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EU-type carbon emissions trade and the distributional impact of overlapping emissions taxes

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Abstract

The European Union fulfills its emissions reductions commitments by means of an emissions trading scheme covering some part of each member state's economy and by national emissions control in the rest of their economies. The member states also levy energy/emissions taxes overlapping with the trading scheme. Restricting our focus on cost-effective policies, this paper investigates the distributive consequences of increasing the overlapping emissions tax that is uniform across countries. For quasi-linear utility functions and for a class of parametric utility and production functions emissions tax increases turn out to be exactly offset by permit price reductions. As a consequence permit-exporting [permit-importing] countries lose [gain] from an increase in the emissions tax. These results are not general, however. By means of a numerical example we show that export-import reversals and welfare reversals are possible.

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1   Introduction

In the Kyoto Protocol the EU committed to reduce its greenhouse gas emissions by 8%
in 2012 from its baseline emissions in 1990. In order to fulfill this commitment the EU
has established an emissions trading scheme (ETS) in 2005 (see EU 2003a) allowing for
EU-wide trade in emissions permits. With respect to emissions control the economies of
all member states are split into two sectors. The installations covered by the ETS, referred
to as the ETS sector, include combustion installations, mineral oil refineries, coke ovens,
installations producing and processing ferrous materials, mineral installations and industrial
plants for the production of pulp and paper. In the rest of the economy, called the non-ETS
sector (that mainly consists of private households and transportation), emissions control is
the national governments' responsibility and is carried out through instruments other than
emissions trading. Another peculiarity of EU emissions control is the existence of emissions
or energy taxes in the ETS sectors overlapping with the ETS (Johnstone 2003, Sorrell and
Sijm 2003). Table 1 lists exemplarily selective overlapping energy taxes in ETS-sectors.[1]

	Coal in industry	Heavy fuel oil for combustion installations	Light fuel oil in industry
Austria	35.9	5.4	24.2
Finland	43.0	-	14.0
France	-	-	11.7
Germany	-	10.1	13.0
Ireland	-	7.0	8.9
Poland	-	6.4	12.5
Spain	-	-	16.6
Sweden	-	-	12.8

Table 1: Percentage of energy taxes[2] in prices in the second quarter 2007 (International Energy Agency 2007)

We aim at modeling a hybrid EU-type policy in a stylized way. Under the simplifying
assumption that in their non-ETS sectors national governments control emissions through
a sectoral emissions tax, box 2 in Table 2 shows the mix of complementary and overlapping
policy instruments that will be focused here. Box 1 in Table 2 represents the limiting case

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[1]The 'Energy Tax Directive' (EU 2003b) sets minimum tax levels on fossil fuel starting in 2004. It
allows for many exemptions and tax reductions, e.g. for energy-intensive business. Art. 14 stipulates that
Member States shall exempt from taxation energy products used to produce electricity but on the other
hand it allows Member States to subject these energy products to taxation without having to respect the
minimum levels of taxation laid down in the Directive. The effect of the Directive is small because most
countries have higher rates than the minimums.

[2]VAT is not included in prices and taxes shown for industry and electricity generation because it is
refunded.

in which no tax is levied in the ETS sector while box 3 can be interpreted as the state in which the rate of the tax in the ETS sector is so high as to 'dry up' the permit market.

	Emissions control in the ETS sector via		
	ETS	ETS and sectoral tax	Sectoral tax
Emissions control in the non-ETS sector via sectoral tax	1	2	3

Table 2: EU-type emissions control in a two-sector economy

To capture such policies in a multi-country model we translate the EU commitment of emissions reductions into an upper bound, say $\bar{c}$, for total emissions in the group of countries. The EU burden sharing agreement (EU 1999) is then interpreted as a political decision to split the overall emissions cap $\bar{c}$ into national caps, c_i, one for each member state $i = 1, \ldots, n$, satisfying $\sum_j c_j = \bar{c}$. Throughout the rest of this paper we will take the partition $(c_1, \ldots, c_n)$ of the overall cap $\bar{c}$ as given. As noted above, the EU ETS covers only part of each member state's economy. Therefore, the national cap c_i needs to be further split into a cap for the ETS and the non-ETS sector. The cap for the non-ETS sector is then implemented by a sectoral emissions tax whereas the cap of the ETS sector defines the country's budget of emissions permits to be issued by the national government and allocated to the installations in its ETS sector.

Obviously, that hybrid policy is susceptible to many pitfalls of cost *in*effectiveness, in particular when overlapping taxes in the ETS sectors are included in the policy mix. To the best of our knowledge, there are only two papers studying international emissions trading with *overlapping* emissions taxes. Böhringer et al. (2007) use a CGE model to study the impact of overlapping regulation through a unilateral carbon tax in one of the countries (Germany). They show that the German ETS sector carries excessive costs while the other EU member states may benefit or lose from that tax. Eichner and Pethig (2008) use a general equilibrium model to assess policy (in)effectiveness when national governments have discretion in fixing both an overlapping tax and their budget of emissions permits. Eichner and Pethig (2008) establish that cost effectiveness for the group of countries is attained if and only if there is a tax in the ETS sector (possibly zero) that is uniform across countries and a tax in the non-ETS sector that is also uniform across countries at a rate equal to the sum of the permit price and the rate of the tax in the ETS sector. The important message is that as long as the emissions tax in the ETS sectors is uniform across countries it can be fixed at different levels without compromising cost effectiveness for the group of countries.

In the present paper we will briefly reconstruct the model developed in Eichner and Pethig (2008), restate their cost-effectiveness result, and base our subsequent analysis on that model. However, we will exclusively consider cost-effective policies disregarding (empirically existing) cost *ineffective* policies and possible corrective actions that are addressed by Böhringer et al. (2007) or Eichner and Pethig (2008). This restriction serves to direct as clearly as possible the focus on the objective of the present paper, namely to study the *distributional* consequences of variations in the overlapping emissions tax.

To our knowledge, this distributional issue has not been analyzed in a *two-sector* general equilibrium framework which captures the EU emissions control in a stylized way. We aim to determine the welfare effects of variations in the overlapping emissions tax. More specifically, denote by $u_i(c, t_y)$ the welfare of country i in the (unique) competitive equilibrium of the multi-country model which prevails when $c = (c_1, \ldots, c_n)$ is the vector of national emissions caps and when t_y is the rate of the emissions tax in the countries' ETS sectors.[3] We are interested in the sign of the derivative $\partial u_i(c, t_y)/\partial t_y$ whose calculation turns out to be non-trivial because it requires carrying out a full-scale comparative static analysis of the multi-country model.

Due to the model's complexity we do not succeed in fully characterizing the distributional impact of variations in t_y (from $t_y = 0$ to some high level of t_y for which the permit price becomes zero). However, we do obtain clear-cut analytical answers for the special case of quasi-linear utility as well as for a class of parametric utility and production functions. In both cases we show that an increase in the tax rate t_y is exactly offset by a reduction in the equilibrium permit price leaving all firms' production unaffected. Yet the value of imports and exports of permits is affected, and it is shown that an individual country loses wealth to the benefit of all other countries, when it exports permits, and gains wealth at the expense of all other countries, when it imports permits.

However, the perfect offset of variations in the tax rate by changes in the equilibrium permit price turns out to be a special case that is typical only in models where the market of emissions permits is isolated from all other markets. In general, market-interdependence effects, i.e. repercussions of tax variations in markets other than the permit market, render imperfect the offset between the tax rate and the permit price. We show this by means of a numerical example in which one of the countries that initially exports permits eventually starts importing permits following successive increases in the tax rate t_y. As a consequence of that export-import reversal the country switches from welfare gains to losses. Therefore, the central message of the paper can be summarized as follows: the distributional impact

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[3]As we restrict our attention to cost-effective policies it is necessary to assume the tax rate to be uniform across countries.

of variations in the emissions tax in the countries' ETS sector is significant, and since in the real world market-interdependence effects are not negligible, that impact is less clear-cut in magnitude and direction than suggested by partial equilibrium analysis.

Those less clear-cut distributional consequences are specific, indeed, to the EU-type emissions control as modeled here. To demonstrate that more clearly, we modify our standard model of two-sector economies with a joint ETS by briefly considering a model of two-sector economies with *separate national ETSs* and, alternatively a model of *one-sector economies* with a joint ETS. In both cases market-interdependence effects turn out to be absent implying that the pertaining allocation and distribution is as in our standard model with quasi-linear preferences.

The paper is organized as follows. Section 2 sets up the model and characterizes the cost-effective allocation which is then decentralized in competitive markets by appropriate emissions control. Section 3 presents the incidence of increasing the uniform overlapping emissions tax in the ETS sector. In Section 4 we analyze modified versions of our standard model and Section 5 concludes.

2 Cost-effective EU-style carbon emissions control

2.1 The model

Consider a group of n countries embedded in the world economy. The economy of each country i $(i = 1, \ldots, n)$ consists of two sectors X^i and Y^i producing two consumption goods x_{si} and y_{si} with the help of fossil fuel, e_{xi} and e_{yi}, by means of the production functions[4]

$$x_{si} = X^i(e_{xi}) \qquad \text{and} \qquad y_{si} = Y^i(e_{yi}) \tag{1}$$

that are increasing and strictly concave. The representative consumer of country i derives utility from consumption x_i and y_i of these goods according to the quasi-concave utility function

$$u_i = U^i(x_i, y_i) \tag{2}$$

that is increasing in both arguments. Good X is nontradable, and hence domestic consumption is required to match domestic production

$$x_i = x_{si}. \tag{3}$$

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[4]Upper case letters denote functions and subscripts attached to them indicate partial derivatives.

Good Y and fossil fuel are traded on world markets at constant prices p_y and p_e, respectively. All fuel input is assumed to be imported from the rest of the world.

Since CO_2 emissions are proportional to the input of fossil fuel, we simply denote both by the same symbols. Modeling the EU's commitment in the Kyoto Protocol to reduce its greenhouse gas emissions by 8 percent from 1990 to 2012 we assume that the group of countries as a whole restricts its total emissions to some level[5] $\bar{c} > 0$. To meet that target each country is assigned a *national emissions cap* $c_i \geq 0$ like in the EU burden-sharing agreement such that $\sum_j c_j = \bar{c}$. In each country the national emissions cap needs to be split up into two *sectoral caps* c_{yi} and c_{xi} satisfying

$$c_i = c_{xi} + c_{yi}. \tag{4}$$

The sectoral caps are assumed to restrain emissions in the following way[6]

$$c_{xi} = e_{xi}, \tag{5}$$

$$\sum_i c_{yi} = \sum_i e_{yi}. \tag{6}$$

2.2 The cost-effective allocation

Consider now a social planner aiming to maximize the weighted sum of the utilities of all countries' representative consumers subject to (1)–(6) and subject to the group's consolidated trade balance

$$\sum_j [p_y(y_{sj} - y_j) - p_e(e_{xj} + e_{yj})] = 0 \tag{7}$$

vis-à-vis the rest of the world. Solving the associated Lagrangean

$$
\begin{aligned}
\mathcal{L} &= \sum_j \alpha_j\, U^j(x_j, y_j) + \sum_j \lambda_{xj} \left[X^j(e_{xj}) - x_j \right] + \sum_j \lambda_{cj}(c_j - c_{yj} - e_{xj}) \\
&+ \lambda_h \sum_j \{ p_y[Y(e_{yj}) - y_j] - p_e(e_{xj} + e_{yj}) \} + \lambda_e \sum_j (c_{yj} - e_{yj}),
\end{aligned}
\tag{8}
$$

a cost-effective allocation is characterized by the marginal conditions

$$\frac{U_y^i}{U_x^i} = \frac{p_y}{\mu_{xi}} \qquad \text{for } i = 1, \ldots, n, \tag{9}$$

$$\mu_{xi} X_e^i = p_y Y_e^i = p_e + \mu_e \qquad \text{for } i = 1, \ldots, n, \tag{10}$$

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[5]Throughout the paper we take $\bar{c}$ to be smaller than total business-as-usual emissions $\sum_j (e_{xj} + e_{yj})$.

[6]Equation (5) is required to hold for all i and is therefore more restrictive than the constraint $\sum_j c_{xj} = \sum_j e_{xj}$. The rationale of the differential treatment of the sectors X and Y is to model in the next section the institutional setting of the EU where the ETS covers the sectors Y of all member states only while each member state is obliged to implement the cap c_{xi} in its sector X.

where $\mu_{xi} := \lambda_{xi}/\lambda_h$ and $\mu_e := \lambda_e/\lambda_h$, and where λ_e, λ_h. The cost-effective allocation requires consumption efficiency (9) by equalizing the marginal rates of substitution and the price ratios across countries, and production efficiency (10) by equalizing marginal abatement costs across sectors and countries.

2.3 Cost-effectiveness in the competitive economy

Having characterized in Proposition 1 the cost-effective allocation as a benchmark, we now introduce into the model (1)-(7) competitive markets for good X with price p_{xi} in all countries $i = 1, \ldots, n$ along with the following emissions control policies: There is an emissions tax on good Y at rate t_{yi}, an emissions tax on good X at rate t_{xi} and the group as a whole operates an emissions trading scheme (ETS) with mandatory participation of all countries' sectors Y. Henceforth we will refer to sector Y as the ETS sector and to sector X as the non-ETS sector. To install the ETS, each country i issues the amount c_{yi} of marketable emissions permits and allocates them to the firms in its ETS sector. A competitive market for permits will arise with the aggregate supply being fixed at $\sum_j c_{yj}$ and with the aggregate demand $\sum_j e_{yj}$ being determined by the permit price π_e as to meet the market-clearing condition (6).

In this institutional setting the profits of the aggregate sectoral firms are[7]

$$p_{xi}X^i(e_{xi}) - (t_{xi} + p_e)e_{xi} \qquad \text{and} \qquad p_y Y^i(e_{yi}) - \pi_e(e_{yi} - c_{yi}) - (t_{yi} + p_e)e_{yi},$$

and the associated first-order conditions for profit maximization read

$$p_{xi}X_e^i(e_{xi}) = t_{xi} + p_e \qquad \text{and} \qquad p_y Y_e^i(e_{yi}) = t_{yi} + p_e + \pi_e. \qquad (11)$$

The consumer of country i maximizes her utility $U^i(x_i, y_i)$ subject to her budget constraint

$$z_i = p_{xi}x_i + p_y y_i, \qquad (12)$$

where $z_i := g_{xi}^* + g_{yi}^* + t_{xi}e_{xi} + t_{yi}e_{yi}$ is her income consisting of the firm's maximum profits, g_{xi}^* and g_{yi}^*, and the tax revenues, $t_{xi}e_{xi} + t_{yi}e_{yi}$, recycled to the consumer in a lumpsum fashion. The first-order conditions for utility maximization yield the demand function for good X,

$$x_i = D^i(p_{xi}, z_i). \qquad (13)$$

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[7]The way profits are defined for the ETS sector implies gratis allocation of permits to that sector. Due to the high level of abstraction of the model under consideration, allocating permits via auction would leave the results unchanged.

Proposition 1. *(Eichner and Pethig 2008)*[8]
The equilibrium allocation of the competitive economy (1)-(6) and (11)-(13) is costeffective, if and only if

$$t_{xi} = t_x \quad and \quad t_{yi} = t_y \qquad for \ all \quad i = 1, \ldots, n \tag{14}$$

and

$$t_x = \pi_e + t_y. \tag{15}$$

The important message of Proposition 1 is that a cost-effective allocation can be attained by means of a policy mix consisting of an ETS and of emissions taxes satisfying (14) and (15). There is a 'degree of freedom' concerning the level of the (uniform) tax rate t_y because for the firms in the ETS sector it is the total price of energy input and emissions, $p_e + \pi_e + t_y$, that matters. The firms' demand for energy and emissions permits depends on that total price irrespective of what its components are.

Casual evidence of carbon emissions control in the EU suggests that none of the three equalities in (14) and (15) are satisfied. There are positive non-uniform tax rates t_{yi} in various member states as shown in Table 1. Yet the rates t_{yi} tend to be low relative to the (implicit) tax rates t_{xi} and the average permit price π_e was also very low during the first trading phase 2005-2007 suggesting that $t_{xi} > \pi_e + t_{yi}$ for many EU member states during the last years.

Although Proposition 1 provides straightforward guidelines for improving the cost-effectiveness of carbon emissions control in the EU, we will not elaborate on that issue but focus, instead, on the distributional impacts of the hybrid EU-style policy mix. To avoid coping with distributional consequences of cost-*in*effective allocations we will restrict our attention to cost-effective policies. In other words, (14) and (15) are assumed to hold throughout the rest of the paper. It follows that for fixed national emissions caps, c_i, the tax rate t_y (with $t_{yi} = t_y$ all i) is the sole autonomous policy instrument. In practice, employing a uniform tax rate t_y presupposes an internationally coordinated tax policy or alternatively, a supranational fiscal authority fixing the tax rate t_y and requiring all governments to set the tax rate t_x in their non-ETS sectors as to satisfy (15). We will make use of the latter interpretation and refer to the supranational fiscal authority as the center.

Observing the cost-effective conditions (14) and (15), the center has some discretion in fixing the tax rate t_y. In fact, there is a range of tax rates t_y supporting cost-effective competitive equilibria with the following polar cases:

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[8]See also the stylized analysis of overlapping regulation in Böhringer et al. (2008).

(i) Suppose the center fixes the tax rate at $t_y = 0$ (box 1 in Table 2). The cost-effective emissions control then consists of a tax-and-cap policy in each country's domestic non-ETS sector and an international ETS covering the ETS sectors of all countries. Although in this case no overlapping regulation is employed both instruments are still linked through the cost-effectiveness condition $t_x = \pi_e$.

(ii) Suppose the center fixes the rate at some high level, say $\bar{t}_y > 0$, such that in the resultant equilibrium total demand for permits equals total supply at price $\pi_e = 0$ (box 3 in Table 2). The cost-effectiveness condition then is $t_x = \bar{t}_y$. A strange feature of this scenario is that in spite of $\pi_e = 0$ the market for emissions permits is still in operation. This polar case will play a benchmark role in the subsequent analysis.

It follows that associated to each $t_y \in [0, \bar{t}_y]$ there is a *cost-effective* competitive equilibrium (box 2 in Table 2). However, we do not yet know how these equilibria differ with respect to the distribution of the countries' income and welfare. Our goal is to explore the distributive impacts of variations in t_y. These effects will be investigated by means of a comparative static analysis of our multi-country model in the next section. To ease the exposition, we will omit some of the tedious calculations referring the reader to the full-scale comparative statics in the Appendix.

3 Incidence of the uniform emissions tax overlapping with the ETS

3.1 Comparative statics using general functional forms

In this section we start from an initial competitive equilibrium for some vector $c = (c_1, \ldots, c_n)$ of national emissions caps and for some $t_y \in [0, \bar{t}_y]$. We will leave the national emissions cap unchanged but will disturb the initial equilibrium by a small (exogenous) variation in t_y and determine the displacement effects characterizing the new cost-effective equilibrium reached after the shock. Ultimately, we are interested in the associated redistribution of national welfare as measured by changes in the utility of the countries' representative consumers which turn out to be (Appendix A)[9]

$$\frac{du_i}{\lambda_i dt_y} = t_y \left(\frac{\alpha_i \delta_i - \beta_i \gamma_i}{\gamma_i} \right) \left(\frac{d\pi_e}{dt_y} + 1 \right) + \left(\frac{\alpha_i t_y D_z^i + \gamma_i}{\gamma_i} \right) \Delta e_{yi} \frac{d\pi_e}{dt_y}, \tag{16}$$

where $\alpha_i := -\frac{X_e^i}{p_{xi} X_{ee}^i} > 0$, $\beta_i := -\left(\frac{1}{Y_{ee}^i} + \frac{1}{p_{xi} X_{ee}^i} \right) > 0$, $\delta_i := \alpha_i - \beta_i t_y D_z^i$, $\gamma_i := \alpha_i X_e^i - D_p^i - (x_i + \alpha_i t_y) D_z^i$. In addition, $\lambda_i > 0$ is the marginal welfare of income in country i and

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[9]For convenience of notation good Y is chosen as numeraire ($p_y \equiv 1$).

$\Delta e_{yi} := c_i - e_{xi} - e_{yi}$ is the amount of permits exported ($\Delta e_{yi} > 0$) or imported ($\Delta e_{yi} < 0$) by country i. Although it can be shown (Appendix A) that $\alpha_i \delta_i > \beta_i \gamma_i$ and that $\gamma_i > 0$ under weak restrictions, (16) only yields limited information on the sign of du_i/dt_y. We are able to infer from (16) that sign $\frac{du_i}{dt_y} = -\text{sign }\Delta e_{yi}$, if $t_y = 0$ and $d\pi_e/dt_y < 0$, and that

$$\frac{du_i}{dt_y} < 0, \quad \text{if} \quad t_y > 0, \Delta e_{yi} > 0, \frac{d\pi_e}{dt_y} \in]-1, 0]$$

and if $\eta^i_{xz} := x_i D^i_z / z_i$ is sufficiently small (see Appendix A). Yet in general, the sign of du_i/dt_y is ambiguous for permit-exporting countries as well as for permit-importing countries. It crucially depends on the sign and magnitude of $d\pi_e/dt_y$ the specification of which requires to explore how the permit market responds to variations in the tax rate t_y. Since the permit market is at the core of the EU-style emissions control we will investigate the determinants of $d\pi_e/dt_y$ in more detail.

Observe first that in the initial equilibrium the equations (6) and (11) hold so that the clearance of the permit market can be expressed by

$$\sum_j \left[E^{xj}(p_{xj}, q_e) + E^{yj}(q_e) \right] = \sum_j c_j, \tag{17}$$

where $q_e := p_e + \pi_e + t_y$, and where $E^{xi}(\cdot)$ and $E^{yi}(\cdot)$ are sectoral demand functions for energy and permits implicitly contained in (11). If in (17) the prices $p_{x1}, \ldots, p_{xn}$ clear the national markets for good X, equation (17) determines the equilibrium permit price, π_e, for some given t_y. Differentiating (17) with respect to t_y yields, after some rearrangement of terms,

$$\frac{d\pi_e}{dt_y} = -1 - \frac{\sum_j \left(E^{xj}_{p_{xj}} \frac{dp_{xj}}{dt_y} \right)}{\sum_j \left(E^{xj}_{q_e} + E^{yj}_{q_e} \right)} = -1 + \frac{\sum_j \left(\alpha_j \frac{dp_{xj}}{dt_y} \right)}{\sum_j \beta_j}. \tag{18}$$

According to (18) changes in the tax rate t_y are exactly offset by opposite changes in the permit price π_e unless $\sum_j [\alpha_j (dp_{xj}/dt_y)] \neq 0$. This term is clearly zero in partial equilibrium models where Y is the only consumer good. However, in market economies with more than one consumer good, the interdependence effects dp_{xi}/dt_y will lead to $d\pi_e/dt_y \neq -1$, in general. In Appendix A these interdependence effects are calculated as

$$\frac{dp_{xi}}{dt_y} = \frac{\delta_i + \Delta e_{yi} D^i_z}{\gamma_i} \frac{d\pi_e}{dt_y} + \frac{\delta_i}{\gamma_i}. \tag{19}$$

From inserting (19) into (18) follows, after some rearrangement of terms,

$$\frac{d\pi_e}{dt_y} = -\frac{1}{1 + \frac{\sum_j \frac{\alpha_j D^j_z}{\gamma_j} \Delta e_{yj}}{\sum_j \frac{\alpha_j \delta_j - \beta_j \gamma_j}{\gamma_j}}}. \tag{20}$$

Not surprisingly, (20) allows for deviations of $d\pi_e/dt_y$ from -1 in either direction as does (18). However, closer inspection of (20) shows that progress can be made in the special case of utility functions taking on the functional form $U^i(x_i, y_i) = V^i(x_i) + y_i$ with V^i being increasing and strictly concave in x_i. For that class of so-called quasi-linear utility functions the income effect of the demand for good X is known to be zero ($D_z^i \equiv 0$) such that (20) turns into $d\pi_e/dt_y = -1$.

Proposition 2.

If the utility functions U^i from (2) are quasi-linear the incidence of the emissions tax is given by Table 3.

	$d\pi_e$	dp_{xi}	de_{yi}	de_{xi}	$d\Delta e_{yi}$	dx_i	dy_i	dz_i	du_i
$dt_y, \Delta e_{yi} > 0$	-1	0	0	0	0	0	$-$	$-$	$-$
$dt_y, \Delta e_{yi} < 0$	-1	0	0	0	0	0	$+$	$+$	$+$

Table 3: Tax incidence in the general model in case of quasi-linear utility functions

It is easy to see that with $D_z^i = 0$ (all i) the ETS and the overlapping (uniform) emissions tax are perfect substitutes in the sense that the total factor price, $q_e = p_e + \pi_e + t_y$, is unaffected by variations in t_y. Since $D_z^i = 0$ and since $d\pi_e/dt_y$ eliminates spillovers between the permit market and the national markets for good X, the demand for permit remains unchanged in all sectors and hence the permit market is unaffected. However, the distributional incidence of tax shifts is pronounced: An increase in the tax rate t_y benefits permit-importing countries but reduces the welfare of permit-exporting countries. Since $de_{xi} = de_{yi} = 0$, $d(\Delta e_{yi})/dt_{yi} = 0$ follows, i.e. a country's permit exports or imports do not depend on the level of t_y.

Zero income elasticity of demand for good X appears to be a restrictive and unrealistic assumption. As pointed out above it eliminates market-interdependence effects and thus "isolates" the permit market which can then be studied as in a partial equilibrium model. To gain further insights in the tax incidence without assuming zero income effects we will resort to parametric functional forms of the Cobb-Douglas type.

3.2 Comparative statics using parametric functional forms

We now parametrize the model by introducing the Cobb-Douglas utility functions

$$U^i(x_i, y_i) = x_i^{h_i} y_i^{1-h_i}, \quad X^i(e_{xi}) = e_{xi}^{a_i}, \quad Y^i(e_{yi}) = e_{yi}^{b_i}. \tag{21}$$

With (21) the impact of variations in the tax rate t_y on the permit price, π_e, can be shown (Appendix B) to be

$$\frac{\mathrm{d}p_{xi}}{\mathrm{d}t_y} = \kappa_i \Delta e_{yi} \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + \mu_i \left(1 + \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} \right), \tag{22}$$

where $\kappa_i := \frac{(1-a_i)p_{xi}a_i}{e_{xi}\left[(p_e+\pi_e)a_i + \bar{h}_i q_e\right]} > 0$, $\bar{h}_i := \frac{(1-h_i)}{h_i} > 0$, $\mu_i := \rho_i p_{xi} \left[\frac{\bar{h}_i q_e + a_i(p_e+\pi_e)}{a_i q_e} - \frac{(1-a_i)}{e_{xi}} \right] \left(\gtreqless 0 \right)$ and $\rho_j := \frac{a_i}{(p_e+\pi_e)a_i + \bar{h}_i q_e} > 0$. According to (22), $\mathrm{d}p_x/\mathrm{d}t_y \neq 0$ is non-zero in general, and this is true even in case of $\mathrm{d}\pi_e/\mathrm{d}t_y = -1$.[10] More specifically, for $\mathrm{d}\pi_e/\mathrm{d}t_y = -1$ it follows from (22) that $\mathrm{d}p_{xi}/\mathrm{d}t_y > 0$ for permit-importing countries and $\mathrm{d}p_{xi}/\mathrm{d}t_y < 0$ for permit-exporting countries. If $\mathrm{d}\pi_e/\mathrm{d}t_y \neq -1$, $\mathrm{d}p_{xi}/\mathrm{d}t_y$ may be positive or negative.

Inserting (22) into (18) does not render the result more informative. However, taking another route of comparative static calculations (Appendix B) we find

$$\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} = -\frac{1}{1 - \frac{\sum_j \rho_j \Delta e_{yj}}{\sum_j \rho_j \sigma_j}}, \tag{23}$$

where $\sigma_i := \frac{\bar{h}_i e_{xi}}{a_i} + \frac{(\bar{h}_i + a_i)e_{yi}}{(1-b_i)a_i} > 0$. As in the model of Section 3.1, the change in the countries' welfare, $\mathrm{d}u_i/\mathrm{d}t_y$, is given by (16). $\mathrm{d}u_i/\mathrm{d}t_y$ remains ambiguous in sign although γ_i is now unequivocally positive. However, closer inspection of (23) reveals that $\mathrm{d}\pi_e/\mathrm{d}t_y = -1$, if

$$a_i = a \quad \text{and} \quad h_i = h \quad \text{for all } i. \tag{24}$$

Note that with (24) countries may still differ with respect to their production functions for good Y ($b_i \neq b_j$) and their national caps ($c_i \neq c_j$) such that net exports and imports of permits will be non-zero, in general.

A remarkable consequence of the assumption (24) is that in contrast to the special case $D_z^i = 0$, all i, of Section 3.1 variations in t_y do affect the market for good X: Combining (22) and (24) shows that a tax hike $\mathrm{d}t_y > 0$ will raise [lower] the equilibrium price p_{xi} if country i imports [exports] permits. Moreover, we know from comparing (18) and (23) that if (24) holds the opposite price changes of permit exporting and importing countries are symmetric in the sense that $\sum_j [\alpha_j(\mathrm{d}p_{xj}/\mathrm{d}t_y)] = 0$.

Since q_e remains unchanged, the demand for energy inputs of the ETS sectors does not change either. On the other hand, the increase [reduction] in the price p_{xi} induced by $\mathrm{d}t_y > 0$ reduces [increases] the demand for energy inputs in the non-ETS sector of permit exporting [importing] countries such that exports as well as imports rise. Since it can be shown that

$$\lim_{\Delta e_{yi} \to 0} \frac{\mathrm{d}\Delta e_{yi}}{\mathrm{d}t_y} = 0,$$

---

[10]Recall that in the previous section $D_z^i = 0$ for all i implied $\mathrm{d}\pi_e/\mathrm{d}t_y = -1$ as well as $\mathrm{d}p_{xi}/\mathrm{d}t_y = 0$. Note also that with the Cobb-Douglas utility function the income effect on the demand for both goods is positive.

we conclude that the subsets of permit-exporting and importing countries are independent of the level of t_y. The comparative statics carried out in Appendix B yield

Proposition 3.
If the functions X^i, Y^i and U^i from (1) and (2) are specified by (21) and if (24) holds, the incidence of the emissions tax is given by Table 4.

	$d\pi_e$	dp_{xi}	de_{yi}	de_{xi}	$d\Delta e_{yi}$	dx_i	dy_i	dz_i	du_i
$dt_y, \Delta e_{yi} > 0$	-1	$-$	0	$-$	$+$	$-$	$-$	$-$	$-$
$dt_y, \Delta e_{yi} < 0$	-1	$+$	0	$+$	$-$	$+$	$+$	$+$	$+$

Table 4: Tax incidence in the parametric model, when technologies of good X and preferences are the same across countries

Comparing Table 4 with Table 3 reveals that in both cases we observe $d\pi_e/dt_y = -1$ and the qualitative changes in de_{yi}, dy_i, dz_i and du_i are the same. However, while in case of $D_z^i \equiv 0$ (Table 3) both endogenous markets, i.e. the market for good X and the permit market, remain unaffected, the parametric model satisfying (24) exhibits repercussions in both markets. As an implication, the cost-effective split of the national emissions caps into two sectoral caps depends on the level of the tax rate t_y in the parametric model satisfying (24) while it is unaffected by t_y in case of $D_z^i = 0$.

Apart from these differences, the restrictions imposed on the model in Propositions 2 and 3 have an important property in common: They imply that the derivatives of all endogenous variables with respect to t_y are either zero or unconditionally positive or negative. In other words, there are functions $v = v(t_y)$ for all endogenous variables $v = \pi_e, p_{xi}, e_{xi}, e_{yi}, \Delta e_{yi}, x_i, y_i, z_i, u_i$ which are monotone or strictly monotone in the tax rate t_y. For that reason the comparative statics analysis does not only yield 'local information' for marginal variations in the tax rate but provides 'global information' about the properties of the functions $v = v(t_y)$. The most relevant properties are highlighted in

Proposition 4.
Denote by $\Delta e_{yi}(t_y)$ and $u_i(t_y)$ country i's permit trade balance and welfare, respectively, when the center has fixed the tax rate at $t_y \in [0, \bar{t}_y]$ and suppose the functional forms are as specified in Proposition 2 or Proposition 3.

(a) If country i exports [imports] permits for some $t_y \in [0, \bar{t}_y]$, it exports [imports] permits for all $t_y \in [0, \bar{t}_y]$.

(b) Permit-exporting [permit-importing] countries lose [gain] whenever the tax rate t_y is

raised such that

$$u_i(0) > u_i(\bar{t}_y), \ if \ \Delta e_{yi} > 0 \ and \ u_i(0) < u_i(\bar{t}_y), \ if \ \Delta e_{yi} < 0. \tag{25}$$

We conclude that under the conditions of Proposition 4 the distributional consequences of variations in the tax rate t_y are unambiguous. Unfortunately this feature does not hold in general, i.e. if $D_z^i \neq 0$ or if (24) does not hold, we cannot draw conclusions from marginal information provided by the comparative-static analysis on the global properties of the functions $v(t_y)$. In the next Section we will therefore resort to numerical analysis of our parametric model aiming at global information on the functions $v(t_y)$ when (24) is *not* satisfied. Particular attention will be placed on whether and how $\mathrm{d}\pi_e/\mathrm{d}t_y$ deviates from minus one, whether non-marginal variations of t_y may lead to export-import reversals and what the associated changes in the distribution of national welfare are.

3.3 Non-monotone changes in welfare: a numerical example

In this section we consider a parametric model of Section 3.2 for which condition (24) is not satisfied. To make progress we consider a three-country model in which the parameters take on the values $a_1 = 0.2$, $a_2 = 0.6$, $a_3 = 0.9$, $b_1 = b_2 = b_3 = 0.5$, $c_1 = 0.605$, $c_2 = 0.6$, $c_3 = 1.3$, $h_1 = h_2 = h_3 = 0.5$, $p_e = 0.2$. For this model we then compute the equilibrium allocation as a function of the tax rate t_y with the help of the tool Mathematica (Appendix C) establishing

Proposition 5.
If in the parametric model of Section 3.2 the tax rate t_y is successively raised, some permit-exporting country may eventually import permits such that its national income and welfare first decline but then increase.

The qualitative properties of the response of the entire equilibrium allocation to successive increases in the tax rate are summarized in Table 5, and the Figures 1, 2 and 3 provide additional illustration of some of the particularly interesting functions.

	$\pi_e(t_y)$	$p_{xi}(t_y)$	$e_{yi}(t_y)$	$e_{xi}(t_y)$	$\Delta e_{yi}(t_y)$	$x_i(t_y)$	$y_i(t_y) \quad z_i(t_y) \quad u_i(t_y)$
country 1 $\Delta e_{y1} \gtreqless 0$	DECR	U-SH CONV	INCR & CONV	INCR CONV	DECR CONC	INCR CONV	U-SH & CONV
country 2 $\Delta e_{y2} < 0$		INCR CONV		CONV	DECR CONV		INCR & CONV
country 3 $\Delta e_{y3} > 0$		DECR CONC		DECR CONC	INCR CONV	DECR CONC	DECR & CONV

(DECR= monotone decreasing, U-SH= u-shaped, INCR= monotone increasing, CONV= strictly convex, CONC= strictly concave)

Table 5: Equilibrium quantities and prices as functions of the tax rate t_y: numerical example
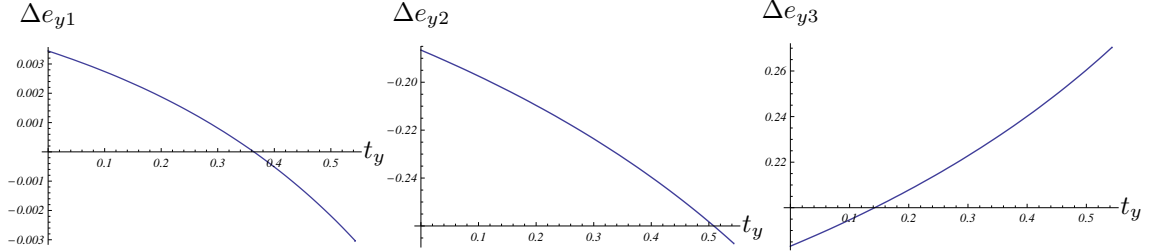

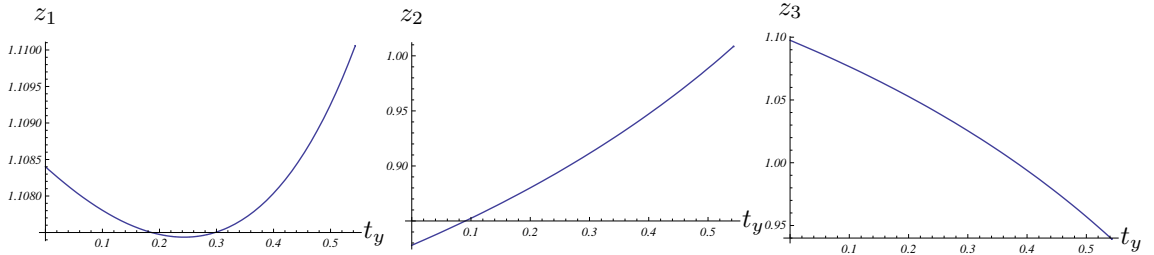Figure 1: Exports and imports of permits

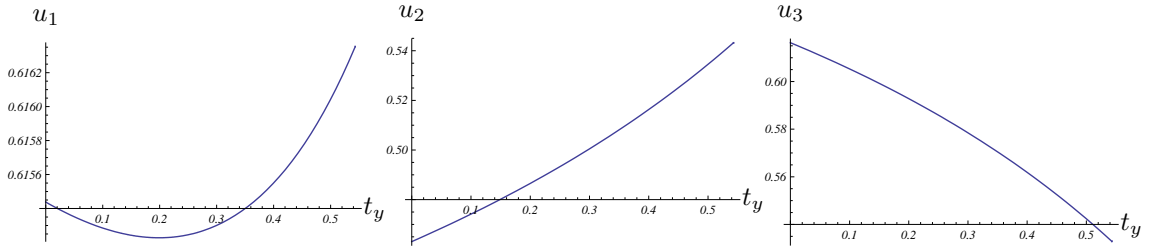

Figure 2: National incomes

Figure 3: National welfare

A few comments on the results are in order. Although the graph of the function $\pi_e(t_y)$ (see Appendix C) is clearly negatively sloped it exhibits little curvature, if any. Nonetheless we safely conclude that $d\pi_e/dt_y < -1$ because $e_{yi}(t_y) = \left(\frac{p_e + \pi_e + t_y}{b_i}\right)^{\frac{1}{b_i-1}}$ is obviously strictly increasing in t_y for $i = 1, 2, 3$ and hence

$$\frac{de_{yi}}{dt_y} = -\frac{e_{yi}}{(1-b_i)q_e}\left(\frac{d\pi_e}{dt_y} + 1\right).$$

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Country 2 imports and country 3 exports permits, and as in the model of Table 4 the levels of imports and exports rise with increasing tax rate (Figure 2). Corresponding to these changes the national income and the welfare in country 2 increase with the tax rate while national income and welfare shrink in country 3 (Figures 2 and 3).

The striking feature of the numerical example under consideration highlighted in Proposition 5 are the effects of successive tax increases on the allocation in country 1. As shown in the left panel of Figure 1 country 1 first exports permits but becomes an importer of permits when the tax continues to increase.[11] Along with that reversal from exports to imports the price of good X, the national income and national welfare of country 1 are u-shaped functions of the tax rate (left panels of Figures 2 and 3). The observation that export-import reversals are feasible and with them non-monotone welfare changes makes it difficult to assess correctly the impact of tax hikes on the international distribution of welfare.

4 Emissions tax incidence in simplified models

It is clearly the EU-type emissions control as modeled above that is crucial for our result that national welfare may be non-monotone in the tax rate t_y (Proposition 5). To further clarify and reinforce that observation we will show in the sequel that non-monotone welfare changes do not occur in modified versions of our standard model in which either the international ETS is replaced by disjoined national ETSs or in which all national economies consist of one sector only. In Section 4.1 we investigate two-sector economies with domestic ETS and in Section 4.2 we turn to one-sector economies with international ETS.

4.1 Two-sector economies with domestic ETSs

We now assume that the group of countries does not achieve its emissions target $\sum_j c_{yi}$ through an international ETS but that in each country a domestic ETS is in operation such that the emissions in country i's sector Y are restrained by

$$c_{yi} = e_{yi}. \tag{26}$$

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[11]The same qualitative result is obtained by Böhringer et al. (2007) in their CGE approach for the case that an overlapping tax is unilaterally levied by one of the EU member states. However, that policy not only places an excess cost on the country levying an overlapping tax but it also is cost-ineffective from the point of view of the group of countries. In the present paper we focus on variations in an overlapping tax that is uniform across countries and hence does compromise cost-effectiveness.

The two-sector model with domestic ETS is specified by (1) - (5) and (26), and the country's trade balance is given by

$$p_y(y_{sj} - y_j) - p_e(e_{xj} + e_{yj}) = 0 \qquad (27)$$

Profit maximization implies (11) and the consumer's budget and demand function are given by (12) and (13), respectively. Without loss of generality we restrict our attention to a representative country and suppress the index i. Then the cost-effective allocation follows from solving the Lagrangian

$$
\begin{aligned}
\mathcal{L} &= U(x,y) + \lambda_x[X(e_x) - x] + \lambda_c(c - c_y - e_x) \\
&+ \lambda_h\{p_y[Y(e_y) - y] - p_e(e_x + e_y)\} + \lambda_e(c_y - e_y).
\end{aligned}
\qquad (28)
$$

It is straightforward to show that the competitive equilibrium of an individual two-sector economy with domestic ETS is cost-effective if and only if $t_x = \mu_e$, $t_y \in [0, \mu_e]$ and $\pi_e = \mu_e - t_y$, where $\mu_e := \lambda_e/\lambda_h$. As in the previous section cost-effectiveness can be achieved by levying an emissions tax in sector X in combination either with an emissions tax in sector Y, or with a pure domestic ETS, or with intermediate policies combining positive tax rates $t_y > 0$ with an ETS.[12]

The comparative statics in Appendix D of parametric changes in t_y reveal

Proposition 6. *Consider a group of countries each of which consists of a two-sector economy and operates a domestic ETS. For such country it is true that*

(i) *an increase in the emissions tax rate t_y is exactly offset by a reduction in the equilibrium permit price* $(\mathrm{d}\pi_e = -\mathrm{d}t_y)$;

(ii) *the cost-effective allocation and the country's welfare are unaffected by small variations in the emissions tax rate t_y.*

Proposition 6 highlights the distributional irrelevance of variations in t_y in the simple economy under consideration. Because the emissions controls of the countries are independent, neither the ETS-only policy nor the tax-only policy nor any policy mix has allocative or distributional displacement effects.

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[12]Note that tax rates will tend to differ across countries in the model under consideration because the countries' emissions controls are no longer interdependent. As a consequence, the overall cap $\bar{c} = \sum_j c_j$ is not implemented in a cost-effective way, in general, although each country's emissions control is cost-effective from the country's domestic perspective.

4.2 One-sector economies with international ETS

In this subsection we deviate from our standard model by assuming that the economy of country i consists only of a single sector, sector Y. Carrying over the notation from the standard model, the model in the present section is given by

$$y_{si} = Y^i(e_{yi}), \tag{29}$$

$$u_i = U^i(y_i), \tag{30}$$

$$\bar{c} = \sum_j c_j = \sum_j e_{yj}, \tag{31}$$

$$\sum_j [p_y(y_{sj} - y_j) - p_e e_{yj}] = 0, \tag{32}$$

$$p_y Y_e^i = \pi_e + t_{yi} + p_e, \tag{33}$$

$$z_i = p_y y_i. \tag{34}$$

(29) and (30) specify the production and utility function, respectively. The market for emissions clears according to (31). (32) is the group's consolidated trade balance, (33) is the aggregate firm's first-order condition of profit maximization and (34) is the consumer's budget constraint.

Determining the cost-effective allocation and decentralizing it in a competitive economy we obtain a special case of Proposition 1 where $t_{xi} = 0$ for all i, which is obvious since we eliminated sector X. In a group of countries with international ETS a precondition for cost-effectiveness is an emissions tax t_y being uniform across countries. Again we get an equivalence regarding cost-effectiveness of the ETS-only policy, the tax-only policy and all "convex combinations". However, unlike in the model of the previous section, variations in the tax rate now do have allocative and distributional effects. To obtain more specific qualitative information on those effects, we investigate the impacts of parametric changes in t_y on cost-effective competitive equilibria in comparative static analysis (Appendix D). The results are summarized in

Proposition 7. *Consider a group of countries each of which consists of a one-sector economy and all countries operate a joint ETS. The incidence of small variations in the uniform emissions tax is given by*

	$d\pi_e$	de_{yi}	$d\Delta e_{yi}$	dy_i	dz_i	du_i
$dt_y, \Delta e_{yi} > 0$	-1	0	0	$-$	$-$	$-$
$dt_y, \Delta e_{yi} < 0$	-1	0	0	$+$	$+$	$+$

Table 6: Tax incidence in one-sector economies; $\Delta e_{yi} := c_i - e_{yi}$

From Proposition 7 we infer that an increase in the tax rate t_y is exactly offset by a reduction in the permit price $(\mathrm{d}\pi_e/\mathrm{d}t_y = -1)$ thus leaving the firms' production and input decisions unaffected $(\mathrm{d}y_{si}/\mathrm{d}t_y = \mathrm{d}e_{yi}/\mathrm{d}t_y = 0)$. Therefore the change in the consumer's income is given by

$$\mathrm{d}z_i = \mathrm{d}y_i = \Delta e_{yi}\mathrm{d}\pi_e, \tag{35}$$

where $\Delta e_{yi} := c_i - e_{yi}$ is the amount of permits exported $(\Delta e_{yi} > 0)$ or imported $(\Delta e_{yi} < 0)$ by country i. According to (35) increasing t_y reduces the permit-exporting country's income. As an immediate consequence, the consumption of good Y declines and with it the country's welfare. All these effects are reversed in permit-importing countries.

To sum up, allocative and distributional effects vanish altogether if economies have two sectors but ETS is domestic. Furthermore, one-sector economies with international ETS exhibit the same allocative and distributional effects as two-sector economies with international ETS and quasi-linear utility functions. It follows that a necessary condition for non-monotone welfare changes is an EU-type emissions control in the sense that an international ETS is established in a group of countries such that only some part of all countries' economies is covered by the ETS.

5   Concluding remarks

In a stylized way, our paper addresses *distributional* consequences of a hybrid regime of CO_2 emissions control designed to capture the basic features of the regime applied in the EU since 2005. Characteristic of the EU regime is an EU-wide international ETS that coexists with national complementary and overlapping national emissions taxes. Restricting our attention to cost-effective competitive equilibria we show that an increase in the rate of the emissions tax levied in the ETS sector is exactly offset by a reduction in the equilibrium permit price in the case of quasi-linear utility functions and for a class of parametric utility and production functions. Since the reduction in the permit price lowers [raises] a permit-exporting [importing] country's income, permit-exporting [importing] countries lose [gain] from an increase in the emissions tax. However, these results are not general, because emissions tax changes may cause ramifications beyond the permit market. With the help of a numerical example we show that an initially permit-exporting country may switch to import permits when the emissions tax is successively increased. Hence, export-import reversals are feasible such that initially welfare-losing countries may turn to welfare-gaining countries and vice versa.

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Appendix A: Comparative statics of the general model

The cost-effective competitive equilibrium of the multi-country economy is completely described by the following equations:

$$\sum_j c_j = \sum_j (e_{xj} + e_{yj}), \tag{A1}$$

$$x_{si} = x_i, \qquad i = 1, \ldots, n, \tag{A2}$$

$$x_{si} = X^i(e_{xi}), \qquad i = 1, \ldots, n, \tag{A3}$$

$$x_i = D^i(p_{xi}, z_i), \qquad i = 1, \ldots, n, \tag{A4}$$

$$z_i = p_{xi}x_{is} + y_{is} - p_e(e_{xi} + e_{yi}) + \pi_e(c_i - e_{xi} - e_{yi}), \quad i = 1, \ldots, n \tag{A5}$$

$$y_{si} = Y^i(e_{yi}), \qquad i = 1, \ldots, n, \tag{A6}$$

$$z_i = p_{xi}x_i + y_i, \qquad i = 1, \ldots, n, \tag{A7}$$

$$p_{xi}X_e^i(e_{xi}) = p_e + t_x, \qquad i = 1, \ldots, n, \tag{A8}$$

$$Y_e^i = p_e + \pi_e + t_y, \qquad i = 1, \ldots, n, \tag{A9}$$

$$t_x = \pi_e + t_y, \tag{A10}$$

where without loss of generality good Y is chosen as numeraire ($p_y \equiv 1$). The variables determined by (A1) - (A10) are $e_{xi}, e_{yi}, x_{si}, x_i, p_{xi}, z_i, y_{si}, y_i$ for $i = 1, \ldots, n$, π_e and t_x. The tax rate t_y is treated here as an exogenous parameter. It is convenient to compress the system of equations (A1) - (A10) as follows:

$$\sum_j c_j = \sum_j (e_{xj} + e_{yj}), \tag{A11}$$

$$X^i(e_{xi}) = D^i(p_{xi}, z_i), \tag{A12}$$

$$z_i = p_{xi}X^i(e_{xi}) + Y^i(e_{yi}) - p_e(e_{xi} + e_{yi}) + \pi_e \Delta e_{yi}, \tag{A13}$$

$$p_{xi}X_e^i(e_{xi}) = Y_e^i(e_{yi}), \tag{A14}$$

$$Y_e^i(e_{yi}) = p_e + \pi_e + t_y, \tag{A15}$$

$$y_i = Y^i(e_{yi}) - p_e(e_{xi} + e_{yi}) + \pi_e \Delta e_{yi}, \tag{A16}$$

where $\Delta e_{yi} := c_i - e_{xi} - e_{yi}$ in (A13) is the amount of permits exported or imported by country i. The equations (A11) - (A15) serve to determine π_e and e_{xi}, e_{yi}, p_{xi} and z_i for $i = 1, \ldots, n$. Equation (A16) represents the current account balances, and it determines y_i after e_{xi}, e_{yi} and π_e are solved via (A11) - (A15). Our aim is to perform a comparative static analysis to specify the impact on the economy of exogenous variations in the uniform

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tax rate t_y. To that end (A11) through (A15) are now totally differentiated.

$$\sum_j \mathrm{d}e_{xj} + \sum_j \mathrm{d}e_{yj} = 0, \tag{A17}$$

$$X_e^i \mathrm{d}e_{xi} - D_p^i \mathrm{d}p_{xi} - D_z^i \mathrm{d}z_i = 0, \tag{A18}$$

$$\mathrm{d}z_i = x_i \mathrm{d}p_{xi} + t_y(\mathrm{d}e_{xi} + \mathrm{d}e_{yi}) + \Delta e_{yi} \mathrm{d}\pi_e, \tag{A19}$$

$$X_e^i \mathrm{d}p_{xi} + p_{xi} X_{ee}^i \mathrm{d}e_{xi} - Y_{ee}^i \mathrm{d}e_{yi} = 0 \tag{A20}$$

$$Y_{ee}^i \mathrm{d}e_{yi} - \mathrm{d}\pi_e - \mathrm{d}t_y = 0 \tag{A21}$$

To obtain (A19) we have differentiated (A13),

$$\mathrm{d}z_i = x_{si} \mathrm{d}p_{xi} + p_{xi} X_e^i \mathrm{d}e_{xi} + Y_e^i \mathrm{d}e_{yi} - p_e(\mathrm{d}e_{xi} + \mathrm{d}e_{yi}) + \Delta e_{yi} \mathrm{d}\pi_e - \pi_e(\mathrm{d}e_{xi} + \mathrm{d}e_{yi}),$$

and then made use of (A14) and (A15).

Next we consider $\mathrm{d}e_{yi} = \frac{1}{Y_{ee}^i}(\mathrm{d}\pi_e + \mathrm{d}t_y)$ from (A21) in (A20) to obtain

$$\mathrm{d}e_{xi} = \frac{1}{p_{xi} X_{ee}^i}(\mathrm{d}\pi_e + \mathrm{d}t_y) - \frac{X_e^i}{p_{xi} X_{ee}^i}\mathrm{d}p_{xi}. \tag{A22}$$

Summation of $\mathrm{d}e_{xi}$ from (A22) and $\mathrm{d}e_{yi}$ from (A21) yields

$$\mathrm{d}e_{xi} + \mathrm{d}e_{yi} = \alpha_i \mathrm{d}p_{xi} - \beta_i(\mathrm{d}\pi_e + \mathrm{d}t_y), \tag{A23}$$

where $\alpha_i := -\frac{X_e^i}{p_{xi} X_{ee}^i} > 0$ and $\beta_i := -\left(\frac{1}{Y_{ee}^i} + \frac{1}{p_{xi} X_{ee}^i}\right) > 0$. Inserting (A23) in (A17) gives

$$\frac{\sum_j \alpha_j \mathrm{d}p_{xj}}{\sum_j \beta_j} - \mathrm{d}\pi_e = \mathrm{d}t_y. \tag{A24}$$

We take advantage of (A23) to turn (A19) into

$$\mathrm{d}z_i = (x_i + \alpha_i t_y)\mathrm{d}p_{xi} - \beta_i t_y(\mathrm{d}\pi_e + \mathrm{d}t_y) + \Delta e_{yi} \mathrm{d}\pi_e. \tag{A25}$$

We make use of (A22) and (A25) to transform (A18) into

$$\mathrm{d}p_{xi} = \frac{\delta_i(\mathrm{d}\pi_e + \mathrm{d}t_y)}{\gamma_i} + \frac{D_z^i \Delta e_{yi} \mathrm{d}\pi_e}{\gamma_i}, \tag{A26}$$

where $\delta_i := \alpha_i - \beta_i t_y D_z^i$ and $\gamma_i := \alpha_i X_e^i - D_p^i - (x_i + \alpha_i t_y)D_z^i$.

We insert (A26) into (A24) to obtain, after some rearrangement of terms,

$$\mathrm{d}\pi_e \sum_j \frac{\alpha_j D_z^j \Delta e_{yj}}{\gamma_j} + (\mathrm{d}\pi_e + \mathrm{d}t_y) \sum_j \left[\frac{\alpha_j \delta_j}{\gamma_j} - \beta_j\right] = 0, \tag{A27}$$

which in turn can be rewritten as

$$\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} = -\frac{1}{1 + \frac{\sum_j \frac{\alpha_j D_z^j}{\gamma_j}\Delta e_{yj}}{\sum_j \frac{\alpha_j \delta_j - \beta_j \gamma_j}{\gamma_j}}}. \tag{A28}$$

Next, we differentiate the utility function (2) to get $\mathrm{d}u_i = U_x^i \mathrm{d}x_i + U_y^i \mathrm{d}y_i$ and use $\frac{U_x^i}{p_{xi}} = \frac{U_y^i}{p_y} = \lambda_i$ to obtain

$$\frac{\mathrm{d}u_i}{\lambda_i} = p_{xi}\mathrm{d}x_i + \mathrm{d}y_i, \tag{A29}$$

where λ_i is the marginal utility of income (i.e. the Lagrange multiplier assigned to the consumer's budget constraint). From (A3), (A8) and (A10) we infer

$$\mathrm{d}x_i = X_e^i \mathrm{d}e_{xi} = \frac{p_e + \pi_e + t_y}{p_{xi}} \mathrm{d}e_{xi}. \tag{A30}$$

From (A16) we obtain with the help of (A15)

$$\mathrm{d}y_i = t_y \mathrm{d}e_{yi} - (p_e + \pi_e)\mathrm{d}e_{xi} + \Delta e_{yi}\mathrm{d}\pi_e. \tag{A31}$$

Inserting (A30) and (A31) in (A29) gives

$$\frac{\mathrm{d}u_i}{\lambda_i} = (p_e + \pi_e + t_y)\mathrm{d}e_{xi} + t_y\mathrm{d}e_{yi} - (p_e + \pi_e)\mathrm{d}e_{xi} + \Delta e_{yi}\mathrm{d}\pi_e = t_y(\mathrm{d}e_{xi} + \mathrm{d}e_{yi}) + \Delta e_{yi}\mathrm{d}\pi_e \tag{A32}$$

or, equivalently,

$$\frac{\mathrm{d}u_i}{\lambda_i \mathrm{d}t_y} = t_y \frac{\mathrm{d}e_{xi} + \mathrm{d}e_{yi}}{\mathrm{d}t_y} + \Delta e_{yi}\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y}. \tag{A33}$$

From (A23) it follows that

$$\frac{\mathrm{d}e_{xi} + \mathrm{d}e_{yi}}{\mathrm{d}t_y} = \alpha_i \frac{\mathrm{d}p_{xi}}{\mathrm{d}t_y} - \beta_i \left(\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + 1 \right). \tag{A34}$$

(A26) yields

$$\frac{\mathrm{d}p_{xi}}{\mathrm{d}t_y} = \frac{\delta_i}{\gamma_i} \left(\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + 1 \right) + \frac{D_z^i \Delta e_{yi}}{\gamma_i} \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y}. \tag{A35}$$

Making use of (A35) in (A34) yields

$$\frac{\mathrm{d}e_{xi} + \mathrm{d}e_{yi}}{\mathrm{d}t_y} = \left(\frac{\alpha_i \delta_i}{\gamma_i} - \beta_i \right) \left(\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + 1 \right) + \frac{\alpha_i D_z^i \Delta e_{yi}}{\gamma_i} \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y}. \tag{A36}$$

We take advantage of (A36) to turn (A33) into

$$\frac{\mathrm{d}u_i}{\lambda_i \mathrm{d}t_y} = t_y \left(\frac{\alpha_i \delta_i - \beta_i \gamma_i}{\gamma_i} \right) \left(\frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} + 1 \right) + \left(\frac{\alpha_i t_y D_z^i + \gamma_i}{\gamma_i} \right) \Delta e_{yi} \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y}. \tag{A37}$$

Lemma 1.

$$\eta_{xz}^i < \frac{p_e + \pi_e + t_y}{t_y} \cdot \frac{z_i}{p_{xi}x_i} \qquad \Longrightarrow \qquad \gamma_i > 0. \tag{A38}$$

Proof. Observe that

$$\gamma_i := \alpha_i X_e^i - D_p^i - (x_i + \alpha_i t_y)D_z^i = \alpha_i X_e^i - \frac{p_{xi}D_p^i}{x_i} \cdot \frac{x_i}{p_{xi}} - (x_i + \alpha_i t_y)\frac{z_i D_z^i}{x_i} \cdot \frac{x_i}{z_i}. \tag{A39}$$

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Making use of the definitions $\eta_{xp}^i := \frac{p_{xi}D_p^i}{x_i}$ and $\eta_{xz}^i := \frac{z_i D_z^i}{x_i}$ (A39) turns into

$$\gamma_i = \frac{x_i}{p_{xi}} \left[\frac{\alpha_i p_{xi}}{x_i} X_e^i - \eta_{xp}^i - \left(\frac{p_{xi}x_i}{z_i} + \frac{\alpha t_y p_{xi}}{z_i} \right) \eta_{xz}^i \right]. \tag{A40}$$

With the help of the Slutzky equation (in elasticity notation), formally $\eta_{xp}^i = \eta_{xp}^{ci} - \frac{p_{xi}x_i}{z_i}\eta_{xz}^i$ with $\eta_{xp}^{ci} := \frac{\partial x_i}{\partial p_{xi}} \cdot \frac{p_{xi}}{x_i}\Big|_{u=const.} < 0$, (A40) can be rearranged to yield

$$\gamma = \frac{x_i}{p_{xi}} \left[\alpha_i p_{xi} \left(\frac{X_e^i}{x_i} - \frac{t_y}{z_i}\eta_{xz}^i \right) - \eta_{xp}^{ci} \right]. \tag{A41}$$

Finally, we consider (A7) to obtain

$$\frac{X_e^i}{x_i} - \frac{t_y}{z_i}\eta_{xz}^i = \frac{p_e + \pi_e + t_y}{p_{xi}x_i} - \frac{t_y}{z_i}\eta_{xz}^i = \frac{t_y}{p_{xi}x_i} \left(\frac{p_e + \pi_e + t_y}{t_y} - \frac{p_{xi}x_i}{z_i}\eta_{xz}^i \right). \tag{A42}$$

Lemma 2. *The term $\alpha_i\delta_i - \beta_i\gamma_i$ is negative.*

Proof. Observe that

$$\begin{aligned}
\alpha_i\delta_i - \beta_i\gamma_i &= \alpha_i \left(\alpha_i - \beta_i t_y D_z^i \right) - \beta_i \left[\alpha_i X_e^i - D_p^i - (x_i + \alpha_i t_y)D_z^i \right] \\
&= \alpha_i \left(\alpha_i - \beta_i X_e^i \right) + \beta_i \left(D_p^i + x_i D_z^i \right).
\end{aligned} \tag{A43}$$

Making use of the definitions of β_i and the elasticities η_{xp}^i and η_{xz}^i, and making use of the Slutzky equation we obtain

$$\alpha_i\delta_i - \beta_i\gamma_i = \alpha_i\frac{X_e^i}{Y_{ee}^i} + \beta_i\frac{x_i}{p_{xi}} \left(\eta_{xp}^i + \frac{p_{xi}x_i}{z_i}\eta_{xz}^i \right) = \alpha_i\frac{X_e^i}{Y_{ee}^i} + \beta_i\frac{x_i}{p_{xi}}\eta_{xp}^{ci}. \tag{A44}$$

Comparative statics for quasi-linear utility functions (Table 3)

While $d\pi_e$ and du_i follows from setting $D_z^i = 0$ in (20) and (16), dp_{xi}, de_{yi}, de_{xi}, dx_i, dy_i, dz_i follows from (A26), $de_{yi} = \frac{d\pi_e + dt_y}{Y_{ee}^i}$, (A22), (A30), (A31) and (A25), respectively.

Appendix B: Comparative statics of the parametric model

For the parametric functional forms $U^i(x_i, y_i) = x_i^{h_i} y_i^{1-h_i}$, $X^i(e_{xi}) = e_{xi}^{a_i}$, $Y^i(e_{yi}) = e_{yi}^{b_i}$ the cost-effective competitive equilibrium is determined by

$$\sum_j c_j = \sum_j (e_{xj} + e_{yj}), \tag{B1}$$

$$e_{xi}^{a_i} = \frac{h_i z_i}{p_{xi}}, \tag{B2}$$

$$z_i = p_{xi} e_{xi}^{a_i} + e_{yi}^{b_i} - (q_e - t_y)(e_{xi} + e_{yi}) + \pi_e c_i, \tag{B3}$$

$$p_{xi} a_i e_{xi}^{a_i-1} = b_i e_{yi}^{b_i-1} = q_e, \tag{B4}$$

$$b_i e_{yi}^{b_i-1} = q_e, \tag{B5}$$

$$y_i = e_{yi}^{b_i} - (q_e - t_y)(e_{xi} + e_{yi}) + \pi_e c_i, \tag{B6}$$

where $q_e := p_e + \pi_e + t_y$. Note that (B2), (B3) and (B6) imply

$$y_i = (1 - h_i) z_i. \tag{B7}$$

Next we rearrange the system of equations (B1)-(B5). We make use of (B2) and (B4) to get

$$z_i = \frac{e_{xi} q_e}{a_i h_i}. \tag{B8}$$

From (B4) and (B5) we obtain

$$p_{xi} e_{xi}^{a_i} = \frac{e_{xi} q_e}{a_i}. \tag{B9}$$

We rearrange (B5) and write

$$e_{yi}^{b_i} = \frac{e_{yi} q_e}{b_i}. \tag{B10}$$

We make use of (B8)-(B10) in (B3) to get

$$\left[\frac{(1 - h_i) + a_i h_i}{a_i h_i} q_e - t_y \right] e_{xi} - \left[\frac{1 - b_i}{b_i} q_e + t_y \right] e_{yi} - \pi_e c_i = 0. \tag{B11}$$

Total differentiation of (B1), (B5), (B6) and (B11) yields

$$\sum_j (\mathrm{d}e_{xj} + \mathrm{d}e_{yj}) = 0, \tag{B12}$$

$$\mathrm{d}e_{yi} = -\frac{e_{yi}}{(1 - b_i) q_e} (\mathrm{d}\pi_e + \mathrm{d}t_y), \tag{B13}$$

$$\mathrm{d}z_i = \frac{\mathrm{d}y_i}{1 - h_i}, \tag{B14}$$

$$\left[\frac{(1 - h_i) + a_i h_i}{a_i h_i} q_e - t_y \right] \mathrm{d}e_{xi} - \left(\frac{1 - b_i}{b_i} q_e + t_y \right) \mathrm{d}e_{yi} - \mathrm{d}\pi_e c_i$$
$$+ e_{xi} \left[\frac{1 - h_i + a_i h_i}{a_i h_i} \mathrm{d}\pi_e + \frac{1 - h_i}{a_i h_i} \mathrm{d}t_y \right] - e_{yi} \left[\frac{1 - b_i}{b_i} \mathrm{d}\pi_e + \frac{\mathrm{d}t_y}{b_i} \right] = 0. \tag{B15}$$

(B15) can be rearranged to read

$$\frac{\mathrm{d}e_{xi}}{\rho_i} = \Delta e_{yi}\mathrm{d}\pi_e - \left(\frac{1-b_i}{b_i}q_e + t_y\right)\mathrm{d}e_{yi} + (\mathrm{d}\pi_e + \mathrm{d}t_y)\left(\frac{\bar{h}_i e_{xi}}{a_i} - \frac{e_{yi}}{b_i}\right), \tag{B16}$$

where $\bar{h}_i := \frac{1-h_i}{h_i}$ and $\rho_i := \frac{a_i}{h_i q_e + a_i(p_e + \pi_e)} > 0$. Inserting $\mathrm{d}e_{yi}$ from (B13) in (B16) yields after some rearrangement of terms

$$\frac{\mathrm{d}e_{xi}}{\rho_i} = \Delta e_{yi}\mathrm{d}\pi_e - (\mathrm{d}\pi_e + \mathrm{d}t_y)\left[\frac{\bar{h}_i e_{xi}}{a_i} + \frac{t_y e_{yi}}{(1-b_i)q_e}\right]. \tag{B17}$$

In view of (B13) and (B17), the sum of $\mathrm{d}e_{xi}$ and $\mathrm{d}e_{yi}$ is equal to

$$\mathrm{d}e_{xi} + \mathrm{d}e_{yi} = -\sigma_i \rho_i(\mathrm{d}\pi_e + \mathrm{d}t_y) + \Delta e_{yi}\rho_i\mathrm{d}\pi_e, \tag{B18}$$

where $\sigma_i := \frac{\bar{h}_i e_{xi}}{a_i} + \frac{(\bar{h}_i + a_i)e_{yi}}{(1-b_i)a_i} > 0$. Next, we insert (B18) in (B12) to obtain

$$\mathrm{d}\pi_e \sum_j \rho_j(\sigma_j - \Delta e_{yj}) = -\mathrm{d}t_y \sum_j \rho_j\sigma_j \quad\Longleftrightarrow\quad \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} = -\frac{1}{1 - \sum_j \frac{\rho_j \Delta e_{yj}}{\rho_j \sigma_j}}. \tag{B19}$$

Totally differentiating (B4) yields

$$\mathrm{d}p_{xi} = \frac{(1-a_i)p_{xi}}{e_{xi}}\mathrm{d}e_{xi} + \frac{p_{xi}}{q_e}(\mathrm{d}\pi_e + \mathrm{d}t_y), \tag{B20}$$

which with the help of (B17) can be turned into

$$\mathrm{d}p_{xi} = \frac{(1-a_i)p_{xi}\rho_i}{e_{xi}}\Delta e_{yi}\mathrm{d}\pi_e + \mu_i(\mathrm{d}\pi_e + \mathrm{d}t_y), \tag{B21}$$

where $\mu_i := \rho_i p_{xi}\left[\frac{\bar{h}_i q_e + a_i(p_e + \pi_e)}{a_i q_e} - \frac{(1-a_i)}{e_{xi}}\right]$. $\mathrm{d}x_i$, $\mathrm{d}y_i$, $\mathrm{d}z_i$ and $\mathrm{d}u_i$ have been calculated in (A30), (A31), (B14) and (A33). We make use of (B13), (B17) and (B18) to transform (A30), (A31), (B14) and (A33) into

$$\mathrm{d}x_i = \frac{q_e\rho_i}{p_{xi}}\Delta e_{yi}\mathrm{d}\pi_e - (\mathrm{d}\pi_e + \mathrm{d}t_y)\frac{q_e\rho_i}{p_{xi}}\left[\frac{\bar{h}_i e_{xi}}{a_i} + \frac{t_y e_{yi}}{(1-b_i)q_e}\right], \tag{B22}$$

$$\mathrm{d}y_i = -(p_e + \pi_e)\rho_i\Delta e_{yi}\mathrm{d}\pi_e - (\mathrm{d}\pi_e + \mathrm{d}t_y)\rho_i\left[\frac{h_i e_{xi}}{a_i} + \frac{t_y}{(1-b_i)q_e}\left(e_{yi} + \frac{1}{\rho_i}\right)\right], \tag{B23}$$

$$\mathrm{d}z_i = -\frac{(p_e + \pi_e)\rho_i\Delta e_{yi}}{(1-h_i)}\mathrm{d}\pi_e - \frac{(\mathrm{d}\pi_e + \mathrm{d}t_y)\rho_i}{(1-h_i)}\left[\frac{h_i e_{xi}}{a_i} + \frac{t_y}{(1-b_i)q_e}\left(e_{yi} + \frac{1}{\rho_i}\right)\right], \tag{B24}$$

$$\frac{\mathrm{d}u_i}{\lambda_i} = -t_y\sigma_i\rho_i(\mathrm{d}\pi_e + \mathrm{d}t_y) + (t_y\rho_i + 1)\Delta e_{yi}\mathrm{d}\pi_e. \tag{B25}$$

The signs in Table 4 follow from setting $\mathrm{d}\pi_e = -\mathrm{d}t_y$ in (B13), (B17), (B21)-(B25).

Appendix C: Numerical example

In this Appendix C we show how we solved the equilibrium equation system in order to simulate the graphs. Recall that for the special functional forms the multi-country equilibrium is determined by (B1)-(B6). In the following, we transform the equations (B1)-(B6). First, we eliminate the variables p_{xi} and z_i through substitution. Invoking $z_i = \frac{e_x^{a_i} p_{xi}}{\sigma_i}$ from (B2) in (B3) we get

$$p_{xi} e_{xi}^{a_i} \frac{(1-\sigma_i)}{\sigma_i} = e_{yi}^{b_i} - (q_e - t_y)(e_{xi} + e_{yi}) + \pi_e c_i. \tag{C1}$$

Next, we insert $p_{xi} = \frac{q_e}{a_i e_{xi}^{a_i-1}}$, which follows from (B4) and (B5), in (C1) to obtain

$$
\begin{aligned}
e_{xi}\left[q_e\frac{(1-\sigma_i)}{a_i\sigma_i} + (q_e - t_y)\right] &= e_{yi}^{b_i} - (q_e - t_y)e_{yi} + \pi_e c_i \\
\Longleftrightarrow e_{xi} + e_{yi} &= \frac{e_{yi}^{b_i} - (q_e - t_y)e_{yi} + \pi_e c_i}{q_e\frac{(1-\sigma_i)}{a_i\sigma_i} + (q_e - t_y)} + e_{yi} \\
&= \frac{e_{yi}^{b_i} + \frac{q_e(1-\sigma_i)}{a_i\sigma_i}e_{yi} + \pi_e.c_i}{q_e\frac{(1-\sigma_i)}{a_i\sigma_i} + (q_e - t_y)}.
\end{aligned}
\tag{C2}
$$

The equations (B1), (B5) and (C2) now determine the equilibrium values of e_{xi}, e_{yi} for all i and π_e. Next, we insert $e_{yi} = \left(\frac{q_e}{b_i}\right)^{\frac{1}{b_i-1}}$ from (B5) in (C2) which yields

$$e_{xi} + e_{yi} = \frac{\left(\frac{q_e}{b_i}\right)^{\frac{b_i}{b_i-1}} + \frac{q_e(1-\sigma_i)}{a_i\sigma_i}\left(\frac{q_e}{b_i}\right)^{\frac{1}{b_i-1}} + \pi_e c_i}{q_e\frac{(1-\sigma_i)}{a_i\sigma_i} + (q_e - t_y)}, \tag{C3}$$

which in turn is inserted into (B1) to obtain

$$\sum_i \frac{\left(\frac{q_e}{b_i}\right)^{\frac{b_i}{1-b_i}} + \frac{q_e(1-\sigma_i)}{a_i\sigma_i}\left(\frac{q_e}{b_i}\right)^{\frac{1}{1-b_i}} + \pi_e c_i}{q_e\frac{(1-\sigma_i)}{a_i\sigma_i} + (q_e - t_y)} = \sum_i c_i. \tag{C4}$$

Equation (C4) implicitly determines π_e as a function of t_y, formally $\pi_e = \pi_e(t_y)$. Then we can compute $e_{yi}(t_y)$ from (B5), $e_{xi}(t_y)$ from (B4), $\Delta e_{yi}(t_y)$ from (C3), $x_i(t_y)$ from $x_i = X^i(e_{xi})$, $y_i(t_y)$ from (B6), $z_i(t_y)$ from (B7) and $u_i(t_y)$ from $u_i = x_i^{h_i} y_i^{1-h_i}$.

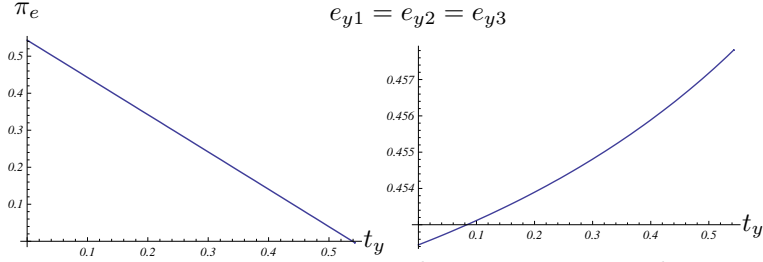The missing graphs of the numerical example are presented in the Figures 4-8.
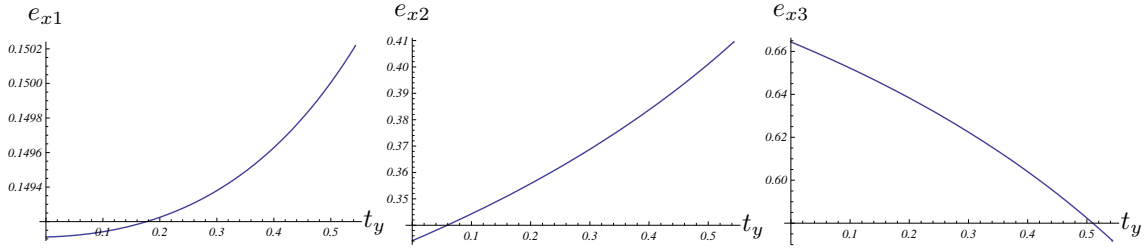
Figure 4: The permit price, the emissions in sector Y
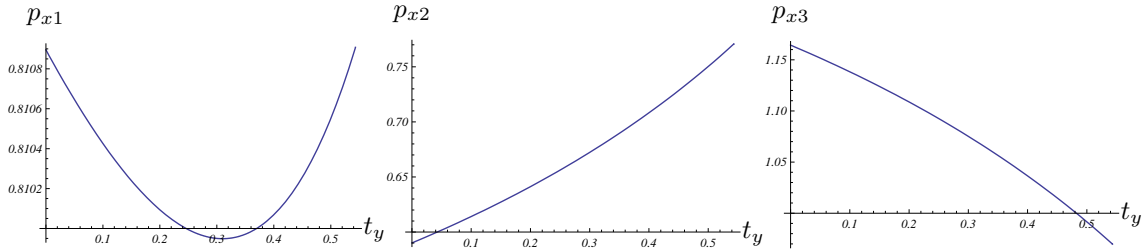

Figure 5: The emissions in sector X

Figure 6: The domestic price of good X

	$i = 1$		$i = 2$		$i = 3$	
	$t_y = 0$	$\pi_e = 0$	$t_y = 0$	$\pi_e = 0$	$t_y = 0$	$\pi_e = 0$
		$\bar{t}_y$		$\bar{t}_y$		$\bar{t}_y$
p_{xi}	0.811	0.811	0.590	0.771	0.149	0.970
e_{xi}	0.149	0.150	0.334	0.409	0.664	0.572
e_{yi}	0.452	0.458	0.452	0.458	0.452	0.458
Δe_{yi}	0.003	-0.003	-0.186	-0.267	0.318	0.270
x_i	0.683	0.684	0.518	0.585	0.692	0.605
y_i	0.554	0.555	0.414	0.504	0.549	0.470
z_i	1.108	1.110	0.828	1.009	1.097	0.939
u_i	0.615	0.533	1.097	0.543	0.616	0.811

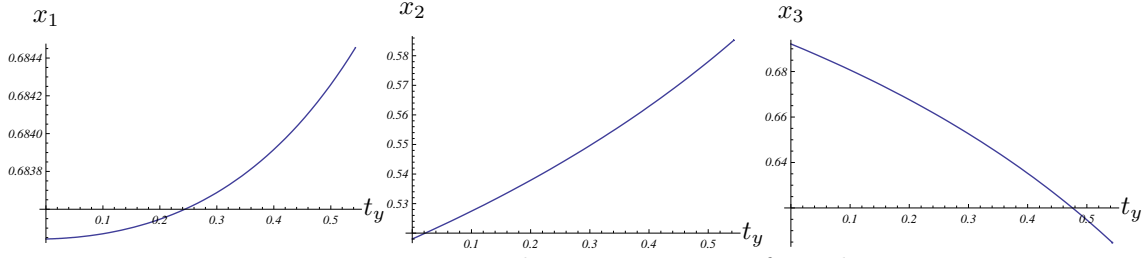Table 6: Numerical values for $t_y = 0$, $\pi_e = 0$
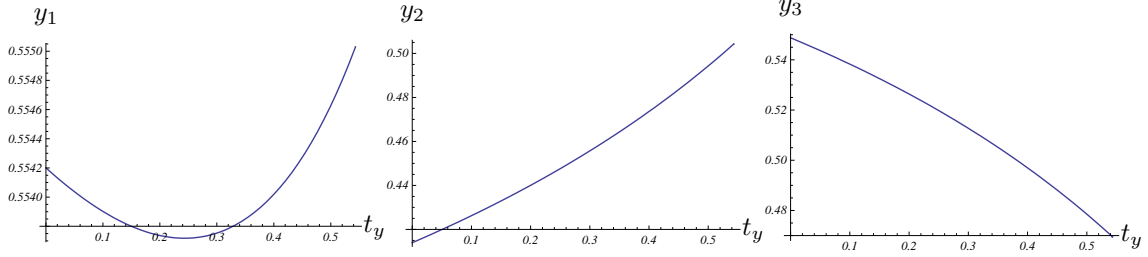
Figure 7: The consumption of good X

Figure 8: The consumption of good Y

Appendix D: Proof of Proposition 6

The cost-effective competitive equilibrium of the two-sector economy with domestic ETS is described by $c_i = e_{xi} + e_{yi}$ and (A2) - (A10). This equation system can be compressed to $c_i = e_{xi} + e_{yi}$ and (A12) - (A16). Total differentiation yields

$$\mathrm{d}e_{xi} = -\mathrm{d}e_{yi}, \tag{D1}$$

$$X_e^i \mathrm{d}e_{xi} - D_p^i \mathrm{d}p_{xi} - D_z^i \mathrm{d}z_i = 0, \tag{D2}$$

$$\mathrm{d}z_i = x_i \mathrm{d}p_{xi}, \tag{D3}$$

$$X_e^i \mathrm{d}p_{xi} + p_{xi} X_{ee}^i \mathrm{d}e_{xi} - Y_{ee}^i \mathrm{d}e_{yi} = 0, \tag{D4}$$

$$Y_{ee}^i \mathrm{d}e_{yi} - \mathrm{d}\pi_e - \mathrm{d}t_y = 0. \tag{D5}$$

Making use of (D1) in (D4) yields

$$\mathrm{d}e_{xi} = -\frac{X_e^i}{p_{xi} X_{ee}^i + Y_{ee}^i} \mathrm{d}p_{xi}. \tag{D6}$$

Making use of (D3) in (D2) we obtain

$$\mathrm{d}e_{xi} = \frac{D_p^i + x_i D_z^i}{X_e^i} \mathrm{d}p_{xi}. \tag{D7}$$

Since in general $-\frac{X_e^i}{p_{xi} X_{ee}^i + Y_{ee}^i} \neq \frac{D_p^i + x_i D_z^i}{X_e^i}$, (D6) and (D7) are satisfied if and only if $\mathrm{d}e_{xi} = \mathrm{d}p_{xi} = 0$. With that observation we infer $\mathrm{d}e_{yi} = \mathrm{d}z_i = 0$ and $\mathrm{d}\pi_e = -\mathrm{d}t_y$.

Appendix E: Comparative statics of one-sector economies

For given prices p_e and p_y and for given emissions tax rate t_y the cost-effective competitive equilibrium is determined by

$$\sum_j c_{yj} = \sum_j e_{yj}, \tag{E1}$$

$$z_i = p_y y_{si} + \pi_e \Delta e_i - p_e e_{yi}, \qquad i = 1, \ldots, n, \tag{E2}$$

$$y_{si} = Y^i(e_{yi}), \qquad i = 1, \ldots, n, \tag{E3}$$

$$z_i = p_y y_i, \qquad i = 1, \ldots, n, \tag{E4}$$

$$Y_e^i = p_e + \pi_e + t_y, \qquad i = 1, \ldots, n, \tag{E5}$$

where $\Delta e_{yi} := c_{yi} - e_{yi}$ is the amount of permits exported or imported by country i. In the following we carry out a comparative static analysis to specify the impact of exogenous variations in the uniform tax rate t_y. Without loss of generality we set $p_y \equiv 1$. Total differentiation of (E1) - (E5) yields

$$0 = \sum_j \mathrm{d}e_{yj}, \tag{E6}$$

$$\mathrm{d}z_i = \mathrm{d}y_{si} + \mathrm{d}\pi_e \Delta e_i - (p_e + \pi_e)\mathrm{d}e_{yi}, \tag{E7}$$

$$\mathrm{d}y_{si} = Y_e^i \mathrm{d}e_{yi}, \tag{E8}$$

$$\mathrm{d}z_i = \mathrm{d}y_i, \tag{E9}$$

$$Y_{ee}^i \mathrm{d}e_{yi} = \mathrm{d}\pi_e + \mathrm{d}t_y. \tag{E10}$$

Inserting (E10) into (E6) we obtain

$$(\mathrm{d}\pi_e + dt_y)\sum_j \frac{1}{Y_{ee}^i} = 0 \quad \Longleftrightarrow \quad \frac{\mathrm{d}\pi_e}{\mathrm{d}t_y} = -1. \tag{E11}$$

We make use of (E11) in (E10) to get

$$\frac{\mathrm{d}e_{yi}}{\mathrm{d}t_y} = 0 \tag{E12}$$

and from (E8) we infer

$$\frac{\mathrm{d}y_{si}}{\mathrm{d}t_y} = 0. \tag{E13}$$

Next, we take advantage of (E11), (E12) and (E13) to rewrite (E7) as

$$\frac{\mathrm{d}z_i}{\mathrm{d}t_y} = -\Delta e_{yi}. \tag{E14}$$

Finally, we insert (E14) into (E9) to get

$$\frac{\mathrm{d}y_i}{\mathrm{d}t_y} = -\Delta e_{yi}, \tag{E15}$$

which in turn is inserted into $\mathrm{d}u_i = U_y^i \mathrm{d}y_i$ to establish

$$\frac{\mathrm{d}u_i}{\mathrm{d}t_y} = -U_y^i \Delta e_{yi}. \tag{E16}$$