Sensorics Exam

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24.08.2022

Name:	
Mat.-No.:	
Grade:	

Task	T1	T2	T3	T4	T5	T6	Sum
Score:	27	11	22	20	23	17	120
Accomplished:							

Exam duration: 2 hours

Permitted Tools: Pocket Calculator and 4 pages of formula collection

Task 1: Short Tasks (27 Points)

a) If you have to buy an A/D converter for a data acquisition task, which are the three most important characteristics of converters that you have to consider. (3 points)

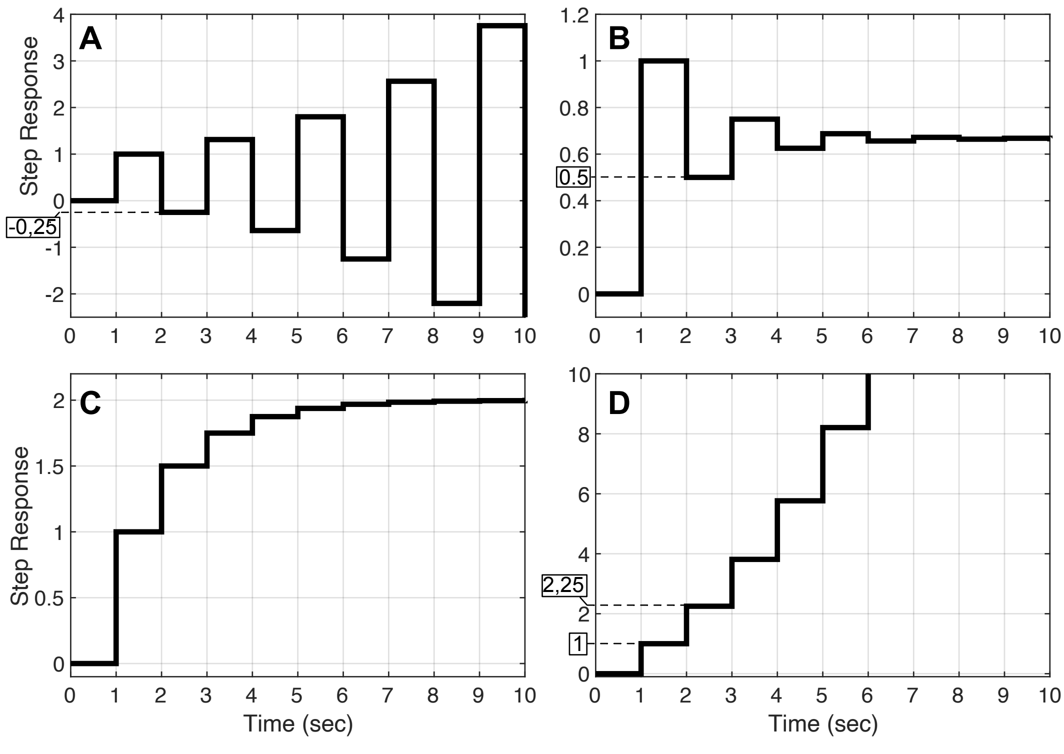
b) Explain the difference between quantization and sampling. Quantization and sampling each can cause one distinct discrepancy between the actual and the measured data. How are these discrepancies called, explain them briefly (5 points).

c) Explain briefly the physical principles behind the three methods to measure temperature: Thermocouples, PTC and NTC sensors. (5 points)

d) Name two methods used to measure volume (or mass) flow. Explain them briefly. (4 points)

e) Given are 4 step responses (diagrams A, B C, D) with different values for the parameter a of the following time-discrete system (10 points):

$$G(z) = \frac{Y(z)}{U(z)} = \frac{b}{z+a}$$



1) Considering the shape of the step responses, in which region of the the z-plane does the pole $p = -a$ lie for each step response (inside or outside the unit circle, on the negative or positive real axis)? Give a brief explanation.

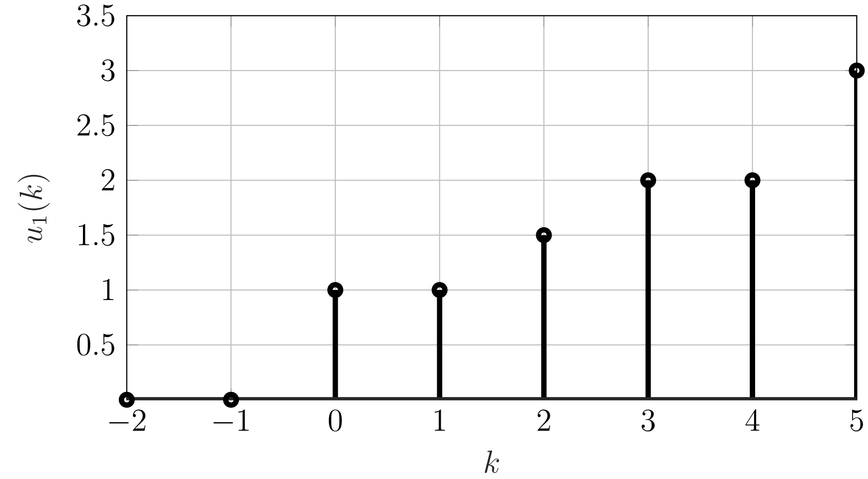
2) Calculate the corresponding differential equation for $G(z)$.

3) All 4 systems have the same value for b. Determine b using the differential equation and one of the step responses.

4) Determine the parameter a (or pole $p = -a$) for all 4 systems using the differential equation and the step responses.

Task 2: Median Filter (11 Points)

The following figure shows the input signal $u_1(k)$.



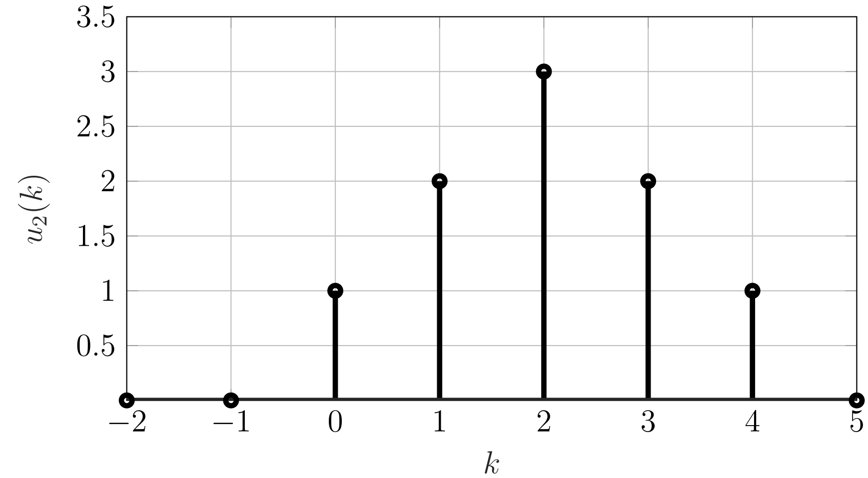
A causal median FIR filter of 3rd order should be used to filter the input signal $u_1(k)$.

a) Write down the equation of this causal 3rd median filter in the time domain.

b) Determine and draw the output signal $y_1(k)$ of the filter for the given input $u_1(k)$ for $k \geq 0$.

c) Determine the transfer function $G_1(z)$ of a linear filter which also calculates the output $y_1(k)$ (task part a)) from the given input $u_1(k)$ (no calculation is required). Explain how it is possible that two different filters yield the same output sequence.

Now another input signal $u_2(k)$ is given.

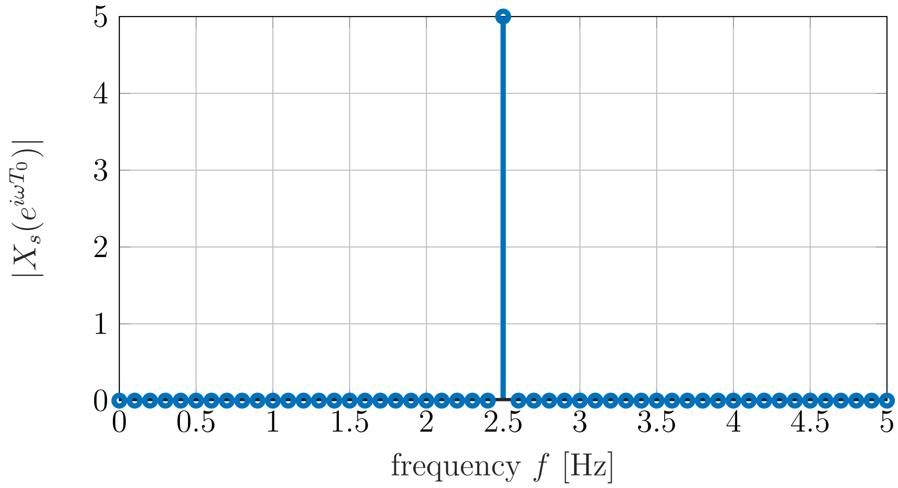


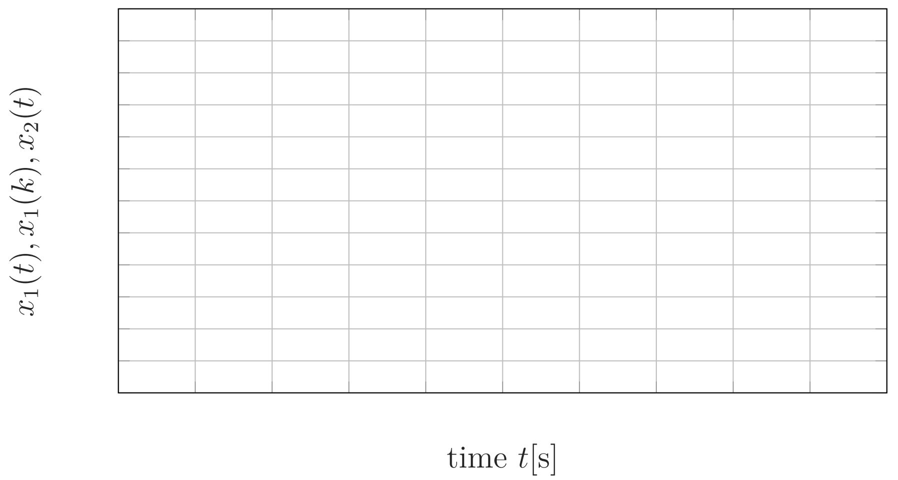
d) Calculate and draw the new output signal $y_2(k)$ of the causal 3rd order median filter applied on $u_2(k)$ for $k \geq 0$.

e) Does the linear transfer function $G_1(z)$ also lead to the output signal $y_2(k)$ (task part d)) for the given input $u_2(k)$. Please give an explanation.

Task 3: **DFT und Aliasing (22 Punkte)**

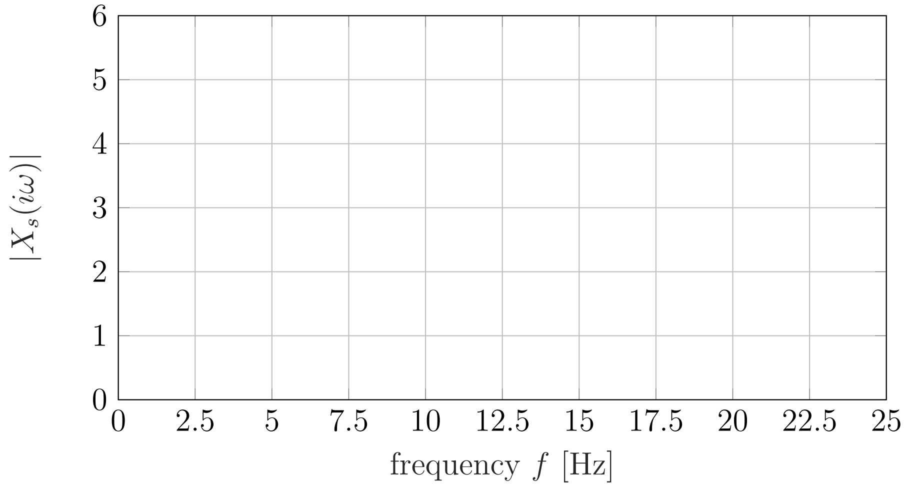
a) The following figure shows the discrete frequency spectrum of a signal recorded with the measurement frequency of $f_0 = 10\text{Hz}$. In the given diagram, draw one period of a continuous-time signal that leads to the frequency spectrum shown. Give the formula for this signal. In addition, draw 5 samples into the continuous-time diagram that match the given measurement frequency.



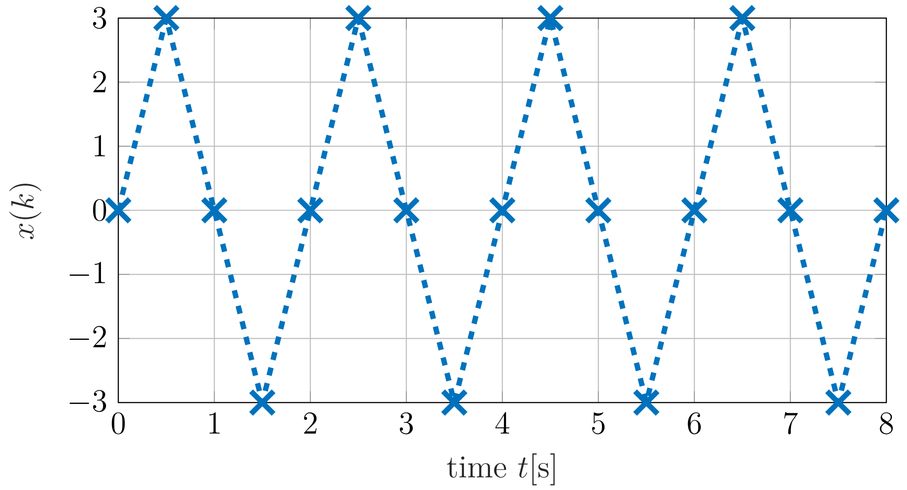


b) Assume that the sampling theorem did not hold during the recording of the signal $x(k)$. Which other frequencies could the presented frequency spectrum from task part a) also belong to. Give a general formula.

c) Draw the Shannon frequency and 2 other possible frequencies that could lead to the frequency spectrum from task a) in the given diagram. In addition, draw another continuous-time signal in the time diagram from task a) that could lead to the specified frequency spectrum.

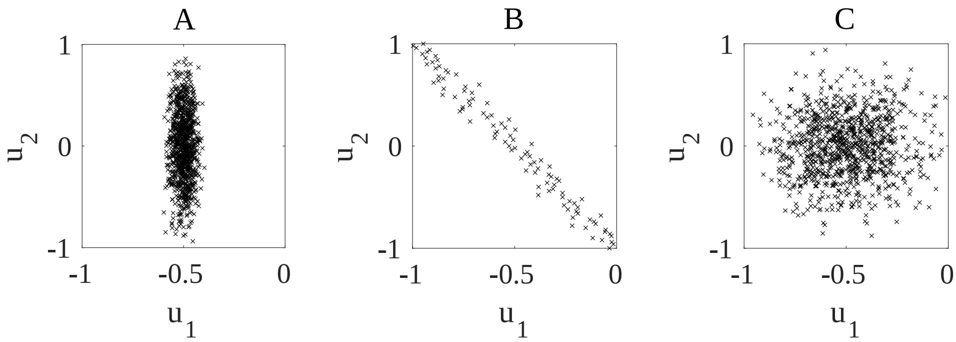


d) The following figure shows the samples of a periodic signal. Specify the period length as well as the measurement frequency of the signal. Calculate a DFT for this signal and draw the discrete frequency spectrum.



Task 4: Principal Component Analysis (20 Punkte)

a) Given are the following three datasets. For each, a principal component analysis (PCA) is performed. Choose for every dataset one of the given sets of singular values and explain your choice. Additionally, draw the corresponding principal axes into the diagrams.



ID	Singular Values
1	$s_1 = 10;\ s_2 = 9.5$
2	$s_1 = 8;\ s_2 = 0$
3	$s_1 = 9;\ s_2 = 0.7$
4	$s_1 = 10;\ s_2 = -1$
5	$s_1 = 9;\ s_2 = 2$
6	$s_1 = -1;\ s_2 = -3$

b) Please mention some possible reasons, why a principal component analysis is performed on a dataset.

c) Describe the terms 'feature selection' and 'feature extraction'. To which term PCA does belong?
What are the differences regarding the construction/measurement of new datasets?
What does this mean for the PCA?

d) A PCA returns two singular values. The first one is larger than 0. The second one is equal to 0. Sketch a possible dataset, that fulfills these requirements.
What is the value of the variance of the second principal axis?

Task 5: Measurement Errors and Statistics (23 Punkte)

The temperature of a fluid in a chemical manufacturing process is measured 5 times. It can be assumed, that there is in fact no change in the temperature of the fluid at all. The standard deviation of the disturbance is determined to be $\sigma_T = 0.6$. The following values are obtained:

$$T\,[^\circ\mathrm{C}] = [45,\ 44.8,\ 45,\ 43.9,\ 43.8]$$

Note that subtasks d) and e) can be solved independently from subtasks a) to c).

a) Calculate the range in which the true temperature will be with an error probability of 4.55% (confidence interval). Use the given table for this task:

Interval	Probability
$\mu_x - 1\sigma_x < x < \mu_x + 1\sigma_x$	68.27%
$\mu_x - 2\sigma_x < x < \mu_x + 2\sigma_x$	95.45%
$\mu_x - 3\sigma_x < x < \mu_x + 3\sigma_x$	99.73%
$\mu_x - 1\sigma_x < x < \mu_x + 1\sigma_x$	99.99%

b) Estimate the instrument's standard deviation s_T due to the temperature disturbance with the help of the measured values from above.

c) Given s_T from subtask b), will anything change regarding the confidence interval calculation compared to subtask a)?

d) The true temperature T_0 is to be estimated from N noisy measurements using three different thermometers. Assume that the measured values are normally distributed. Sketch the probability densities of the estimated temperature for three different N ($N_1 = 100$, $N_2 = 200$, $N_3 = 400$) for the three following cases:

- 1.) Unbiased and consistent estimator.
- 2.) Asymptotically unbiased and consistent estimator.
- 3.) Biased and non-consistent estimator.

Sketch each case in an individual plot. Mark the true value of the temperature T_0 and make sure to label the axes and label the sketched probability densities with the respective number of measurement data points.

Hint: Asymptotically biased means that the estimated parameter has a bias for small values of N, but no bias for $N \to \infty$.

e) How does the number of measurements N influence the estimation of (i) the variance and (ii) the mean value?

Task 6: Time-Discrete Systems (17 Points)

IIR System Analysis

A system is described by the difference equation

$$y(k) = b_0 u(k) - a_1 y(k-1) \ .$$

Additionally, two constants, $\alpha = -0.6$ and $\beta = 2$, are given.

a) Calculate the gain of the system for $b_0 = \beta$ and $a_1 = \alpha$.

b) Which property of the system changes, if the algebraic sign of b_0 is changed to $b_0 = -\beta$? How does the property change?

c) Now, the algebraic sign of a_1 is changed to $a_1 = -\alpha$ (b_0 returns to it's original state $b_0 = \beta$). Which property of the system changes in contrast to the original system (original system: $a_1 = \alpha$ and $b_0 = \beta$)? Why does this happen?

d) Now, let $a_1 = \frac{1}{\alpha}$ (and $b_0 = \beta$). Which property of the system changes in contrast to the original system (original system: $a_1 = \alpha$ and $b_0 = \beta$)? Why does this happen?

FIR System Analysis

A first-order FIR filter is described by the difference equation

$$y(k) = b_0 u(k) + b_1 u(k-1) \ .$$

Additionally, two constants $\beta_0 = 0.5$ and $\beta_1 = 0.5$ are given.

e) What kind of filter characteristic does the FIR filter have with $b_0 = \beta_0$ and $b_1 = \beta_1$?

f) What kind of filter characteristic does the FIR filter have with $b_0 = \beta_0$ and $b_1 = -\beta_1$?

Solution:

<u>Task 1:</u> Short Tasks (27 Points)

a) If you have to buy an A/D converter for a data acquisition task, which are the three most important characteristics of converters that you have to consider. (3 points)

<u>Answer</u>: Resolution, speed and cost (complexity). 3

b) Explain the difference between quantization and sampling. Quantization and sampling each can cause one distinct discrepancy between the actual and the measured data. How are these discrepancies called, explain them briefly (5 points).

<u>Answer</u>: Sampling is the discretization of time, quantization the discretization of the amplitude of a continuous signal. Sampling causes discrepancies in the frequency content of a signal, this is called aliasing and depends on the sampling frequency. Quantization introduces the quantization error, which depends on the resolution of the A/D converter. 5

c) Explain briefly the physical principles behind the three methods to measure temperature: Thermocouples, PTC and NTC sensors. (5 points)

<u>Answer</u>: Thermocouples use the effect that two wires made from different materials that are connected at one end produce a voltage proportional to the temperature at their unconnected ends. PTC and NTC sensors use the effect that electrical resistance depends on the temperature. PTC (positive temperature coefficient) sensors use the property of ohmic conductors to increase their resistance with temperature, while it is the other way round using NTC (negative temperature coefficient) sensors, which are semiconductors. 5

d) Name two methods used to measure volume (or mass) flow. Explain them briefly. (4 points)

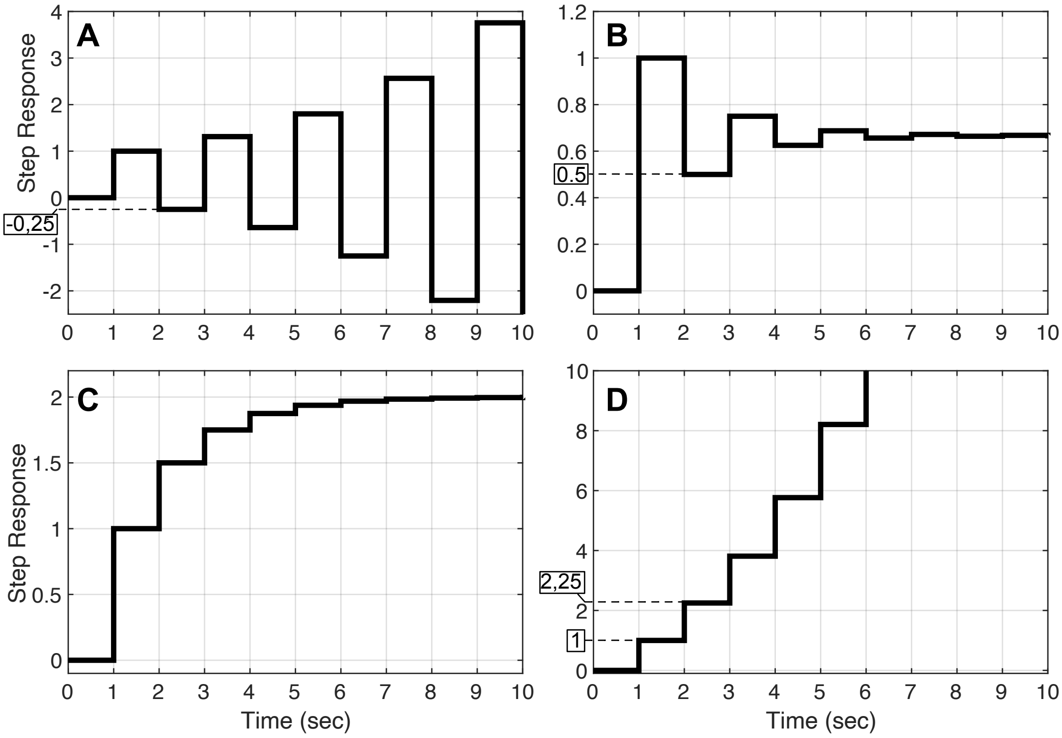
<u>Answer</u>: Possible answers (only two needed):

- **Differential pressure method:** Indirect measurement using the pressure change in a fluid if the cross section of the flow is changed (usually narrowed by an orifice, nozzle etc.).
- **Volume counter method:** The revolutions of a mechanism that is turned by the flow are counted. If the volume that passes during one revolution (chamber volume) is known the flow can be calculated.
- **Float measurement:** The position of a body that is is lifted up by the forces caused by the the flow is measured. The cross section of the flow increases when the body is lifted, leading to a relation between volume flow and the (squared) lifting height.
- **Magnetic inductive measurement:** For all conducting fluids a voltage can be induced (and measured) by applying a magnetic field orthogonal to the flow direction. The induced voltage is proportional to the speed of the flow.
- **Coriolis-based measurement:** Using a rotating or vibrating pipe Coriolis forces are generated if the fluid moves perpendicular to the rotation axis of the pipe. The force is proportional to the speed of the fluid and can therefore be used to calculate it.

- **Hot wire measurement:** An electrically heated wire is cooled by the flowing medium. The temperature difference between the wire and the fluid depends proportionally on the mass flow and can therefore be used to determine it. 4

e) Given are 4 step responses (diagrams A, B C, D) with different values for the parameter a of the following time-discrete system (10 points):

$$G(z) = \frac{Y(z)}{U(z)} = \frac{b}{z+a}$$



1) Considering the shape of the step responses, in which region of the the z-plane does the pole $p = -a$ lie for each step response (inside or outside the unit circle, on the negative or positive real axis)? Give a brief explanation.
2) Calculate the corresponding differential equation for $G(z)$.
3) All 4 systems have the same value for b. Determine b using the differential equation and one of the step responses.
4) Determine the parameter a (or pole $p = -a$) for all 4 systems using the differential equation and the step responses.

Answer:

1) A and D lie outside the unit circle (unstable), B and C inside (stable). A and B alternate from every sample to the next. This happens if the pole lies on the negative real axis. Consequently C and D lie on the positive real axis. 4
2) The differential equation for $G(z)$ is:

$$Y(z+a) = bU \Leftrightarrow y(k+1) + ay(k) = bu(k) \Leftrightarrow \boxed{y(k+1) = bu(k) - ay(k)}$$ 1

3) Calculating the value for $k = 1$:

$$y(1) = bu(0) - ay(0) \Leftrightarrow 1 = b \cdot 1 - a \cdot 0 \Leftrightarrow \boxed{b = 1}$$ 2

4) Calculating the value for $k = 2$:

A: $y(2) = bu(1) - ay(1) \Leftrightarrow -0.25 = 1 \cdot 1 - a \cdot 1 \Leftrightarrow \boxed{a = 1.25}$ or $p = -1.25$

B: $y(2) = bu(1) - ay(1) \Leftrightarrow 0.5 = 1 \cdot 1 - a \cdot 1 \Leftrightarrow \boxed{a = 0.5}$ or $p = -0.5$

C: $y(2) = bu(1) - ay(1) \Leftrightarrow 1.5 = 1 \cdot 1 - a \cdot 1 \Leftrightarrow \boxed{a = -0.5}$ or $p = 0.5$ $\boxed{3}$

D: $y(2) = bu(1) - ay(1) \Leftrightarrow 2.25 = 1 \cdot 1 - a \cdot 1 \Leftrightarrow \boxed{a = -1.25}$ or 1.25

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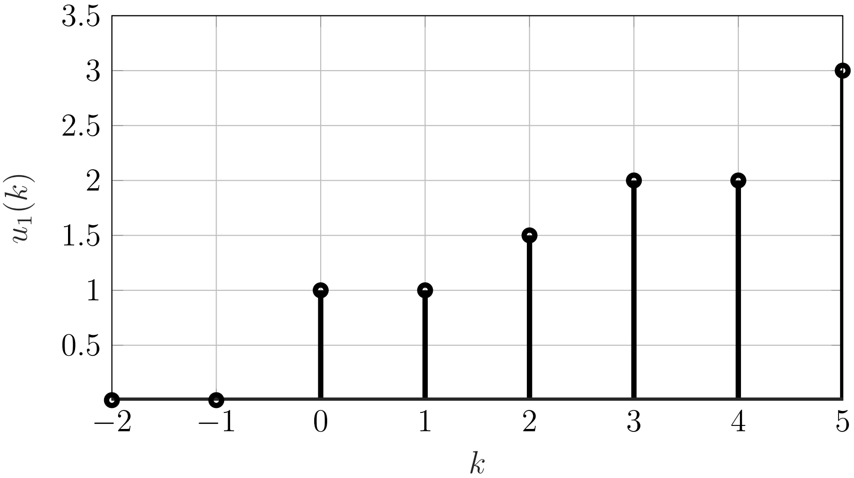
Task 2: Median Filter (11 Points)

The following figure shows the input signal $u_1(k)$.



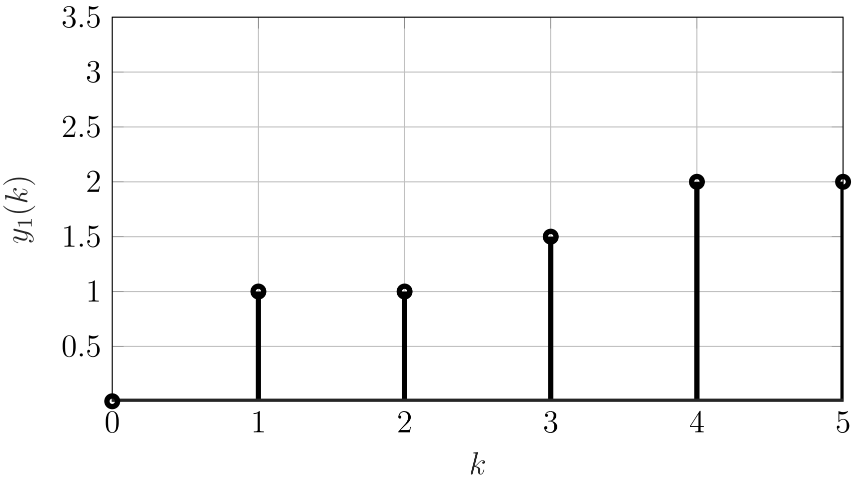
A causal median FIR filter of 3rd order should be used to filter the input signal $u_1(k)$.

a) Write down the equation of this causal 3rd median filter in the time domain.
Answer: The equation of a median filter 3rd is given by

$$y_1(k) = \text{median}(u(k), u(k-1), u(k-2)).$$

1

b) Determine and draw the output signal $y_1(k)$ of the filter for the given input $u_1(k)$ for $k \geq 0$.
Answer:



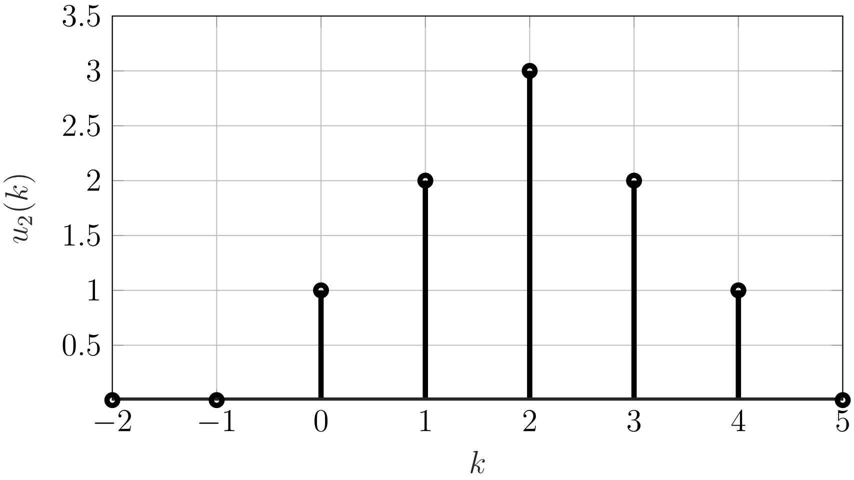
3

c) Determine the transfer function $G_1(z)$ of a linear filter which also calculates the output $y_1(k)$ (task part a)) from the given input $u_1(k)$ (no calculation is required). Explain how it is possible that two different filters yield the same output sequence.
Answer: Due to the monotonically increasing input sequence, the input value of $u_1(k-1)$ is determined as output $y_1(k)$ in the median filter. This results in the transfer function:

$$G_1(z) = z^{-1}.$$

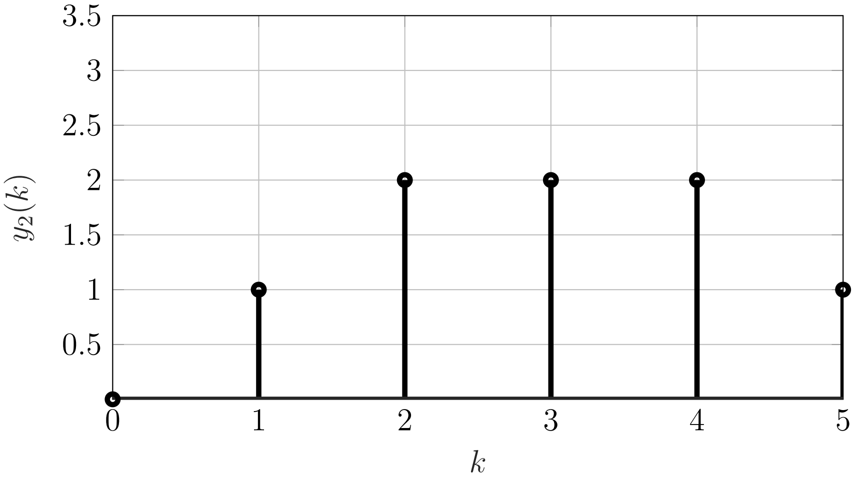
Two different linear filters cannot have the same output response to the same input signal. This is only possible because the median filter is a nonlinear filter. 2

Now another input signal $u_2(k)$ is given.



d) Calculate and draw the new output signal $y_2(k)$ of the causal 3$^{\text{rd}}$ order median filter applied on $u_2(k)$ for $k \geq 0$.

Answer: Median filter equation can be found in task a).



3

e) Does the linear transfer function $G_1(z)$ also lead to the output signal $y_2(k)$ (task part d)) for the given input $u_2(k)$. Please give an explanation.

Answer: No, the transfer function $G_1(z)$ does not lead to the the output $y_2(k)$. This only holds for monotonic increasing or decreasing input trajectories. 2

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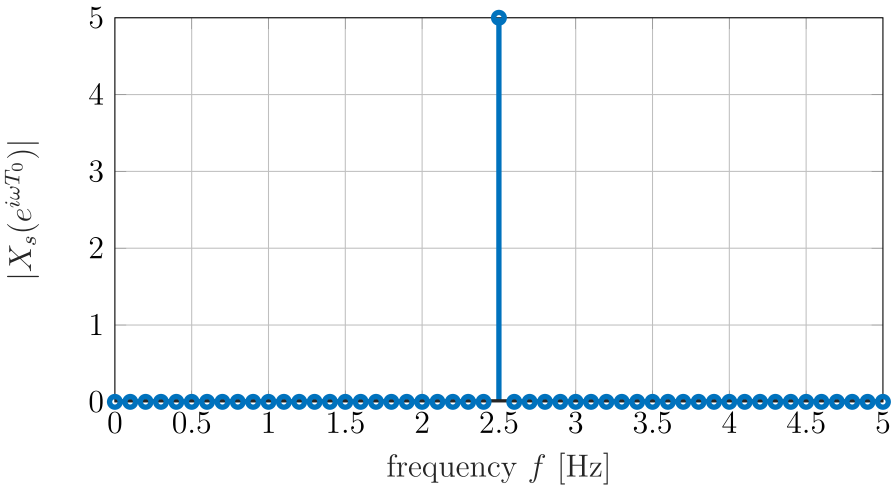
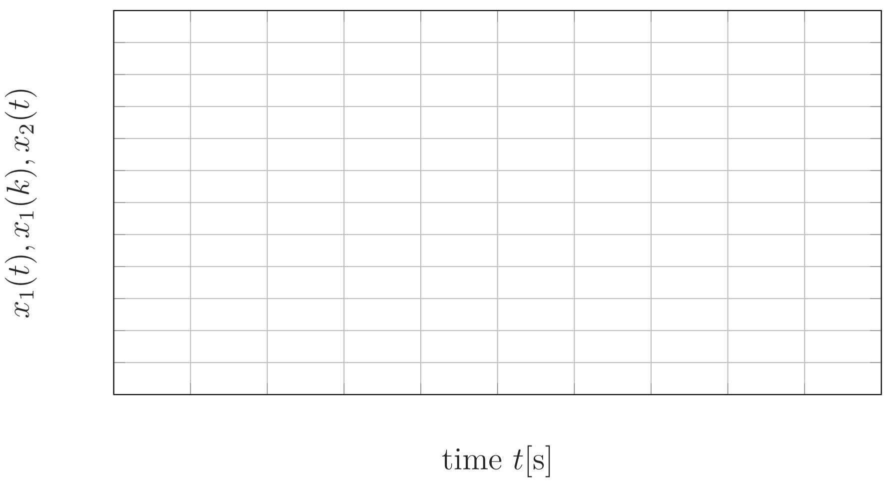
Task 3: DFT und Aliasing (22 Punkte)

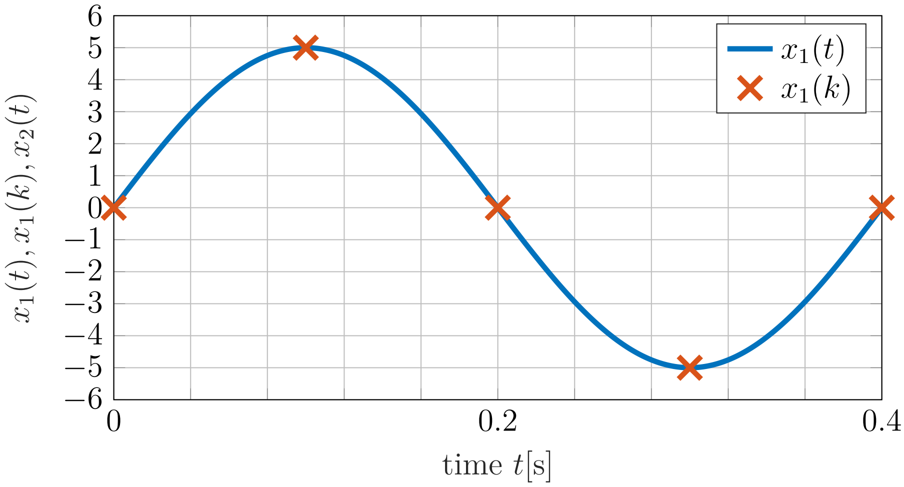
a) The following figure shows the discrete frequency spectrum of a signal recorded with the measurement frequency of $f_0 = 10\text{Hz}$. In the given diagram, draw one period of a continuous-time signal that leads to the frequency spectrum shown. Give the formula for this signal. In addition, draw 5 samples into the continuous-time diagram that match the given measurement frequency.





Answer: From the figure it can be seen that the signal consists only of one frequency component with 2.5Hz. The amplitude at this frequency is 5. From this, the general formula of a sinusoidal signal with the frequency of $f_1 = 2.5\text{Hz}$ and amplitude $A_1 = 5$ can be derived.

$$x(t) = A_1 \sin(2\pi f_1 t) = 5 \sin(2\pi \cdot 2.5\text{Hz} \cdot t)$$



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b) Assume that the sampling theorem did not hold during the recording of the signal $x(k)$. Which other frequencies could the presented frequency spectrum from task part a) also belong to. Give a general formula.

Answer: If the sampling theorem does not hold when recording a signal, the aliasing effect can occur. With this effect, frequencies that lie outside the frequency spectrum from 0Hz to $0.5f_0 = 5\text{Hz}$ are mirrored in that range. These additional frequency spectra are also called shadow spectra. In general, the following shadow spectra are generated by the sampling process:

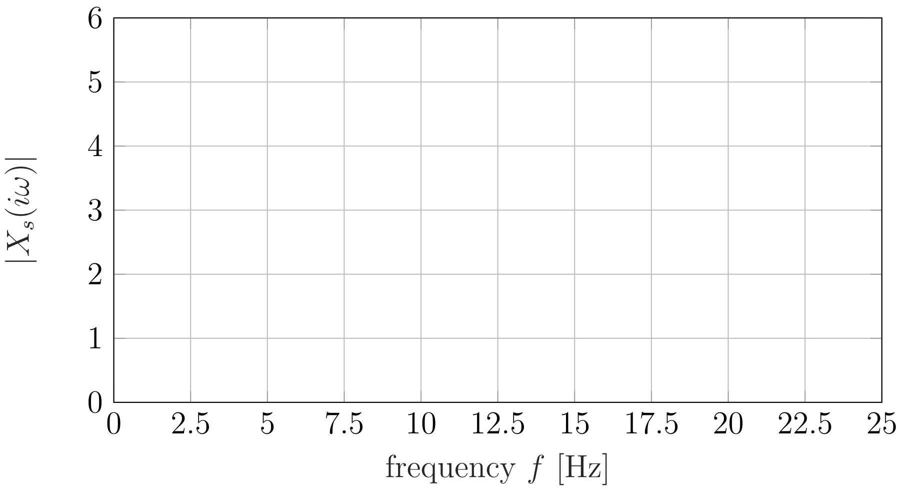
$$f_l = f_1 + l f_0 \quad \text{mit } l = \ldots -2, -1, 0, 1, 2 \ldots$$

For this reason, the frequency spectrum shown may also belong to the following frequencies:

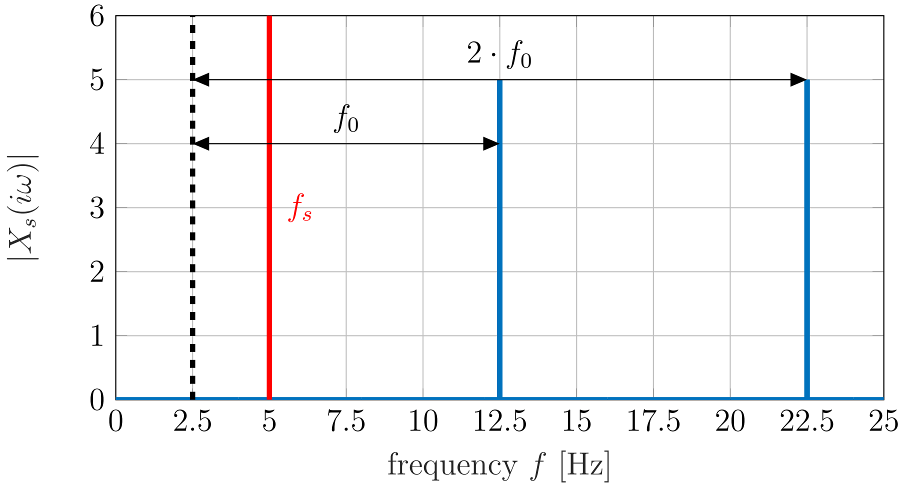
$$f_2 = f_1 + l f_0 = 2.5\text{Hz} + l \cdot 10\text{Hz} \quad \text{mit } l = \ldots -2, -1, 0, 1, 2 \ldots$$

2

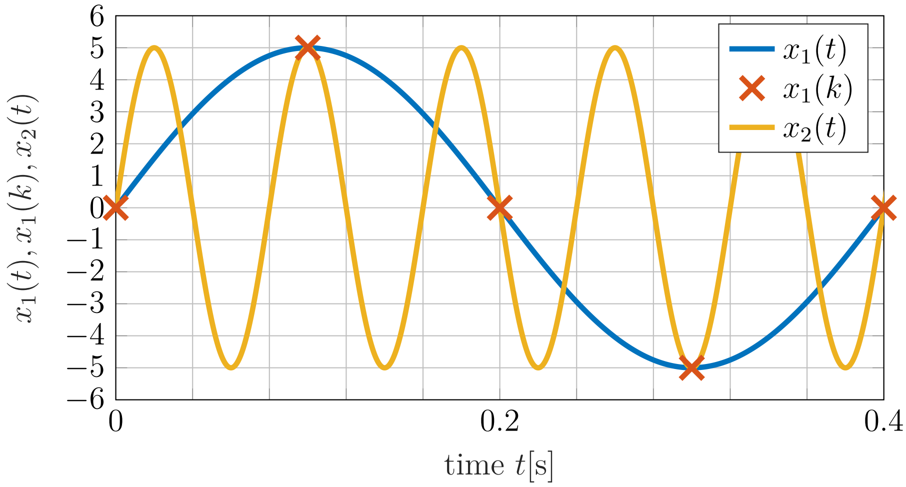
c) Draw the Shannon frequency and 2 other possible frequencies that could lead to the frequency spectrum from task a) in the given diagram. In addition, draw another continuous-time signal in the time diagram from task a) that could lead to the specified frequency spectrum.



Answer: The Shannon frequency is half the measurement frequency $f_s = 0.5 f_m = 5\text{Hz}$. Two possible frequencies can be derived from the solution of task b).

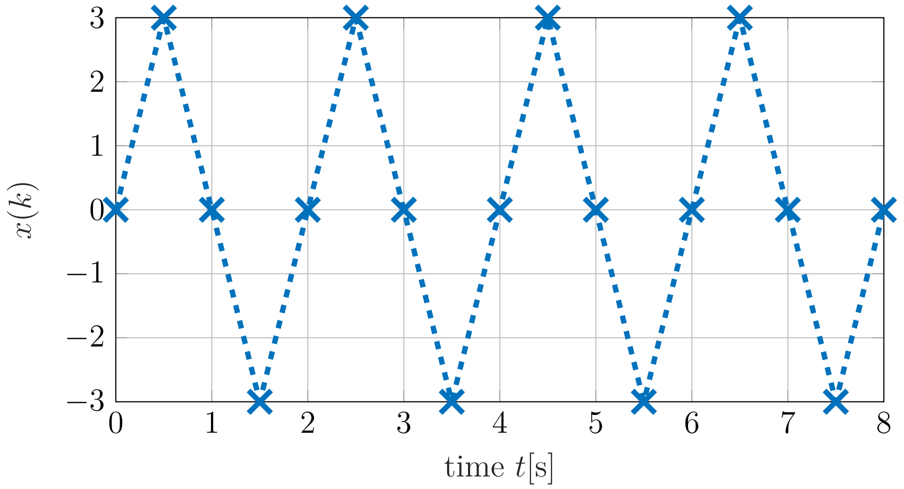


The frequency of $f_2 = 12.5$Hz is $\frac{12.5}{2.5} = 5$ times the original frequency:



6

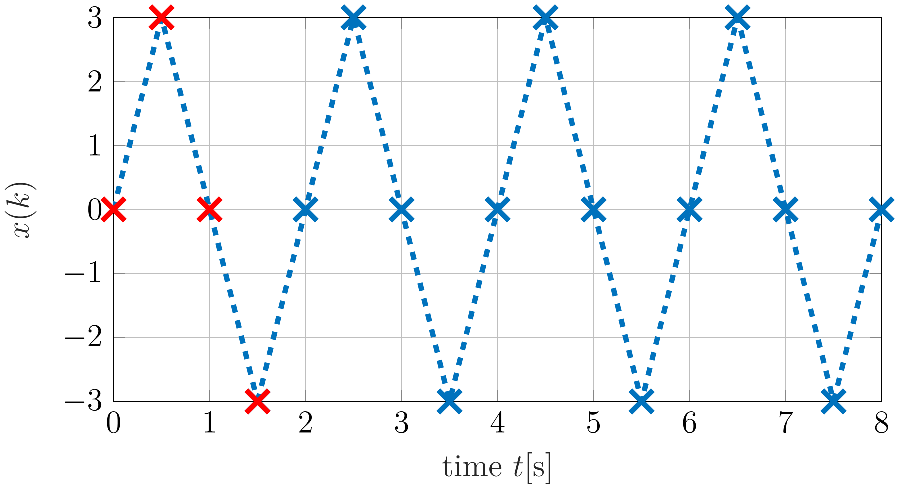
d) The following figure shows the samples of a periodic signal. Specify the period length as well as the measurement frequency of the signal. Calculate a DFT for this signal and draw the discrete frequency spectrum.



Answer: Since the signal repeats after 2s seconds, the period of the signal is $T = 2$s. The measuring points have been recorded at a time-distance of $T_0 = 0.5$s. Thus the measurement frequency $f_0 = \frac{1}{T_0} = 2$Hz can be calculated.

For the DFT of a periodic signal only one period of the signal has to be considered.

With a measurement frequency of $f_0 = \frac{1}{T_0} = 2\text{Hz}$ and the period $T = 2\text{s}$ of the signal, it can be determined that $N = T \cdot f_0 = 4$ samples must be considered. For this purpose, the first 4 samples can be used.



In general, the DFT of a discrete signal can be calculated as follows:

$$\text{DFT}(x(k)) = \underline{\text{X}}(n) = \sum_{k=0}^{N-1} x(k) e^{\frac{-i2\pi nk}{N}}$$

With 4 data points to consider, the following matrix equation can be formulated:

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_N^1 & W_N^2 & W_N^3 \\ 1 & W_N^2 & W_N^4 & W_N^6 \\ 1 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} \quad \text{mit } W_N = e^{\frac{-i2\pi}{N}}$$

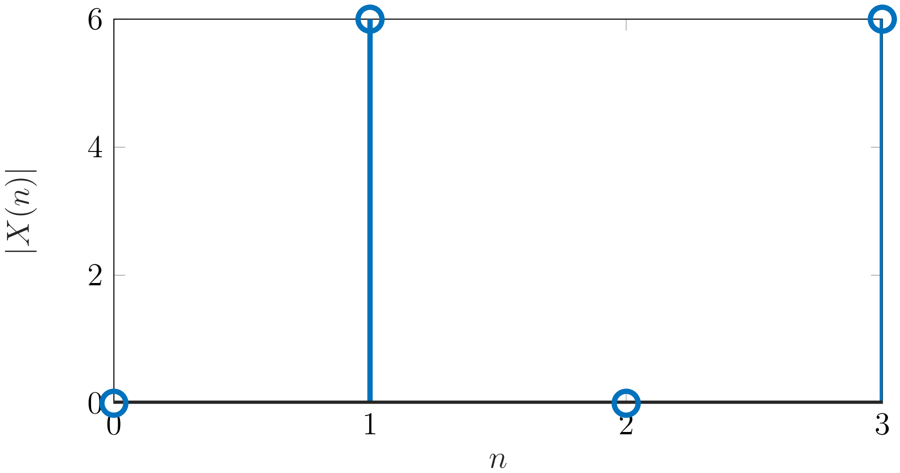
mit $N = 4$

$$\underline{\text{X}} = \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 0 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ -6i \\ 0 \\ 6i \end{bmatrix}$$

For the frequency spectrum, only the magnitude of $\underline{\text{X}}$ is crucial:

$$|\underline{\text{X}}| = \begin{bmatrix} 0 \\ 6 \\ 0 \\ 6 \end{bmatrix}$$

This results in the following frequency spectrum:



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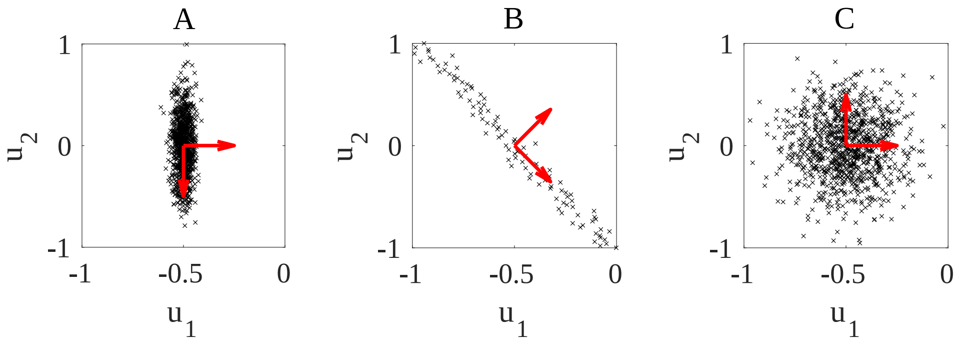
Task 4: Principal Component Analysis (20 Punkte)

a) Given are the following three datasets. For each, a principal component analysis (PCA) is performed. Choose for every dataset one of the given sets of singular values and explain your choice. Additionally, draw the corresponding principal axes into the diagrams.



ID	Singular Values
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4	$s_1 = 10;\ s_2 = -1$
5	$s_1 = 9;\ s_2 = 2$
6	$s_1 = -1;\ s_2 = -3$

Answer:

Dataset	Singular Value ID
A	5
B	3
C	1

- No negativ singular values
- A: Small variance along u_1.
- B: In comparision to A, larger variance along first principal axis. Therfore, ratio between singluar values larger compared to A.
- C: Similiar variance along both dimensions.

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b) Please mention some possible reasons, why a principal component analysis is performed on a dataset.
Answer:
- Reduction of dimensions
- Easier description of a dataset

2

c) Describe the terms 'feature selection' and 'feature extraction'. To which term PCA does belong?
What are the differences regarding the construction/measurement of new datasets? What does this mean for the PCA?
Answer:
Feature Selection:
- From p original inputs, a smaller number of q features is selected.
- Only the selected features are required afterwards.

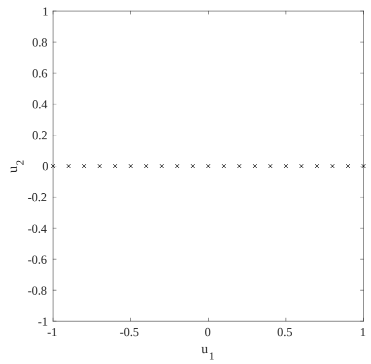
Feature Extraction:
- From p original inputs (features), a smaller number of new features is created.
- All new created features are dependend of all p original inputs.

The PCA belongs to the methods of feature extraction. Therefore, all original inputs are required. If a new dataset should be created, all original inputs must be measured or simulated.

6

d) A PCA returns two singular values. The first one is larger than 0. The second one is equal to 0. Sketch a possible dataset, that fulfills these requirements.
What is the value of the variance of the second principal axis?

Answer:
If the second singular value is equal to 0, the second principal component has no influence on the data. The variance of this dimension is zero.



3

$\sum 20$

Task 5: Measurement Errors and Statistics (23 Punkte)

The temperature of a fluid in a chemical manufacturing process is measured 5 times. It can be assumed, that there is in fact no change in the temperature of the fluid at all. The standard deviation of the disturbance is determined to be $\sigma_T = 0.6$. The following values are obtained:

$$T\,[°\text{C}] = [45,\ 44.8,\ 45,\ 43.9,\ 43.8]$$

Note that subtasks d) and e) can be solved independently from subtasks a) to c).

a) Calculate the range in which the true temperature will be with an error probability of 4.55% (confidence interval). Use the given table for this task:

Interval	Probability
$\mu_x - 1\sigma_x < x < \mu_x + 1\sigma_x$	68.27%
$\mu_x - 2\sigma_x < x < \mu_x + 2\sigma_x$	95.45%
$\mu_x - 3\sigma_x < x < \mu_x + 3\sigma_x$	99.73%
$\mu_x - 1\sigma_x < x < \mu_x + 1\sigma_x$	99.99%

Answer:
Sample mean: $\bar{T} = \frac{1}{5}\sum_{i=1}^{5} T(i) = \frac{222.5}{5} = 44.5°\text{C}$.
Standard deviation of the sample mean: $\sigma_{\bar{T}} = \frac{\sigma_T}{\sqrt{N}} = \frac{0.6}{2.24} = 0.27$.
From the accepted error probability follows a requested confidence level of 95.45%. This corresponds to $c = 2$ for the range calculation. The range is

$$\begin{aligned} \bar{T} - c \cdot \sigma_{\bar{T}} < T < \bar{T} + c \cdot \sigma_{\bar{T}} \\ 44.5°\text{C} - 2 \cdot 0.27 < T < 44.5 + 2 \cdot 0.27 \\ 43.96°\text{C} < T < 45.04°\text{C} \end{aligned}$$

$\boxed{3}$

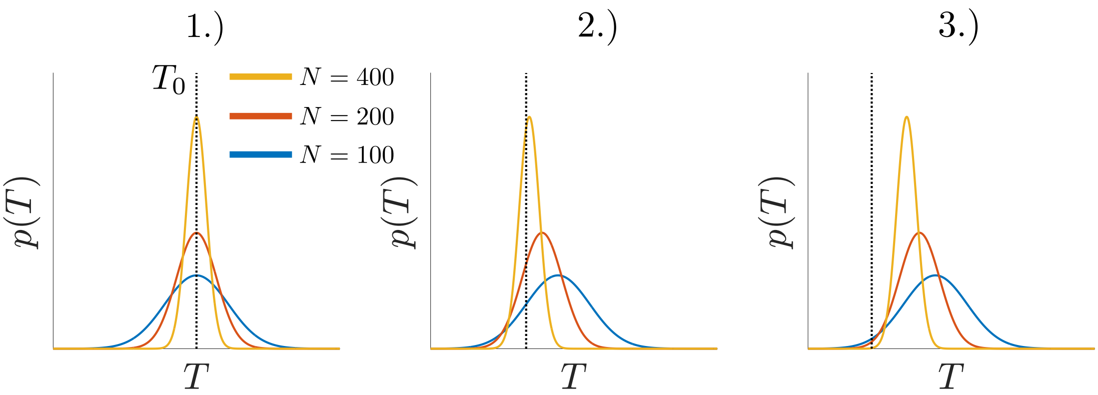
b) Estimate the instrument's standard deviation s_T due to the temperature disturbance with the help of the measured values from above.
Answer:
$s_T = \sqrt{\frac{1}{4}\sum_{i=1}^{5}\left[T(i) - \bar{T}\right]} = 0.6.$ $\boxed{3}$

c) Given s_T from subtask b), will anything change regarding the confidence interval calculation compared to subtask a)?
Answer:
Even though the estimated and the true standard deviation are equal, the resulting interval will be different. In the case, where the estimated standard deviation is used to calculate the range, the factor c has to be determined from a t-distribution, and therefore c will be a greater value. $\boxed{3}$

d) The true temperature T_0 is to be estimated from N noisy measurements using three different thermometers. Assume that the measured values are normally distributed. Sketch the probability densities of the estimated temperature for three different N ($N_1 = 100$, $N_2 = 200$, $N_3 = 400$) for the three following cases:

- 1.) Unbiased and consistent estimator.
- 2.) Asymptotically unbiased and consistent estimator.
- 3.) Biased and non-consistent estimator.

Sketch each case in an individual plot. Mark the true value of the temperature T_0 and make sure to label the axes and label the sketched probability densities with the respective number of measurement data points.
Hint: Asymptotically biased means that the estimated parameter has a bias for small values of N, but no bias for $N \to \infty$.
Answer:



12

e) How does the number of measurements N influence the estimation of (i) the variance and (ii) the mean value?
Answer:
The number of samples influences the mean linearly and the variance quadratically.

2

$\sum 23$

Task 6: Time-Discrete Systems (17 Points)

IIR System Analysis

A system is described by the difference equation

$$y(k) = b_0 u(k) - a_1 y(k-1) \ .$$

Additionally, two constants, $\alpha = -0.6$ and $\beta = 2$, are given.

a) Calculate the gain of the system for $b_0 = \beta$ and $a_1 = \alpha$.
Answer:
Calculate the transfer function

$$y(k) = b_0 u(k) - a_1 y(k-1)$$

$$\circ\!\!-\!\!\bullet$$

$$Y(z)(1 + a_1 z^{-1}) = b_0 U(z)$$

$$G(z) = \frac{Y(z)}{U(z)} = \boxed{\frac{b_0}{1 + a_1 z^{-1}}}$$

3

Calculate the gain

$$\lim_{z\to 1}(z-1)H(z) = G(z=1) = \frac{b_0}{1+a_1} = \frac{\beta}{1+\alpha} = \frac{2}{1-0.6} = \boxed{5}$$ 2

b) Which property of the system changes, if the algebraic sign of b_0 is changed to $b_0 = -\beta$? How does the property change?
Answer:

$$\lim_{z\to 1}(z-1)H(z) = G(z=1) = \frac{b_0}{1+a_1} = \frac{-\beta}{1+\alpha} = \frac{-2}{1-0.6} = \boxed{-5}$$

The gain of the system changes. The absolute value remains but the **algebraic sign of the gain changes**. 1 1

c) Now, the algebraic sign of a_1 is changed to $a_1 = -\alpha$ (b_0 returns to it's original state $b_0 = \beta$). Which property of the system changes in contrast to the original system (original system: $a_1 = \alpha$ and $b_0 = \beta$)? Why does this happen?
Answer:
The original system does not yield alternating behavior. This is because there is only one positive real pole ($p_1 = -a_1 = 0.6$), which is located in the complex plane on the real axis. When the sign of a_1 is changed, the pole of the system is negative and still on the real axis ($p_{\text{new}} = -0.6$). All systems with a real and negative pole possess **alternating dynamic behavior**. 3

d) Now, let $a_1 = \frac{1}{\alpha}$ (and $b_0 = \beta$). Which property of the system changes in contrast to the original system (original system: $a_1 = \alpha$ and $b_0 = \beta$)? Why does this happen?
Answer:
The original system with $a_1 = \alpha = -0.6$ has a stable pole at $p_1 = 0.6$ (pole inside the unit circle in the complex plane). The inverse of any stable pole is unstable (outside the unit circle in the complex plane). Therefore, the system with $p_{\text{new}} = -a_1 = -\frac{1}{\alpha} = 1\frac{2}{3}$ is **unstable**. 3

FIR System Analysis

A first-order FIR filter is described by the difference equation

$$y(k) = b_0 u(k) + b_1 u(k-1) \ .$$

Additionally, two constants $\beta_0 = 0.5$ and $\beta_1 = 0.5$ are given.

e) What kind of filter characteristic does the FIR filter have with $b_0 = \beta_0$ and $b_1 = \beta_1$?
Answer: The FIR filter has a low-pass characteristic (mean filter). 2

f) What kind of filter characteristic does the FIR filter have with $b_0 = \beta_0$ and $b_1 = -\beta_1$?
Answer:The filter is now a difference filter. Therefore, it has high-pass characteristic. 2

$\sum$ 17