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Lehrstuhl Theoretische Informatik
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Compilerbau I
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Übungsblatt 8

Aufgabe 1 Sei $\Sigma = \{a, +\}$ und $G_i = (\{S\}, \Sigma, P_i, S)$, $i \in \{1, 2\}$, wobei P_1 und P_2 gegeben sind durch:

$$P_1\colon S \to SS+ \mid a$$
$$P_2\colon S \to +SS \mid a$$

(a) Konstruieren Sie die Shift-Reduce-Parser $M^{(1)}_{G_i}$ zu G_i, $i \in \{1, 2\}$.

Lösung:
$M^{(1)}_{G_1} = (Q, \Sigma, \delta, q_0, F)$ mit

$$\begin{aligned}
Q &= \{a, +, S, q_0, f\}\\
F &= \{f\}\\
\delta &= \{(a, a, aa), (+, a, +a), (q_0, a, q_0a), (f, a, fa), (S, a, Sa)\}\\
&\cup \{(a, +, a+), (+, +, ++), (q_0, +, q_0+), (f, +, f+), (S, +, S+)\}\\
&\cup \{(aa, \varepsilon, aS), (+a, \varepsilon, +S), (q_0a, \varepsilon, q_0S), (fa, \varepsilon, fS), (Sa, \varepsilon, SS)\}\\
&\cup \{(aSS+, \varepsilon, aS), (+SS+, \varepsilon, +S)\}\\
&\cup \{(q_0SS+, \varepsilon, q_0S), (fSS+, \varepsilon, fS), (SSS+, \varepsilon, SS)\}\\
&\cup \{(q_0S, \varepsilon, f)\}
\end{aligned}$$

$M^{(1)}_{G_2} = (Q, \Sigma, \delta, q_0, F)$ mit

$$\begin{aligned}
Q &= \{a, +, S, q_0, f\}\\
F &= \{f\}\\
\delta &= \{(a, a, aa), (+, a, +a), (q_0, a, q_0a), (f, a, fa), (S, a, Sa)\}\\
&\cup \{(a, +, a+), (+, +, ++), (q_0, +, q_0+), (f, +, f+), (S, +, S+)\}\\
&\cup \{(aa, \varepsilon, aS), (+a, \varepsilon, +S), (q_0a, \varepsilon, q_0S), (fa, \varepsilon, fS), (Sa, \varepsilon, SS)\}\\
&\cup \{(a+SS, \varepsilon, aS), (++SS, \varepsilon, +S)\}\\
&\cup \{(q_0+SS, \varepsilon, q_0S), (f+SS, \varepsilon, fS), (S+SS, \varepsilon, SS)\}\\
&\cup \{(q_0S, \varepsilon, f)\}
\end{aligned}$$

(b) Konstruieren Sie die Item-Kellerautomaten $M^{(2)}_{G_i}$ zu G_i, $i \in \{1, 2\}$.

Lösung:
$M_{G_1}^{(2)} = (Q, \Sigma, \delta, q_0, F)$ mit

$$
\begin{aligned}
Q &= \{[S' \to \bullet S], [S' \to S\bullet]\}\\
&\quad \cup \{[S \to \bullet a], [S \to a\bullet]\}\\
&\quad \cup \{[S \to \bullet SS+], [S \to S\bullet S+], [S \to SS\bullet +], [S \to SS+\bullet]\}\\
q_0 &= [S' \to \bullet S]\\
F &= \{[S' \to S\bullet]\}\\
\delta &= \{([S' \to \bullet S], \varepsilon, [S' \to \bullet S][S \to \bullet a])\}\\
&\quad \cup \{([S' \to \bullet S], \varepsilon, [S' \to \bullet S][S \to \bullet SS+])\}\\
&\quad \cup \{([S \to \bullet SS+], \varepsilon, [S \to \bullet SS+][S \to \bullet a])\}\\
&\quad \cup \{([S \to \bullet SS+], \varepsilon, [S \to \bullet SS+][S \to \bullet SS+])\}\\
&\quad \cup \{([S \to S\bullet S+], \varepsilon, [S \to S\bullet S+][S \to \bullet a])\}\\
&\quad \cup \{([S \to S\bullet S+], \varepsilon, [S \to S\bullet S+][S \to \bullet SS+])\}\\
&\quad \cup \{([S \to \bullet a], a, [S \to a\bullet])\}\\
&\quad \cup \{([S \to SS\bullet +], +, [S \to SS+\bullet])\}\\
&\quad \cup \{([S' \to \bullet S][S \to a\bullet], \varepsilon, [S' \to S\bullet])\}\\
&\quad \cup \{([S' \to \bullet S][S \to SS+\bullet], \varepsilon, [S' \to S\bullet])\}\\
&\quad \cup \{([S \to \bullet SS+][S \to a\bullet], \varepsilon, [S \to S\bullet S+])\}\\
&\quad \cup \{([S \to \bullet SS+][S \to SS+\bullet], \varepsilon, [S \to S\bullet S+])\}\\
&\quad \cup \{([S \to S\bullet S+][S \to a\bullet], \varepsilon, [S \to SS\bullet +])\}\\
&\quad \cup \{([S \to S\bullet S+][S \to SS+\bullet], \varepsilon, [S \to SS\bullet +])\}
\end{aligned}
$$

$M_{G_2}^{(2)} = (Q, \Sigma, \delta, q_0, F)$ mit

$$\begin{aligned}
Q &= \{[S' \to \bullet S], [S' \to S\bullet]\} \\
&\quad \cup \{[S \to \bullet a], [S \to a\bullet]\} \\
&\quad \cup \{[S \to \bullet{+}SS], [S \to +\bullet SS], [S \to +S\bullet S], [S \to +SS\bullet]\} \\
q_0 &= [S' \to \bullet S] \\
F &= \{[S' \to S\bullet]\} \\
\delta &= \{([S' \to \bullet S], \varepsilon, [S' \to \bullet S][S \to \bullet a])\} \\
&\quad \cup \{([S' \to \bullet S], \varepsilon, [S' \to \bullet S][S \to \bullet{+}SS])\} \\
&\quad \cup \{([S \to \bullet{+}SS], \varepsilon, [S \to \bullet{+}SS][S \to \bullet a])\} \\
&\quad \cup \{([S \to \bullet{+}SS], \varepsilon, [S \to \bullet{+}SS][S \to \bullet{+}SS])\} \\
&\quad \cup \{([S \to +S\bullet S], \varepsilon, [S \to +S\bullet S][S \to \bullet a])\} \\
&\quad \cup \{([S \to +S\bullet S], \varepsilon, [S \to +S\bullet S][S \to \bullet{+}SS])\} \\
&\quad \cup \{([S \to \bullet a], a, [S \to a\bullet])\} \\
&\quad \cup \{([S \to +SS\bullet], +, [S \to +SS\bullet])\} \\
&\quad \cup \{([S' \to \bullet S][S \to a\bullet], \varepsilon, [S' \to S\bullet])\} \\
&\quad \cup \{([S' \to \bullet S][S \to +SS\bullet], \varepsilon, [S' \to S\bullet])\} \\
&\quad \cup \{([S \to \bullet{+}SS][S \to a\bullet], \varepsilon, [S \to +S\bullet S])\} \\
&\quad \cup \{([S \to \bullet{+}SS][S \to +SS\bullet], \varepsilon, [S \to +S\bullet S])\} \\
&\quad \cup \{([S \to +S\bullet S][S \to a\bullet], \varepsilon, [S \to +SS\bullet])\} \\
&\quad \cup \{([S \to +S\bullet S][S \to +SS\bullet], \varepsilon, [S \to +SS\bullet])\}
\end{aligned}$$

(c) Geben Sie jeweils für $M_{G_1}^{(1)}$ und $M_{G_1}^{(2)}$ eine akzeptierende Konfigurationsfolge für $aa{+}a{+}$ an.

Lösung:
Konfigurationsfolge für $M_{G_1}^{(1)}$:

$$\begin{aligned}
(q_0, aa{+}a{+}) &\vdash (q_0 a, a{+}a{+}) \vdash (q_0 S, a{+}a{+}) \vdash (q_0 Sa, {+}a{+}) \vdash (q_0 SS, {+}a{+}) \\
&\vdash (q_0 SS{+}, a{+}) \vdash (q_0 S, a{+}) \vdash (q_0 Sa, {+}) \vdash (q_0 SS, {+}) \\
&\vdash (q_0 SS{+}, \varepsilon) \vdash (q_0 S, \varepsilon) \vdash (f, \varepsilon)
\end{aligned}$$

Konfigurationsfolge für $M_{G_1}^{(2)}$:

$$
\begin{aligned}
&([S' \to \bullet S], aa{+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}], aa{+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to \bullet SS{+}], aa{+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to \bullet SS{+}][S \to \bullet a], aa{+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to \bullet SS{+}][S \to a\bullet], a{+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to S\bullet S{+}], a{+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to S\bullet S{+}][S \to \bullet a], a{+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to S\bullet S{+}][S \to a\bullet], {+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to SS\bullet{+}], {+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to SS{+}\bullet], a{+}) \\
&\vdash ([S' \to \bullet S][S \to S\bullet S{+}], a{+}) \\
&\vdash ([S' \to \bullet S][S \to S\bullet S{+}][S \to \bullet a], a{+}) \\
&\vdash ([S' \to \bullet S][S \to S\bullet S{+}][S \to a\bullet], {+}) \\
&\vdash ([S' \to \bullet S][S \to SS\bullet{+}], {+}) \\
&\vdash ([S' \to \bullet S][S \to SS{+}\bullet], \varepsilon) \\
&\vdash ([S' \to S\bullet], \varepsilon)
\end{aligned}
$$

(d) Geben Sie jeweils für $M_{G_2}^{(1)}$ und $M_{G_2}^{(2)}$ eine akzeptierende Konfigurationsfolge für $+a+aa$ an.

Lösung:
Konfigurationsfolge für $M_{G_2}^{(1)}$:

$$
\begin{aligned}
(q_0, {+}a{+}aa) &\vdash (q_0{+}, a{+}aa) \vdash (q_0{+}a, {+}aa) \vdash (q_0{+}S, {+}aa) \vdash (q_0{+}S{+}, aa) \\
&\vdash (q_0{+}S{+}a, a) \vdash (q_0{+}S{+}S, a) \vdash (q_0{+}S{+}Sa, \varepsilon) \\
&\vdash (q_0{+}S{+}SS, \varepsilon) \vdash (q_0{+}SS, \varepsilon) \vdash (q_0S, \varepsilon) \vdash (f, \varepsilon)
\end{aligned}
$$

Konfigurationsfolge für $M_{G_2}^{(2)}$:

$$
\begin{aligned}
&([S' \to \bullet S], +a+aa)\\
&\vdash ([S' \to \bullet S][S \to \bullet +SS], +a+aa)\\
&\vdash ([S' \to \bullet S][S \to +\bullet SS], a+aa)\\
&\vdash ([S' \to \bullet S][S \to +\bullet SS][S \to \bullet a], a+aa)\\
&\vdash ([S' \to \bullet S][S \to +\bullet SS][S \to a\bullet], +aa)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S], +aa)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to \bullet +SS], +aa)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to +\bullet SS], aa)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to +\bullet SS][S \to \bullet a], aa)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to +\bullet SS][S \to a\bullet], a)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to +S\bullet S], a)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to +S\bullet S][S \to \bullet a], a)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to +S\bullet S][S \to a\bullet], \varepsilon)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to +SS\bullet], \varepsilon)\\
&\vdash ([S' \to \bullet S][S \to +SS\bullet], \varepsilon)\\
&\vdash ([S' \to S\bullet], \varepsilon)
\end{aligned}
$$