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Abstract:

In this paper Tabellini's and Alesina's (1990) median voter model for the explanation of budget deficits is modified by endogenizing the private sector. Debt finance is supplemented by taxing a private consumption which serves as an additional source of revenue for funding the public sector. The introduction of the private sector enables us to explain the budget balance as a result of political polarization with a left-wing party and a right-wing party having different preferences for the size of the public sector.

JEL: H61, H62

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1. Introduction

Public debt has a long history in many countries. Since long time many economic explanations of public debt have been offered. Recently, political-economic explanations have gained more and more interest. In those models politicians, voters, voting procedures and institutional settings of democratic countries are analyzed with respect to their inherent tendency to run debts. Surveys are provided by Alesina and Perotti (1995) and Persson and Tabellini (1999 and 2000).

One of the more recent theoretical papers on that issue is Tabellini and Alesina (1990) which presents a very interesting explanation of public deficits in a democracy. In their two-period model Tabellini and Alesina explore a first-period median voter's incentive to run a budget deficit when there is uncertainty about the second-period median voter's preferences or when it is certain that the second-period median voter will favour another composition of government consumption. Under certain conditions the first period median voter is shown to alter the second period's allocation of two public goods to better fit her own preferences by issuing a budget deficit or a surplus. Decisive for this result are the assumptions that outstanding debt needs to be served at the end of the second period i. e. that repudiation is ruled out effectively.

Tabellini and Alesina do not model the private sector at all. In addition to issuing or repaying public debt each government is assumed to have at its disposal a lump sum amount of money (tax revenues) in order to finance its expenditures. Such a setting allows determining the allocation of both public goods but not the allocation of resources between the public and the private sector; more specifically, it does not allow determining the size of the public and the private sector.

The following analysis aims at explaining budget deficits in a democracy by means of a two-period median voter model under certainty. In contrast to the work of Tabellini and Alesina (1990) a private sector will be endogenized which is taxed in order to finance a public good. This assures that both the public budget balance and the size of the public sector are determined endogenously.

The paper is organized as follows: first, the general properties of the models will be introduced in chapter 2. Chapter 3 examines the model under several assumptions about the median voters' preferences. Chapter 4 concludes.

2. The model

2.1 Basic Assumptions

g_i is the amount of a public good consumed, which is provided for free charge and x_i is the consumption of a private good. Let χ_i be the size of the private sector and g_i the size of the public sector in period i, $i=1,2$ with $\chi_i=\sum_{j=1}^{n} x_i^j$. $j=1,\ldots,n$ stands for consumer j. The number of consumers is the same in both periods. x_i^j is consumer j's share of the private sector in period i. This share is assumed to equal $x_i^j=\frac{\chi_i}{n}$ in both periods and for simplicity we set $n=1$ and $j\in[0,1]$ which is equivalent to $x_i^j=x_i=\chi_i \quad \forall j$. Thus the superscript j can be suppressed in the following analysis. In revealing her preference for the size of the public sector consumer j also reveals her preferences for private consumption and vice versa. Consumer j, characterized by the preference parameter α^j has the intertemporal utility

$$(1) \qquad W\left(\alpha^j, g_1, x_1, g_2, x_2\right)=\alpha^j U\left(g_1\right)+\left(1-\alpha^j\right)U\left(x_1\right)+\alpha^j U\left(g_2\right)+\left(1-\alpha^j\right)U\left(x_2\right),$$

where $U_k\left(k\right)>0$, $U_{kk}\left(k\right)<0$ and $\lim_{k\to 0}U_k\left(k\right)=\infty$ has to be satisfied for $k=g,x$.

The consumer's income is 1 in each period. With costless consumption of the public good and in the absence of a private capital market the consumer's budget constraint is

$$(2) \qquad \left(1+\tau_i\right)x_i=1 \quad i=1,2,$$

where $1+\tau_i$ is the consumer price of the private good and τ_i is the tax rate on private consumption. As is obvious from (2), for given τ_i each consumer consumes the same amount $x_i=\frac{1}{1+\tau_i}$ independent of her preference parameter α^j. Now the public sector is to be modelled. In the first period the government's supply of the public good, g_1, is financed by consumption tax revenues, $\tau_1 x_1$, corrected by the budget balance (surplus or deficit) generated by a borrowing or lending on a foreign capital market , b. Hence the public budget constraint is

(3) $$g_1 = \tau_1 x_1 + b\,.$$

The period 2 budget constraint is:

(4) $$g_2 = \tau_2 x_2 - b\,.$$

The supply of the public good in period 2 is also financed by a consumption tax, with the qualification that the funds borrowed by the government of period 1 have to be paid back or that the public savings from running a budget surplus in period 1 need to be spent in period 2. When (2), (3) and (4) are considered in (1) the consumer's utility is directly determined by the public decision variables τ_1, b and τ_2. We assume that every consumer is perfectly informed about the government's options of taxing and spending and hence she knows that (2) and (3) as well as (2) and (4) lead to

(5) $$g_1 = \frac{\tau_1}{1+\tau_1} + b \;\wedge\; g_2 = \frac{\tau_2}{1+\tau_2} - b\,.$$

(2) and (5) are now inserted in (1) to yield the indirect utility function

(6) $$\begin{aligned}\tilde{W}^1\left(\alpha^j, \tau_1, \tau_2\right) = {} & \alpha^j U\left(\frac{\tau_1}{1+\tau_1} + b\right) + \left(1-\alpha^j\right) U\left(\frac{1}{1+\tau_1}\right) \\ & + \alpha^j U\left(\frac{\tau_2}{1+\tau_2} - b\right) + \left(1-\alpha^j\right) U\left(\frac{1}{1+\tau_1}\right).\end{aligned}$$

At the beginning of each period a government is elected via majority voting. In view of (6) the activities of these governments can be completely described as follows: The government of period 1 determines the values of τ_1 and b while the government of period 2 fixes τ_2.

2.2 The second period

First, the majority vote of period 2 is analyzed. We examine which policy τ_2 is chosen by a voter when a budget deficit b has to be served. Her utility maximization is

$$(7) \qquad \max_{\tau_2} \; \tilde{W}^2\left(\alpha_2^j, \tau_2, b\right) = \alpha_2^j U\left(\frac{\tau_2}{1+\tau_2} - b\right) + \left(1-\alpha_2^j\right) U\left(\frac{1}{1+\tau_2}\right).$$

The first order condition

$$(8) \qquad \alpha_2^j U_{g_2}\left(\frac{\tau_2}{1+\tau_2} - b\right) - \left(1-\alpha_2^j\right) U_{x_2}\left(\frac{1}{1+\tau_2}\right) = 0,$$

characterizes an interior maximum, because the function $\tilde{W}^2$ in (7) is strictly concave in τ_2. $\tau_2^j := \arg\max \tilde{W}^2\left(\alpha_2^j, \tau_2, b\right)$ is the tax rate the voter α_2^j prefers to all other tax rates. Obviously (8) implies a functional relationship between τ_2^j, b and α_2^j, which we describe by a function $\tau_2^j = \mathrm{T}^2\left(\alpha^j, b\right)$. The partial derivatives of that function are

$$(9) \qquad \mathrm{T}^2_{\alpha_2^j} = \frac{-\left(U_{g_2} + U_{x_2}\right)\left(1+\tau_2\right)^2}{\alpha_2^j U_{g_2 g_2} + \left(1-\alpha_2^j\right) U_{x_2 x_2}} > 0 \;\wedge\; \mathrm{T}^2_b = \frac{\alpha_2^j U_{g_2 g_2}\left(1+\tau_2\right)^2}{\alpha_2^j U_{g_2 g_2} + \left(1-\alpha_2^j\right) U_{x_2 x_2}} > 0.$$

Having identified all voters by their most preferred tax rates, we want to know which programme is realised by the elected government. With each voter's utility function $\tilde{W}^2$ being single peaked in τ_2 the median voter's favourite programme is realised (Black, 2nd ed.,1969). The median voter is that voter whose favourite tax rate $\tau_2 = \tau_2^m$ is the median of the favourite tax rates of all voters. Since $\mathrm{T}^2_{\alpha_2^j}$ (as established in (9)) the median voter can also be identified unambiguously by her preference parameter α_2^m. Combining (5) and (9) yields the supply of the public good as a function $g_2^m = G^2\left(\alpha_2^m, b\right) := \dfrac{\mathrm{T}^2\left(\alpha_2^m, b\right)}{1+\mathrm{T}^2\left(\alpha_2^m, b\right)} - b$ whose first derivatives are

$$(10)\qquad G^2_{\alpha_2^m} = \frac{-\left(U_{g_2} + U_{x_2}\right)}{\alpha_2^m U_{g_2 g_2} + \left(1-\alpha_2^m\right) U_{x_2 x_2}} > 0 \ \wedge\ G^2_b = \frac{-\left(1-\alpha_2^m\right) U_{x_2 x_2}}{\alpha_2^m U_{g_2 g_2} + \left(1-\alpha_2^m\right) U_{x_2 x_2}} < 0 .$$

Involving (2) and (9) we specify the preferred size of the private sector as $x_2^m = X^2\left(\alpha_2^m, b\right) := \frac{1}{1+\mathrm{T}^2\left(\alpha_2^m, b\right)}$. The derivatives of the function X^2 are

$$(11)\qquad X^2_{\alpha_2^m} = \frac{-\left(U_{g_2} + U_{x_2}\right)\left(1+\tau_2\right)^2}{\alpha_2^m U_{g_2 g_2} + \left(1-\alpha_2^m\right) U_{x_2 x_2}} < 0 \ \wedge\ X^2_b = -\frac{\alpha_2^m U_{g_2 g_2}}{\alpha_2^m U_{g_2 g_2} + \left(1-\alpha_2^m\right) U_{x_2 x_2}} < 0 .$$

Closer inspection of (10) and (11) shows that

$$(12)\qquad G^2_{\alpha_2^m} + X^2_{\alpha_2^m} = 0 \ \wedge\ G^2_b + X^2_b = -1 .$$

We have established that the amount of the private and the public good provided depend on the budget balance and the median voter's preferences. These functions G^2 and X^2 are strictly monotone in all their variables.

Next we investigate the polar cases in which the median voter's preferences take on either the value $\alpha_2^m = 1$ or the value $\alpha_2^m = 0$:

For $\alpha_2^m = 1$, the function $\tilde{W}^2$ from (7) simplifies to $\tilde{W}^2\left(1, \tau_2, b\right) = U\left(\frac{\tau_2}{1+\tau_2} - b\right)$. Since $\tilde{W}^2_{\tau_2}\left(1, \tau_2, b\right) = U_{g_2} G^2_{\tau_2} > 0$ for all τ_2, the median voter's utility function has no maximum in τ_2. In order to obtain a well defined optimization problem nonetheless, we introduce a ceiling[1] $\bar{\tau}_2 > 0$ such that the tax rate in the second period is effectively restricted to $\tau_2 \in \left[0, \bar{\tau}_2\right]$. The consequence of restricting τ_2 to the interval $\left[0, \bar{\tau}_2\right]$ is that there is a critical

[1] Such a ceiling could be introduced by law or constitution, like Proposition 13 of the California Constitution for example. An increase of the state's consumption tax must be approved by at least two-thirds of all members of both houses of the Legislature (Article 13A, Section 3, http://www.leginfo.ca.gov/.const./article_13a). An increase in local taxes must be approved by at least two-thirds of the qualified electors in the affected region. (Article 13A, Section 4, http://www.leginfo.ca.gov/.const./article_13a).

value $\bar{\alpha}_2^m \in \left]0,1\right[$, such that $\tau_2^m \begin{Bmatrix} = \\ < \end{Bmatrix} \bar{\tau}_2$ for all $\alpha_2^m \begin{Bmatrix} = \\ < \end{Bmatrix} \bar{\alpha}_2^m$. $\alpha_2^m \geq \bar{\alpha}_2^m$ implies $\mathrm{T}_b^2 = 0$ and we conclude that

(13) $$g_2^m = \frac{\bar{\tau}_2}{1+\bar{\tau}_2} - b \;\wedge\; x_2^m = \frac{1}{1+\bar{\tau}_2} > 0 \text{ if}^2\; \alpha_2^m \in \left[\bar{\alpha}_2^m, 1\right].$$

It is interesting to observe that $x_2^m > 0$ rather than $x_2^m = 0$ in case of the median voter's extreme preferences for the public sector. The reason for not completely abolishing private consumption is that the taxation of private consumption is the only source of finance for the provision of the public good in the second period.

Suppose now $\alpha_2^m = 0$. In that case (7) simplifies to $\tilde{W}^2(0, \tau_2, b) = U\left(\frac{1}{1+\tau_2}\right)$ such that the median voter's favourite program is

(14) $$g_2^m = 0 \;\wedge\; x_2^m = 1-b \;\wedge\; \tau_2 = \frac{b}{1-b} \text{ if } \alpha_2^m = 0.$$

With the median voter having extreme preferences for the private good, her optimal choice is $x_2^m = 1$ and $\tau_2^m = 0$ if $b = 0$. In case of a predetermionated positive value of b it is necessary to raise a tax with rate $\tau_2^m = \frac{b}{1-b} > 0$ in order to serve the debt incurred in period 1. If a surplus was run in period 1 (e. g. $b < 0$) a "tax" with a negative rate $\tau_2^m \in \left]-\frac{1}{2}, 0\right[$ (i. e. a. subsidy) needs to be implemented.

[2] Note that $\bar{\alpha}_2^m$ depends on b, as total differentiation of $\mathrm{T}^2(\alpha_2^m, b) = \bar{\tau}_2$ with respect to α_2^m and b yields $\frac{d\alpha_2^m}{db} = -\frac{\mathrm{T}_b^2}{\mathrm{T}_{\alpha_2^m}^2} < 0$. In the various cases of the model it is assumed that the values of α_2^m and b prevent the appearance of that problem. See also footnote 3.

2.3 The first period

In the first period each voter drives utility from consuming both goods in both periods. But the consumption of the second period is determined by the second period's median voter with preference parameter α_2^m. By assumption every voter who participates in the election of the government of the first period knows the preference parameter α_2^m (and the policy of the second period) and thus she solves

$$
(15)\qquad \begin{aligned} \max_{\tau_1, b}\ \tilde{W}^1\left(\alpha_1^j, \alpha_2^m, \tau_1, b\right) &= \alpha_1^j U\left(\frac{\tau_1}{1+\tau_1}+b\right)+\left(1-\alpha_1^j\right)U\left(\frac{1}{1+\tau_1}\right) \\ &+\alpha_1^j U\left(\frac{\mathrm{T}^2\left(\alpha_2^m, b\right)}{1+\mathrm{T}^2\left(\alpha_2^m, b\right)}-b\right)+\left(1-\alpha_1^j\right)U\left(\frac{1}{1+\mathrm{T}^2\left(\alpha_2^m, b\right)}\right). \end{aligned}
$$

$\tilde{W}^1$ from (15) is defined over a two-dimensional policy space $\left\{\left(\tau_1, b\right) \middle| \tau_1 \in \left[-\frac{1}{2}, \overline{\tau}_1\right] \wedge b \in \left[-1, 1\right]\right\}$. With a two-dimensional policy space, the existence of a voting equilibrium cannot be established unless rather specific conditions are satisfied. To make progress the two-dimensional policy space will be reduced to the dimension b only by partial maximization of $\tilde{W}^1$ with respect to τ_1, hoping at the same time, that the resultant function is single peaked in b.

The first order condition for maximizing $\tilde{W}^1$ with respect to τ_1 is

$$
(16)\qquad \alpha_1^j U_{g_1} - \left(1-\alpha_1^j\right)U_{x_1} = 0 .
$$

As the second derivative has a negative sign, (16) characterizes a maximum. Total differentiation of (16) with respect to τ_1, α_1^j and b yields a function $\tau_1^j = \mathrm{T}^1\left(\underset{+}{\alpha_1^j}, \underset{-}{b}\right)$ whose first derivatives are

$$
(17)\qquad \mathrm{T}^1_{\alpha_1^j} = \frac{-\left(U_{g_1}+U_{x_1}\right)\left(1+\tau_1\right)^2}{\alpha_1^j U_{g_1 g_1}+\left(1-\alpha_1^j\right)U_{x_1 x_1}} > 0 \ \wedge\ \mathrm{T}^1_b = -\frac{\alpha_1^j U_{g_1 g_1}\left(1+\tau_1\right)^2}{\alpha_1^j U_{g_1 g_1}+\left(1-\alpha_1^j\right)U_{x_1 x_1}} < 0 .
$$

Because of (5) the public good consumption is then given by $g_1^j = G^1\left(\alpha_1^j, b\right) := \frac{\mathrm{T}^1\left(\alpha_1^j, b\right)}{1+\mathrm{T}^1\left(\alpha_1^j, b\right)} + b$ where

(18) $$G^1_{\alpha_1^j} = \frac{-\left(U_{g_1} + U_{x_1}\right)}{\alpha_1^j U_{g_1 g_1} + \left(1-\alpha_1^j\right) U_{x_1 x_1}} > 0 \ \wedge \ G_b^1 = \frac{\left(1-\alpha_1^j\right) U_{x_1 x_1}}{\alpha_1^j U_{g_1 g_1} + \left(1-\alpha_1^j\right) U_{x_1 x_1}} > 0 .$$

Invoking (2) we get $x_1^j = X^1\left(\alpha_1^j, b\right) := \frac{1}{1+\mathrm{T}^1\left(\alpha_1^j, b\right)}$ and the derivatives of the function X^1 are

(19) $$X^1_{\alpha_1^j} = \frac{U_{g_1} + U_{x_1}}{\alpha_1^j U_{g_1 g_1} + \left(1-\alpha_1^j\right) U_{x_1 x_1}} < 0 \ \wedge \ X_b^1 = \frac{\alpha_1^j U_{g_1 g_1}}{\alpha_1^j U_{g_1 g_1} + \left(1-\alpha_1^j\right) U_{x_1 x_1}} > 0 .$$

(18) and (19) imply

(20) $$G^1_{\alpha_1^j} + X^1_{\alpha_1^j} = 0 \ \wedge \ G_b^1 + X_b^1 = 1 .$$

Repeating our procedure in analyzing the second period we now calculate the implications of the extreme preference parameters $\alpha_1^j = 1$ and $\alpha_1^j = 0$.

For $\alpha_1^j = 1$ $\tilde{W}^1$ from (15) simplifies to $\tilde{W}^1\left(1, b, \tau_1\right) = U\left(\frac{\tau_1}{1+\tau_1} - b\right)$. Since $\tilde{W}^1_{\tau_1} = U_{g_1} G^1_{\tau_1} > 0$ for all $\tau_1 > 0$, the voter's utility function has no maximum in τ_1. To obtain a well-defined maximization problem we introduce again a ceiling on the tax rate, denoted $\overline{\tau}_1$, and we set $\overline{\tau}_1 = \overline{\tau}_2$ for simplicity. Similar as in the context of the second period we find that there is a critical value $\overline{\alpha}_1^j \in \left]0, 1\right[$, such that $\tau_1^j \begin{Bmatrix} = \\ < \end{Bmatrix} \overline{\tau}_1$ for all $\alpha_1^j \begin{Bmatrix} = \\ < \end{Bmatrix} \overline{\alpha}_1^j$ with $\overline{\mathrm{T}}_b^1 = 0$ for all $\alpha_1^j \geq \overline{\alpha}_1^j$.

According to (6) the optimal values of public and private consumption are given by

(21) $g_1^j = \frac{\overline{\tau}_1}{1+\overline{\tau}_1} + b \ \wedge \ x_1^j = \frac{1}{1+\overline{\tau}_1} > 0$ if[3] $\alpha_1^j \in \left[\overline{\alpha}_1^j, 1\right]$.

For $\alpha_1^j = 0$ (15) simplifies to $\tilde{W}^1(0, \tau_1, b) = U\left(\frac{1}{1+\tau_1}\right)$. Hence the voter's favours

(22) $g_1^j = 0 \ \wedge \ x_1^j = 1+b \ \wedge \ \tau_1^j = \frac{-b}{1+b} \ \wedge \ b = \frac{-\tau_1^j}{1+\tau_1^j}$ if $\alpha_1^j = 0$.

If the voter has an extreme preference for the private good and if $b = 0$ she clearly opts for $x_2^j = 1$ and $\tau_1^j = 0$ if $b = 0$. In case of a deficit the private sector can be subsidized by choosing a negative tax rate $\tau_1^j < 0$. On the other hand a budget surplus $b \in \left]-1, 0\right[$ requires a tax rate $\tau_1 > 0$.

Now we replace in (1) the variables g_1, x_1, g_2 and x_2 by $G^1\left(\alpha_1^j, b\right)$, $X^1\left(\alpha_1^j, b\right)$, $G^2\left(\alpha_2^m, b\right)$ and $X^2\left(\alpha_2^m, b\right)$ respectively, and consider the utility maximization problem

$$\text{(23)} \quad \begin{aligned} \max_b \ V^1\left(\alpha_1^j, \alpha_2^m, b\right) &:= \alpha_1^j U\left[G^1\left(\alpha_1^j, b\right)\right] + \left(1-\alpha_1^j\right) U\left[X^1\left(\alpha_1^j, b\right)\right] \\ &+ \alpha_1^j U\left[G^2\left(\alpha_2^m, b\right)\right] + \left(1-\alpha_1^j\right) U\left[X^2\left(\alpha_2^m, b\right)\right]. \end{aligned}$$

After consideration of (16) and (20) the first order condition can be rearranged to read

(24) $\alpha_1^j U_{g_1} - v\left(\alpha_1^j, \alpha_2^m, b\right) = 0$ with $v\left(\alpha_1^j, \alpha_2^m, b\right) := -\alpha_1^j U_{g_2} G_b^2 - \left(1-\alpha_1^j\right) U_{x_2} X_b^2$.

In (24) $v\left(\alpha_1^j, \alpha_2^m, b\right)$ is the marginal damage of the budget deficit or the marginal utility of the budget surplus, respectively, in the second period.

Denote by b^j the value of b satisfying (24). b^j is clearly a maximum of V^1 from (23) if V^1 is strictly concave (and if an interior maximum exists). Tabellini and Alesina (1990) show

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[3] Here, $\overline{\alpha}_1^m$ depends on b, and following the methodology in footnote 2 we get $\frac{d\alpha_1^m}{db} = -\frac{\mathrm{T}_b^1}{\mathrm{T}_{\alpha_1^m}^1} > 0$. Again, we assume that this problem does not appear in the various cases of the model.

that strict concavity of V^1 can be secured by imposing some 'mild' restriction[4] on the functional form U. In our further analysis we assume that this condition is satisfied. Now each voter with preference parameter α_1^j can be identified by her preferred budget balance. Under these conditions a majority vote ensures that the first period median voter's preferred budget balance b^m is realised. Throughout the following analysis this median voter is identified by her preference parameter α_1^m.

By using (10) and (11) and setting $\alpha_1^j = \alpha_1^m$ we turn $v\left(\alpha_1^j, \alpha_2^m, b\right)$ into

$$(25) \qquad v\left(\alpha_1^m, \alpha_2^m, b\right) = \frac{\left[U_{g_2}\right]^2 \left[U_{x_2}\right]^2 \left(\frac{\alpha_1^m U_{x_2 x_2}}{\left[U_{x_2}\right]^2} + \frac{\left(1-\alpha_1^m\right) U_{g_2 g_2}}{\left[U_{g_2}\right]^2} \right)}{\left[U_{g_2}\right]^2 \left[U_{x_2}\right]^2 \left(\frac{U_{g_2 g_2}}{\left[U_{g_2}\right]^2 U_{x_2}} + \frac{U_{x_2 x_2}}{U_{g_2} \left[U_{x_2}\right]^2} \right)}.$$

The quotient

$$(26a) \qquad \lambda(k) := -\frac{U_{kk}(k)}{\left[U_k(k)\right]^2},$$

is called concavity index of the utility function $U(k)$ with $k = g, x$. Furthermore, it is helpful to consider a function R defined by

$$(26b) \qquad R(k) := -\frac{U_{kk}(k)}{U_k(k)}.$$

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[4] According to Tabellini and Alesina (1990) the following condition is sufficient for strict concavity of V^1 in b:

$$(1') \qquad \begin{aligned} &R(x_2)^3 R(g_2) + R(g_2)^2 R(x_2)^2 + (1-\gamma) R_{g_2} R(x_2)^2 + \gamma R(g_2)^3 R(x_2) + \\ &\gamma R(g_2)^2 R(x_2)^2 + (\gamma - 1) R_{x_2} R(g_2)^2 > 0 \end{aligned}.$$

γ is defined as $\gamma := \frac{1-\alpha_1^m}{\alpha_1^m}\frac{1-\alpha_2^m}{\alpha_2^m}$ and $R(k) := -\frac{U_{kk}(k)}{U_k(k)}$ is the degree of absolute risk aversion in case of uncertainty. We assume that (1') is fulfilled throughout the paper.

Like in the model of Tabellini and Alesina (1990) (26a) and (26b) enable us to transform (25) into

(27) $$v\left(\alpha_1^m, \alpha_2^m, b\right) = \frac{\alpha_1^m U_{g_2} R\left(x_2^m\right) + \left(1-\alpha_1^m\right) U_{x_2} R\left(g_2^m\right)}{R\left(g_2^m\right) + R\left(x_2^m\right)}.$$

Owing to the strict concavity of V^1 in b, the sign of b^m can be easily determined by the sign of $V_b^1\left(\alpha_1^m, \alpha_2^m, 0\right)$. More specifically, we have $b^m \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$ if and only if $V_b^1\left(\alpha_1^m, \alpha_2^m, 0\right) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$.

Differentiating (27) with respect to α_2^m leads to

(28) $$v_{\alpha_2^m}\left(\alpha_1^m, \alpha_2^m, b\right) = U_{g_2} G_{\alpha_2^m}^2 \frac{\left\{\frac{\alpha_2^m - \alpha_1^m}{1-\alpha_2^m}\right\}\left\{R\left(x_2^m\right)\left[\left[R\left(g_2^m\right)\right]^2 + R_{g_2}\right] + R\left(g_2^m\right)\left[\left[R\left(x_2^m\right)\right]^2 + R_{x_2}\right]\right\}}{\left[R\left(g_2^m\right) + R\left(x_2^m\right)\right]^2}.$$

Combining (26a) and (26b) yields

(29) $$\left[R(k)\right]^2 + R_k = \lambda_k(k) U_k.$$

Recalling (11) and the assumption $U_{g_2} > 0$ we find that the sign $v_{\alpha_2^m}$ from (28) depends on the signs of $\lambda_k(k)$ and the difference $\alpha_1^m - \alpha_2^m$ as shown in table 1.

	$\alpha_2^m < \alpha_1^m$	$\alpha_1^m = \alpha_2^m$	$\alpha_2^m > \alpha_1^m$
$\lambda_k(k) < 0$	$v_{\alpha_2^m}\left(\alpha_1^m, \alpha_2^m, b\right) > 0$	$v_{\alpha_2^m}\left(\alpha_1^m, \alpha_2^m, b\right) = 0$	$v_{\alpha_2^m}\left(\alpha_1^m, \alpha_2^m, b\right) < 0$
$\lambda_k(k) = 0$	$v_{\alpha_2^m}\left(\alpha_1^m, \alpha_2^m, b\right) = 0$	$v_{\alpha_2^m}\left(\alpha_1^m, \alpha_2^m, b\right) = 0$	$v_{\alpha_2^m}\left(\alpha_1^m, \alpha_2^m, b\right) = 0$
$\lambda_k(k) > 0$	$v_{\alpha_2^m}\left(\alpha_1^m, \alpha_2^m, b\right) < 0$	$v_{\alpha_2^m}\left(\alpha_1^m, \alpha_2^m, b\right) = 0$	$v_{\alpha_2^m}\left(\alpha_1^m, \alpha_2^m, b\right) > 0$

Table 1: Sign of $v_{\alpha_2^m}\left(\alpha_1^m, \alpha_2^m, b\right)$ depending on $\alpha_1^m - \alpha_2^m$ and $\lambda_k(k)$

Inspection of table 1 reveals that $v\left(\alpha_1^m, \alpha_2^m, b\right)$ attains a maximum or a minimum, if $\alpha_1^m = \alpha_2^m$.

3. Solutions for various combinations of α_1^m and α_2^m

If $\alpha_1^j = \alpha_1^m$ the first period's median voter maximizes V^1 from (23) with respect to b and (24) then becomes

(30) $\alpha_1^m U_{g_1} - v\left(\alpha_1^m, \alpha_2^m, b\right) = 0$.

A popular (though probably oversimplified) thesis is that a median voter with a high preference for the public sector is associated with a left-wing party and a median voter with little preference for the public sector is associated with a right-wing party. Throughout the paper we will follow that assumption, associating $\alpha_i^m = 0$ with a right-wing government, $\alpha_i^m \in \left]0, \bar{\alpha}_i^m\right[$ with a moderate government and $\alpha_i^m \in \left[\bar{\alpha}_i^m, 1\right]$ with a left-wing government in period $i = 1, 2$.

Case (i): $\alpha_2^m \in \left]0, \bar{\alpha}_2^m\right[$

Case (i, a): $\alpha_2^m \in \left]0, \bar{\alpha}_2^m\right[$ and $\alpha_1^m \in \left]0, \bar{\alpha}_1^m\right[$

According to table 1 $v\left(\alpha_1^m, \alpha_2^m, b\right)$ reaches its maximum value if $\alpha_1^m = \alpha_2^m$. (30) simplifies to $\left(1-\alpha_1^m\right)U_{x_1} - \left(1-\alpha_1^m\right)U_{x_2} = 0$, which implies $U_{x_1} = U_{x_2}$ as well as $U\left(x_1^m\right) = U\left(x_2^m\right)$, $x_1^m = x_2^m$ and $\tau_1^m = \tau_2^m$ because of (2). Moreover, $U_{g_1} = U_{g_2}$ holds if and only if $U\left(g_1^m\right) = U\left(g_2^m\right)$ and hence $g_1^m = g_2^m$. Finally, equation (5) and $\tau_1^m = \tau_2^m$ yield $b^m = 0$ and $\tau_1^m = \tau_2^m \geq 0$.

Proposition 1:

A median voter of period 1 with preferences $\alpha_1^m \in \left]0, \bar{\alpha}_1^m\right[$ *who is certain to be the second period's median voter favours a balanced budget to smooth out intertemporal tax rates. Those tax rates are always non-negative.*

Table 1 also shows that for $b = 0$ and $\alpha_1^m \in \left]0, \bar{\alpha}_1^m\right[$ the second period's marginal utility is $v\left(\alpha_1^m, \alpha_2^m, 0\right) \begin{Bmatrix} < \\ > \end{Bmatrix} v\left(\alpha_1^m, \alpha_1^m, 0\right)$ if and only if $\lambda_k\left(k\right) \begin{Bmatrix} < \\ > \end{Bmatrix} 0$ for all $\alpha_1^m \neq \alpha_2^m$. Thus we have $V_b^1\left(\alpha_1^m, \alpha_2^m, 0\right) \begin{Bmatrix} > \\ < \end{Bmatrix} 0$ if and only if $\lambda_k\left(k\right) \begin{Bmatrix} < \\ > \end{Bmatrix} 0$ and also $b^m \begin{Bmatrix} > \\ < \end{Bmatrix} 0$ if and only if $\lambda_k\left(k\right) \begin{Bmatrix} < \\ > \end{Bmatrix} 0$.

Proposition 2:

A median voter with preferences $\alpha_1^m \in \left]0, \bar{\alpha}_1^m\right[$ *who knows that she will not remain the median voter in the second period chooses a budget deficit in case of a falling concavity index and a budget surplus in case of an increasing concavity index.*

Another straightforward result contained in table 1 is that $\lambda_k(k) = 0$ implies $v_{\alpha_2^m}(\alpha_1^m, \alpha_2^m, b) = 0$.

Proposition 3:

If the concavity index is constant, a median voter with preferences $\alpha_1^m \in \left]0, \bar{\alpha}_1^m\right[$ *chooses a balanced budget.*

It is worthwhile noting that although the budget is balanced in case of $\lambda_k(k) = 0$ all other variables differ in both periods if $\alpha_1^m \neq \alpha_2^m$. Thiss can be verified by inspection of the equations (9) – (11) and (17) – (19). For example, the utility function $U(k) = \ln(k+1)$ leads to the optimal values $b^m = 0, \tau_1^m = \frac{\alpha_1^m}{1-\alpha_1^m}$ and $\tau_2^m = \frac{\alpha_2^m}{1-\alpha_2^m}$. Hence the tax rates τ_1^m and τ_2^m are identical if and only if $\alpha_1^m = \alpha_2^m$.

Summing up, a nonzero budget balance does not depend on the preferences of the first period's median voter but on the difference of the preference parameters of both median voters. It is irrelevant whether the median voter prefers a large or a small public sector.

The thesis of left-wing governments being more deficit prone than right-wing government is not supported by the results reported above.

Case (i, b): $\alpha_2^m \in \left]0, \bar{\alpha}_2^m\right[$ and $\alpha_1^m = 0$

If $\alpha_1^m = 0$, (23) simplifies to $\max_b V^1(0, \alpha_2^m, b) = U\left[X^1(1+b)\right] + U\left[X^2(\alpha_2^m, b)\right]$. The first derivative of V^1 with respect to b is

(31) $V_b^1\left(0, \alpha_2^m, b\right) = U_{x_1}\left(1+b\right) + U_{x_2}\left(\frac{1}{1+\tau_2}\right) X_b^2 .$

As $\alpha_2^m > \alpha_1^m = 0$ implies $U_{x_2}\left(\frac{1}{1+\tau_2}\right) > U_{x_1}\left(1+b\right)$ and $X_b^2 \in \left]-1, 0\right[$ holds (see (11)), the sign of (31) is ambiguous. Therefore the sign of b^m cannot be determined. With $\alpha_1^m U_{g_2}$ being independent of α_2^m the cross derivative $V^1_{b\alpha_2^m}$ satisfies

(32) $V^1_{b\alpha_2^m}\left(0, \alpha_2^m, b\right) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \text{ if } \lambda_k\left(k\right) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 .$

The greater the value of α_2^m, the greater (smaller) is the marginal utility of a rising deficit if the concavity index is increasing (decreasing). Thus V_b^1 increases (decreases) when α_2^m increases and b is fixed. If $\lambda_k\left(k\right) = 0$, V_b^1 remains constant and the first period's median voter chooses her preferred allocation without regard of α_2^m.

We established the following result on the budget policy of a right-wing government with extreme preference for the private sector, that is in power in period 1 and that knows that its successor will increase the public and decrease the private sector. Whether that right-wing government runs a surplus, a deficit or a balanced budget depends on the properties of the utility function.

Case(i, c) : $\alpha_2^m \in \left]0, \bar{\alpha}_2^m\right[$ and $\alpha_1^m \in \left[\bar{\alpha}_1^m, 1\right]$

If $\alpha_1^m \in \left[\bar{\alpha}_1^m, 1\right]$, the first derivative is $V_b^1\left(\alpha_1^m, \alpha_2^m, b\right) = \alpha_1^m U_{g_1} - v\left(\alpha_1^m, \alpha_2^m, b\right)$. The sign of V_b^1 evaluated at $b=0$ can be determined by consulting table 1 for $\alpha_1^m \neq \alpha_2^m$. The optimal budget balance is $b^m \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$ if and only if $\lambda_k\left(k\right) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0$. As in the case $\alpha_1^m \in \left]0, \bar{\alpha}_1^m\right[$ the sign of b^m therefore depends on the properties of the concavity index only.

Hence we established the following result on the budget policy of a left-wing government, strongly in favour of the largest possible public sector and knowing that it will be replaced by a more moderate government: Depending on the properties of the concavity index such a government generates a surplus, a deficit or a balanced budget.
Possible interpretations of this behaviour are as follows:

1) In case of a budget deficit the left-wing government seems to aim at increasing the public sector in the first period. At the same time it cuts its successors budget needed to boost the private sector.
2) Though a surplus puts pressure on the public sector in the first period it can be used as a kind of insurance against a larger cut-off in the second period. The inherited surplus enables the moderate government in the second period to enlarge the private sector without a complete loss of the public sector.

$\alpha_1^m = 0$ (right-wing)	$\alpha_1^m \in \left]0, \bar{\alpha}_1^m\right[$ (moderate)		$\alpha_1^m \in \left[\bar{\alpha}_1^m, 1\right]$ (left-wing)
	$\left(\alpha_1^m = \alpha_2^m\right)$	$\left(\alpha_1^m \neq \alpha_2^m\right)$	
$b^m = ?$ $g_1^m = 0$ $x_1^m = 1 + b$ $sign\left\{V^1_{b\alpha_2^m}\right\} =$ $sign\left\{\lambda_k(k)\right\}$	$b^m = 0$ $\tau_1^m = \tau_2^m \geq 0$ $g_1^m = g_2^m = \frac{\tau_1^m}{1+\tau_1^m}$ $x_1^m = x_2^m = \frac{1}{1+\tau_1^m}$	$\left.\begin{array}{l} b^m = 0 \\ sign\left\{\alpha_1^m - \alpha_2^m\right\} \\ = sign\left\{\tau_1^m - \tau_2^m\right\} \\ = sign\left\{g_1^m - g_2^m\right\} \\ = sign\left\{x_2^m - x_1^m\right\} \end{array}\right\}$ if $\lambda_k(k) = 0$ $sign\left\{b^m\right\} = sign\left\{-\lambda_k(k)\right\}$ if $\lambda_k(k) \neq 0$	$b^m = 0$ if $\lambda_k(k) = 0$ $\tau_1^m = \bar{\tau}_1$ $g_1^m = \frac{\bar{\tau}_1}{1+\bar{\tau}_1}$ $x_1^m = \frac{1}{1+\bar{\tau}_1}$ $sign\left\{b^m\right\} = sign\left\{-\lambda_k(k)\right\}$ if $\lambda_k(k) \neq 0$

Table 2: Possible allocations in case of $\alpha_2^m \in \left]0, \bar{\alpha}_2^m\right[$ (moderate)

Case (ii): $\alpha_2^m \in \left[\bar{\alpha}_2^m, 1\right]$

Case (ii, a): $\alpha_2^m \in \left[\bar{\alpha}_2^m, 1\right]$ and $\alpha_1^m \in \left]0, \bar{\alpha}_1^m\right[$

The first order condition for a maximum is

$$(33) \qquad V_b^1\left(\alpha_1^m, \alpha_2^m, b\right) = \alpha_1^m U_{g_1} - \alpha_1^m U_{g_2}\left(\frac{\bar{\tau}_2}{1+\bar{\tau}_2} - b\right) = 0 \qquad .$$

As the second derivative V_{bb}^1 is negative[5], it characterizes an interior maximum of V^1. (33) implies $G^1\left(\alpha_1^m, b\right) = G^2\left(1, b\right)$ and $b = \frac{1}{2}\left(\frac{\bar{\tau}_2}{1+\bar{\tau}_2} - \frac{\tau_1}{1+\tau_1}\right)$ because of (5) and (13). As $\tau_1 < \bar{\tau}_1 = \bar{\tau}_2$ for all $\alpha_1^m \in \left]0, \bar{\alpha}_1^m\right[$, b^m is positive. One also has $\frac{db}{d\tau_1} < 0$, which leads to $\frac{db}{d\alpha_1^m} = \frac{db}{d\tau_1} \cdot \frac{d\tau_1}{d\alpha_1^m} < 0$. Taking (5) into account we obtain $g_1^m = \frac{\bar{\tau}_2}{1+\bar{\tau}_2} - b^m$ after some transformations. The size of the public sector is the same in both periods. The first period's tax rate is $\tau_1^m = \frac{\bar{\tau}_2 - 2b^m\left(1+\bar{\tau}_2\right)}{1+2b^m\left(1+\bar{\tau}_2\right)}$ and the size of the private sector is $x_1^m = \frac{1}{1+\bar{\tau}_2} + 2b^m$.

This result fits the following scenario: A left-wing government which favours an extremely large public sector will be in office in the second period. The moderate first period government will run a deficit, which is the larger the more right-wing the government is. As a consequence, the left wing government is forced to reduce its spending for the public sector and the moderate government succeeds in equalizing the size of the public sector in both periods and in boosting the private sector in the first period by reducing taxes.

[5] $V_{bb}^1 < 0$ follows from $X_b^1 = 1 - G_b^1$ and $G_{bb}^1 + X_{bb}^1 = 0$.

Case (ii, b): $\alpha_2^m \in \left[\bar{\alpha}_2^m, 1\right]$ and $\alpha_1^m = 0$

(33) reduces to $V_b^1\left(0, \alpha_2^m, b\right) = U_{x_1}\left(1+b\right) > 0$. Thus V^1 has no maximum with respect to b. The first periods median voter chooses $b^m = \frac{-\tau_1^m}{1+\tau_1^m} = \frac{\bar{\tau}_2}{1+\bar{\tau}_2}$ and $\tau_1^m = -\frac{\bar{\tau}_2}{1+2\bar{\tau}_2}$. Her successor realises $g_2^m = \frac{\bar{\tau}_2}{1+\bar{\tau}_2} - b = \frac{\bar{\tau}_2}{1+\bar{\tau}_2} - \frac{\bar{\tau}_2}{1+\bar{\tau}_2} = 0$.

This is a case of extreme political polarization. An extremely right-wing government knows that it will be replaced by an extremely left-wing government. The debt financed, extreme size of the private sector transfers all available resources from the second to the first period.

Contrary to the popular theses that right-wing governments are less prone to budget deficits than left-wing governments, in the present scenario a right-wing government induces a deficit and the repayment forces its left-wing successor to forego all public expenditures. A similar interpretation was given by Persson and Svensson (1989) in the context of a similar two-period-model. Persson and Svensson (1989) developed a model which allowed public consumption in the second period only. The first period's surplus was smallest under a conservative government which knew that it would be replaced by a more liberal one[6].

Case (ii, c): $\alpha_2^m \in \left[\bar{\alpha}_2^m, 1\right]$ and $\alpha_1^m \in \left[\bar{\alpha}_1^m, 1\right]$

If $\alpha_1^m \in \left[\bar{\alpha}_1^m, 1\right]$ the median voters of both periods choose the allocation of private and public goods given by (13) and (21). Then the first order condition for a maximum is $V_b^1\left(\alpha_1^m, \alpha_2^m, b\right) = \alpha_1^m U_{g_1}\left(\frac{\bar{\tau}_1}{1+\bar{\tau}_1} + b\right) - \alpha_1^m U_{g_2}\left(\frac{\bar{\tau}_2}{1+\bar{\tau}_2} - b\right) = 0$. It implies $g_1^m = g_2^m$ and $\frac{\bar{\tau}_1}{1+\bar{\tau}_1} + b = \frac{\bar{\tau}_2}{1+\bar{\tau}_2} - b$ which, in term, yields $b^m = 0$ owing to $\bar{\tau}_1 = \bar{\tau}_2$. Furthermore the private sector must be given by $x_1^m = x_2^m = \frac{1}{1+\bar{\tau}_1}$.

[6]Furthermore, Wagschal (1998) argues that conservative governments are more debt prone as they are more willing to reduce taxes.

In this case a left-wing government is in office in both periods. It has no incentive to generate an unbalanced budget for this would lead to an undesirable intertemporal distortion of consumption.

$\alpha_1^m = 0$ (right-wing)	$\alpha_1^m \in \left]0, \bar{\alpha}_1^m\right[$ (moderate)	$\alpha_1^m \in \left[\bar{\alpha}_1^m, 1\right]$ (left-wing)
$b^m = \frac{\bar{\tau}_2}{1+\bar{\tau}_2} > 0$ $\tau_1^m = -\frac{\bar{\tau}_2}{1+2\bar{\tau}_2} < 0,\ \tau_2^m = \bar{\tau}_2$ $g_1^m = g_2^m = 0$ $x_1^m = \frac{1+2\bar{\tau}_2}{1+\bar{\tau}_2},\ x_2^m = \frac{1}{1+\bar{\tau}_2}$	$b^m > 0$ $\frac{db}{d\alpha_1^m} < 0$ $\tau_1^m = \frac{\bar{\tau}_2 - 2b^m\left(1+\bar{\tau}_2\right)}{1+2b^m\left(1+\bar{\tau}_2\right)},\ \tau_2^m = \bar{\tau}_2$ $g_1^m = g_2^m = \frac{\bar{\tau}_2}{1+\bar{\tau}_2} - b^m$ $x_1^m = \frac{1}{1+\bar{\tau}_2} + 2b^m,\ x_2^m = \frac{1}{1+\bar{\tau}_2}$	$b^m = 0$ $\tau_1^m = \tau_2^m = \bar{\tau}_1 = \bar{\tau}_2 \geq 0$ $g_1^m = g_2^m = \frac{\bar{\tau}_1}{1+\bar{\tau}_1},$ $x_1^m = x_2^m = \frac{1}{1+\bar{\tau}_1}$

Table 3: Possible allocations in case of $\alpha_2^m \in \left[\bar{\alpha}_2^m, 1\right]$ (left-wing)

Case (iii): $\alpha_2^m = 0$

If $\alpha_2^m = 0$, first period's median voter solves

$$(34) \qquad \max_b\ V^1\left(\alpha_1^m, 0, b\right) = \alpha_1^m U\left[G^1\left(\alpha_1^m, b\right)\right] + \left(1-\alpha_1^m\right)U\left[X^1\left(\alpha_1^m, b\right)\right] + \left(1-\alpha_1^m\right)U\left(1-b\right).$$

Case (iii, a): $\alpha_2^m = 0$ and $\alpha_1^m \in \left]0, \bar{\alpha}_1^m\right[$

If $\alpha_1^m \in \left]0, \bar{\alpha}_1^m\right[$, V^1 from (34) is strictly concave in b and therefore an interior maximum b^m is fully determined by the first order condition

$$(35) \qquad V_b^1\left(\alpha_1^m, 0, b\right) = \alpha_1^m U_{g_1} - \left(1-\alpha_1^m\right)U_{x_2} = 0.$$

Combining (35) and (16) yields $\left(1-\alpha_1^m\right)U_{x_1}\left(\frac{1}{1+\tau_1}\right)=\left(1-\alpha_1^m\right)U_{x_2}\left(1-b\right)$, which yields $b^m=\frac{\tau_1^m}{1+\tau_1^m}$ and $\tau_1^m=\frac{b^m}{1-b^m}$. (10) implies $g_1^m>0$, and regarding (5) we get $\tau_1^m>0$, $g_1^m=\frac{2\tau_1^m}{1+\tau_1^m}=2b^m$, $b^m>0$ and $x_1^m=1-b^m$.

In this scenario, the public sector in the first period is funded in equal share by taxes and by debt. The size of the private sector is the same in both periods. With the same tax rate being chosen in both periods, an intertemporal smoothing of taxes rates, described by Barro (1979), can be observed.

The right-wing government in office in the second period has a predecessor which is more left-wing. The predecessor has two incentives to generate a budget deficit. First, the right wing government will not be able to promote the private sector through subsidies, because it needs to levy a tax for serving the debt. Second the first period's government knows that the public sector will be completely shut down in the second period, regardless of the sign or size of the debt. It therefore creates a budget deficit to transfer resources from the second to the first period in order to enlarge the public sector in the first period.

This scenario supports the popular thesis that left-wing governments are more deficit prone. Martimort (2001) argues that left-wing governments prefer higher deficits in order to defend their redistributive goals against upcoming conservative governments.

Case (iii, b): $\alpha_2^m=0$ and $\alpha_1^m=0$

Under these conditions (34) reduces to $\max_b V^1\left(0,0,b\right)=U\left(1+b\right)+U\left(1-b\right)$. The associated first order condition is $V_b^1\left(0,0,b\right)=U_{x_1}\left(1+b\right)-U_{x_2}\left(1-b\right)=0$ which immediately yields $b^m=0$. From $\tau_1^m=\frac{-b^m}{1+b^m}$ we infer $\tau_1^m=0, x_1^m=1$ and $g_1^m=0$.

In this scenario the extreme right wing government being in office in the first period knows that it will also be in office in the second period. It therefore does not want to place a tax burden on the higher valued private sector.

Case (iii, c): $\alpha_2^m = 0$ and $\alpha_1^m \in \left[\bar{\alpha}_1^m, 1\right]$

If $\alpha_1^m \in \left[\bar{\alpha}_1^m, 1\right]$, g_2^m and x_2^m are determined as in (13) and the first order condition (35) turns into

(36) $$V_b^1\left(\alpha_1^m, 0, b\right) = \alpha_1^m U_{g_1}\left(\frac{\bar{\tau}_1}{1+\bar{\tau}_1} + b\right) - \left(1-\alpha_1^m\right) U_{x_2}\left(1-b\right) = 0,$$

and the second derivative is negative. Total differentiation of (36) with respect to b and α_1^m leads to $\frac{db}{d\alpha_1^m} = -\frac{U_{g_1}\left(\frac{\bar{\tau}_1}{1+\bar{\tau}_1} + b\right) + U_{x_2}\left(1-b\right)}{\alpha_1^m U_{g_1 g_1}\left(\frac{\bar{\tau}_1}{1+\bar{\tau}_1} + b\right) + U_{x_2 x_2}\left(1-b\right)} > 0$.

Thus b^m increases with α_1^m. Next we maximize the right side of (36) for $b = 0$. b^m satisfies $b^m \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$ if and only if $\alpha_1^m U_{g_1}\left(\frac{\bar{\tau}_1}{1+\bar{\tau}_1}\right) - \left(1-\alpha_1^m\right) U_{x_2}\left(1\right) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$. The term on the right side of (36) is equivalent to $\alpha_1^m \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \frac{U_{x_2}\left(1\right)}{U_{x_2}\left(1\right) + U_{g_1}\left(\frac{\bar{\tau}_1}{1+\bar{\tau}_1}\right)}$. From $\frac{\bar{\tau}_1}{1+\bar{\tau}_1} < 1$ follows $\frac{U_{x_2}\left(1\right)}{U_{g_2}\left(\frac{\bar{\tau}_2}{1+\bar{\tau}_2}\right)} < 1$ and thus $\alpha_1^m > \frac{U_{x_2}\left(1\right)}{U_{x_2}\left(1\right) + U_{g_1}\left(\frac{\bar{\tau}_1}{1+\bar{\tau}_1}\right)}$ holds for $\alpha_1^m \geq \frac{1}{2}$. As a consequence $b^m > 0$ if $\alpha_1^m \geq \frac{1}{2}$.

In the present scenario, the first period government prefers the largest possible public sector and its successor has extreme preferences for the private sector. The budget balance decreases with the first period's government's preferences for the private sector. The first period government opts for a resource transfer via a budget deficit from the second period into the first period's public sector, provided that the first period's government political position is left from the centre. If its position is right from the centre the sign of the budget balance depends on the exogenously given values of $\bar{\tau}_1$ und α_1^m and on the properties of the utility function.

<table>
<tr>
<td rowspan="2">$\alpha_1^m = 0$
(right-wing)</td>
<td rowspan="2">$\alpha_1^m \in \left]0, \overline{\alpha}_1^m\right[$ (moderate)</td>
<td colspan="2">$\alpha_1^m \in \left[\overline{\alpha}_1^m, 1\right]$ (left-wing)</td>
</tr>
<tr>
<td>$\alpha_1^m < \frac{1}{2}$</td>
<td>$\alpha_1^m \geq \frac{1}{2}$</td>
</tr>
<tr>
<td rowspan="2">$b^m = 0$
$\tau_1^m = \tau_2^m = 0$
$g_1^m = g_2^m = 0$
$x_1^m = x_2^m = 1$</td>
<td rowspan="2">$b^m = \frac{\tau_1^m}{1+\tau_1^m} > 0,$
$\tau_1^m = \tau_2^m = \frac{b^m}{1-b^m} > 0,$
$\frac{db}{d\alpha_1^m} = \frac{T_{\alpha_1^m}^1}{\left(1+\tau_1\right)^2} > 0,$
$g_1^m = \frac{2\tau_1^m}{1+\tau_1^m} = 2b^m,\ g_2^m = 0,$
$x_1^m = x_2^m = 1-b^m$</td>
<td>$sign\left\{b^m\right\} =$
$sign\left\{\alpha_1^m - \frac{U_{x_2}\left(1\right)}{U_{x_2}\left(1\right) + U_{g_1}\left(\frac{\overline{\tau}_1}{1+\overline{\tau}_1}\right)}\right\}$</td>
<td>$b^m > 0$</td>
</tr>
<tr>
<td colspan="2">$\frac{db}{d\alpha_1^m} > 0,\ g_1^m = \frac{\overline{\tau}_1}{1+\overline{\tau}_1} + b^m,\ g_2^m = 0$
$x_1^m = \frac{1}{1+\overline{\tau}_1},\ x_2^m = 1-b^m$</td>
</tr>
</table>

Table 4: Possible allocations in case of $\alpha_2^m = 0$ (right-wing)

4. Conclusions

The general result is that an unbalanced budget can occur when the median voters' preferences in both periods diverge. These diverging preferences can also be interpreted as a political party system with two parties favouring a different size of the public sector and thus a different fiscal policy. According to their preferences for the size of the public sector we talk about a left-wing party or government and about a conservative / right-wing party or government. The government in office in the first period knows whether it will be in office in the second period or not.

Depending on the properties of the model several results were possible:

1) The parties favour a certain budget policy regardless of their ideological preference for the size of public sector. Then their policy aims at protecting their goals against their successor's policy. The sign of the budget balance depends on the concavity properties of the utility function.

2) If at least one government has extreme preference for either the public or the private sector the sign of the budget balance is independent of the concavity properties of the utility function. Special cases occurred, which could be interpreted by the common assumptions of partisan theory:

   i) Left-wing governments are more deficit prone because of their redistributive goals and their preference for a larger public sector.
   ii) Right-wing governments can generate deficits in order to enforce more fiscal discipline to their left-wing successor. Furthermore deficit finance enables a right-wing government to reduce taxes imposed on the high-valued private sector.

Finally, table 5 sums up the results derived from the various cases.

		Government in period 1		
		Left-wing	**Moderate**	**Right-wing**
Government in period 2	**Left-wing**	balanced budget	deficit	deficit
	Moderate	Deficit, balanced budget and surplus possible	Deficit, balanced budget and surplus possible	undeterminable
	Right-wing	Deficit, balanced budget and surplus possible	deficit	balanced budget

Table 5: Possible budget policies under various government sequences

The results presented here have all been generated under the assumption that the first period's government acts under certainty. It would be interesting to expand this model to the case of uncertainty e. g. when the first period's government does not know whether it will stay in office in the second period and what the policy of its successor will be.

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