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Efficient management of insecure fossil fuel imports through taxing (!) domestic green energy?[*]

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Abstract

A small open economy produces a consumer good along with green and black energy and imports fossil fuel for black-energy production at an uncertain world market price. Efficient risk management requires curbing fuel consumption, and hence carbon emissions, when consumers are prudent. Moreover, if consumer preferences display constant absolute risk aversion (implying prudence), an efficient response to increasing risk is promoting green energy and reducing total energy production. Unregulated competitive markets are inefficient when consumers are risk averse. With the plausible assumption of prudent consumers and risk neutral producers, taxing both fossil fuel and green energy restores efficiency.

JEL-classification: F18, Q42, Q48
Keywords: Price uncertainty, black energy, green energy, fossil fuel

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1 Introduction

Many countries, notably the Annex I countries of the Kyoto Protocol, take action to curb carbon dioxide emissions, and many OECD countries also have policies to promote energy from renewable energy sources (OECD/IEA 2008). From the economists' perspective, such regulation is warranted, if it serves to correct for significant market imperfections. While the case is strong for using carbon cutting policies to cope with global change externalities, the economic rationale for supporting the (domestic) production of green energy is less clear. The theoretical *economic* literature on green energy support focuses on learning-by-doing and technological spillovers (e.g. Fischer and Newell 2008, Fischer 2008) as well as on externalities combined with various other imperfections such as imperfect property rights or information (e.g. Bennear and Stavins 2007). Yet there appears to be little agreement on whether such market imperfections are empirically relevant enough to provide a convincing rationale for promoting green energy.

Nonetheless, the *political* support for promoting green energy is still strong in many countries, if not growing. The reasons policymakers put forward for that support tend to differ from the economists' arguments alluded to above. For example, in the recently amended German Renewable Energies Act, the purpose of that act is described as the sustainable development of energy provision especially in the interest of using fossil resources carefully and *reducing the dependence from energy imports*.[1] The European Commission (Com 2007) acknowledges serious energy challenges concerning security of supply and import dependence and argues that the promotion of renewable energies plays a part in securing energy supply. The EU Renewable Energies Roadmap aims at enabling the EU to meet the 'twin objectives' of increasing security of energy supply and reducing greenhouse gas emissions.

As for the objective of fighting global change, green energy promotion as well as emissions reduction schemes clearly curb emissions and thus both of them contribute to climate stabilization. However, there is ample evidence and theoretical support for the proposition that promoting green energy is less cost-effective as a means of fighting climate change than the reduction of carbon emissions through instruments targeting those emissions directly. Consequently, if fighting global change is considered the only political goal, there is no role for green energy promotion.[2] But in the present paper we will consider, as many policymak-

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[1] Federal Government of Germany/Bundesregierung (2008), Gesetz zur Förderung erneuerbarer Energien im Wärmebereich, Bundesgesetzblatt Jg. 2008 Teil I Nr. 36 vom 18.8.2008.

[2] In a report to the German Federal Ministry of Affairs in 2004 the scientific council to that ministry recommended discontinuing the promotion of green energy in Germany on the grounds that the introduction of the European emissions trading scheme has turned the promotion of green energy into an ecologically

ers do, energy security as a political goal in its own right (in countries that heavily depend on the import of fossil energy resources). It is then clear that this goal is also promoted by both types of instruments, i.e. by green energy promotion as well as by emissions reduction schemes. Yet the decisive questions are whether the degree of energy security is inefficiently low in the absence of regulation and if so which instrument is more effective in correcting for that inefficiency. If supporting green energy should turn out to be necessary for efficient risk management, one would have a theoretical foundation for the observed green energy promotion with a rationale different from fighting global change and from other reasons mentioned above.

The present paper aims at exploring the role and effectiveness of curbing emissions and promoting green energy as alternative or joint instruments for the efficient management of risk from energy insecurity in countries that depend on the fossil fuel imports. To our knowledge that issue has not yet been addressed in the analytical literature which is remarkable given the prominence policymakers assign to the energy security goal and their confidence that green energy needs to be promoted for improving energy security. A key feature of our analytical approach will be uncertainty with respect to the price of imported fossil fuels. Among the various reasons for such uncertainties are political instability in fuels exporting countries, market power or cartels of these countries and perhaps sharp price fluctuations due to large-scale speculation.[3]

To tackle the implications of fossil fuel price uncertainty we consider a small open economy which imports fossil fuel at an uncertain price to produce black energy. In addition to black energy the economy produces green energy. The first part of the paper studies how the efficient allocation changes when the price risk increases. Following Feder, Just and Schmitz (1977) we assume that the social planner makes all decisions on production, consumption and trade before the uncertainty about the fossil fuel price is resolved. Turning to the competitive market economy the producer of black energy now faces input price uncertainty.[4] Again, decisions (now producers' and consumers') on production, consumption and trade are made before the true value of the international fossil fuel price is known (see also Batra and Russel 1974). In doing so, we implicitly assume there does not exist a future market for the input and hence there is no hedging opportunity for producers.

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useless and economically expensive instrument.

[3] Outstanding empirical examples of such price uncertainty (and volatility) are the massive supply-side induced oil price shocks in the 1970s. Quantity uncertainty, i.e. the risk of delivery falling short of ordered fossil fuel imports (which currently appears to exist, e.g., with respect to Russian natural gas exports) is another aspect of energy insecurity. However, with fully flexible prices quantity uncertainty necessarily translates into price uncertainty. The present paper focuses on flexible prices.

[4] Our treatment of the competitive firm under price uncertainty goes back to Sandmo (1971) and Batra and Ullah (1974).

When economic agents, or the social planner, make decisions under uncertainty, the resultant allocation depends on the agents' attitude toward risk. We will focus on risk aversion and risk neutrality in alternative scenarios and show that it crucially depends on the assumptions regarding the agents' risk attitutes whether taxes or subsidies on imported fossil fuels and/or on domestic green production are effective means of risk management. In general, regulation of black and green energy is necessary for efficient risk management implying allocative inefficiency in the absence of regulation. More specifically, we show that increases in the price risk induce the social planner to reduce black energy production, if she is prudent. She will also increase green energy production and decrease total energy production, if she is constant absolute risk averse. There is some literature and evidence, which considers as plausible the assumptions of producers being risk neutral and consumers being risk averse and prudent. If we take that scenario as the most relevant one, the striking message of the present paper is that efficient risk management requires

(i) curbing fossil fuel imports (and thus curbing carbon emissions) and

(ii) taxing (!) rather than subsidizing green energy production.

Hence policies of reducing the use of imported fossil fuels promote two ends, fighting climate change and promoting energy security. In contrast, supporting green energy is counterproductive with regard to both ends. Even worse, (in the scenario under consideration) efficient risk management requires to discourage green energy production.

The paper is organized as follows. Section 2 outlines the model. In Section 3 we derive the properties of the efficient allocation and present the comparative statics of the price risk on the efficient allocation. Section 4 investigates various corrective tax-subsidy schemes in case competitive markets operate efficiently. Section 5 provides some concluding remarks.

2 The model

Consider the economy of a small open country that generates energy according to

$$b = B(e), \qquad g = G(r_g) \quad \text{and} \quad z = b + g. \tag{1}$$

Fossil fuel, e, is used as an input in the production of 'black' energy b. 'Green' energy, g, is produced by means of the domestic (composite) production factor r_g. Both kinds of energy, b and g, are perfect substitutes. In addition to energy the country produces the amount

$$x = X(r_x) \tag{2}$$

of some (composite) consumption good X with input r_x. The production functions B, G and X are increasing and strictly concave. All fossil fuel needs to be imported at uncertain world market price $p_e + q$. The price p_e is constant, whereas q is a risky mark-up representing a random variable with support $[0, \infty[$, with mean $\mu_q \geq 0$ and with standard deviation $\sigma_q \geq 0$. The country pays for its imports with the revenues from exporting good X that is traded at the constant world market price $p_x \equiv 1$. The trade balance reads

$$x - x_d - (p_e + q)e = 0, \tag{3}$$

where x_d denotes the domestic consumption of good X. Since the trade balance contains the random variable q, the consumption of good X, x_d, then turns out to be a random variable with the moments

$$\mu_x = x - (p_e + \mu_q)e \quad \text{and} \quad \sigma_x = \sigma_q e. \tag{4}$$

Supply and demand match for both capital and energy

$$r_g + r_x = \bar{r} \quad \text{and} \quad z = z_d, \tag{5}$$

where $\bar{r}$ denotes the country's endowment of the production factor and z_d is the domestic consumption of energy. The model is closed by introducing the representative consumer's utility function

$$u = \tilde{U}(x_d, z_d) = \mathcal{V}(x_d) + \mathcal{U}(z_d). \tag{6}$$

In (6), the consumer derives utility from consumption of good X and from consumption of energy. The subutility functions $\mathcal{V}$ and $\mathcal{U}$ are increasing in their argument and concave. Since the set of distributions of the random variable x_d implied by (4) forms a linear class, expected utility and mean-variance preferences are perfect substitutes (Meyer 1987). It follows that any given von Neumann-Morgenstern function $\mathcal{V}$ can be represented in terms of mean-variance preferences without loss of generality. Therefore, we write the expected utility from the random variable x_d in terms of mean μ_x and standard deviation σ_x as

$$\mathbf{E}\mathcal{V}(x_d) = \int_a^b \mathcal{V}(\mu_x + \sigma_x n) \mathrm{d}F(n) =: V(\mu_x, \sigma_x), \tag{7}$$

where a and b define the interval containing the support of the standardized random variable n, and F is the distribution function of n. Due to that standardization, the mean and the standard deviation of n are, respectively, zero and one. Denoting by $A\left(x_d\right) := -\frac{\mathcal{V}_{xx}(x_d)}{\mathcal{V}_x(x_d)}$ the Arrow-Pratt measure of absolute risk aversion and by $M(\mu_x, \sigma_x) := -\frac{V_\sigma(\mu_x, \sigma_x)}{V_\mu(\mu_x, \sigma_x)}$ the marginal rate of substitution between μ_x and σ_x, Meyer (1987) and Lajeri and Nielsen

(2000) have shown that identity (7) gives rise to the following equivalences between von Neumann-Morgenstern utility functions and two-parameter functions[5]:

$$\begin{aligned}
\mathcal{V}_x(x_d) > 0 \quad &\Longleftrightarrow \quad V_\mu(\mu_x, \sigma_x) > 0, && \text{(8a)}\\
\mathcal{V}_{xx}(x_d) < 0 \quad &\Longleftrightarrow \quad V_\sigma(\mu_x, \sigma_x) < 0, && \text{(8b)}\\
&\Longleftrightarrow \quad V_{\mu\mu} < 0, V_{\sigma\sigma} < 0, V_{\mu\mu}V_{\sigma\sigma} - V_{\mu\sigma}^2 > 0, && \text{(8c)}\\
\mathcal{V}_{xxx}(x_d) \gtreqless 0 \quad &\Longleftrightarrow \quad V_{\mu\sigma} \gtreqless 0, && \text{(8d)}\\
A_x(x_d) \gtreqless 0 \quad &\Longleftrightarrow \quad M_\mu \gtreqless 0 && \text{(8e)}
\end{aligned}$$

for all μ_x and $\sigma_x \geq 0$. (8b) reflects risk aversion which also corresponds to the concavity of $V(\mu_x, \sigma_x)$, see (8c). Following Kimball (1990) we call an agent prudent [imprudent] if and only if her preferences display $\mathcal{V}_{xxx} > [<]0$. In view of (8d) and as identified by Lajeri and Nielsen (2000) prudence translates into $V_{\mu\sigma} > 0$ for mean-variance preferences. Finally, an agent is said to be decreasing [increasing] absolute risk averse if her mean-variance preferences exhibit $M_\mu < [>]0$. Decreasing absolute risk aversion (DARA) and prudence are related as follows:

$$M_\mu = -\frac{V_{\sigma\mu}V_\mu - V_\sigma V_{\mu\mu}}{V_\mu^2} < 0 \qquad \Longleftrightarrow \qquad -\frac{V_{\mu\sigma}}{V_{\mu\mu}} > -\frac{V_\sigma}{V_\mu}. \tag{9}$$

In case of risk neutrality, the von Neumann-Morgenstern utility function is linear, and it is straightforward to show

$$\mathcal{V}_{xx} = 0 \quad \Longleftrightarrow \quad V_\sigma = V_{\mu\sigma} = V_{\mu\mu} = V_{\sigma\sigma} \equiv 0. \tag{10}$$

3 The efficient allocation

Consider a benevolent planner who maximizes the representative consumer's expected utility

$$\mathbf{E}\tilde{U}(x_d, z_d) \equiv V(\mu_x, \sigma_x) + \mathcal{U}(z_d)$$

subject to (1), (2), (4), (5). Solving the associated Lagrangian

$$\begin{aligned}
\mathcal{L} \;=\; & V(\mu_x, \sigma_x) + \mathcal{U}(z_d) + \lambda_r(\bar{r} - r_g - r_x) + \lambda_z\left[B(e) + G(r_g) - z_d\right]\\
+\; & \lambda_\mu\left[X(r_x) - (p_e + \mu_q)e - \mu_x\right] + \lambda_\sigma(\sigma_q e - \sigma_x)
\end{aligned} \tag{11}$$

yields the first-order conditions listed in the first column of Table 1.

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[5] For notational convenience we suppress the arguments of the function $V(\mu_x, \sigma_x)$ when there is no risk of confusion.

		Pareto efficiency 1	Markets 2
Consumption	1	$\frac{V_\sigma(\mu_x^*,\sigma_x^*)}{V_\mu(\mu_x^*,\sigma_x^*)} = \varphi_\sigma$	
	2	$\frac{\mathcal{U}_z(z_d^*)}{V_\mu(\mu_x^*,\sigma_x^*)} = \varphi_z$	$\frac{\mathcal{U}_z(z_d^o)}{\mathcal{V}_x(x_d^o)} = p_z$
Production	3	$X_r(r_x^*) = \varphi_r$	$X_r(r_x^o) = p_r$
Energy	4	$\varphi_z G_r(r_g^*) = \varphi_r$	$(p_z - s)G_r(r_g^o) = p_r$
Production	5	$\varphi_z B_e(e^*) + \varphi_\sigma \sigma_q = p_e + \mu_q$	$p_z B_e(e^o) + \frac{W_\sigma(\mu_\pi,\sigma_\pi)}{W_\mu(\mu_\pi,\sigma_\pi)}\sigma_q = p_e + \mu_q + t$

Table 1: Efficiency and markets with price uncertainty
(Notation: $\varphi_z = \lambda_z/\lambda_\mu$ and $\varphi_r = \lambda_r/\lambda_x$)

Combined with (1), (2), (4), (5), the first-order conditions determine the efficient allocation $(e^*, b^*, g^*, r_g^*, r_x^*, x^*, \mu_x^*, \sigma_x^*, z^*, z_d^*)$. The terms in column 1 can be rearranged to read

$$\frac{X_r^*}{G_r^*} = \frac{\mathcal{U}_z^*}{V_\mu^*} = \frac{p_e + \mu_q}{B_e^*} - \frac{V_\sigma^* \sigma_q}{V_\mu^* B_e^*}. \tag{12}$$

The first term in (12) is the marginal rate of transformation between x_s and g. Since $x_s^* = X(r_x^*) = X(\bar{r} - r_g^*) = X[\bar{r} - G^{-1}(g^*)]$, the value of X_r^*/G_r^* uniquely determines x_s^* and g^*. Suppose (12) (with stars attached) represents the optimality condition for $\sigma_q > 0$ and denote by

$$\frac{X_r^n}{G_r^n} = \frac{\mathcal{U}_z^n}{V_\mu^n} = \frac{p_e + \mu_q}{B_e^n}$$

the optimality condition in the absence of risk ($\sigma_q = 0$). X_r^n/G_r^n clearly determines x_s^n and g^n which gives rise to the question whether g^* is greater or smaller than g^n. Simple calculations show that

$$g^* \gtreqless g^n \iff \frac{X_r^*}{G_r^*} \gtreqless \frac{X_r^n}{G_r^n} \iff B_e(e^n) - B_e(e^*) \gtreqless \frac{V_\sigma^* \sigma_q B_e(e^n)}{(p_e + \mu_q)V_\mu^*}(< 0). \tag{13}$$

Hence there is $\tilde{e} < e^n$ such that $g^* \gtreqless g^n \iff e^* \gtreqless \tilde{e}$. In other words, as long as in the transition from $\sigma_q = 0$ to $\sigma_q > 0$ the reduction in the use of fossil fuel is not to strong it is optimal to produce more green energy under uncertainty than under certainty. Yet we cannot infer from (13) whether e is decreasing and g is increasing in risk. These mappings may as well be non-monotone because general equilibrium effects need to be accounted for and the sign and size of second derivatives of the utility function V may play a role. To further clarify the impact of risk on the optimal allocation we carry out a full-scale comparative-static analysis (Appendix A) and report the results in

Proposition 1. *If the efficient allocation of the model (1), (2), (4)-(6) is disturbed by a small variation in the risk parameter* σ_q,[6] *the direction of change in the efficient values* e, r_g, r_x, x *is as shown in Table 2. The direction of change in all other variables is ambiguous.*

	$\frac{\mathrm{d}b}{\mathrm{d}\sigma_q}, \frac{\mathrm{d}e}{\mathrm{d}\sigma_q}$	$\frac{\mathrm{d}g}{\mathrm{d}\sigma_q}, \frac{\mathrm{d}r_g}{\mathrm{d}\sigma_q}$	$\frac{\mathrm{d}x}{\mathrm{d}\sigma_q}, \frac{\mathrm{d}r_x}{\mathrm{d}\sigma_q}$	$\frac{\mathrm{d}z}{\mathrm{d}\sigma_q}$
$V_{\mu\sigma} \geq 0$	$-$	?	?	?
$-\frac{V_{\mu\sigma}}{V_{\mu\mu}} \geq -\frac{V_\sigma}{V_\mu} \geq \frac{1}{\varepsilon} \cdot \left(-\frac{V_{\mu\sigma}}{V_{\mu\mu}}\right)$	$-$	$+$	$-$	?

Table 2: Impact of variations in risk on the efficient allocation
(Notation: $\varepsilon := -\frac{B_e}{eB_{ee}} > 0$)

To interprete Proposition 1 note first that all results reported in Table 2 refer to the case of risk aversion and prudence ($V_{\mu\sigma} > 0$). Our focus on prudence is warranted because empirical evidence (Charas and Holt 1996, Guiso et al. 1996) and experimental evidence (Binswanger 1981, Levy 1994) suggest that utility functions are decreasing absolute risk averse ($M_\mu < 0$) which in turn implies prudence ($V_{\mu\sigma} > 0$).[7] Unfortunately, in Proposition 1 the only clear-cut and intuition-conforming information about an efficient response to increasing risk is that fuel imports need to be reduced. As the change in the provision of green energy can assume either sign we get no answer to our central question whether expanding green energy is an efficient response to increasing risk.[8] Under additional sufficient conditions listed in the second row of Table 2 we attain the clear result $\mathrm{d}g/\mathrm{d}\sigma_q > 0$. These conditions do not seem to be very restrictive. In view of (9) the first inequality $-\frac{V_{\mu\sigma}}{V_{\mu\mu}} > -\frac{V_\sigma}{V_\mu}$ turns out to be DARA which is not a controversial assumption in the pertaining literature. The second inequality, $-\frac{V_\sigma}{V_\mu} \cdot \varepsilon > \left(-\frac{V_{\mu\sigma}}{V_{\mu\mu}}\right)$, is satisfied if the price elasticity of demand for black energy is sufficiently large, i.e. if the production function $B(e)$ has little curvature. A large value of ε can be considered an approximation to linear cost functions (with setup costs) of power plants, an assumption that is not uncommon in the energy economics literature. The observation that $\mathrm{d}g/\mathrm{d}\sigma_q > 0$ for sufficiently large ε nicely reconfirms the last inequality in (13). We know from (13) that the difference $B_e(e^n) - B_e(e^*)$ tends to zero for $\varepsilon \to \infty$ and hence renders positive the difference $g^* - g^n$.

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[6] Increasing the standard deviation σ_q is equivalent to a mean preserving spread of the random variable q.

[7] There is also a strong theoretical argument for prudence. Menegatti (2001) has proven that $\mathcal{V}_x > 0$, $\mathcal{V}_{xx} < 0$ and sign $\mathcal{V}_{xxx}$ being the same for all $x_d \geq 0$ is sufficient for $\mathcal{V}_{xxx} > 0$ for all $x_d \geq 0$.

[8] It is interesting to note that even if $\mathrm{d}q/\mathrm{d}\sigma_q < 0$ in case of $V_{\mu\sigma} \geq 0$, the ratio of green to black energy, g/b, will increase if and only if $|\mathrm{d}g/\mathrm{d}\sigma_q| < |\mathrm{d}b/\mathrm{d}\sigma_q|$. Changing the composition of total energy in favor of green energy can then be considered as an expansion of green energy in *relative* rather than in *absolute* terms.

For the sake of more specific results we parametrize the utility function and the production function by

$$\mathcal{V}(x_d) = -\frac{1}{a} \exp^{-ax_d}, \tag{14}$$

$$B(e) = e^{\theta}, \tag{15}$$

where $a > 0$ and $\theta \in]0,1[$. The utility function (14) belongs to the class of hyperbolic absolute risk averse functions and displays constant absolute risk aversion (CARA). Since utility functions of type (14) are mathematically convenient representations and simplify comparative static analyses considerably, they are the most commonly used functional forms in the expected utility approach (for applications see Cass and Stiglitz 1970, Hens et al. 2002 or Gollier and Schlesinger 2003). Wagener (2005) shows that the utility function (14) translates into the mean-variance utility function

$$V(\mu, \sigma) = -\frac{1}{a} H(\sigma) \exp^{-a\mu}, \tag{16}$$

with $H(0) = 0$, and $H_\sigma > 0$ for all $\sigma > 0$. It is worth mentioning that prudence is not only necessary for DARA functions but also for CARA functions (16). Hence the result $\mathrm{d}e/\mathrm{d}\sigma_q < 0$ from Table 2 is valid for CARA functions. In addition, Appendix B proves:

Proposition 2. *Suppose the mean-variance utility function $V(\mu, \sigma)$ is speciefied by (16) and the production function $B(e)$ is specified by (15). If the efficient allocation of the model (1), (2), (4)-(6) is disturbed by a small variation in the risk parameter σ_q, then the efficient response is*

(i) to reduce black energy production b,

(ii) to increase green energy production g,

(iii) to reduce total energy consumption z and

(iv) to reduce consumer good consumption x.

Under the conditions of Proposition 2 that are slightly more restrictive than those of Proposition 1 an efficient response to increasing energy insecurity consists in curbing black as well as total energy while expanding green energy. That involves a shift in the composition of total energy toward green energy which we have already identified in Proposition 1 under the conditions of the second row of Table 2. The observation that the use of fossil fuel is monotone decreasing in risk under conditions of both Propositions 1 *and* Proposition 2 suggests that this result appears to be quite robust.

Having characterized the social planner's efficient solution as a benchmark we will now turn to the decentralized economy with perfectly competitive markets for the consumption good, the resource and for energy. The government has at its disposal two instruments whose rates are not sign-constrained to regulate fossil-fuel use and/or green-energy production. In the remainder of the paper we seek to answer the following questions:

(i) Does the allocation of the no-tax competitive equilibrium deviate from the social planner's solution?

(ii) If it deviates, is it possible to characterize corrective tax-subsidy policies?

4 The competitive economy and corrective taxation

To prepare for tackling these core questions we first need to specify the competitive economy with fossil fuel price uncertainty and taxation. Then we present the main result of decentralizing the efficient allocation by prices and taxes. Next we assess the capacity of green subsidies and black taxes as means to cope with energy insecurity. Finally, we infer backward from the competitive equilibrium with corrective taxes the characteristics of market failure in the absence of taxation.

We denote the market prices associated to the perfectly competitive markets for the consumption good, the resource and for energy by $p_x \equiv 1, p_r$ and p_z, respectively. The government has at its disposal tax policies[9] (s, t) where s is the rate of a tax on green energy production and t is the rate of a tax on fossil fuel input;[10] both rates are unconstrained in sign. In this setup, the profits of the three industries are given by

$$\pi_g = (p_z - s)G(r_g) - p_r r_g, \tag{17a}$$

$$\pi_x = X(r_x) - p_r r_x, \tag{17b}$$

and $\pi_b = p_z B(e) - (p_e + q + t)e$, respectively.

Inspection of the profits π_g, π_x and π_b reveals that it is the producer of black energy who is exposed to and has to cope with price uncertainty while the other producers and the consumer are not subject to any uncertainty.[11] Hence the profit of the producer of black energy becomes a random variable such that she needs to determine her production plan under input price uncertainty. However, her (ex ante) supply of black energy is deterministic

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[9] We refrain from considering other taxes such as a tax on total energy consumption and a tax on black energy production because they would complicate the analysis without providing additional insights.

[10] In our simple model, this tax is equal to an import tariff as well as a carbon emissions tax.

[11] Note the decisive difference between the risk management of the social planner and of the agents in the market economy. The former does not account for (domestic) markets and profits and thus rightly identifies the consumption of good X as a random variable derived from the price uncertainty in the trade balance (see Section 3).

which means that the uncertainty is not passed on to the consumer. As will be shown below, that difference in risk management of the social planner and the agents in the market economy will lead to market failure which will then give rise to the question whether suitable taxes and/or subsidies are available to correct for those failures.

The manager of the black energy firm is assumed to be either risk neutral or risk averse. Her preferences are represented by the two-moment utility function $W(\mu_\pi, \sigma_\pi)$, with the function W posessing the same properties as the function V in (8a)-(8e) and (10). The manager's decision problem is

$$\begin{aligned} \max_e W(\mu_\pi, \sigma_\pi) \quad \text{s.t.} \quad \mu_\pi &= p_z B(e) - (p_e + \mu_q + t)e, \\ \sigma_\pi &= \sigma_q e. \end{aligned} \tag{18}$$

For any tax policy (s, t), a *competitive ex ante equilibrium* of the economy (1) - (3) and (5) is attained if the prices p_r and p_z are market clearing, if firms maximize profits (17a), (17b), (18), and if the representative consumer maximizes her utility (6) subject to the budget constraint[12]

$$\phi + p_r \bar{r} = p_z z_d + x_d, \tag{19}$$

where $\phi := \mu_\pi^o + \pi_g^o + \pi_x^o + te + sg$ is a lumpsum transfer of profits and net tax revenues to the consumer. $\mu_\pi^o + \pi_g^o$ and π_x^o denote maximum profits. The first-order conditions listed in the second column of Table 1 determine the equilibrium allocation $(e^o, b^o, g^o, r_g^o, r_x^o, x^o, z^o, z_d^o)$ for some predetermined tax policy (s, t), where the superscript o indicates the market equilibrium. We now wish to determine that particular tax policy (s, t) which makes the corresponding equilibrium allocation coincide with the social planner's optimum. To that end we compare the columns 1 and 2 of Table 1 and obtain

Proposition 3. *A competitive ex ante equilibrium exists and the pertinent equilibrium allocation is efficient, if the (endogenous) prices are given by*

$$p_r = \varphi_r \quad \textit{and} \quad p_z = \frac{V_\mu(\mu_x, \sigma_x)}{\mathcal{V}_x(x_d)} \varphi_z \tag{20}$$

and if the fiscal policy (s, t) satisfies

$$s = \frac{(V_\mu - \mathcal{V}_x)\varphi_z}{\mathcal{V}_x} \quad \textit{and} \quad t = sB_e + \left(\frac{W_\sigma}{W_\mu} - \frac{V_\sigma}{V_\mu} \right) \sigma_q. \tag{21}$$

In (20) - (21), $\varphi_r, \varphi_z, B_e, V_\mu$ and V_σ are evaluated at the solution of (11) and $\mathcal{V}_x, W_\sigma$ and W_μ are evaluated at the agents' optimal programs in the market economy.

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[12] Observe that (19) is implied by (1) - (4) and recall that the consumer acts under certainty.

In (20) - (21) the sign of the difference $V_\mu(\mu_x, \sigma_x) - \mathcal{V}_x(x_d)$ is important for the result as well as for the interpretation of Proposition 3. It is therefore useful to begin with investigating the determinants of that sign.

Recall that (7) links the mean-variance utility function $V(\mu_x, \sigma_x)$ and the von Neumann-Morgenstern utility function $\mathcal{V}(x_d)$. Differentiation of (7) with respect to μ_x yields

$$V_\mu(\mu_x, \sigma_x) = \int_a^b \mathcal{V}_x(\mu_x + \sigma_x n)\mathrm{d}F(n). \tag{22}$$

An immediate implication of (22) is $V_\mu(\mu_x, 0) = \mathcal{V}_x(\mu_x)$, which gives rise to

$$V_\mu(\mu_x, \sigma_x) \gtreqless \mathcal{V}_x(\mu_x) \quad \Longleftrightarrow \quad V_{\mu\sigma}(\mu_x, \sigma_x) \gtreqless 0. \tag{23}$$

for $\sigma_x > 0$. The right side of the equivalence (23) is linked, in turn, via (8d) to the concepts of prudence and imprudence as defined in our remarks on (8d) in Section 2.

The equivalence (23) and its relation to (8d) suggest to make more transparent the implications of Proposition 3 by distinguishing the consumer's (and hence the benevolent planner's) and the black energy producer's attitudes toward risk according to whether they are risk neutral ($V_\sigma = 0, W_\sigma = 0$) or risk avers ($W_\sigma < 0, V_\sigma < 0$) *and* - in the latter case - whether the consumer's von Neumann-Morgenstern utility function displays prudence ($V_{\mu\sigma} > 0$) or imprudence ($V_{\mu\sigma} < 0$). This distinction of preference attributes gives rise to the following three scenarios:[13]

Scenario 1: The consumer is risk neutral ($V_\sigma = 0$) and the black energy producer is risk avers ($W_\sigma < 0$) or risk neutral ($W_\sigma = 0$).

Scenario 2: The consumer is risk avers ($V_\sigma < 0$) and imprudent ($V_{\mu\sigma} < 0$) and the black energy producer is risk avers ($W_\sigma < 0$) or risk neutral ($W_\sigma = 0$).

Scenario 3: The consumer is risk avers ($V_\sigma < 0$) and prudent ($V_{\mu\sigma} > 0$) and the black energy producer is risk avers ($W_\sigma < 0$) or risk neutral ($W_\sigma = 0$).

Although these scenarios differ with respect to their empirical relevance,[14] we will explore the implications of each of them to see what drives the results. The issue of empirical relevance will be addressed later.

For Scenario 1, (21) readily yields the corrective policy

$$s = 0 \quad \text{and} \quad t = \frac{W_\sigma}{W_\mu}\sigma_q \leq 0.$$

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[13] Observe that the sign of $W_{\mu\sigma}$ is irrelevant for the qualitative results of Proposition 3.

[14] Recall our remarks following Proposition 1 in Section 3.

Note first that any regulation of green energy, taxing as well as subsidizing, would render the risk management inefficient in Scenario 1. If $W_\sigma < 0$, the efficient regulation consists of subsidizing (!) fossil fuel. At first glance that result may appear puzzling but its logic is straightforward. If society, represented by the consumer, is risk neutral and the producer is risk avers, the latter needs to receive an incentive in form of a subsidy to overcome her reluctance to take some risk in production. Curbing carbon emissions ($t > 0$) would reduce rather than enhance welfare.

Suppose next that $V_\sigma = W_\sigma = 0$, i.e. that both the consumer and the black energy producer are risk neutral. The straightforward implication is that $(s = 0, t = 0)$ is the optimal policy. No tax policy is needed at all to correct for allocative distortions because there is no such distortion. Although risk exists, the agents essentially behave as under certainty. Scenario 1 with $V_\sigma = W_\sigma = 0$ can therefore - and will later - be considered as the benchmark case of certainty. We conclude that in Scenario 1 neither curbing emissions via t nor promoting green energy via s can be rationalized as a means for enhancing energy security.

Consider next the Scenario 2 which requires

$$s = \frac{(V_\mu - \mathcal{V}_x)\varphi_z}{\mathcal{V}_x} < 0 \quad \text{and} \quad t = sB_e + \left(\frac{W_\sigma}{W_\mu} - \frac{V_\sigma}{V_\mu}\right)\sigma_q$$

as a corrective policy. In this case, promoting green energy ($s < 0$) is an appropriate means to cope with energy insecurity. To understand the rationale of that policy we first assume that $\left(\frac{W_\sigma}{W_\mu} - \frac{V_\sigma}{V_\mu}\right) = 0$. According to (23) for imprudent consumers ($V_{\mu\sigma} < 0$) the marginal utility of an additional unit of μ_x under uncertainty is lower than an additional unit of $x_\mathrm{d}(= \mu_x)$ under certainty, in formal terms $V_\mu(\mu_x, \sigma_x) < \mathcal{V}(\mu_x)$. With this information we infer from (20) that the market price p_z is lower than the associated shadow price φ_z. Comparing column 1 and 2 in rows 4 and 5, respectively, of Table 1 and accounting for $p_z < \varphi_z$ we conclude that both the producer of green energy and the producer of black energy receive in the market too weak price signals for producing energy if $s = t = 0$. This market failure is corrected by subsidizing green energy ($s < 0$) *and* subsidizing fossil fuel ($t < 0$). The green energy subsidy stimulates the production of green energy, while the fossil fuel subsidy fosters the production of black energy.

Suppose now that $\left(\frac{W_\sigma}{W_\mu} - \frac{V_\sigma}{V_\mu}\right) \neq 0$. This term introduces an additional effect caused by the difference in the consumer's and producer's risk aversion. If the consumer is more risk averse than the producer, fossil fuel use has to be taxed, ceteris paribus, since the producer is too lax in coping with risk. In contrast, if the consumer is less risk averse than the producer, the producer is too anxious dealing with the risk and fossil fuel use has to be subsidized. Therefore, the corrective tax rate can attain either sign irrespective of whether

$W_\sigma < 0$ or $W_\sigma = 0$. $t > 0$ is the more likely the greater is the consumer's as compared to the producer's risk aversion. $\left(\left|\frac{V_\sigma}{V_\mu}\right| > \left|\frac{W_\sigma}{W_\mu}\right|\right)$. In conclusion, in Scenario 2 green energy promotion ($s < 0$) is an indispensible instrument for coping with energy insecurity in an efficient way. Under certain conditions, this holds for emissions reduction policies ($t > 0$) as well but the case of welfare-enhancing fossil fuel subsidies cannot be ruled out.

Suppose finally, Scenario 3 prevails. In that scenario the policy (s, t) is corrective, if and only if

$$s = \frac{(V_\mu - \mathcal{V}_x)\varphi_z}{\mathcal{V}_x} > 0 \quad \text{and} \quad t = sB_e + \left(\frac{W_\sigma}{W_\mu} - \frac{V_\sigma}{V_\mu}\right)\sigma_q.$$

The striking result is that efficiency requires discouraging (i.e. taxing) green energy production rather than promoting (subsidizing) it. Using the same arguments as in Scenario 2 it is now straightforward to show that $p_z > \varphi_z$ for prudent consumers. Hence, if $\frac{W_\sigma}{W_\mu} - \frac{V_\sigma}{V_\mu} = 0$ both green energy and the fossil fuel use needs to be taxed in order to manage the risk in an efficient way. Accounting for $\frac{W_\sigma}{W_\mu} - \frac{V_\sigma}{V_\mu} \neq 0$, the efficient fuel tax rate is unambiguously positive, if the black energy producer is risk neutral. Otherwise it may be negative but only if the producer's risk aversion is sufficiently stronger than that of the consumer (which does not seem to be plausible).

We conclude that promoting green energy in Scenario 3 is not suitable as an instrument to cope with energy insecurity. It is even welfare reducing and therefore harmful. Moreover, except for cases of strongly risk avers black energy producers, taxing fuel is also a necessary instrument for efficient risk management.

	Instrument for efficient risk management	
	fossil fuel tax	green energy subsidy
Scenario 1	NO	NO
Scenario 2	NO*/YES	YES
Scenario 3	YES*/NO	NO

* under plausible conditions

Table 3: Assessment of instruments for risk management

Our preceding discussion of the Scenarios 1-3 and its summary in Table 3 show that the effectiveness of the tax instruments for an efficient risk management crucially depends on the agents' attitudes toward risk. The appropriate choice of instruments is therefore an empirical issue. Consumers use to be portrayed as being risk avers while producers are usually considered as risk neutral. If producers are risk averse they are likely less risk averse than consumers suggesting that $\left|\frac{W_\sigma}{W_\mu}\right| < \left|\frac{V_\sigma}{V_\mu}\right|$. Moreover, as we mentioned before, empirical as well as experimental studies suggest that preferences exhibiting DARA are realistic. Since DARA implies prudence, Scenario 3 appears to be more realistic than the other scenarios. We highlight that main result of our policy analysis in

Proposition 4. *Suppose consumers are prudent and more risk averse than producers. Then efficient risk management requires taxing both green energy and fossil fuels.*

Recall that in the Introduction of the present paper we started out on the intuition or conjecture that efficient management of risk from energy insecurity might turn out to be a rationale for subsidizing green energy. Subject to the qualification that the behavioral assumptions of Proposition 4 are empirically relevant we now find the contrary. Not only is green energy promotion ineffective as a means of coping with energy insecurity, it even renders inefficient the risk management.

The information on corrective regulation (s^*, t^*) we gained in Proposition 3 and the subsequent discussion of the Scenarios 1-3 leaves unanswered the question what the qualitative difference is between the no-policy allocation (e_o, g_o) and the efficient allocation (e^*, g^*). It is tempting to argue that $e_o \lesseqqgtr e^*$ if $t^* \gtreqqless 0$ and $g_o \lesseqqgtr g^*$ if $s^* \lesseqqgtr 0$. However, since both tax instruments have an impact on both fossil fuel consumption and the production of green energy, the 'backward inference' from (s^*, t^*) to $\text{sign}\,(e_o - e^*)$ and $\text{sign}\,(g_o - g^*)$ is not that simple. To see this, take the puzzling observation that efficiency requires taxing green energy in Proposition 4 while according to Proposition 2 and one part of Proposition 1 the efficient production of green energy is strictly increasing in risk. For resolving that seeming 'contradiction' we ease the exposition by restricting our attention to the black energy producer being risk neutral. If in that case the consumer is risk neutral as well we get the benchmark scenario (of risk neutral agents) which yields the same market allocation as in the absence of risk. (See our discussion of Scenario 1 above). That, in turn, allows us to draw on Pethig and Wittlich (2009) who analyze the model consisting of the equations (1)-(3), (5) and (6) *in the absence of uncertainty* and characterize the equilibrium values (e, g) for alternative policies $(s \leq 0, t \geq 0)$. They illustrate their result in a graph which we have reproduced here in Figure 1 and extended to include $s \geq 0$ and $t \leq 0$.

Point A in Figure 1 represents the levels of green energy, g_o, and fossil fuel e_o, in the no-policy competitive equilibrium $(s = 0, t = 0)$. If we keep s constant at $s = 0$ but successively increase t we move on the line AB from A toward B. During that move fuel consumption declines and the production of green energy increases. Alternatively if we keep t constant at $t = 0$ and successively increase $|s|$, where $s \leq 0$, we move on the line AC from A toward C, and we thus also curb the use of fuel and expand green energy. However, in the latter case the increase in green energy is larger and the emissions reduction is smaller

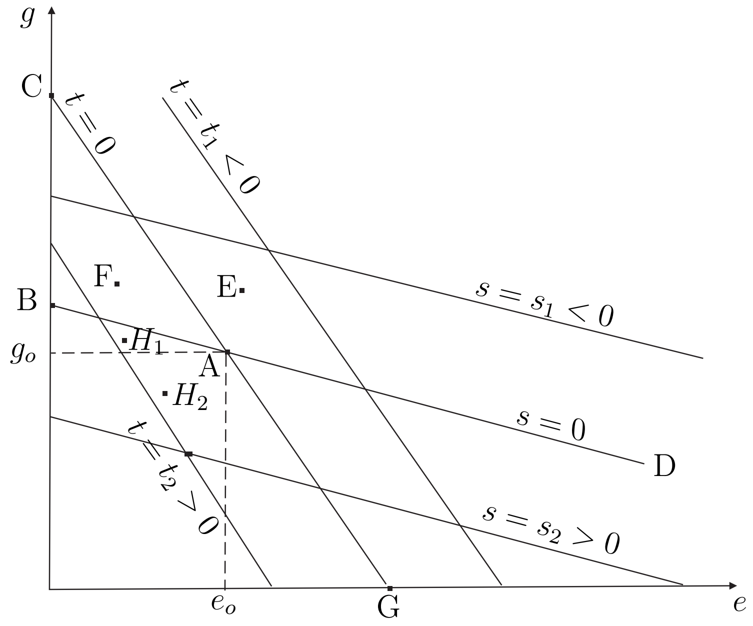

Figure 1: Allocations (e, g) of fossil fuel and green energy attained through policies (s, t)

than in the former case.[15] Thus the area ABC in Figure 1 is the set of all equilibrium allocations (e, g) attainable through tax policies $(s \leq 0, t \geq 0)$. Moreover, each point in that area is uniquely associated with a tuple $(s \leq 0, t \geq 0)$ that supports the corresponding competitive equilibrium.

Figure 1 is a convenient device to illustrate how the efficient allocation (e^*, g^*) deviates from the no-policy market allocation (e_o, g_o). Obviously, in Scenario 1 the efficient allocation coincides with the no-policy allocation in point A in Figure 1. If in Scenario 2 the corrective policy is $(s^* < 0, t^* < 0)$, the efficient allocation is a point in the area above the line CAD, e.g. point E. In this case we cannot exclude any divergence between (e_o, g_o) and (e^*, g^*) other than $e^* < e_0$ *and* $g^* < g_0$. If in Scenario 2 the corrective policy turns out to be $(s^* < 0, t^* > 0)$, the efficient allocation lies in the interior of the triangle ABC at a point such as F which implies $e^* < e_0$ and $g^* > g_0$. While the information on the divergence of (e_o, g_o) and (e^*, g^*) has been limited in the case $(s^* < 0, t^* < 0)$ of Scenario 2 we now have clear qualitative information on the kind of inefficiency of the unregulated economy. Thus in the scenario under consideration the information of Figure 1 is more specific than that from the 'marginal' comparative statics of increasing the risk σ_q presented in Appendix A.

Finally, we turn to risk averse and prudent consumers (Scenario 3). With producers being risk neutral, the corrective policy is characterized by $(s^* > 0, t^* > 0)$ and the efficient allocation is a point below the line BAG. Inspection of Figure 1 shows that we cannot exclude any divergence between (e_o, g_o) and (e^*, g^*) other than $e^* > e_o$ *and* $g^* > g_o$.

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[15] In other words, all lines in Figure 1 with constant t are steeper than the lines with constant s.

However, we know from Proposition 1 that in the transition from efficiency under certainty to efficiency under uncertainty the fossil fuel use decreases monotonely. Hence, the efficient tuple (e^*, g^*) can only be a point in the interior of the area $\mathrm{BA}e_o$, e.g. the point H_1 or H_2. As Proposition 1 shows the sign of $g_o - g^*$ remains unclear under risk aversion and prudence (first row of Table 2) so that we cannot discriminate between H_1 and H_2. However, with some further qualifications (second row of Table 2 and Proposition 2) we know that (e^*, g^*) is a point such as H_1 in the interior of the triangle $g_o\mathrm{BA}$. Observe that in this case $g^* > g_o$ while it is efficient, at the same time, to tax green energy ($s^* > 0$). We have thus demonstrated that $g^* > g_o$ and $s^* > 0$ is not an incompatible constellation. For prudent consumers we summarize the results of the tax incidence in

Proposition 5. *Suppose the consumer is prudent. Then in the transition from laissez-faire* ($s = 0, t = 0$) *to efficient regulation* $\left(s^* = \frac{V_\mu - \mathcal{V}_x}{\mathcal{V}_x}\varphi_z > 0,\ t^* = sB_e > 0\right)$ *the consumption of fossil fuel and black energy decreases. Moreover, if the utility function* $\mathcal{V}(x_\mathrm{d})$ *is specified by (14) and if the production function* $B(e)$ *is specified by (15), in this transition the green energy production increases.*

5 Concluding remarks

The present paper addressed price uncertainty in a small open economy. Using mean-variance preferences which in our model are equivalent to expected utility preferences we point out that increases in the variance of the fossil fuel price (= increasing risk) reduce the efficient black energy production for prudent consumers and enhance the efficient green energy production for constant absolute risk averse consumers. These results are intuitive. Turning to competitive markets we get at first glance counterintuitive results. If consumers are prudent and producers are risk neutral both fossil fuel and green energy have to be taxed to implement the efficient allocation.

Closer inspection shows that in our framework the social planner faces uncertainty with respect to a composite consumer good. The import price risk affects the national income via the trade balance and causes uncertainty with respect to consumption. In contrast, in the market the uncertainty hits the producer of black energy while the consumption of the consumer good is certain. Since the marginal valuation of an additional unit of good X is greater under uncertainty than under certainty, and the world market price p_x is fix, the market price of energy turns out to be greater than the social planner's shadow price of energy. As a consequence, in the market the producers of energy perceive too high market price signals. To correct for this market failure both fossil fuel and green energy has to be taxed. There is an additional effect induced by the divergence between the consumer's (social planner's) and producer's risk attitudes. If the social planner is more risk averse than the producer, the former wants to curb black energy and fossil fuel consumption more strongly than the latter. This additional effect ceteris paribus also calls for taxing fossil fuel (but not for regulating green energy).

Our paper leaves open some issues for future research. First, it is unclear whether our results also hold for import quantity uncertainty with rigid import prices. Second, in an alternative set-up one could assume that consumers purchase fossil fuel directly and are thus directly exposed to price risk. Third, one could introduce forward markets and investigate hedging of the price risk. These topics are beyond the scope of the present paper but appear to be interesting for future research.

References

Batra, R.N. and A. Ullah (1974): Competitive firm and the theory of input demand under price uncertainty, *Journal of Political Economy* 82, 537-548.

Batra, R.N. and W.R. Russel (1974): Gains from trade under uncertainty, *American Economic Review* 64, 1040-1048.

Binswanger, H.P. (1981): Attitudes towards risk: Theoretical implications of an experiment in rural India, *Economic Journal* 91, 867-890.

Bennear, L.S. and R.N. Stavins (2007): Second-best theory and the use of multiple policy instruments, *Environmental and Resource Economics* 37, 111-129.

Cass, D. and J.E. Stiglitz (1970): The structure of investor preferences and asset returns, and separability in portfolio allocation, *Journal of Economic Theory* 2, 122-160.

Chavas, J.P. and M.T. Holt (1996): Economic behavior under uncertainty: A joint analysis of risk preferences and technology, *Review of Economics and Statistics* 78, 329-335.

COM (2007): Communication from the Commission to the European Council and the European Parliament of 10 January 2007, An energy policy for Europe, http://europa.eu /legislation_summaries/energy/european_energy_policy/27067_en.htm.

Feder, G., Just, R.E. and A. Schmitz (1977): Storage with price uncertainty in international trade, *International Economic Review* 18, 553-568.

Fischer, C. (2008): Emissions pricing, spillovers, and public investment in environmentally friendly technologies, *Energy Economics* 30, 487-502.

Fischer, C. and R.G. Newell (2008): Environmental and technology policies for climate mitigation, *Journal of Environmental Economics and Management* 55, 142-162.

Gollier, C. and H. Schlesinger (2003): Preserving preference rankings under background risk, *Economics Letters* 80, 337-341.

Guiso, L., Jappelli, T. and D. Terlizzese (1996): Income risk, borrowing constraints, and portfolio choice, *American Economic Review* 86, 158-172.

Hens, T., Laitenberger, J. and A. Löffler (2003): Two remarks on the uniqueness of equilibria in the CAPM, *Journal of Mathematical Economics* 37, 123-132.

Kimball, M.S. (1990): Precautionary saving in the small and in the large, *Econometrica* 58, 53-73.

Lajeri, F. and L.T. Nielsen (2000): Parametric characterizations of risk aversion and prudence, *Economic Theory* 15, 469-476.

Levy, M. (1994): Absolute and relative risk aversion: An experimental study, *Journal of Risk and Uncertainty* 8, 452-467.

Menegatti, M. (2001): On the conditions for precautionary saving, *Journal of Economic Theory* 98, 189-193.

Meyer, J. (1987): Two moment decision models and expected utility maximization, *American Economic Review* 77, 421-430.

OECD/IEA (2008): Deploying Renewables. Principles for Effective Policies, Paris.

Pethig, R. and C. Wittlich (2009): Interaction of carbon reduction and green energy promotion in a small fossil-fuel importing country. CESifo Working Paper No. 2749.

Sandmo, A. (1971): On the theory of the competitive firm under price uncertainty, *American Economic Review* 61, 65-73.

Wagener, A. (2005): Linear risk tolerance and mean-variance preferences, *Economics Bulletin* 4, 1-8.

Appendix A: Proof of Proposition 1

Comparative statics of the social planner's solution with respect to σ_q: Maximizing $V\left[X(\bar{r}-r_g)-(p_e+\mu_q)e, e\sigma_q\right]+\mathcal{U}\left[B(e)+G(r_g)\right]$ yields the first order conditions

$$-(p_e+\mu_q)-\sigma_q M+\frac{\mathcal{U}_z}{V_\mu}B_e=0 \;\equiv\; \Phi, \tag{A1}$$

$$-X_r+\frac{\mathcal{U}_z}{V_\mu}G_r=0 \;\equiv\; \Omega, \tag{A2}$$

where $M:=-\frac{V_\sigma}{V_\mu}$. Total differentiation of (A1) and (A2) yields

$$\begin{pmatrix}\Phi_e & \Phi_{r_g}\\ \Omega_e & \Omega_{r_g}\end{pmatrix}\begin{pmatrix}\mathrm{d}e\\ \mathrm{d}r_g\end{pmatrix}=\begin{pmatrix}-\Phi_{\sigma_q}\\ -\Omega_{\sigma_q}\end{pmatrix}, \tag{A3}$$

where

$$\begin{aligned}
\Phi_e &= -\sigma_q^2(M_\mu M+M_\sigma)+\frac{2MM_\mu\sigma_q B_e\mathcal{U}_z}{V_\mu}+\frac{B_e^2\mathcal{U}_{zz}}{V_\mu}+\frac{B_e^2\mathcal{U}_z^2V_{\mu\mu}}{V_\mu^3}+\frac{B_{ee}U_z}{V_\mu}, && \text{(A4)}\\
\Phi_{r_g} &= \sigma_q X_r M_\mu+\frac{B_eG_r\mathcal{U}_{zz}}{V_\mu}+\frac{B_e\mathcal{U}_zX_rV_{\mu\mu}}{V_\mu^2}, && \text{(A5)}\\
\Phi_{\sigma_q} &= -M-\sigma_q eM_\sigma-\frac{eB_e\mathcal{U}_zV_{\mu\sigma}}{V_\mu^2}, && \text{(A6)}\\
\Omega_e &= \frac{B_eG_r\mathcal{U}_{zz}}{V_\mu}+\frac{\sigma_q M_\mu G_r\mathcal{U}_z}{V_\mu}+\frac{G_rB_e\mathcal{U}_z^2V_{\mu\mu}}{V_\mu^3}, && \text{(A7)}\\
\Omega_{r_g} &= X_{rr}+\frac{\mathcal{U}_zG_{rr}}{V_\mu}+\frac{G_r^2\mathcal{U}_{zz}}{V_\mu}+\frac{X_rG_r\mathcal{U}_zV_{\mu\mu}}{V_\mu^2}, && \text{(A8)}\\
\Omega_{\sigma_q} &= -\frac{eG_r\mathcal{U}_zV_{\mu\sigma}}{V_\mu^2}. && \text{(A9)}
\end{aligned}$$

Solving the equation system (A3) by using Cramer's rule we obtain

$$\begin{aligned}
\frac{\mathrm{d}e}{\mathrm{d}\sigma_q} &= \frac{-\Phi_{\sigma_q}\Omega_{r_g}+\Omega_{\sigma_q}\Phi_{r_g}}{D}, && \text{(A10)}\\
\frac{\mathrm{d}r_g}{\mathrm{d}\sigma_q} &= \frac{-\Phi_e\Omega_{\sigma_q}+\Omega_e\Phi_{\sigma_q}}{D}, && \text{(A11)}
\end{aligned}$$

where $D=\Phi_e\Omega_{r_g}-\Omega_e\Phi_{r_g}>0$ via the assumption that the second-order condition for a maximum is satisfied. Making use of (A5), (A6), (A8), (A9) in (A10) we get after rearrangement of terms

$$\begin{aligned}
\frac{\mathrm{d}e}{\mathrm{d}\sigma_q}\cdot D &= (M+\sigma_q eM_\sigma)\left(X_{rr}+\frac{\mathcal{U}_zG_{rr}}{V_\mu}+\frac{G_r^2\mathcal{U}_{zz}}{V_\mu}+\frac{X_rG_r\mathcal{U}_zV_{\mu\mu}}{V_\mu^2}\right)\\
&+\frac{e\mathcal{U}_zV_{\mu\sigma}}{V_\mu^2}\left(B_eX_{rr}+\frac{B_e\mathcal{U}_zG_{rr}}{V_\mu}-M_\mu\sigma_qG_rX_r\right). && \text{(A12)}
\end{aligned}$$

Accounting for $M_\sigma V_{\mu\mu} - M_\mu V_{\mu\sigma} = -\frac{1}{V_\mu}(V_{\mu\mu}V_{\sigma\sigma} - V_{\mu\sigma}^2)$ in (A12) yields

$$\begin{aligned}\frac{\mathrm{d}e}{\mathrm{d}\sigma_q}\cdot D &= (M+\sigma_q e M_\sigma)\left(X_{rr}+\frac{\mathcal{U}_z G_{rr}}{V_\mu}+\frac{G_r^2\mathcal{U}_{zz}}{V_\mu}\right)+\frac{e\mathcal{U}_z V_{\mu\sigma}}{V_\mu^2}\left(B_e X_{rr}+\frac{B_e\mathcal{U}_z G_{rr}}{V_\mu}\right)\\ &\quad -\frac{e\sigma_q X_r G_r\mathcal{U}_z}{V_\mu^3}(V_{\mu\mu}V_{\sigma\sigma}-V_{\mu\sigma}^2).\end{aligned}\tag{A13}$$

Next, we insert (A4), (A6), (A7), (A9) into (A11) and rearrange terms to get

$$\begin{aligned}\frac{\mathrm{d}r_g}{\mathrm{d}\sigma_q}\cdot D &= \frac{\mathcal{U}_z^2 e\sigma_q B_e G_r}{V_\mu^3}(M_\mu V_{\mu\sigma}-M_\sigma V_{\mu\mu})+\frac{\mathcal{U}_z^2}{V_\mu^3}G_r\left(eV_{\mu\sigma}B_{ee}-MB_eV_{\mu\mu}\right)\\ &\quad -\frac{\mathcal{U}_z G_r\sigma_q^2 e}{V_\mu}\left[\frac{V_{\mu\sigma}}{V_\mu}(M_\mu M+M_\sigma)+M_\sigma M_\mu\right]-\frac{MM_\mu\sigma_q\mathcal{U}_z G_r}{V_\mu}\\ &\quad -\frac{(M+e\sigma_q M_\sigma)\mathcal{U}_{zz}B_eG_r}{V_\mu}.\end{aligned}\tag{A14}$$

Observe that $M_\mu V_{\mu\sigma} - M_\sigma V_{\mu\mu} = \frac{1}{V_\mu}(V_{\mu\mu}V_{\sigma\sigma} - V_{\mu\sigma}^2)$ and $\frac{V_{\sigma\mu}}{V_\mu}(MM_\mu + M_\sigma) + M_\sigma M_\mu = \frac{M}{V_\mu^2}(V_{\mu\mu}V_{\sigma\sigma} - V_{\mu\sigma}^2)$. Using this information in (A14) we get

$$\begin{aligned}\frac{\mathrm{d}r_g}{\mathrm{d}\sigma_q}\cdot D &= \left(\frac{\mathcal{U}_z^2 e\sigma_q B_e G_r}{V_\mu^4}-\frac{M\mathcal{U}_z G_r\sigma_q^2 e}{V_\mu^3}\right)(V_{\mu\mu}V_{\sigma\sigma}-V_{\mu\sigma}^2)\\ &\quad +\frac{\mathcal{U}_z^2 G_r}{V_\mu^3}(eV_{\mu\sigma}B_{ee}-MB_eV_{\mu\mu})-\frac{MM_\mu\sigma_q\mathcal{U}_zG_r}{V_\mu}\\ &\quad -\frac{(M+e\sigma_qM_\sigma)\mathcal{U}_{zz}B_eG_r}{V_\mu}.\end{aligned}\tag{A15}$$

Finally, using the first-order condition (A1) in (A15) establishes

$$\begin{aligned}\frac{\mathrm{d}r_g}{\mathrm{d}\sigma_q}\cdot D &= \frac{\mathcal{U}_z^2\sigma_q eG_r}{V_\mu^3}(p_e+\mu_q)(V_{\mu\mu}V_{\sigma\sigma}-V_{\mu\sigma}^2)\\ &\quad +\frac{\mathcal{U}_z^2B_eG_rV_{\mu\mu}}{V_\mu^3}\left(\frac{V_{\mu\sigma}}{V_{\mu\mu}}\frac{B_{ee}\cdot e}{B_e}+\frac{V_\sigma}{V_\mu}\right)-\frac{MM_\mu\sigma_q\mathcal{U}_zG_r}{V_\mu}\\ &\quad -(M+e\sigma_qM_\sigma)\frac{\mathcal{U}_{zz}B_eG_r}{V_\mu}.\end{aligned}\tag{A16}$$

According to (9) prudence ($V_{\mu\sigma} > 0$) implies decreasing absolute risk aversion ($M_\mu < 0$). In addition, the concavity of V ($V_{\mu\mu}V_{\sigma\sigma} - V_{\mu\sigma}^2$) implies convex indifference curves, i.e. $m_\sigma + mm_\mu > 0$. Then $V_{\mu\sigma} > 0$ and the concavity of V are sufficient for $M_\sigma > 0$. Using these properties in (A13) we immediately get $\mathrm{d}e/\mathrm{d}\sigma_q < 0$ if $V_{\mu\sigma} > 0$. Closer inspection of (A16) reveals that all sum terms on the right side of (A16) are positive for $V_{\mu\sigma} > 0$ except for $\frac{\mathcal{U}_z^2 B_e V_{\mu\mu}}{V_\mu^3}\left(\frac{V_{\mu\sigma}}{V_{\mu\mu}}\frac{B_{ee}e}{B_e}+\frac{V_\sigma}{V_\mu}\right)$. To ensure that this term is also non-negative it must hold

$$-\frac{V_{\mu\sigma}}{V_{\mu\mu}}\cdot\frac{1}{\varepsilon}\leq-\frac{V_\sigma}{V_\mu},\tag{A17}$$

where $\varepsilon := -\frac{B_e}{eB_{ee}} > 0$, and hence we get $\frac{\mathrm{d}r_g}{\mathrm{d}\sigma_q} > 0$. Next, observe that

$$\frac{\mathrm{d}b}{\mathrm{d}\sigma_q} = B_e \frac{\mathrm{d}e}{\mathrm{d}\sigma_q}, \quad \frac{\mathrm{d}g}{\mathrm{d}\sigma_q} = G_r \frac{\mathrm{d}r_g}{\mathrm{d}\sigma_q}, \quad \frac{\mathrm{d}x}{\mathrm{d}\sigma_q} = -X_r \frac{\mathrm{d}r_g}{\mathrm{d}\sigma_q}, \quad \frac{\mathrm{d}r_x}{\mathrm{d}\sigma_q} = -\frac{\mathrm{d}r_g}{\mathrm{d}\sigma_q}. \tag{A18}$$

The comparative static effect $\mathrm{d}z/\mathrm{d}\sigma_q$ is ambiguous in sign.

Appendix B: Proof of Proposition 2

Observe that the utility function (16) satisfies $M_\mu = 0$ or equivalently $-\frac{V_\sigma}{V_\mu} = -\frac{V_{\sigma\mu}}{V_{\mu\mu}}$. Hence, it holds $V_{\mu\sigma} > 0$ and using the same arguments as in the proof of Proposition 1 we get $\frac{\mathrm{d}e}{\mathrm{d}\sigma_q} < 0$ and $\frac{\mathrm{d}b}{\mathrm{d}\sigma_q} < 0$. Next, observe that $\varepsilon = \frac{1}{1-\theta} > 1$ ensures that (A17) holds and we obtain $\frac{\mathrm{d}r_g}{\mathrm{d}\sigma_q} > 0$ and $\frac{\mathrm{d}g}{\mathrm{d}\sigma_q} > 0$. Finally, we insert (A13) and (A15) in

$$\frac{\mathrm{d}z}{\mathrm{d}\sigma_q} = B_e \frac{\mathrm{d}e}{\mathrm{d}\sigma_q} + G_r \frac{\mathrm{d}r_g}{\mathrm{d}\sigma_q}, \tag{B1}$$

use $\frac{eB_{ee}}{B_e} = \theta - 1$, $X_r = \frac{U_z}{V_\mu} G_r$ and rearrange terms to get

$$\begin{aligned} \frac{\mathrm{d}z}{\mathrm{d}\sigma_q} \cdot D \;\; = \;\; & (M + \sigma_q e M_\sigma) \left(B_e X_{rr} + \frac{\mathcal{U}_z B_e G_{rr}}{V_\mu} \right) - \frac{M M_\mu \sigma_q \mathcal{U}_z G_r^2}{V_\mu} \\ & - \frac{M \mathcal{U}_z G_r \sigma_q^2 e}{V_\mu^3} (V_{\mu\mu} V_{\sigma\sigma} - V_{\mu\sigma}^2) + \frac{e B_e^2 U_z V_{\mu\sigma}}{V_\mu^2} \left(X_{rr} + \frac{\mathcal{U}_z G_{rr}}{V_\mu} \right) \\ & + \frac{\mathcal{U}_z^2 G_r^2 V_{\mu\sigma} \theta}{V_\mu^3}. \end{aligned} \tag{B2}$$

Accounting for $M_\mu = 0$ and $V_{\mu\sigma} > 0$ establishes $\frac{\mathrm{d}z}{\mathrm{d}\sigma_q} < 0$.