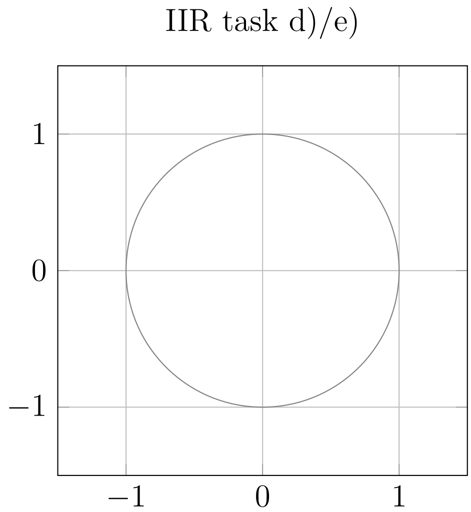
a) Calculate in the time domain for the dynamic system $y(k) = 0.5y(k-1) + u(k)$ the FIR system that approximates the dynamic behaviour. Therefore use the coefficients that are larger than 0.2.

b) Transform both the IIR and the calculated FIR system from a) in the z-domain.

c) Plot the poles $\times$ and the zeros $\circ$ for the IIR system in the left diagram and for the FIR system in the right diagram. Mark multiple poles or zeros by writing the number beside the corresponding pole or zero.



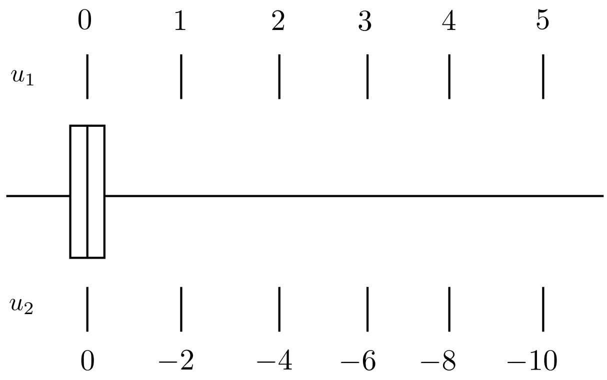
d) Calculate for the dynamic system $y(k) = 1.5y(k-1) - 0.5y(k-2) + u(k)$ poles and zeros and plot them in the diagram below. Mark multiple poles or zeros by writing the number beside the corresponding pole or zero.



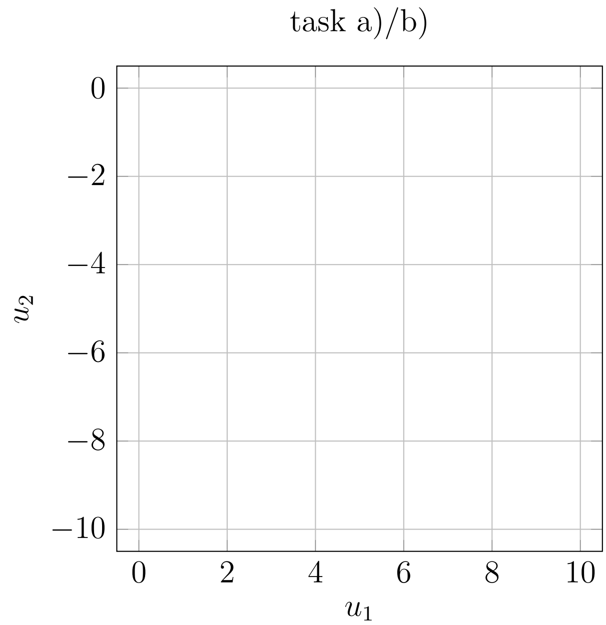
e) Can the system from d) be approximated by an FIR system satisfactorily.

Task 8: Singular Value Decomposition (14 Points)

The inputs of a plant u_1 and u_2 can be changed with the depicted slide control. At the shown position $u_1 = u_2 = 0$ holds.

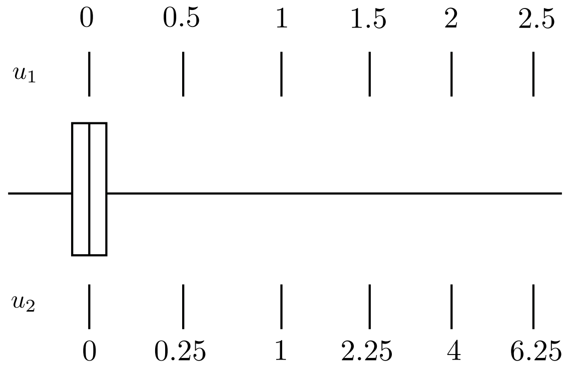


a) Plot the possible input combinations for $u_1 = 1, 2, 3, 4, 5$ in the diagram.

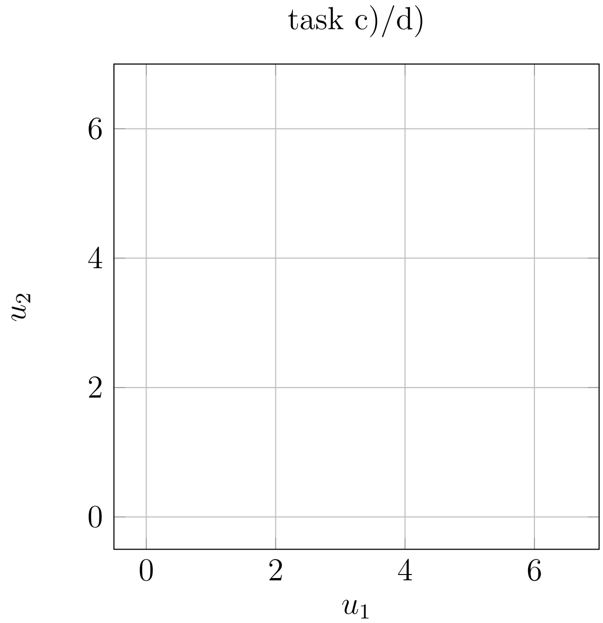


b) Perform a PCA geometrically. Now draw the principal axes and mark the principal axis with the singular value 0.

c) At another plant the inputs u_1 and u_2 can be modified with the slide control shown below.



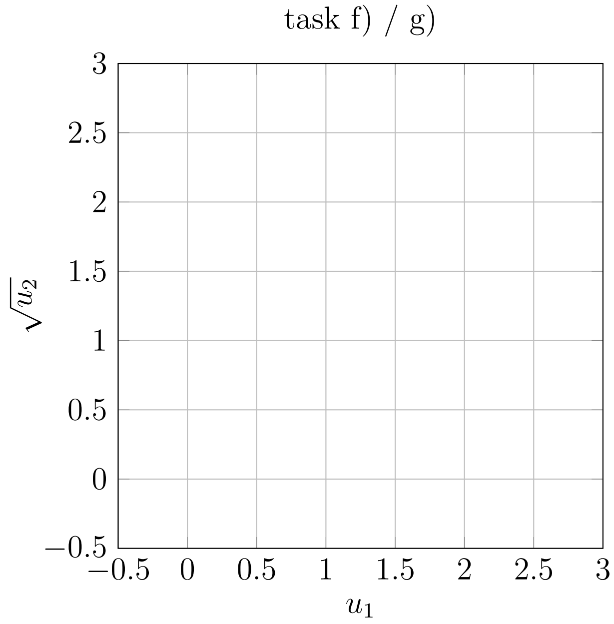
Plot the possible input combinations for $u_1 = 0, 0.5, 1, 1.5, 2, 2.5$ in the diagram.



d) Perform a PCA geometrically. Now draw the principal axes qualitatively.

e) Why is none of the singular vectors 0 although due to the slide controler the input dimension is 1.

f) Now plot in the diagram shown below u_1 on the one axis and $\sqrt{u_2}$ on the other axis.



g) Perform a PCA geometrically. Now draw the principal axes. Why is now one of the singular values 0 again?

Solutions:

Task 1: Comprehension Questions

a) How large should the internal resistance of a voltage meter be?

- ■ Very large, in the ideal case infinite large.
- □ Very small, in the ideal case zero.
- □ It depends on the electrical circuit, what internal resistance is ideal.

b) A non-causal filter...

- □ ...requires only current and previous input values.
- ■ ...requires future input values.
- □ ...requires future output values.

c) Operational amplifier circuits ...

- ■ ... can be used to realize a PID controller.
- ■ ... need an external energy source.
- □ ... do not need an external energy source.

d) Bridge circuits ...

- □ ... measure current and voltage in order to determine purely ohmic impedances.
- □ ... can only be used to measure purely ohmic impedances.
- ■ ... can be used to determine the value of impedances.

e) If you differentiate a noisy signal with respect to time t ...

- □ ... this can be realized in the Laplace domain by a multiplication with $1/s$.
- ■ ... the noise is amplified.
- □ ... the system becomes unstable.

f) Assess the following statements regarding the Student's t-distribution.

- ■ Its confidence interval $1 - \alpha$ is wider than the confidence interval of the corresponding normal distribution.
- □ If the size N of the regarded data set is big enough ($N \to \infty$), it converges to the cauchy distribution.
- ■ It is used if the real standard deviation has to be estimated from data.

$\boxed{\sum 8}$

Task 2: Temperature Measurement

a) What type of error occurs through the usage of the approximate relationship $\hat{T}(R)$?

A systematic, static error. 2

b) Determine up to which resistance the relative absolute error between the approximate relationship $\hat{T}(R)$ and the true relationship $T(R)$ is less or equal to 5 %.

The relative absolute error is defined as:

$$e_{rel}(R) = \frac{|\hat{T}(R) - T(R)|}{T(R)} \,. \tag{5}$$

Because the true temperature is always above its approximation, we can write (5) as:

$$e_{rel}(R) = \frac{T(R) - \hat{T}(R)}{T(R)} \,.$$ 1

This relative absolute error should be less or equal to 0.05:

$$\begin{aligned} e_{rel}(R) &\leq 0.05 \\ \frac{T(R) - \hat{T}(R)}{T(R)} &\leq 0.05 \\ \frac{24 + 2.4R + 0.001R^2 - 24 - 2.4R}{24 + 2.4R + 0.001R^2} &\leq 0.05 \\ 0.001R^2 &\leq 0.05 \cdot (24 + 2.4R + 0.001R^2) \\ 9.5 \cdot 10^{-4}R^2 - 0.12R + 1.2 \leq 0 \end{aligned}$$ 6

Solving the quadratic equation leads to two results from which only the positive one makes sense in this context $\rightarrow R_{5\%} \approx 135.63\,\Omega$. Therefore the relative absolute error is smaller than 5 % as long as the resistance $R < 135.63\,\Omega$. 2

c) What temperature would be measured if a wire from the PTC sensor to the measuring instrument is broken? Explain your answer briefly.

Because of the wire damage no current is able to flow, which would correspond to an infinite hight resistance $\rightarrow$ The measured temperature also tends to infinity.

Alternative answer:
If there is no voltage drop over the PTC, one might conclude that the resistance is zero $\rightarrow$ The measured temperature should then be 24 Kelvin. 2

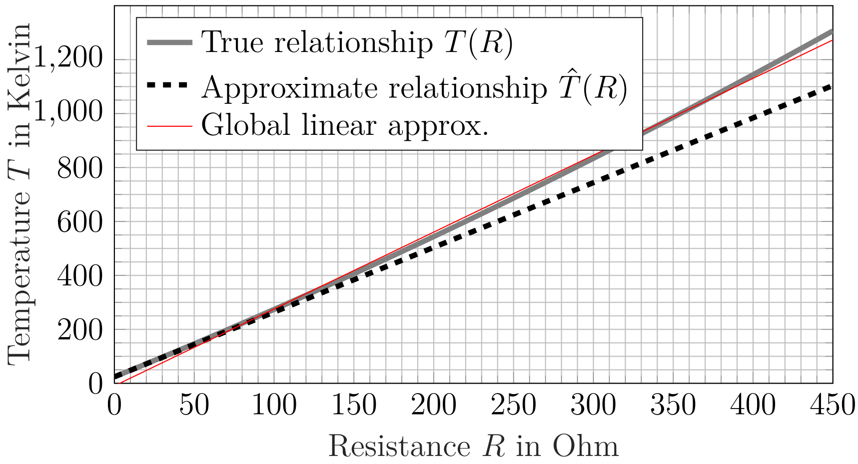
d) Use the Taylor series expansion to linearize the true relationship $T(R)$ for a operating point R_0. How are the linearization through the Taylor series expansion and the one shown in Fig. 1 ($\hat{T}(R)$) related?

For the linearization all terms with a higher order than one are neglected:

$$\begin{aligned} Tf(R; R_0) &\approx \frac{T^{(0)}(R_0)}{0!}(R - R_0)^0 + \frac{T^{(1)}(R_0)}{1!}(R - R_0)^1 \\ &\approx \underbrace{24 + 2.4 \cdot R_0 + 0.001 \cdot R_0^2}_{T(R_0)} + \underbrace{(2.4 + 0.002 \cdot R_0)}_{T^{(1)}(R_0)}(R - R_0) \end{aligned} \tag{6}$$ 3

Connection to $\hat{T}(R)$: If the operating point is chosen to be $R_0 = 0$, $\hat{T}(R)$ is obtained. 1

e) Sketch into Fig. 1 a good linear approximation, such that the mean error is minimized.



1

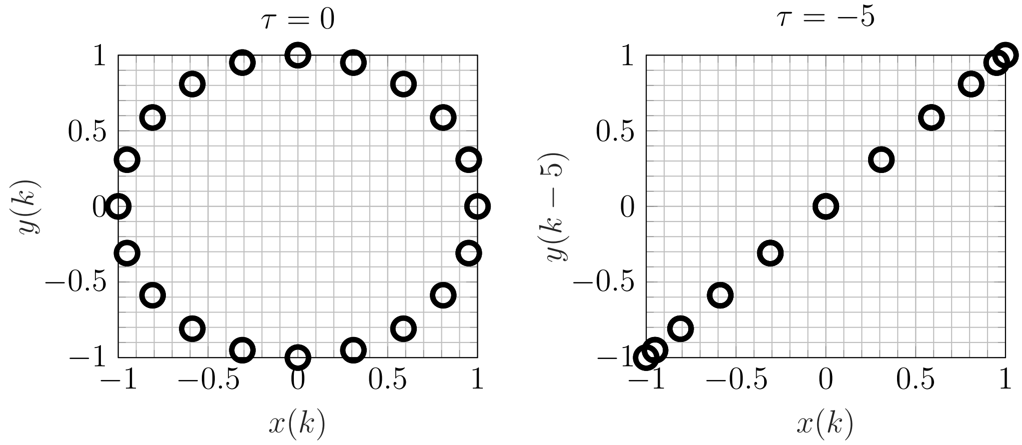
$\sum$ 18

Task 3: Cross-Correlation

a) Which of the cross-correlation functions shown in Fig. 3 belongs to the signals $x(k)$ and $y(k)$ shown in Fig. 2?

Cross-correlation b) belongs to the signals $x(k)$ and $y(k)$ shown in Fig. 2, because for a time shift of $\tau = -5$ the two signals lie upon each other. Therefore the maximum cross-correlation value has to be reached for that time shift. 6

b) Sketch the time shifted signal $y(k-\tau)$ versus the signal $x(k)$ in the graphs below for $\tau = 0$ and $\tau = -5$.



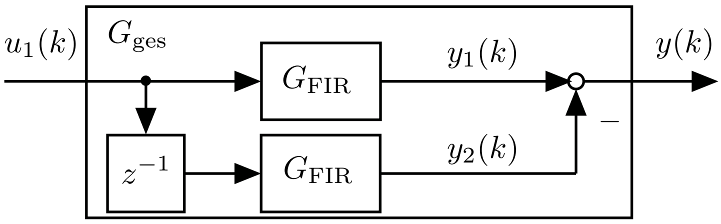
12

c) Now assume that the sampling frequency is doubled. How would the time shift change at which the cross-correlation reaches its maximum?

The time shift at which the maximum cross-correlation reaches its maximum value will be doubled $\tau_{new} = 2\tau_{old}$, because the number of samples is increased by a factor of two. 2

$\sum$ 20

Task 4: **FIR System (16 points)**



a) How is a general FIR system with order m defined (time domain)?

$$y(k) = b_0 u(k) + b_1 u(k-2) + b_2 u(k-2) + \ldots + b_m u(k-m) \tag{7}$$

$$y(k) = \sum_{i=0}^{m} b_i u(k-i) \tag{8}$$

1

b) The FIR coefficients are set to $b_0 = 4$, $b_1 = 2$, $b_2 = 1$ and $b_3 = 1$ ($b_i = 0$ for all $i > 3$). Calculate the step response for the outputs $y_1(k)$ and $y_2(k)$ for $k = 0, 1, \ldots, 4$.

	$h_1(k)$	$h_2(k)$	$h(k)$
$k=0$	b_0 $h_1(0) = 4$	0 $h_2(0) = 0$	b_0 $h(0) = 4$
$k=1$	$b_0 + b_1$ $h_1(1) = 6$	b_0 $h_2(1) = 4$	b_1 $h(1) = 2$
$k=2$	$b_0 + b_1 + b_2$ $h_1(2) = 7$	$b_0 + b_1$ $h_2(2) = 6$	b_2 $h(2) = 1$
$k=3$	$b_0 + b_1 + b_2 + b3$ $h_1(3) = 8$	$b_0 + b_1 + b_2$ $h_2(3) = 7$	b_3 $h(3) = 1$
$k=4$	$b_0 + b_1 + b_2$ $h_1(4) = 8$	$b_0 + b_1 + b_2 + b_3$ $h_2(4) = 8$	0 $h(4) = 0$

3

c) Determine the transfer function $G_{\text{ges}}(z)$ in the z-domain. Use the given coefficients!

$$G_{\text{ges}} = G_{\text{FIR}} - G_{\text{FIR}} z^{-1} \tag{9}$$

$$G_{\text{ges}} = b_0 + b_1 z^{-1} + b_2 z^{-2} - \left(b_0 + b_1 z^{-1} + b_2 z^{-2}\right) z^{-1} \tag{10}$$

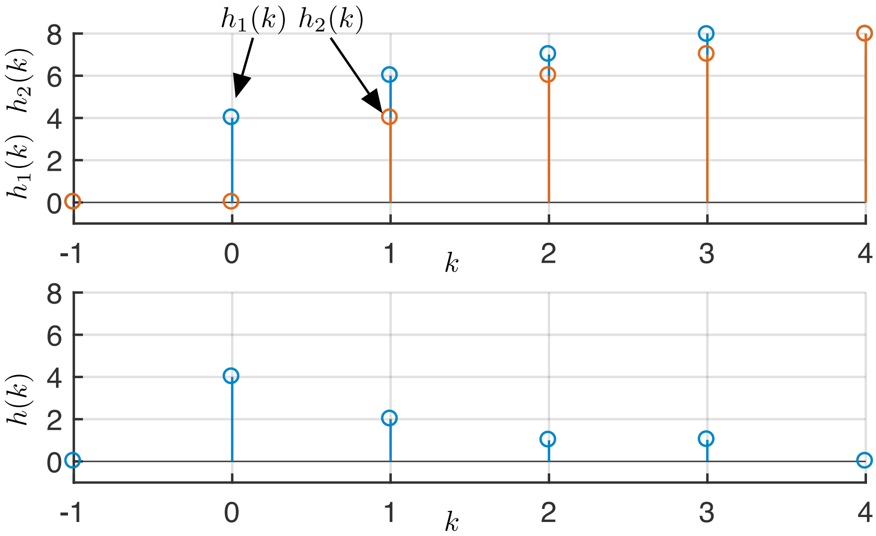
$$G_{\text{ges}} = b_0 + (b_1 - b_0) z^{-1} + (b_2 - b_1) z^{-2} - b_2 z^{-3} \tag{11}$$

$$\tag{12}$$

2

d) Calculate the step response of the system $G_{\text{ges}}(z)$ for $k = 0, 1, \ldots, 4$. (Solution in subtask b))

1



e) Sketch the step responses for the outputs $y_1(k)$, $y_2(k)$ and $y(k)$. Use the give diagrams.

3

f) What input signal $u_2(k)$ has to be chosen, if the output $y_3(k)$ should be identical with the step response of $G_{\text{ges}}(z)$ ($y_3(k) = y(k)$ with $u_1(k) = \sigma(k)$)?
Here: $y(k) = b_k$ for all positive k. This is the impulse response of G_{FIR}. So the input signal $u_2(k)$ should be an impulse.

2

g) Determine the relationship between the input signals $u_1(k)$ and $u_2(k)$, when the output signals $y_3(k)$ and $y(k)$ are identical. Determine the relationship $\frac{U_1(z)}{U_2(z)}$ in the z-domain.

$$Y(z) = U_1(z)(G_{\text{FIR}} - G_{\text{FIR}} z^{-1}) \tag{13}$$

$$Y_3(z) = G_{\text{FIR}} U_2(z) \quad \text{gleichsetzen} \tag{14}$$

$$U_1(z)(G_{\text{FIR}} - G_{\text{FIR}} z^{-1}) = G_{\text{FIR}} U_2(z) \tag{15}$$

$$U_1(z)(1 - z^{-1}) = U_2(z) \tag{16}$$

$$\Rightarrow \quad \frac{U1(z)}{U_2(z)} = \frac{1}{1 - z^{-1}} \tag{17}$$

2

h) How is the transfer function $\frac{U_1(z)}{U_2(z)}$ called?
step function / integrator.

1

i) Transform the relationship $\frac{U_1(z)}{U_2(z)}$ in the time-domain.

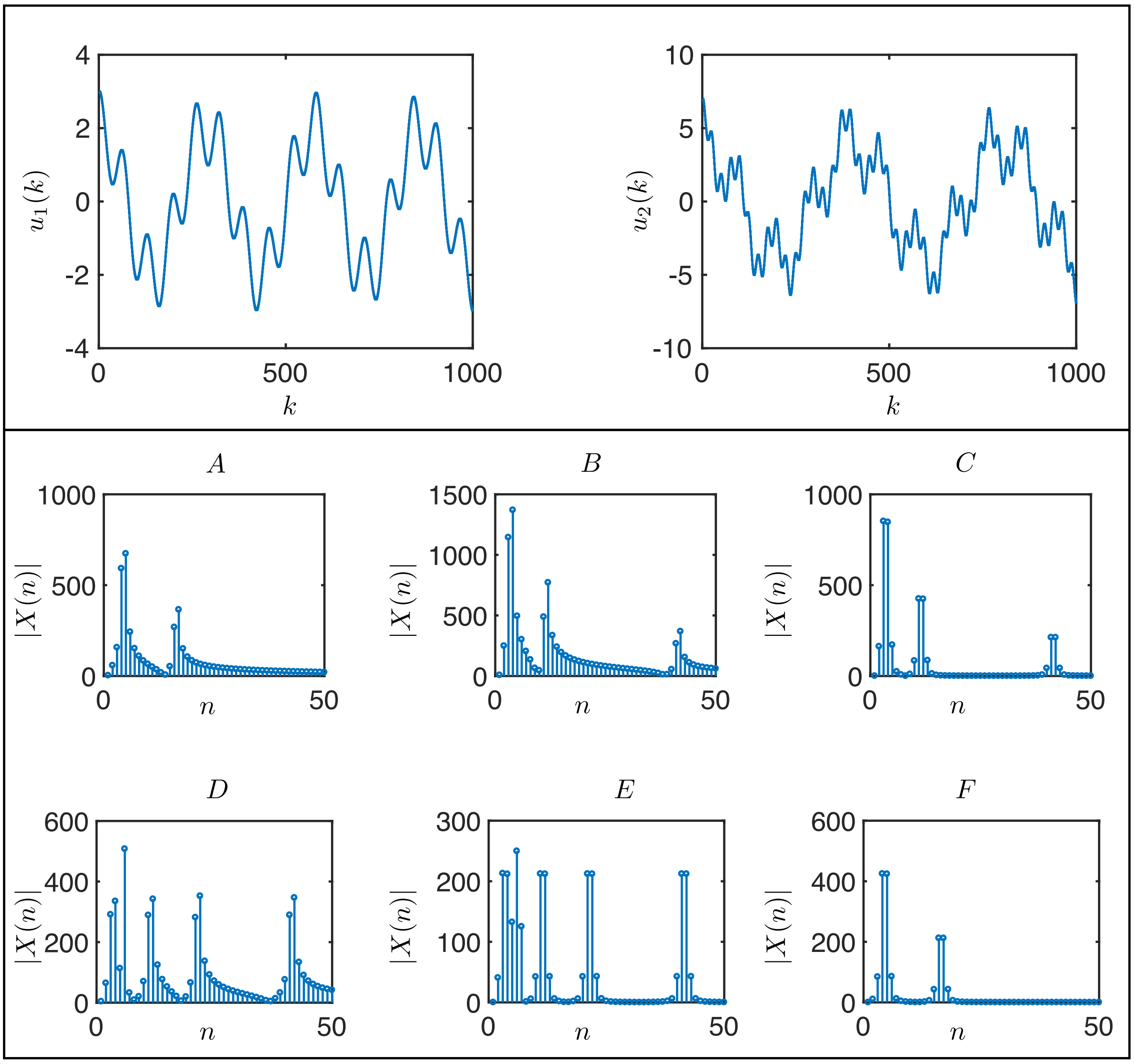
$$u_2(k) = u_1(k) - u_1(k-1) \tag{18}$$

1

$\sum 16$

Task 5: DFT und Fensterung (11 points)

a) The signals $u_1(k)$ and $u_2(k)$ are fourier transformed. Which problem may arise? Describe the problem and the reason why it exists.
The disconituity of the signals are the reson for the appearing leackage effect. 2



b) Assign $u_1(k)$ and $u_2(k)$ to one DFT respectively.
Note: Only the first 50 values of $|X(n)|$ are shown.

	$u_1(k)$	$u_2(k)$
DFT	A	B

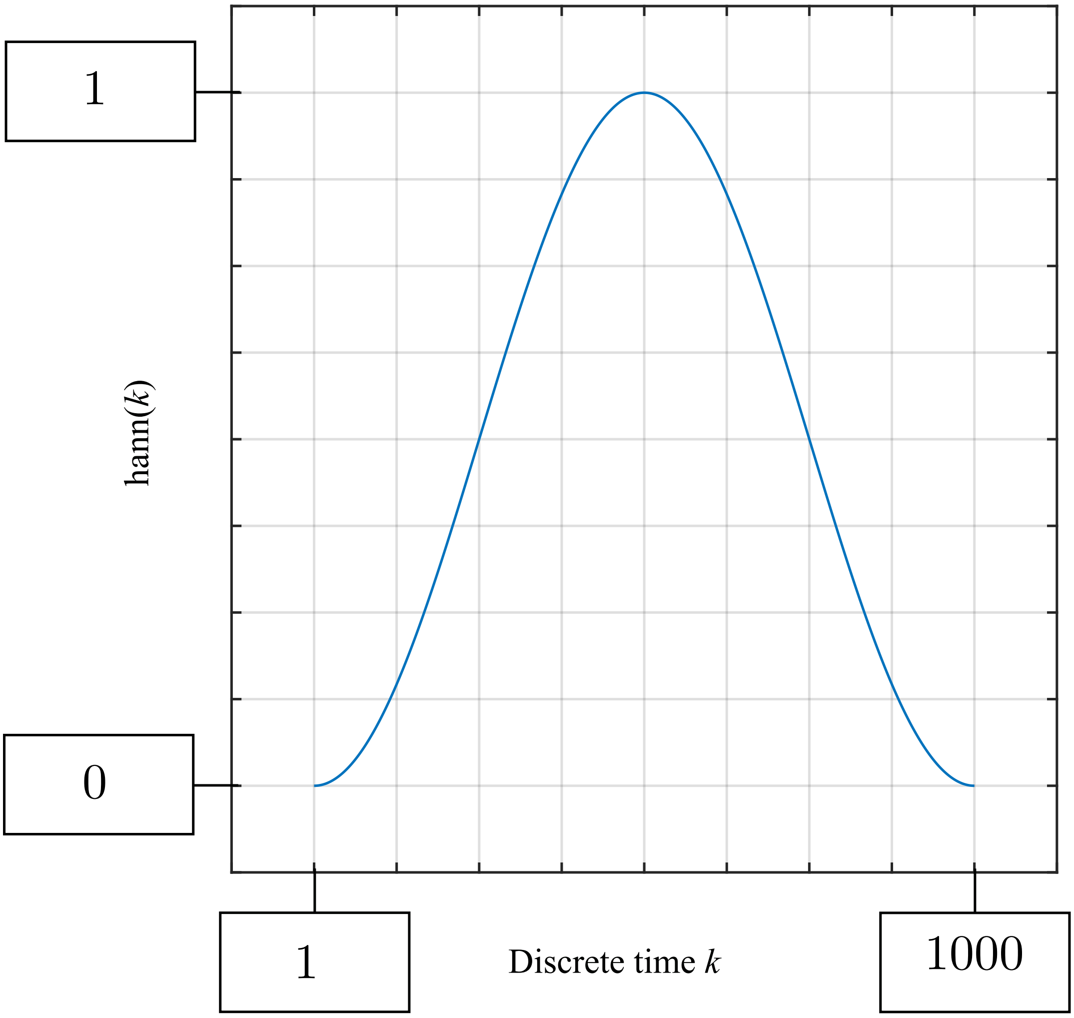
2

c) Sketch the hann window for the signals $u_1(k)$ and $u_2(k)$ in the given diagram. Fill the boxes with the minimum and maximum values of the axes. 3

d) The signals $u_1(k)$ and $u_2(k)$ are multiplied with a hann window. The signals $u_{1,h}(k)$ and $u_{2,h}(k)$ are the result. Assign $u_{1,h}(k)$ and $u_{2,h}(k)$ to one DFT respectively.

	$u_{1,h}(k)$	$u_{2,h}(k)$
DFT	F	C

2

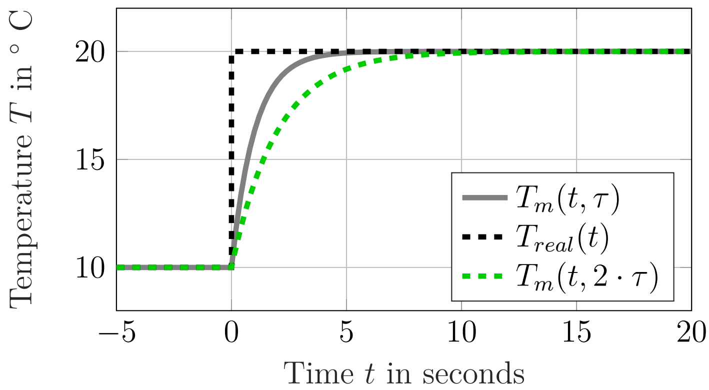


e) How do the amplitudes of the spectrum change if a hann window is applied? Explain why this effect occurs.
The values of the hann window is between 0 and 1. Thus multiplying the signal with the hann window decreases the power of the signal as well. Hence, the amplitudes appearing in the DFT are smaller compared to the DFT of the signal without widowing. 2

$\sum 11$

Task 6: Static and Dynamic Behavior of Sensors (18 Points)

a) Sketch the time course of the measured temperature $T_m(t, \tau)$ **qualitatively**, if a step in the true temperature T_{real} from $10\,°$ C to $20\,°$ C. Assume that the steady state of $T_{real} = 10\,°$ is already reached, when the step happens.



6

The curve $T_m(t, 2\cdot\tau)$ is not part of the solution to subtask a. It serves as visualization for subtask c.

b) How is the error called that arises due to the transient behavior of the sensor and what can be done to avoid this error?

Dynamic error. 1

Can be avoided if one waits long enough. 1

c) Due to external influences the time constant τ changes to $\tau_{new} = 2\tau$. Calculate at which time the maximum error between $T_m(t, \tau)$ and $T_m(t, \tau_{new})$ occurs.

In order to find the point in time, where the maximum error occurs, the time derivative of the error

$$e(t) = T_m(t, \tau) - T_m(t, 2 \cdot \tau)$$

is set to zero and the resulting equation is solved for t:

$$0 = \frac{de(t)}{dt}$$

$$0 = \frac{d}{dt}\left(T_{real}(1 - e^{-t/\tau})\right) - \frac{d}{dt}\left(T_{real}(1 - e^{-t/(2\tau)})\right)$$

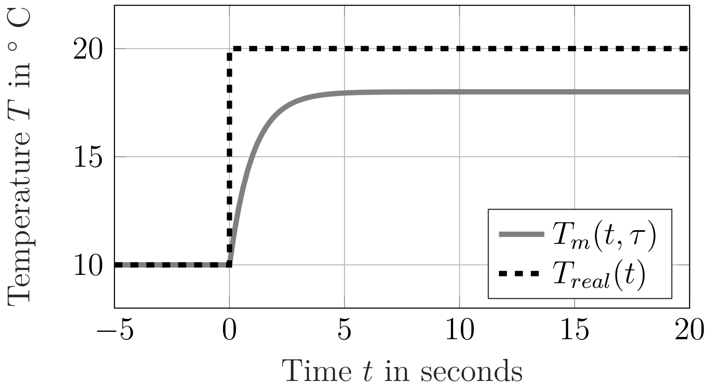
$$0 = \frac{\cancel{T_{real}}}{\cancel{\tau}} e^{-t/\tau} - \frac{\cancel{T_{real}}}{2\cancel{\tau}} e^{-t/(2\tau)}$$

$$\frac{1}{2} e^{-t/(2\tau)} = e^{-t/\tau}$$

$$\ln(0.5) - \frac{t}{2\tau} = \frac{-t}{\tau}$$

$$-2 \cdot \tau \cdot \ln(0.5) = t \text{ with } \tau = 1 \rightarrow t \approx 1.39 \text{ seconds}$$ 6

d) Now assume, that due to wear the gain of the sensor has changed. Sketch the step response of a sensor with a gain-error **qualitatively** if the step in the true temperature goes T_{real} from $10\,^\circ$ C to $20\,^\circ$ C. Again, assume that the steady state of $T_{real} = 10\,^\circ$ is already reached, when the step happens.



$\boxed{2}$

e) How is the error called that arises due to the wrong sensor gain and what can be done to compensate this error?

Static error. $\boxed{1}$

A correction factor is needed to compensate this error. $\boxed{1}$

$\boxed{\sum 18}$

Task 7: FIR and IIR

a) With the impulse

$$u(k) = \begin{cases} 1 & k = 0 \\ 0 & k = 1 \end{cases} \tag{19}$$

the impulse response is calculated according to

$$y(0) = 1 \tag{20}$$
$$y(1) = 0.5 \tag{21}$$
$$y(2) = 0.25 \tag{22}$$

For the approximating system with coefficients greater than 0.2 it holds that

$$y(k) = u(k) + 0.5u(k) + 0.25u(k). \tag{23}$$

$\boxed{3}$

b) Transformation in the z-domain (FIR)

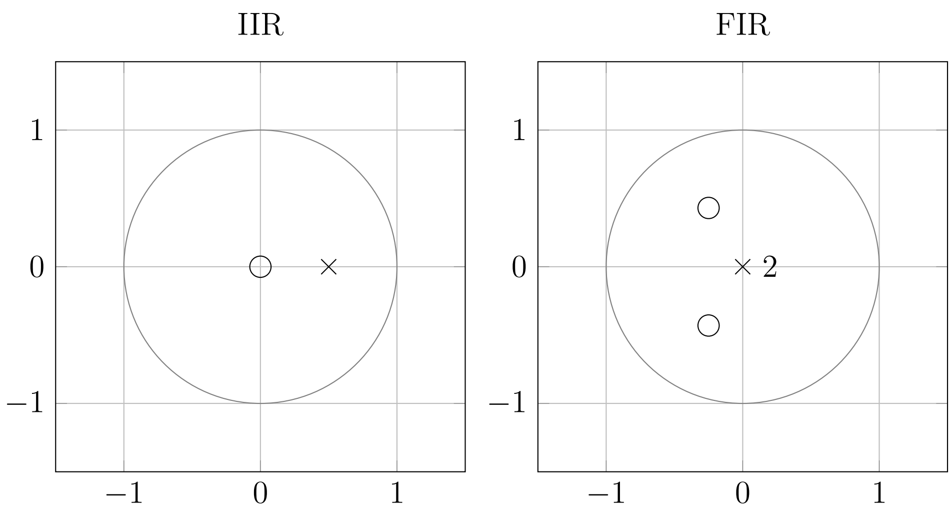
$$\frac{Y_{\text{app}}}{U_{\text{app}}} = 1 + 0.5z^{-1} + 0.25z^{-2} = \frac{z^2 + 0.5z + 0.25}{z^2}. \tag{24}$$

and IIR

$$\frac{Y(z)}{U(z)} = \frac{z}{0.5 + z} \tag{25}$$

$\boxed{2}$

c) From the transfer functions the poles and zeros are calculated
Poles IIR System: $p_1 = 0.5$.
Poles FIR System: $p_1 = p_2 = 0$
Zeros FIR System: $n_{1,2} = -\frac{1}{4} \pm i\frac{\sqrt{3}}{4} \approx -0.25 \pm 0.43i$.

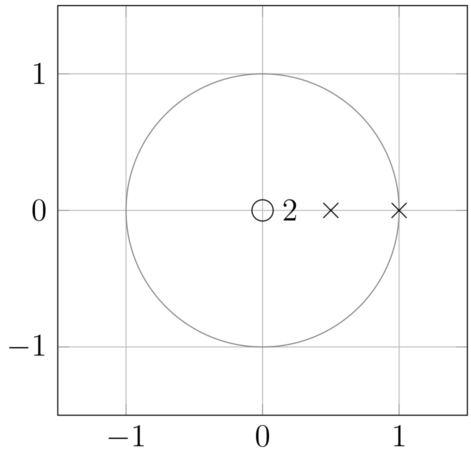


$\boxed{5}$

d) The transfer function of the system in the z-domain is

$$\frac{Y(z)}{U(z)} = \frac{z^2}{z^2 - 1.5z + 0.5} = \frac{z^2}{(z-0.5)(z-1)}. \tag{26}$$

IIR task d)/e)



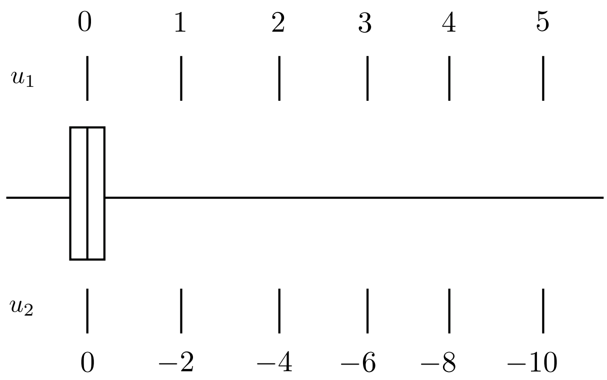
4

e) This is not possible since the system has a pole at 1 and thus only marginal stable.

1

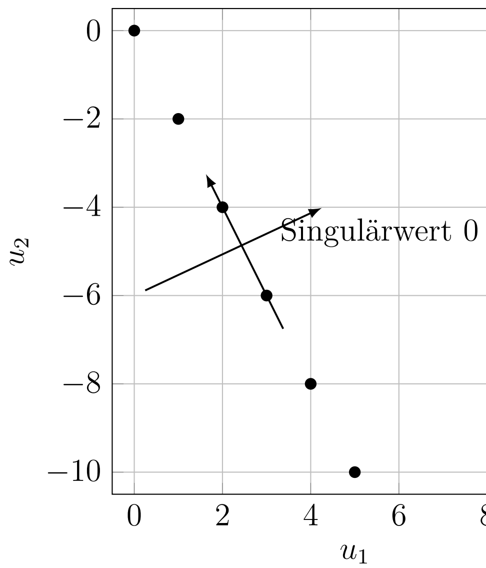
$\sum$ 15

<u>Task 8:</u> **Singular value decomposition**

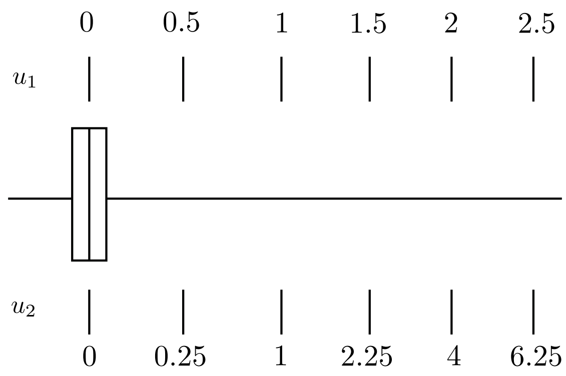


a) see diagram [1]

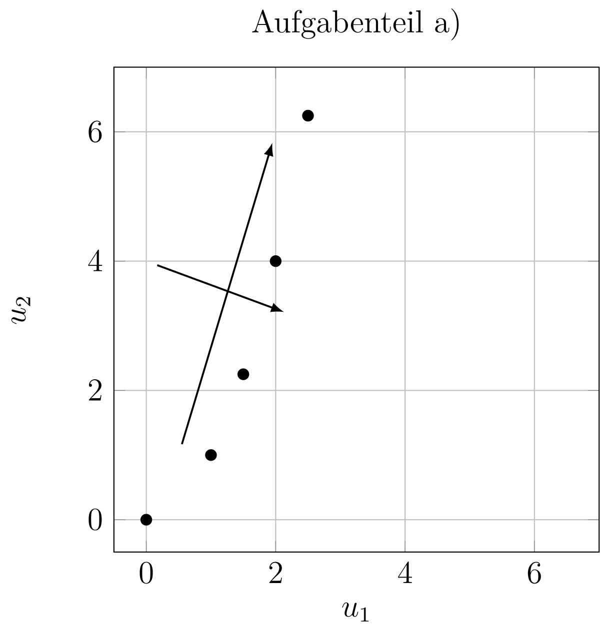
Aufgabenteil a)



b) see diagram [3]

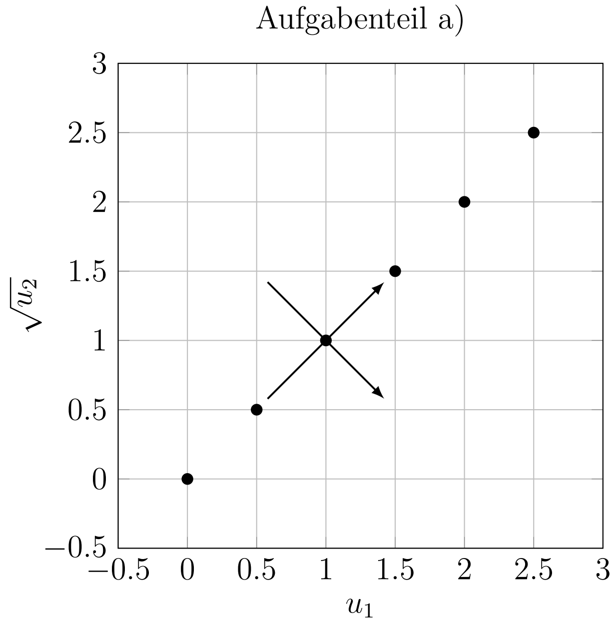


c) see diagramm 1



d) see diagram 1

e) The relation between $u_2 = u_1^2$ ist nolinear. Thus there is a variance orthogonal to the first singular vector. 3 1



f) see diagramm $\boxed{1}$

g) Because due to the nonlinear transformation$u_3^* = \sqrt{u_2}$ a linear relationship between u_1 und u_3^* holds. $\boxed{3}$

$\boxed{\sum 14}$