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Buy coal for preservation and act strategically on the fuel market*

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Abstract

In Harstad's (2012) model, climate damage only hits one group of countries, called the coalition, and the coalition's climate policy consists of capping own fuel demand and supply combined with the purchase of fossil fuel deposits for preservation. Harstad's Theorem 1 states that if the deposit market clears the coalition's strategic fuel-cap policy implements the first-best. The present paper reconstructs that efficiency result and argues that the deposit market equilibrium as defined in Harstad (2012) fails to be attained, unless the non-coalition countries act cooperatively on the deposit market. Without such cooperation, the coalition's strategic action on the fuel market distorts the allocation to its own favor.

JEL classification: Q31, Q38, Q55

Key words: climate coalition, fossil fuel, deposits, extraction, fuel caps

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1 Introduction

Compelling scientific evidence suggests that greenhouse gas emissions, notably carbon emissions, generate severe negative climate externalities that can be internalized by global cooperative action. The experience from the Kyoto protocol and the climate change summits of recent years to reach a post-Kyoto agreement are disappointing. It is true that several countries have increased their efforts to curb emissions, notably the (Annex 1) countries that committed to emissions reductions in the Kyoto Protocol. Yet many small and large countries still refrain from taking (strong) action, and most of them have expanded their emissions significantly since 1990. That raises the question of what the chances are of a climate coalition to reduce carbon emissions efficiently by unilateral action.

Environmental economists have intensively analyzed this question. There is a literature that shows that unilateral environmental or trade policy is distortionary in the presence of trade and transboundary pollution (Markusen 1975, Hoel 1994, Copeland 1996).¹ In these second-best settings, the unilateral policy causes carbon leakage, which renders global emissions inefficiently high. The inefficiency aggravates, if the climate coalition implements its environmental policy strategically by influencing the terms of trade to its own favor.

Most of the aforementioned studies investigate demand-side climate policies. Bohm (1993), Harstad (2012) and Asheim (2013) are the only studies we know with an analytical approach to supply-side policies in which countries suffering from climate damage purchase or lease fossil energy deposits ('buy coal') to prevent their extraction. In a stylized parametric model, Bohm (1993) derives conditions under which a special policy mix consisting of the purchase or lease of deposits and a fuel-demand cap implements the emission cap at lower costs than the stand-alone fuel-demand-cap policy. Asheim (2013) makes the case for deposit policies as a distributional instrument in a growth model à la Dasgupta-Heal-Solow-Stiglitz. Harstad (2012) considers a world of heterogeneous countries in which all countries' carbon emissions generate climate damage in some group of countries. This group forms a coalition to mitigate that damage with a policy mix of deposit purchases and caps on own demand and supply of fuel. He shows that if the deposit market is in equilibrium the coalition implements the first-best by acting strategically in the sense that it has the power to manipulate the fuel price in its favor (ibidem, Theorem 1). That result is surprising in our view, because it runs counter to the 'standard' outcome in various fields of economics, that exerting market power always makes the strategically acting agent better off than price taking.

¹Not only unilateral environmental policy is inefficient, but also non-cooperative environmental policy (Ludema and Wooton 1994, Copeland and Taylor 1995, Kiyono and Ishikawa 2013) and the formation of self-enforcing international environmental agreements (Barrett 1994, Rubio and Ulph 2006, Eichner and Pethig 2013).

The present paper aims to get a better understanding of the power and limits of Harstad's Theorem 1. The crucial question we wish to answer is why the coalition refrains from using – or prefers not to use – its option to influence the fuel price. The answer is not obvious because if the coalition would make use of that option, the intuition from the literature is that it would benefit from strategic action at the cost of rendering the allocation inefficient.

Harstad's theorem relies on two key assumptions. The first is the concept of deposit market and deposit market equilibrium. The market consists of a set of bilateral trades with prices that may differ between each pair of traders and the "... market clears when there exists no pair of countries that would both strictly benefit from trading some of their deposits at some price" (Harstad 2012, p. 92). The second key assumption is that Harstad sets up a three-stage game in which countries trade fuel deposits before the coalition chooses its unilateral climate policy mix. Specifically, at the first stage deposits are traded, at the second stage the coalition chooses its fuel caps and at the third stage the non-coalition countries choose their fuel demand and supply and the fuel market clears. To reconstruct Harstad's efficiency theorem it is convenient to consider two types of deposit markets. The deposit market *I* is one-directional in the sense that the only deposit trades are the coalition's purchases of deposits from non-coalition countries. The deposit market *II* is Harstad's (2012) deposit market. In that market concept all countries may buy and/or sell any number of deposits and hence it contains the deposit market *I* as a special case.

Playing the three-stage game with deposit market *I* we find that if the coalition refrains from strategic action the outcome is first-best. However, if it acts strategically, it chooses inefficient fuel caps which increase its welfare compared to the efficient fuel caps it chooses in case of non-strategic (price taking) action. The coalition's benefit from strategic action translates into a welfare loss of all non-coalition countries. The outcome of the game with deposit market *II* is markedly different. We show that for any given initial endowment of deposits there exist purchases of deposits, additional to those in market *I*, such that the deposit market *II* clears and the coalition's fuel supply exactly matches its fuel demand in the fuel market equilibrium. For that specific set of deposit trade one gets Harstad's efficiency result, since the coalition refrains from strategically setting its fuel caps and implements the efficient caps.

Our reconstruction of Harstad's efficiency theorem offers interesting new insights. In order to carry out the additional trades in deposit market *II* referred to in the last paragraph, the coalition has to be compensated such that it secures (at least) the welfare it attains with strategic action in deposit market *I*. If the non-coalition countries do not cooperate, then all of them have strong free-rider incentives to let some other country purchase the additional

deposits in market I and we are in a prisoner's dilemma. The coalition will not be paid for the additional deposit purchases, the equilibrium of the deposit market II is not reached and the coalition will set strategically (and inefficiently) its caps as on the deposit market I . Only if all non-coalition countries cooperate on the deposit market II , the non-coalition countries may prevent the coalition from setting the fuel caps strategically.

The remainder of the paper is organized as follows. Section 2 briefly presents the model and characterizes the social optimum without and with deposit trading. Section 3 investigates the three-stage game with deposit market I and Section 4 turns to the three-stage game with deposit market II . Section 4 provides an assessment of Harstad's Theorem 1. Section 5 concludes.

2 The pollutee-pays approach to restore efficiency

The basic analytical framework. Harstad (2012) considers a world economy with two groups of countries, M and N . The members of group M participate in an international climate agreement and group - or coalition - M acts as one agent. Each country produces and consumes fuel. Country $i \in \{M\} \cup N =: \Omega$ derives the benefit $B_i(y_i)$ from consuming y_i units of fuel (with $B'_i > 0$ and $B''_i < 0$) and produces the quantity x_i of fuel from the fossil fuel deposits it owns. Fuel generates the greenhouse gas carbon dioxide proportional to fuel production and the carbon dioxide emissions cause climate damage $H(\sum_{\Omega} x_j)$ in the coalition M . At the beginning of the game, the cost of extracting fuel is $C_i(x_i)$ with $C'_i > 0$ and $C''_i > 0$.

The marginal extraction function C'_i defines country i 's endowment of deposits where deposits are characterized by the amount of fuel stored in them and by the cost of extracting that fuel. Specifically, the function C'_i "... is a mapping from country i 's deposits, ordered according to costs, to the marginal extraction cost of these deposits" (Harstad 2012, p. 85).² We express the (conventional) case that country $i \in \Omega$ owns all deposits specified by the marginal extraction cost function C'_i by saying that i owns the deposits $[0, \infty[_{C'_i}$.

Social optimum versus market failure. In the conventional textbook approach the social planner solves the Lagrangean

$$\mathcal{L}(x_1, \dots, x_N, x_M, y_1, \dots, y_N, y_M, \lambda_f) = \sum_{\Omega} [B_j(y_j) - C_j(x_j)] - H\left(\sum_{\Omega} x_j\right) + \lambda_f \sum_{\Omega} (x_j - y_j) \quad (1)$$

²For the concept of deposit endowments in the formal model see Appendix A1.

and obtains the first-order condition

$$B'_i(y_i) = \lambda_f \quad \text{and} \quad C'_i(x_i) = \lambda_f - H' \left(\sum_{\Omega} x_j \right) \quad \forall i \in \Omega. \quad (2)$$

Denote the efficient values in (2) by $y_i = y_i^*$, $x_i = \sigma_i^*$, and $\lambda_f = \lambda_f^*$. According to (2) efficiency is attained if and only if all deposits³ $[0, \sigma_i^*]_{C'_i}$, $\forall i \in \Omega$, are exploited. The outcome fails to be efficient in a world economy with a competitive fuel market when no cooperation and no deposit trade takes place. The absence of deposit trading simply means that each country sticks to its initial endowment of deposits, $[0, \infty]_{C'_i}$. In that case, the fuel supplies and demands are implicitly determined by

$$B'_i(y_i) = p \quad \text{and} \quad C'_i(x_i) = p - \delta(i)H' \left(\sum_{\Omega} x_j \right) \quad \forall i \in \Omega, \quad (3)$$

where $\delta(M) = 1$ and $\delta(i) = 0$ for $i \in N$. Comparing (2) and (3) reveals that the climate damage is excessive in the unregulated market economy without deposit trading because the non-coalition countries disregard the climate damage generated by their fuel supply.

The standard procedure to implement the first-best allocation (2) in a market economy is the Pigouvian polluter-pays solution.⁴ However, we leave that approach to efficiency aside in the present paper because we follow Harstad in assuming that the countries unaffected by climate damage refrain from any mitigation policy that makes them worse off.

Social optimum and tradable deposits. The characterization of the social optimum in (2) implicitly assumes that each country's initial endowment of deposits is $[0, \infty]_{C'_i}$, $\forall i \in \Omega$, and the ownership of deposits remains unchanged. In order to characterize efficiency in case of tradable deposits, we assume, as before, that country i 's initial endowment of deposits is $[0, \infty]_{C'_i}$. However, now imagine a social planner who takes away from each country $i \in \Omega$ all deposits in some interval $[\sigma_i, \xi_i]_{C'_i}$ and transfers them to the coalition obliging it to preserve the deposits it received. Then the question arises how to choose the boundary points σ_i and ξ_i of the interval $[\sigma_i, \xi_i]_{C'_i}$ that maximize global welfare. We denote the 'number' of deposits in the interval $[\sigma_i, \xi_i]_{C'_i}$ by $z_i^s := \xi_i - \sigma_i$, the total number of deposits transferred to the coalition by z_M^d and answer that question by solving the Lagrangean

$$\begin{aligned} \mathcal{L}(x_M, y_1, \dots, y_N, y_M, \xi_1, \dots, \xi_N, z_1^s, \dots, z_N^s, z_M^d, \lambda_f, \lambda_z) = \\ \sum_N [B_j(y_j) - C_j(\xi_j - z_j^s)] + B_M(y_M) - C_M(x_M) - H \left(x_M + \sum_N \xi_j - z_M^d \right) \\ + \lambda_f \left[x_M - y_M + \sum_N (\xi_j - z_j^s - y_j) \right] + \lambda_z \left(\sum_N z_j^s - z_M^d \right). \end{aligned} \quad (4)$$

³Recall that the subscript C'_i attached to $[0, \sigma_i^*]$ indicates that $C'_i(x)$ is the cost of extracting $x \in [0, \sigma_i^*]_{C'_i}$.

⁴That solution consists of setting the fuel price $p^* = \lambda_f^*$ and a fuel supply tax equal to $H'(\sum_{\Omega} \sigma_j^*)$ to be levied by the coalition and all countries.

The first-order conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial y_i} &= B'_i - \lambda_f = 0, \quad \forall i \in \Omega, & \frac{\partial \mathcal{L}}{\partial x_M} &= -C'_M - H' + \lambda_f = 0, & \frac{\partial \mathcal{L}}{\partial z_M^d} &= H' - \lambda_z = 0, \\ \frac{\partial \mathcal{L}}{\partial \xi_i} &= -C'_i - H' + \lambda_f = 0, & \text{and} & & \frac{\partial \mathcal{L}}{\partial z_i^s} &= C'_i + \lambda_z - \lambda_f = 0, \quad \forall i \in N. \end{aligned} \quad (5)$$

Recalling that $\sigma_i = \xi_i - z_i^s$, it is easy to see that the solution of (5) exhibits the same values σ_i^* , y_i^* , all $i \in \Omega$, and λ_f^* as the solution of (1). The additional information (5) provides is that $\lambda_z = \lambda_z^*$ is the shadow price of deposits and that $z_i^s = z_i^{s*} := \xi_i^* - \sigma_i^*$ are the deposits initially owned by the countries $i \in N$ and saved from exploitation by the coalition.

Concepts of deposit market and efficiency. The next question is how the social optimum with tradable deposits can be implemented in the market economy. We will answer that question for the following different concepts of deposit market.

- *Competitive deposit market.* All countries choose their welfare-maximizing supply and demand of deposits taking the (unitary) deposit price as given. The market is in equilibrium, when the price is such that aggregate demand matches aggregate supply.
- *Deposit market I.* The deposit market *I* is a market on which the coalition purchases deposits from other countries for the purpose to prevent the exploitation of the acquired deposits. The market clears, when there exist no deposit purchases for preservation that would be strictly beneficial for both the coalition and the seller.
- *Deposit market II.* On the deposit market *II*, all countries and the coalition may buy and/or sell deposits. Hence, market transactions include those on the market *I* but may be more complex. The market clears, when "... there exists no pair of countries that would both strictly benefit from trading some of their deposits at some price" (Harstad 2012, p. 92).

The concept of deposit market *II* is due to Harstad (2012). The difference between the deposit markets *I* and *II* is that the deposit trade in market *I* is one-directional by assumption, while in the deposit market *II* all countries and the coalition may be active on both sides of the market. The purpose of distinguishing the markets *I* and *II* is to clarify Harstad's analysis and results, which we will discuss in detail in the Sections 4 and 5 below. Consideration of these deposit market concepts yields⁵

Proposition 1. *Suppose (a) the coalition suffers, but the other countries do not suffer from climate damage; (b) the non-coalition countries refrain from mitigating carbon emissions; and (c) deposits can be traded internationally.*

⁵The proof of Proposition 1 is delegated to the Appendix A2.

- (i) *The equilibrium of the world economy with a perfectly competitive fuel market is efficient, when the deposit market is*
- (ia) *either a competitive market,*
 - (ib) *or a deposit market I,*
 - (ic) *or a deposit market II.*
- (ii) *The pattern of equilibrium deposit trades is the same in the cases (ia) and (ib) and may but need not be the same in case (ic).*

Proposition 1(i) formalizes the well-known Coasean insight that the failure to internalize an externality is equivalent to the lack of incentives for the creation of a suitable market. In the sequel, we focus exclusively on the deposit markets *I* and *II*. It is worth emphasizing, however, that the competitive deposit market is a relevant benchmark, because the (unique) shadow price of deposits also guides the clearance of the more general markets *I* and *II*.

Proposition 1(ib) obviously is an example of Proposition 1(ic). However, it is useful for later reference to demonstrate that bilateral deposit trades in the Propositions 1(ia) and 1(ib), in which the coalition only buys and the countries only sell deposits, are not the only pattern that sustains efficiency. The key to understand the existence of more complex but still efficient deposit trade structures is the observation we already made above that efficiency requires exploiting a deposit, if and only if it is contained in $\bigcup_{\Omega} [0, \sigma_j^*]_{C_j'}$. Put differently, efficiency requires exploiting all deposits - and only those - whose extraction costs are lower than or equal to $C_1'(\sigma_1^*) = C_2'(\sigma_2^*) = \dots = C_M'(\sigma_M^*)$. However, efficiency does not depend on which country owns and exploits a deposit with extraction costs lower than or equal to $C_i'(\sigma_i^*)$. Hence, some of those low-cost deposits may be traded between two countries or between a country and the coalition without causing allocative distortions. To fix our ideas, suppose such trades take place in addition to the trades $z_1^{s*}, \dots, z_N^{s*}$ and those additional trades are priced at the profit foregone. Then both trading partners are indifferent with respect to conducting the additional deal. Consequently, if we consider the trades $z_1^{s*}, \dots, z_N^{s*}$ and the additional trades in bilateral deposit trade packages, such packages can be traded without violating the requirement of mutual gains from trade.

The common features of the parts (ia), (ib) and (ic) of Proposition 1 are that the fuel market is perfectly competitive, i.e. that all countries and the coalition take the fuel price as given, and that the markets for fuel and deposits clear simultaneously. In the remainder of the paper, we follow Harstad in assuming that the deposit market clears prior to the fuel market and that the coalition exerts market power on the fuel market. The market power takes the form of influencing the terms of international fuel trade, i.e. the fuel price, by

the coalition's strategic choice of fuel supply and demand. The analysis of these features requires setting up a game model with three stages. The timing of the game is as follows. At stage 1, the deposit market clears. The coalition determines its fuel supply and demand at stage 2, and at stage 3, the fuel market equilibrates. In the following Section 3, we analyze and solve that three-stage game applying the concept of the deposit market *I*. In section 4, we replace the deposit market *I* by the deposit market *II*.

3 The three-stage game with deposit market *I*

We follow the standard procedure of solving the game via backward induction.

Stage 3. At stage 3, M has already chosen its fuel supply and demand, x_M and y_M . The representative consumer of country $i \in N$ determines its fuel demand by maximizing with respect to y_i

$$B_i(y_i) - K_i(x_i, p_a, \pi_z) - p(y_i - x_i) + R_i(p_a, \pi_z) \quad \forall i \in N.$$

p_a is the fuel price anticipated at stage 1; p is the fuel price prevailing at stage 3; π_z is the marginal climate damage determined at stage 1;⁶ K_i is country i 's extraction cost function *after the deposit sales at the first stage*; and $R_i(p_a, \pi_z)$ is i 's revenue from selling deposits at stage 1.⁷ The first-order condition readily yields

$$B'_i(y_i) = p \quad \text{and hence} \quad y_i = B_i^{-1}(p) =: D_i(p) \quad \forall i \in N, \quad (6)$$

where B_i^{-1} is the inverse of the marginal benefit function B'_i . Next, consider the fuel supply of country $i \in N$. At stage 3, i recalls that it sold at stage 1 the deposits $[\sigma_i(p_a, \pi_z), \xi_i(p_a)]_{C'_i}$, where

$$\xi_i = \xi_i(p_a) = C_i'^{-1}(p_a), \quad \sigma_i = \sigma_i(p_a, \pi_z) := C_i'^{-1}(p_a - \pi_z) \quad (7)$$

and where $C_i'^{-1}$ is the inverse of the marginal cost function C'_i . The deposit sale at stage 1 changed i 's endowment of deposits such that i 's initial marginal cost function C'_i turned

⁶For details of the role and determination of π_z see Lemma 1 and footnote 16.

⁷Due to the quasi-linearity of the utility function, the fuel demand is independent of both the extraction costs $K_i(\cdot)$ and the revenues $R_i(\cdot)$. We provide the definition of these terms in the appropriate context below.

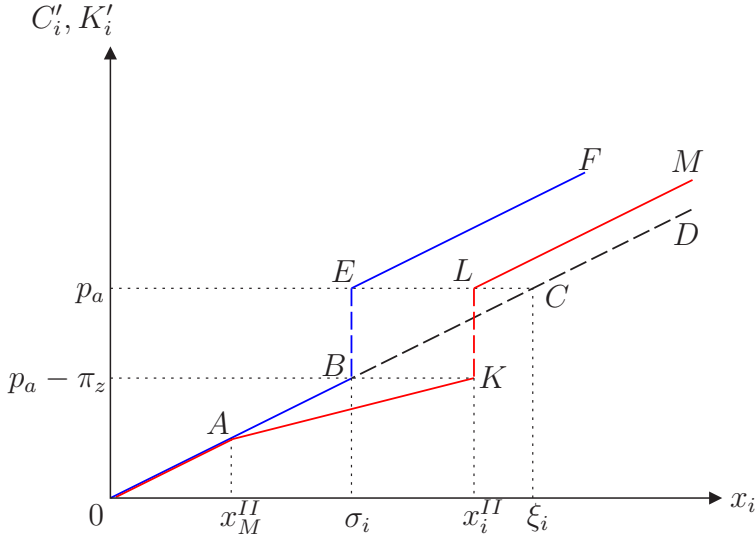


Figure 1a

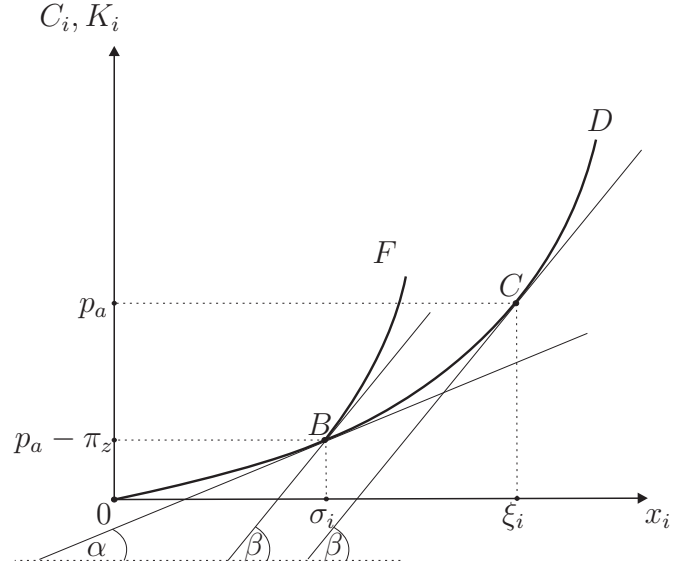


Figure 1b

Figure 1: Marginal and total cost curves of country $i \in N$ before and after deposit trading $[\sigma_i, \xi_i]_{C'_i}$ at stage 1¹⁰

into the marginal cost function K'_i defined by⁸

$$K'_i(x_i, p_a, \pi_z) := \begin{cases} C'_i(x_i) & \text{for } x_i \leq \sigma_i, \\ C'_i(\xi_i) - C'_i(\sigma_i) + C'_i(x_i) & \text{for } x_i \geq \sigma_i, \end{cases} \quad \forall i \in N. \quad (8)$$

Figure 1 illustrates the marginal cost functions C'_i and K'_i (Figure 1a) and the total cost functions C_i and K_i (Figure 1b). The straight line $0D$ in Figure 1a is the graph of C'_i . After having sold the deposits $[\sigma_i, \xi_i]_{C'_i}$ at stage 1, country i 's marginal cost function K'_i , is represented by the line $0BEF$. We derive that line from $0D$ by shifting the line segment CD to the left by the amount $\xi_i - \sigma_i$ such that CD becomes EF . Thus, country i 's endowment of deposits changed from $0ABCD$ to $0ABEF$. The function K'_i is discontinuous at $x_i = \sigma_i$, as reflected in the gap BE of the graph $0BEF$. In Figure 1b, $0BCD$ is the graph of the cost function C_i . After the deposit sale at stage 1, the curve $0BEF$ represents country i 's new cost function K_i . The curve segment BF of K_i results from moving the curve segment CD from its base point C to the new base point B . The gap BE of the graph of K'_i in Figure 1a translates into a kink of the cost curve $0BEF$ at $x_i = \sigma_i$ (= at point B) in Figure 1b. Figure 1b illustrates that if σ_i is approached from above, the marginal extraction cost

⁸To avoid clutter, we write σ_i, ξ_i etc. for the terms $\sigma_i(p_a, \pi_z), \xi_i(p_a)$ etc. unless it is useful to emphasize their dependence on the variables which were determined at earlier stages of the game. Throughout the paper a "prime" indicates the partial derivative with respect to the first argument of a function.

¹⁰The line $0BEF$ in Figure 1a and the line $0BGH$ in Figure 2 are constructed as in Harstad's (2012) Figure 1.

is $K'_i(\sigma_i) = \tan \alpha = p_a$, and it is $K'_i(\xi_i) = \tan \beta = p_a - \pi_z < p_a$, if σ_i is approached from below.

The Appendix A2 shows that maximizing with respect to x_i the welfare $U_i = B_i(y_i) - K_i(x_i, p_a, \pi_z) - p(y_i - x_i) + R_i(p_a, \pi_z)$ yields the fuel supply function S_i with the properties

$$S_i(p, p_a, \pi_z) = \begin{cases} C'_i{}^{-1}(p) & \text{for } p \leq p_a - \pi_z, \\ \sigma_i & \text{for } p \in [p_a - \pi_z, p_a], \\ C'_i{}^{-1}[p - C'_i(\xi_i) + C'_i(\sigma_i)] & \text{for } p \geq p_a, \end{cases} \quad \forall i \in N. \quad (9)$$

In view of (6) and (9), the fuel market clearing condition is

$$x_M + \sum_N S_j(p, p_a, \pi_z) = y_M + \sum_N D_j(p). \quad (10)$$

Equation (10) yields the equilibrium fuel price as a function of x_M , y_M , p_a and π_z , all of which have been determined earlier in the game. We denote that price function as

$$p = P(x_M, y_M, p_a, \pi_z). \quad (11)$$

Stage 2. M 's deposit purchases at stage 1 turned its initial extraction cost function C'_M into the cost function K'_M defined by

$$K'_M(x_M, p_a, \pi_z) = \begin{cases} C'_M(x_M) & \text{for } x_M \leq \sigma_M, \\ \tilde{K}'_M(x_M) & \text{for } x_M \in [\sigma_M, \tilde{\sigma}_M], \\ \tilde{K}'_M(\tilde{\sigma}_M) - C'_M(\tilde{\sigma}_M) + C'_M(x_M) & \text{for } x_M \geq \tilde{\sigma}_M, \end{cases} \quad (12)$$

where

$$\begin{aligned} \tilde{\sigma}_M &= \sigma_M + \sum_{\Omega} (\xi_j - \sigma_j), \quad \tilde{K}'_M(x_M) := C'_M(\sigma_M) - \tilde{C}'(\sigma_M) + \tilde{C}'(x_M) \quad \text{and} \\ \zeta &= \tilde{C}'(x) \iff x = \sum_{\Omega} C'_j{}^{-1}(\zeta) \end{aligned}$$

Figure 2 illustrates the marginal cost functions C'_M and K'_M .¹¹ The straight line $0D$ in Figure 2 is the graph of C'_M . After having purchased the deposits $\bigcup_N [\sigma_j, \xi_j]_{C'_j}$ at stage 1, M 's marginal cost function K'_M is represented by the graph $0BGH$. The line segment BG on that graph, which is flatter than the segments $0B$ and GH , contains both M 's deposits $[\sigma_M, \xi_M]_{C'_M}$ and all acquired deposits $\bigcup_N [\sigma_j, \xi_j]_{C'_j}$ reordered according to extraction costs. The line segment GH of the graph $0BGH$ results from shifting the line segment CD to

¹¹The comparison of the Figures 2 and 1a reveals that they (approximately) illustrate a world economy with the coalition and one country ($N = \{i\}$); the initial deposit endowments the coalition and the country own are the same ($C'_i = C'_M$).

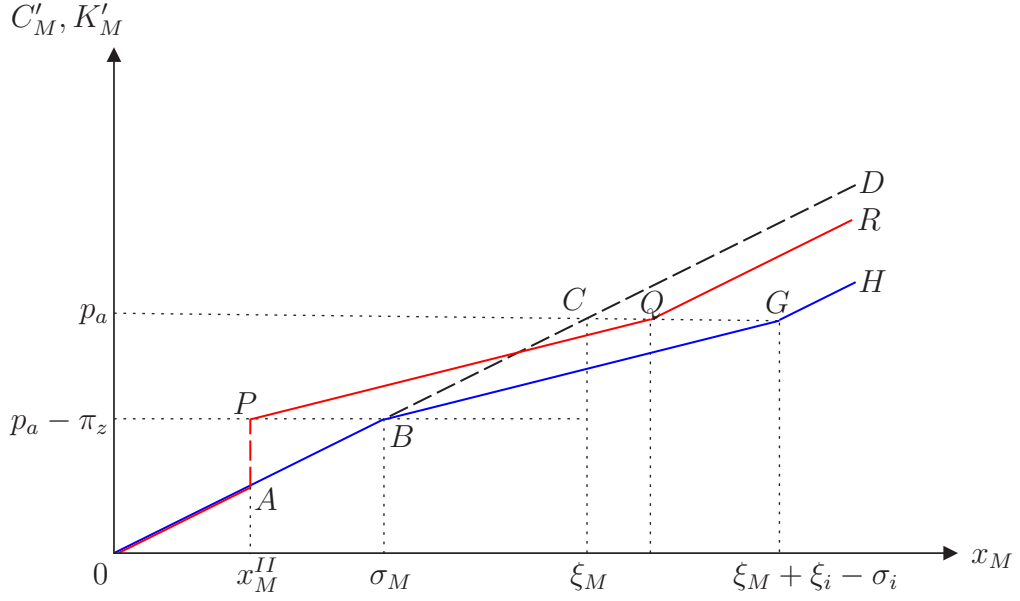


Figure 2: Extraction cost curves of coalition M before and after its purchase of the deposits $\bigcup_N[\sigma_j, \xi_j]C'_j$ at stage 1

the right by the amount $\sum_N(\xi_j - \sigma_j)$. Thus, M 's purchase of deposits changes its deposit endowment from $0D$ to $0BGH$. M chooses its fuel supply and demand by maximizing with respect to x_M and y_M its welfare

$$U_M(x_M, y_M, p_a, \pi_z) = B_M(y_M) - K_M(x_M, p_a, \pi_z) - p(y_M - x_M) - H \left[x_M + \sum_N S_j(p, p_a, \pi_z) \right] - R_M(p_a, \pi_z) \quad (13)$$

subject to (11). The first-order conditions

$$\frac{\partial U_M}{\partial y_M} = B'_M - p - \left(y_M - x_M + H' \sum_N S'_j \right) \frac{\partial P}{\partial y_M} = 0, \quad (14)$$

$$\frac{\partial U_M}{\partial x_M} = -K'_M + p - H' - \left(y_M - x_M + H' \sum_N S'_j \right) \frac{\partial P}{\partial x_M} = 0 \quad (15)$$

coincide with Harstad's (2012) equations (6) and (7). Implicitly, these equations determine M 's optimal choice of x_M and y_M as functions of p_a and π_z . We denote the solution of (14) and (15) by

$$x_M = X_M(p_a, \pi_z) \quad \text{and} \quad y_M = Y_M(p_a, \pi_z). \quad (16)$$

While at stage 3 the equilibrium fuel price depends on x_M , y_M , p_a and π_z , as shown in (11), it now depends on p_a and π_z only,

$$p = P[X_M(p_a, \pi_z), Y_M(p_a, \pi_z), p_a, \pi_z]. \quad (17)$$

Stage 1. We have to determine those deposits $[\underline{x}_i, \bar{x}_i]_{C'_i}$ which each country $i \in N$ sells to the coalition with mutual gains from trade and which clear the deposit market I . M aims at buying deposits the preservation of which fully reduces the climate damage. Hence, M only buys some of those deposits, which are profitable, that is, which country i would have extracted in the absence of deposit trading. Given the anticipated fuel price p_a , the interval with profitable deposits is $[0, \xi_i]_{C'_i}$, where $\xi_i = \xi_i(p_a) := C'^{-1}(p_a)$.¹² Hence M 's purchase and subsequent preservation of $[\underline{x}_i, \bar{x}_i]_{C'_i}$ secures full climate damage reduction only if $[\underline{x}_i, \bar{x}_i]_{C'_i} \subset [0, \xi_i]_{C'_i}$.¹³ Moreover, the inequality $\bar{x}_i \leq \xi_i$ must hold as equality, because there is no other interval of deposits in $[0, \xi_i]_{C'_i}$ of the same size as $[\underline{x}_i, \xi_i]_{C'_i}$, whose economic value is smaller than that of $[\underline{x}_i, \xi_i]_{C'_i}$.¹⁴ These considerations make M 's purchase (and country i 's sale) of deposits equivalent to the choice of \underline{x}_i . To put it differently, we have to determine $z_i^s = \xi_i - \underline{x}_i$, the 'number' of deposits M buys in the interval $[\underline{x}_i, \xi_i]_{C'_i}$. In the Appendix A2 we prove

Lemma 1. *(Equilibrium of the deposit market I)*

Suppose p_a , π_z , $\xi(p_a)$, $\sigma_i(p_a, \pi_z)$ from (7) and $x_M = X_M(p_a, \pi_z)$ from (16) are given and define

$$\hat{\sigma}_i(p_a) := \sigma_i(p_a, \pi_z(p_a)) = C'_i{}^{-1}(p_a - \pi_z(p_a)), \quad (18)$$

where $\pi_z = \pi_z(p_a)$, if and only if

$$\pi_z = H' \left[X_M(p_a, \pi_z) + \sum_N \sigma_j(p_a, \pi_z) \right]. \quad (19)$$

Contingent on the anticipated fuel price p_a , the deposit market I is in equilibrium, if and only if the coalition purchases the deposits $\{[\hat{\sigma}_i(p_a), \xi_i(p_a)]_{C'_i}\}_{i \in N}$.

Lemma 1 establishes that the deposit market is cleared,¹⁵ if $\underline{x}_i = \hat{\sigma}_i(p_a)$ and therefore $z_i^s = \xi_i(p_a) - \hat{\sigma}_i(p_a)$ is satisfied for all $i \in N$. Equation (19) provides the reason for our interpretation of π_z as the shadow price of climate damage and it implicitly specifies the

¹²See equation (7) above.

¹³Here we presuppose w.l.o.g. that the price p_a is so low that the aggregate fuel supply $X_M(p_a, \pi_z) + \sum_N \xi_j(p_a)$ leads to excessive climate damage.

¹⁴The economic value of the deposits in the interval $[\underline{x}_i, \xi_i(p_a)]_{C'_i}$ is the profit $p(\xi_i(p_a) - \underline{x}_i) - C_i(\xi_i(p_a)) + C_i(\underline{x}_i)$ that would accrue to country i if it would extract and sell the fuel from these deposits instead of selling the unexploited deposits to M .

¹⁵The countries in group N are price takers on the fuel market. They leave the decision about which and how many deposits to buy to the coalition, but they do not sell deposits unless the sales price exceeds the profits they could have made from exploiting instead of selling their deposits (profits foregone). If a deal enhances the joint welfare of the trading partners, an agreement about their shares of the surplus is always reached.

shadow price π_z , which we treated as given up to now, as a function of p_a .¹⁶ The specification of π_z by $\pi_z(p_a)$ in (18) not only yields (19), but also determines the equilibrium values

$$\begin{aligned} x_M &= X_M(p_a, \pi_z(p_a)) := \hat{X}_M(p_a), & y_M &= Y_M(p_a, \pi_z(p_a)) := \hat{Y}_M(p_a) \\ \text{and } p &= \hat{P}(p_a) := P[X_M(p_a, \pi_z(p_a)), Y_M(p_a, \pi_z(p_a)), p_a, \pi_z(p_a)]. \end{aligned} \quad (20)$$

Consistency requires equality of the fuel price p_a that is anticipated at stage 1 and the fuel price p that clears the fuel market at stage 3. Assuming that the price function \hat{P} from (20) possesses a fixed point, we set $p = p_a$. That completes the characterization of the solution to the three-stage game.

It remains to examine the efficiency properties of the outcome. Efficiency requires $B'_i = B'_j$ for all $i, j \in \Omega$ and $B'_i - K'_i - H' = 0$ for all $i, j \in \Omega$. These equations are satisfied for all $i \in N$ due to $p = p_a$, (6), (7) and (19). In view of (14) and (15), the equation $B'_M - K'_M - H' = 0$ is also satisfied, if and only if

$$\left(y_M - x_M + H' \sum_N S'_j \right) \frac{\partial P}{\partial y_M} = 0. \quad (21)$$

If the coalition acts strategically on the fuel market, as assumed by design of the three-stage game, we have $\frac{\partial P}{\partial x_M} = -\frac{\partial P}{\partial y_M} \neq 0$. Hence $B'_M - K'_M - H' = 0$, if and only if $y_M - x_M + H' \sum_N S'_j = 0$. This condition is violated, in general. However, if the exceptional case $y_i = x_i$ for all $i \in \Omega$ and $\sum_N S'_j = 0$ holds, the coalition prefers acting as a price taker - and thus secures efficiency - although it has the option to exert market power.

Finally, we consider the interesting special case in which we drop the second stage by assuming that the coalition acts as a price taker on the fuel market along with all other countries. We simply generate that case by setting $\frac{\partial P}{\partial x_M} = -\frac{\partial P}{\partial y_M} = 0$ in (21) and find that (14) and (15) then yield the efficiency condition $B'_M - K'_M - H' = 0$. We conclude that the outcome is efficient if the coalition acts as a price taker on the fuel market and that conclusion is in line with our result in Proposition 1(ib).

The equilibrium fuel supplies can conveniently be illustrated in the Figures 1a and 2. Suppose first, the coalition acts as a price taker in the fuel market, denote by an asterisk the corresponding equilibrium, and substitute in both figures p_a and $p_a - \pi_z$ with p^* and $p^* - \pi_z^*$, respectively. Then country i 's fuel production is point E in Figure 1a and M 's fuel production is point B in Figure 2. When M acts strategically, the equilibrium values of p and $p - \pi_z$ differ from p^* and $p^* - \pi_z^*$. If we replace in both figures p_a and $p_a - \pi_z$ by the new equilibrium prices the production points are again point E in Figure 1a and point B in Figure 2.

¹⁶Obviously, $\pi_z = \pi_z(p_a)$ is the equilibrium deposit price of a perfectly competitive deposit market (contingent on the anticipated fuel price p_a).

If M is a price taker on the fuel market, efficiency is secured no matter how different the countries' fuel supplies and demands are. In contrast, the outcome is inefficient, in general, if the coalition acts strategically at stage 2 of the three-stage game. That observation raises the question whether strategic action is beneficial from the coalition's viewpoint. The answer is in the affirmative because an agent who has the option to act strategically can always choose that value of her policy parameter (here: the fuel supply) which she would have chosen as a price taker. If we find that this agent does not choose the price-taking option, we know that she is better off acting strategically than acting as a price-taker. We summarize the conclusions in

Proposition 2. *(Three-stage game with deposit market I)*

Consider the world economy with deposit market I.

- (i) *The deposit market clears, if and only if the coalition purchases the deposits $\{[\hat{\sigma}_i(p_a), \xi_i(p_a)]_{C'_i}\}_{i \in N}$.*
- (ii) *If the coalition takes the fuel price as given, the outcome of the game is first-best. Denoting the efficient equilibrium fuel price by p^* , the corresponding fuel supplies and demands are $x_i^* = \hat{\sigma}_i(p^*)$ and $y_i = D_i(p^*)$ for all $i \in \Omega$.*
- (iii) *Denote by \mathcal{E} the set of economies $E = [B_i, C'_i, H]_{i \in \Omega}$ that are under consideration in the present paper and by $\mathcal{E} \subset \mathcal{E}$ the set of economies satisfying $y_M - x_M + H' \sum_N S'_j = 0$ in the equilibrium of the game. The coalition's strategic action implements the first-best, if and only if $E \in \mathcal{E}$.*
- (iv) *If the coalition's strategic action fails to implement the first-best, the coalition is better off than in the first-best.*

In spirit, Proposition 2(iii) is similar to Harstad's (2012) Theorem 1. We will offer a detailed discussion of the similarities and differences in Section 5 below. However, for the benefit of our later comparison of the game with deposit market I and Harstad's game, we rewrite and interpret the message of Proposition 2(iii) in a more formal way by means of the following definitions

Ω_I^{eq} = set of equilibrium allocations on the deposit market I ;

Ω^{sa} = set of allocation implemented through the coalition's strategic action (*sa* stands for strategic action);

Ω^* = set of first-best allocations;

$\mathcal{E} \setminus \mathcal{E}$ set of economies in \mathcal{E} that are not contained in \mathcal{E} .

Using these definitions, Proposition 2(iii) consists of the statements

$$\forall E \in \mathcal{E} : \quad \Omega_I^{eq} \Rightarrow \Omega^{sa} \subset \Omega^* \quad \text{and} \quad \forall E \in \mathcal{E} \setminus \mathcal{E} : \quad \Omega_I^{eq} \Rightarrow \Omega^{sa} \cap \Omega^* = \emptyset. \quad (22)$$

The second statement in (22) is as interesting as the first one. Since \mathcal{E} is a very small subset of \mathcal{E} , one can say that, *in general*, the coalition's strategic action fails to implement the first-best solution in the game with deposit market I . That failure is in the coalition's interest because it benefits from its distortionary strategic action (Proposition 2(iv)).

4 The three-stage game with deposit market II

The previous section demonstrated that the coalition benefits from strategic action on the fuel market compared to price taking behavior in the market I (on which the coalition only buys and the other countries only sell deposits). The present section aims to reconstruct Harstad's (2012) efficiency-despite-strategic-action result in a three-stage game with the more general deposit market II .

Recall that according to Proposition 1(ic) the outcome of the game with the deposit market II is efficient, if the coalition refrains from exerting market power and the markets for fuel and deposits clear simultaneously. The challenge of the present section is to understand why the coalition cannot benefit from strategic action, when a more general deposit market concept is applied and market clearance is sequential. We will show that the key to Harstad's efficiency result is a pattern of deposit transactions, which 'redistributes' the ownership of deposits such that in the subsequent fuel market equilibrium the coalition's fuel supply exactly matches its fuel demand.

We follow the standard procedure of solving the game via backward induction.

Stage 3. As in the game of the previous section, M has already chosen its fuel supply and demand x_M and y_M , and the fuel demand of the representative consumer of country $i \in N$ is $y_i = D_i(p)$ from (6). At stage 3, $i \in N$ is aware that its sales and/or purchases of deposits at stage 1 changed its initial endowment of deposits from $[0, \infty]_{C'_i}$ to $[0, \infty]_{K'_i}$, where K'_i , $\forall i \in \Omega$, denotes the marginal cost function after clearance of the deposit market.¹⁷ Choosing an arbitrary country $k \in N$, there exist functions K'_i after stage 1 that satisfy equation (8) for all $i \in N, i \neq k$, and

$$K'_k(x_k, p_a, \pi_z) := \begin{cases} \tilde{K}'_k(x_k) & \text{for } x_k \leq \tilde{\sigma}_k, \\ C'_k(\xi_k) - C'_k(\tilde{\sigma}_k) + C'_k(x_k) & \text{for } x_k \geq \tilde{\sigma}_k, \end{cases} \quad k \in N, \quad (23)$$

¹⁷We use the same functional sign K'_i as in the previous section for notational relief.

where $\tilde{\sigma}_k := \sigma_k + (\sigma_M - D_M)$ and where \tilde{K}'_k is a weakly monotone increasing function on the domain $[0, \tilde{\sigma}_k]$ that results from reordering the marginal extraction costs of the deposits contained in $[0, \sigma_k]_{C'_k} \cup [D_M, \sigma_M]_{C'_M}$. The corresponding fuel supply functions satisfy (10) for $i \in N, i \neq k$, and

$$S_k(p, p_a, \pi_z) = \begin{cases} \tilde{K}'_i^{-1}(p) & \text{for } p \leq C'_k(\sigma_k), \\ \tilde{\sigma}_k & \text{for } p \in [C'_k(\sigma_k), p_a], \\ C'^{-1}_k [p - C'_k(\xi_k) + C'_k(\tilde{\sigma}_k)] & \text{for } p \geq p_a, \end{cases} \quad k \in N. \quad (24)$$

In view of (6), (9) and (24), the fuel market equilibrium condition is

$$x_M + \sum_N S_j(p, p_a, \pi_z) = y_M + \sum_N D_j(p). \quad (25)$$

Equation (25) yields the equilibrium fuel price as a function of x_M, y_M, p_a and π_z , all of which have been determined earlier in the game. We denote that price function as

$$p = P(x_M, y_M, p_a, \pi_z). \quad (26)$$

Stage 2. The analysis of stage 2 is the same, in qualitative terms, as in the last section. The only difference is that we replace the function K'_M from (12) with the coalition's marginal extraction cost function¹⁸

$$K'_M(x_M, p_a, \pi_z) = \begin{cases} C'_M(x_M) & \text{for } x_M \leq D_M, \\ \tilde{K}'(x_M) & \text{for } x_M \in [D_M, \tilde{x}_M], \\ \tilde{K}'(\tilde{x}_M) - C'_M(\tilde{x}_M) + C'_M(x_M) & \text{for } x_M \geq \tilde{x}_M, \end{cases} \quad (27)$$

where

$$\begin{aligned} \tilde{K}'(x_M) &:= C'_M(\sigma_M) - \tilde{C}'_M(D_M) + \tilde{C}'(x_M), \quad \tilde{x}_M := \sigma_M + \sum_{\Omega} (\xi_j - \sigma_j) \quad \text{and} \\ \zeta &= \tilde{C}'(x) \iff x = \sum_{\Omega} C'^{-1}_j(\zeta) \end{aligned}$$

The stage 2 of the present game is fully characterized by the equations (14) through (17).

Stage 1. At stage 1, the following set of bilateral deposit transactions is carried out.

Deposit purchase A: The coalition buys the deposits $[\sigma_i, \xi_i]_{C'_{1i}}$ from each country $i \in N$.

¹⁸With a slight abuse of notation, we use the same functional sign K'_M for the functions in (12) and (27).

Deposit purchase B: The coalition sells the deposits $[D_M, \sigma_M]_{C'_M}$ to some country $k \in N$, if $\sigma_M > D_M$; otherwise it buys the amount $D_M - \sigma_M > 0$ of deposits with extraction costs less than or equal to $p_a - \pi_z$ from some country $k \in N$.^{19,20}

The deposit purchases *A* are the coalition's deposit purchases in the game of the last section. These deposits $[\sigma_i, \xi_i]_{C'_i}$, $\forall i \in N$, are profitable but socially inefficient, and they therefore ought to be preserved. In contrast, the deposits *B* are profitable and socially efficient and hence ought to be exploited.²¹ With regard to efficiency, it is irrelevant, however, whether the coalition or any other country $k \in N$ exploits them.²² We have shown in Section 3 that it is possible to carry out the deposit purchases *A* with mutual gains from trade. This is slightly different with the deposits *B*. If $\sigma_M > D_M$, country k is willing to buy the deposits *B* when the price is slightly lower than the profit it can make by exploiting them itself. If $\sigma_M < D_M$, the coalition is willing to buy the amount $D_M - \sigma_M$ of deposits with extraction costs less than or equal to $p_a - \pi_z$ at a price that is slightly higher than the selling country's profits foregone.²³

To ease the exposition, we restrict our focus on cases in which the deposit purchase *B* is a sale (which we nevertheless denote as purchase *B* for convenience of notation). The inequality $\sigma_M > D_M$, or more precisely $\sigma_M(p_a, \pi_z) > D_M(p_a)$, will be satisfied in economies characterized by $\sigma_M(p^*, \pi_z(p^*)) > D_M(p^*)$, if the anticipated values (p_a, π_z) are sufficiently close to $(p^*, \pi_z(p^*))$.

The Figures 1a and 2 illustrate the marginal extraction cost curves after the deposit purchases *A* and *B* in an economy with the coalition and $N = \{i\}$. As described above, the curves *OBEF* in Figure 1a and *OBGH* in Figure 2 represent the marginal extraction cost curves of country i and the coalition, respectively, after the coalition has purchased the deposits $[\sigma_i, \xi_i]_{C'_i}$ (=deposits *A*) from country i . By presupposition, the purchase *A* results

¹⁹Suppose that $\sigma_M < D_M$ and that there is a country $k \in N$ with a sufficiently large endowment of deposits with costs equal to or less than $p_a - \pi_z$. Then one possible purchase *B* is that the coalition buys the deposits $[\underline{x}, \sigma_k]_{C'_k} \subset [0, \sigma_k]_{C'_k}$, where $\underline{x} := \sigma_k + \sigma_M - D_M$.

²⁰Our specification of purchase *B* follows Harstad's (2012, p. 93) observation that efficiency is attained if we make the coalition a nontrader of fuel through a suitable deposit purchase or sale. In his Lemma 2, Harstad requires all non-coalition countries to become nontraders as well. For details about how one can extend purchase *B* to make all countries nontraders, see Appendix A3.

²¹To avoid clumsy wording, we refer to the deposits in the deposit purchase $A[B]$ also as deposits $A[B]$.

²²Consequently, there may be more than one country k to whom the coalition sells some of the deposits *B* (if $\sigma_M < D_M$) or from whom the coalition buys some of the deposits *B* (if $\sigma_M > D_M$). For convenience of exposition, we keep supposing that the deposit transactions *B* are between one country $k \in N$ and the coalition.

²³Sellers and buyers of deposits with extraction costs less than or equal to $p_a - \pi_z$ are indifferent with respect to making the deal, if the price is exactly equal to the seller's profits foregone.

in $\sigma_i < D_i = x_i^{II}$ and $\sigma_M > D_M = x_M^{II}$. Hence, the purchase B requires the coalition to sell its deposits $[D_M, \sigma_M]_{C'_M}$ to country i . After that transaction, the marginal extraction curves are $0AKLM$ in Figure 1a for country i and $0APQR$ in Figure 2 for the coalition.

Consistency requires equality of the fuel price p_a anticipated at stage 1 and the fuel price p that clears the fuel market at stage 3. We assume that the price function \hat{P} possesses a fixed point, i.e. we set $p = p_a$, and denote the equilibrium price by $p = p^o$. Appendix A4 shows that the equilibrium fuel price is $p = p_a = p^*$ implying that the equilibrium allocation is first-best in the game with deposit market of type II and a price-taking coalition. This 'Coasean' efficiency result remains valid with any number and size of additional bilateral deposit trades between the countries in group N as long as the deposits traded have (marginal) extraction costs that are lower than or equal to $p_a - \pi_z$ (Harstad 2012, p. 104). Harstad's Theorem 1 also answers in the affirmative the question whether efficiency can be retained when the deposit purchases A and B are made and the coalition acts strategically on the fuel market. We summarize his results in

Proposition 3. (*Harstad 2012*)

- (i) *If the coalition takes the fuel price as given in the world economy with deposit market II , the equilibrium of the game is efficient (Harstad 2012, p. 104).*
- (ii) *The set of equilibrium allocations on the deposit market II , Ω_{II}^{eq} , is non-empty in every economy satisfying the assumptions of the present paper: $\forall E \in \mathcal{E} : \Omega_{II}^{eq} \neq \emptyset$.*
- (iii) *In every equilibrium of the deposit market II the coalition's strategic action implements the first-best: $\Omega_{II}^{eq} \Rightarrow \Omega^{sa} \subset \Omega^*$ (Harstad 2012, Theorem 1).*

5 An assessment of Harstad's Theorem 1

By definition, the deposit market II is in equilibrium, if the climate damage externality is fully internalized. This is true independent of whether the coalition seeks to influence the fuel price. In Harstad's Theorem 1 the equilibrium of the deposit market II is a necessary condition for the solution of the three-stage game. It follows that if the coalition does influence the fuel price and thus distorts the allocation, the outcome is obviously incompatible with the presupposition that the deposit market II is in equilibrium. The definitional link between equilibrium and efficiency excludes the possibility of an inefficient equilibrium and hence an inefficient outcome of the game. As our game model of Section 3 shows, there is no compelling reason for an equilibrium concept that requires efficiency by definition.

For the better understanding of Harstad's Theorem 1, it is helpful to compare the

games with deposit market I from Section 3 and deposit market II from Section 4. In formal terms, Harstad's Theorem 1 - as reconstructed in our Propositions 3(ii) and 3(iii) - states that

$$\forall E \in \mathcal{E} : \quad \Omega_{II}^{eq} \Rightarrow \Omega^{sa} \subset \Omega^*, \quad (28)$$

whereas Proposition 2(iii) states

$$\forall E \in \mathcal{E} : \quad \Omega_I^{eq} \Rightarrow \Omega^{sa} \subset \Omega^* \quad \text{and} \quad \forall E \in \mathcal{E} \setminus \mathcal{E} : \quad \Omega_I^{eq} \Rightarrow \Omega^{sa} \cap \Omega^* = \emptyset,$$

as observed above in (22). At first sight, the comparison of (28) and (22) suggests that the market concept II is superior to the market concept I with respect to efficiency, because the former concept prevents inefficiencies of strategic action in every economy, while the latter achieves that result only in the small set of economies \mathcal{E} . In all economies $\mathcal{E} \setminus \mathcal{E}$, the outcome of the game is inefficient, if the deposit market is of type I , and it is efficient, if the market is of type II . As shown in Section 4, the deposit market II cannot attain an equilibrium unless the coalition makes a purchase (or sale), denoted deposit purchase B , in addition to its deposit purchases it makes in the deposit market I (denoted deposit purchases A in Section 4). The crucial question is whether it is in the coalition's self-interest to carry out that extra deposit purchase B .

To answer that question we consider and compare the following games.

Game $G(A_{ns})$: The coalition purchases the deposit A and implements the first-best by refraining from strategic action on the fuel market.

Game $G(A_s)$: The coalition purchases the deposit A and distorts the allocation to its own favor by acting strategically on the fuel market.

Game $G(AB)$: The coalition purchases the deposit A and B and implements the first-best, because the purchase B rendered its strategic action ineffective.

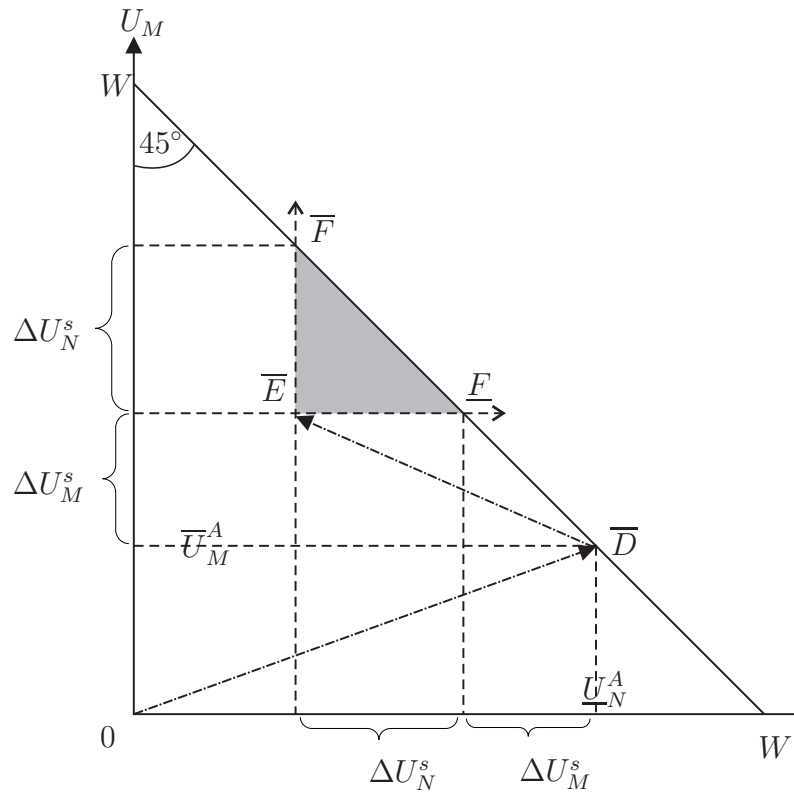


Figure 3a

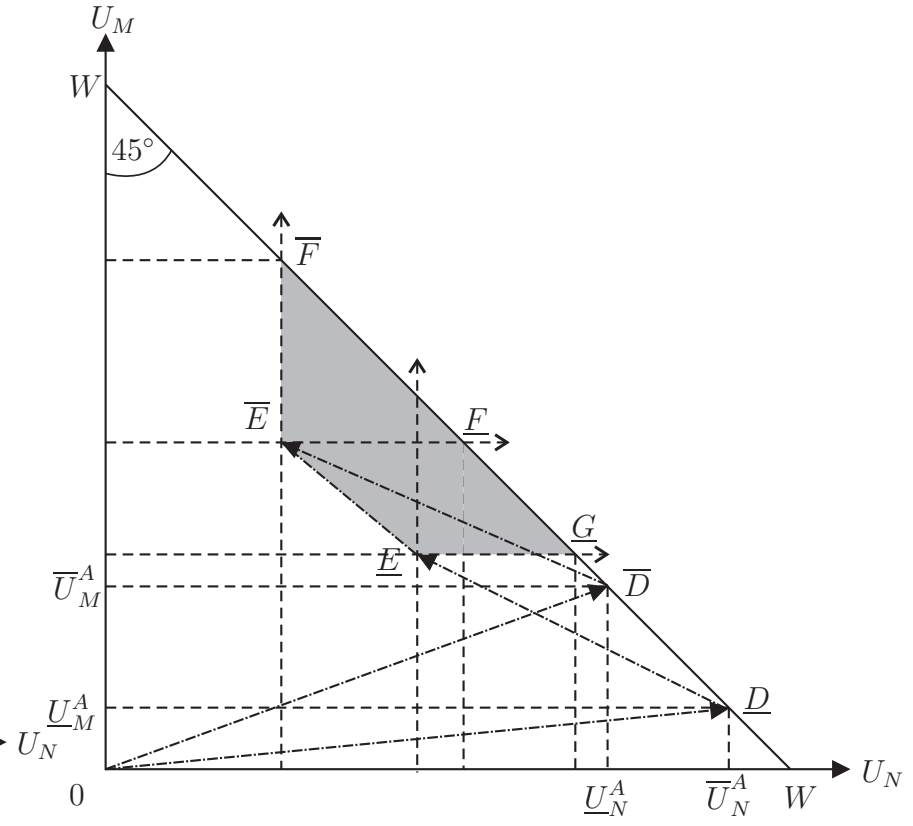


Figure 3b

Figure 3: Welfare implications of different games

The games $G(A_{ns})$ and $G(A_s)$ have been analyzed in Section 3 and the game $G(AB)$ has been analyzed in Section 4. In what follows we will discuss and compare the welfare implications of these games by means of the Figures 3a and 3b.²⁴ The vertical axis of these figures measures the coalition's welfare, U_M , and the horizontal axis measures the *aggregate* welfare, U_N , of all non-coalition countries. The negatively sloped straight line WW is the welfare frontier.

Consider first the game $G(A_{ns})$. Since we require the gains from bilateral deposit trades to be shared among the trading partners, we consider two polar cases. In the first case, the coalition reaps (almost) all trade gains by purchasing the deposits A for (slightly more than) the seller country's profits foregone. In Figure 3a, this case corresponds to the move from the origin 0 (*laissez-faire*) to the point \overline{D} on the welfare frontier WW . In the second polar case, (almost) all gains from deposit trades go to the non-cooperative countries. That case corresponds to a move from the origin 0 to a point such as \underline{D} on the welfare frontier in Figure 3b. All intermediate cases lie on the line segment $\overline{D}\underline{D}$ of the welfare frontier.

If we take the polar solution \overline{D} of the game $G(A_{ns})$ as our point of departure, the outcome of game $G(A_s)$ is illustrated in Figure 3a by the move from \overline{D} to \overline{E} . As argued in Section 3, that move is welfare-increasing for the coalition, welfare-reducing for the non-coalition countries, and it is wasteful because the point \overline{E} lies below the welfare frontier. Figure 3b generalizes the outcome of the game $G(A_s)$. If all gains from deposit trade in game $G(A_{ns})$ accrue to the non-coalition countries, the result of strategic action in game $G(A_s)$ is the move from \underline{D} to \underline{E} . Consequently, the line segment $\underline{E}\overline{E}$ represents all possible outcomes of game $G(A_s)$.

Next, assume the economy is in the equilibrium point \overline{D} of the game $G(A_{ns})$ in Figure 3a and suppose some country considers buying the deposits B . The minimum price the coalition demands is ΔU_M^s in Figure 3a, because it will not strike any bargain in addition to its purchases A that makes it worse off than in the outcome of game $G(A_s)$.²⁵ More generally, the range of feasible prices for the purchase B is the interval $]\Delta U_M^s, \Delta U_M^s + \Delta U_N^s[$, which translates in outcomes on the line segment $\overline{F}\underline{F}$ on the welfare frontier of Figure 3a. Any such outcome is better than the outcome at the equilibrium \overline{E} of game $G(A_s)$ for both the coalition and for the group of non-coalition countries. However, the question is whether an individual country is willing to pay a price in the range $]\Delta U_M^s, \Delta U_M^s + \Delta U_N^s[$, which must be paid in order to induce the coalition to sell the deposits B .

²⁴Table 1 in the Appendix A5 presents the welfare implications for the coalition and the group of non-coalition countries in a systematic way.

²⁵If the deposit purchase B requires the coalition to buy deposits, then the selling country likely needs to sell deposits at some negative price. Recall from Section 4 from the definition of the deposit purchase B .

This scenario gives rise to an interesting strategic 'sub-game' among the non-coalition countries. Suppose first - as we did up to now - that all countries act independently without coordination or cooperation. Then each country has strong free-rider incentives to let some other country purchase the deposits B and benefit without contributing. If the number of non-coalition countries is large, it is plausible that the price the coalition demands for selling the deposits B is so high that the buying country is worse off than in the equilibrium of the game $G(A_s)$. Put differently, it is plausible that abstaining from buying the deposits B is the dominant strategy for all countries (prisoner's dilemma). Even if the coalition's price for selling the deposits B is lower than the buying country's welfare gains from restoring efficiency, the country will be reluctant to buy because of strong free-rider incentives. Each country would benefit from the purchase without contributing to the costs. Our conclusion therefore is that an equilibrium of the game $G(AB)$ cannot be attained under the assumption that the countries outside the coalition act non-cooperatively.

Hence the only way to secure an equilibrium of the game $G(AB)$ is to assume cooperation among all non-coalition countries. At stage 1 we then have a bargaining game between two groups of countries. One group is the 'climate coalition' we have considered throughout the paper, namely the group of countries that suffer from climate damage and pursue a cooperative climate policy of purchasing deposits for preservation. All countries not suffering from climate damage now also form a group, treated as a single agent, whose only purpose is to reap some benefits from preventing allocative distortions, which would result from the climate coalition's strategic action in game $G(A_s)$.

The bargaining set of the 'sub-game' between these two groups is illustrated by the shaded area $\underline{E}\overline{E}\overline{F}\underline{G}$ in Figure 3b the construction of which is straightforward from our preceding discussion of the Figures 3a and 3b. One can then apply a standard solution concept of the cooperative game theory, e.g. the Nash bargaining solution, which selects some equilibrium point on the segment $\overline{F}\underline{G}$ of the welfare frontier in Figure 3b. The closer the equilibrium point is to the point \overline{F} [\underline{G}], the larger is the climate coalition's [the other group's] share of the gains from deposit trade. An interesting additional observation is that in order to make an agreement about some equilibrium point on the line segment $\overline{F}\underline{G}$ in Figure 3b the groups need not carry out the deposit purchase B . One reason why the group of non-coalition countries might want to insist on the purchase B is uncertainty about the coalition's willingness to comply with the agreement. However, compliance is not an issue in Harstad's analysis.

Summing up, we find it highly implausible to argue that the game $G(AB)$ can be played without coordination and cooperation among the non-coalition countries. It is also highly implausible, however, to assume that the non-coalition countries move from independent

action to full cooperation to reap the benefits from restoring efficiency. While this is a possibility in a frictionless world, that assumption is not common practice in economic analyses of markets with strategically acting agents.²⁶ We summarize our results in

Proposition 4.

- (i) *Suppose the non-coalition countries act non-cooperatively in the deposit market II. Then that market does not clear and the coalition's strategic action fails to implement the first-best.*
- (ii) *Suppose the non-coalition countries cooperate and bargain with the coalition in the deposit market II. If the bargaining solution is Pareto-optimal, then the coalition's strategic action implements the first-best.*

6 Concluding remarks

As pointed out in the introduction, the motivation for the present paper is to understand better Harstad's (2012) 'efficiency-despite-strategic-action result'. His theorem appears to contradict conventional wisdom according to which exerting market power always makes the strategically acting agent better off than price taking. Our strategy of analysis was to decompose the transactions on Harstad's deposit market (market *II*) into the sequential purchases *A* and *B*. That enabled us to show that making the deposit purchase *B* is in the coalition's interest only, if it is fully compensated for the welfare increase it would have experienced in case of strategic action. It is grossly implausible that any individual independently acting non-coalition country would make such a deal which likely reduces its welfare while it makes all other non-coalition countries better off. We conclude, therefore, that the only way to rationalize Harstad's efficiency result is to assume full cooperation on the part of non-coalition countries along with the standard idea that two bargaining partners always find a way to exhaust all gains from bargaining.

It is a long standing insight that allocative inefficiencies resulting from non-cooperative behavior can be fixed by adopting a cooperative approach. Evidence from economic activities in the real world also suggests, however, that such cooperation often fails due to various barriers that admittedly are not captured in the formal model. We therefore consider it more realistic that an agent makes use of – and benefit from – her market power at the cost of efficiency than assuming a collective bargaining approach to implement the first-best.

²⁶For example, take the textbook (partial equilibrium) monopoly. The demanders could cooperate and induce the monopolist to produce the perfectly competitive allocation by paying her (slightly more than) the monopoly profit foregone.

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Appendix

A1: Extraction costs and endowment of deposits

For analytical convenience, we assume that each deposit contains a single (small) unit of fuel. The ordering of country i 's deposits according to costs results in a step function, say $\mathcal{C}_i : \mathbb{N} \rightarrow \mathbb{R}_+$, such that $\mathcal{C}_i(x_i)$ is the cost of extracting the unit of fuel from the x_i^{th} deposit²⁷ and $\mathcal{C}_i(x_i) \leq \mathcal{C}_i(x_i+1)$ for all $x_i \in \mathbb{N}$ (where we exclude the equality sign for analytical relief). Finally, we replace the step function $\mathcal{C}_i(x_i)$ by its real-number approximation, denoted $C'_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+$. With a slight abuse of notation we refer to $C'_i(x_i)$ as the extraction cost of country i 's x_i^{th} deposit - which is the deposit with the x_i^{th} lowest extraction cost.²⁸

A2: Proofs

Proof of Proposition 1:

To prove Proposition 1 consider first the case of competitive markets for fuel and deposits with prices p and p_z , respectively. If the countries $i \in N$ would ignore the market for deposits, they would choose their fuel supply as in (2) such that $C'_i(x_i) = p$ or

$$x_i = \xi_i(p) := C'^{-1}_i(p). \quad (\text{A1})$$

If they take advantage of the deposit market, they maximize with respect to y_i and z_i^s

$$U_i = B_i(y_i) - C_i(\xi_i(p) - z_i^s) - p(y_i - \xi_i(p) + z_i^s) + p_z z_i^s \quad (\text{A2})$$

and obtain

$$y_i = B'^{-1}_i(p) =: D_i(p) \quad \text{and} \quad z_i^s = Z_i^s(p, p_z) = \xi_i(p) - \sigma_i(p, p_z) \quad \forall i \in N, \quad (\text{A3})$$

where $\sigma_i(p, p_z) := C'^{-1}_i(p - p_z)$. The coalition maximizes with respect to x_M , y_M and z_M^d

$$U_M = B_M(y_M) - C_M(x_M) - p(y_M - x_M) - H \left(x_M + \sum_N \xi_j(p) - z_M^d \right) - p_z z_M^d \quad (\text{A4})$$

which yields

$$\begin{aligned} y_M &= B_M'^{-1}(p) = D_M(p), \quad x_M = \sigma_M(p, H') := C_M'^{-1}(p - H') \\ \text{and} \quad z_M^d &= Z_M^d(p, p_z) = \sigma_M(p, H') + \sum_N \xi_j(p) - H'^{-1}(p_z). \end{aligned} \quad (\text{A5})$$

²⁷We need not care about an upper bound of the domain of the function \mathcal{C}_i because deposits with extremely high extraction costs will never be exploited under realistic conditions.

²⁸Hence the primary concept is the marginal cost function C'_i rather than the total cost function C_i . Differently put, in the deposit-trading perspective we derive C_i from C'_i rather than C'_i from C_i .

It is straightforward that setting $p = p^* = \lambda_f^*$ and $p_z = p_z^* = \lambda_z^* = H'^*$ in (A3) and (A5) lets the equations in (A3) and (A5) coincide with the equations (5). In particular, we have

$$\begin{aligned} y_i^* &= D_i(p^*), \quad \sigma_i^* = \sigma_i(p^*, p_z^*) \quad \forall i \in \Omega, \quad \xi_i^* = \xi_i(p^*) \quad \forall i \in N, \\ z_i^{s*} &= Z_i^s(p^*, p_z^*) = \xi_i^* - \sigma_i^* \quad \forall i \in N, \quad z_M^{d*} = Z_M^d(p^*, p_z^*) = \sigma_M^* + \sum_N \xi_j^* - H'^{-1}(p_z^*). \end{aligned} \quad (\text{A6})$$

To prove Proposition 1(ib), suppose the first-best allocation (5) is given and assume the price p^* clears the fuel market. We have to show that the coalition can buy the deposits $z_i^{s*} = \xi_i^* - \sigma_i^*$ from country $i \in N$ at a price that is strictly beneficial for both. Country i is obviously willing to sell, if it receives a payment that is slightly higher than the profit $p_z^* z_i^* - C_i(\xi_i^*) + C_i(\sigma_i^*)$ it would have received in case of keeping its deposits $[\sigma_i^*, \xi_i^*]_{C'_i}$. That profit foregone is smaller than the payment $p_z^* z_i^{s*}$ country i gets for its deposit sale in the competitive world economy of Proposition 1(ia), if and only if the functions C'_i are strictly increasing. The coalition's maximum willingness-to-pay for the deposits $z_M^{d*} = \sum_N z_j^{s*}$ is the value of the climate damage reduction $H(\sigma_M^* + \sum_N \xi_j^*) - H(\sigma_M^* + \sum_N \xi_j^* - z_M^{d*})$. That value exceeds M 's expenditure $p_z^* z_M^{d*}$ in the competitive world economy, if and only if the damage function H is strictly increasing. We conclude that there is room for agreements on prices for selling/buying z_i^{s*} such that the deal is mutually advantageous for both the coalition and country i . The deposit market I is cleared (as well as the deposit market II), because when all countries $i \in N$ have sold their deposits $[\sigma_i^*, \xi_i^*]_{C'_i}$ there exists no pair of countries that would both strictly benefit from trading some additional deposits at some price.

Proposition 1(ic) is a special case of Proposition 1(ib). ■

Proof of Lemma 1:

Solve the Lagrangean

$$\begin{aligned} \mathcal{L}(z_1^s, \dots, z_N^s, z_M^d, \lambda_z) &= \sum_N [B_j(y_j) - C_j(\xi_j(p_a) - z_j^s) - p_a(y_j - \xi_j(p_a) + z_j^s)] \\ &\quad + B_M(y_M) - C_M(X_M(p_a, \pi_z)) - p_a(y_M - X_M(p_a, \pi_z)) \\ &\quad - H \left[X_M(p_a, \pi_z) + \sum_N \xi_j(p_a) - z_M^d \right] - \pi_z \left(z_M^d - \sum_N z_j^s \right) \end{aligned} \quad (\text{A7})$$

with respect to $z_1^s, \dots, z_N^s, z_M^d$ and λ_z for predetermined p_a, π_z, y_M and $y_i \forall i \in \Omega$. The first-order conditions yield

$$C'_i(x_i) = p_a - \lambda_z \quad \text{and hence} \quad x_i = \sigma_i(p_a, \lambda_z) := C_i'^{-1}(p_a - \lambda_z) \quad \forall i \in N \quad (\text{A8})$$

$$\text{and} \quad \lambda_z = H' \left[X_M(p_a, \pi_z) + \sum_N \sigma_j(p_a, \lambda_z) \right]. \quad (\text{A9})$$

(A9) implicitly characterizes λ_z as a function of p_a and π_z , and this function possesses a fixed point, denoted $\lambda_z = \pi_z = \pi_z(p_a)$, for all p_a in the relevant sub-domain. Under consideration of $\lambda_z = \pi_z = \pi_z(p_a)$, we determine the solution of (A7) as

$$z_i^s = Z_i^s(p_a) := \xi_i(p_a) - \hat{\sigma}_i(p_a) \quad \forall i \in N, \quad (\text{A10})$$

where $\hat{\sigma}_i(p_a) := \sigma_i(p_a, \pi_z(p_a))$. That M is able to purchase $Z_i(p_a)$ at a price that makes both parties better off as described in the paragraph following the equations (A6). ■

Derivation of (9):

Maximizing $u_i = B_i(y_i) - K_i(x_i, p_a, \pi_z) - p(y_i - x_i) + R_i(p_a, \pi_z)$ with respect to x_i yields

$$K'_i(x_i, p_a, p_z) = p.$$

Suppose that $x_i \leq \sigma_i(p_a, \pi_z) = C_i'^{-1}(p_a - \pi_z)$, then we obtain

$$C'_i(x_i) = p \quad \iff \quad x_i = C_i'^{-1}(p)$$

for $p \leq p_a - \pi_z$.

Suppose that $x_i \geq \sigma_i(p_a, \pi_z) = C_i'^{-1}(p_a - \pi_z)$, then we get

$$C'_i(\xi_i) - C'_i(\sigma_i) + C'_i(x_i) = p \quad \iff \quad x_i = C_i'^{-1}[p - C'_i(\xi_i) + C'_i(\sigma_i)]$$

for $p \geq p_a$ (due to $C'_i(\sigma_i) - C'_i(\xi_i) = -\pi_z$). ■

A3: Bilateral deposit trades that make all countries nontraders of fuel in the equilibrium of the game

Suppose all deposit purchases A and B are made. Then each country $i \in N$ has sold the deposits $[\sigma_i, \xi_i]_{C'_i}$ and country $k \in N$ has purchased the deposits $[D_M, \sigma_M]_{C'_M}$. The profitable and socially efficient deposits are now allocated to the countries as follows. Each

country $i \in N, i \neq k$, owns $[0, \sigma_i]_{C'_i}$ and country $k \in N$ owns $[0, \sigma_k]_{C'_k} \cup [D_M, \sigma_M]_{C'_M}$ of these deposits. It is convenient to choose a different but equivalent representation of the set $[0, \sigma_k]_{C'_k} \cup [D_M, \sigma_M]_{C'_M}$ by reordering the deposits in that set according to extraction costs. We thus generate a weakly increasing marginal extraction cost function, say \tilde{K}'_k , on the domain $[0, \sigma_k + \sigma_M - D_M]_{\tilde{K}'_k}$. Using that notation the set of profitable and socially efficient deposits owned by all countries of group N is $S := [0, \sigma_k + \sigma_M - D_M]_{\tilde{K}'_k} \cup \bigcup_{j \in N, j \neq k} [0, \sigma_j]_{C'_j}$. The fuel price is unaffected by any relocation of these deposits between the countries and all countries are indifferent with respect to such relocations as long as the sales are priced at profits foregone. Hence, any partition of S into N subsets can be attained as the result of (additional) deposit exchanges priced at profits foregone. Harstad's Lemma 2 requires the very specific redistribution of ownership, which allocates to each $i \in N$ a subset of S which contains

$$\tilde{\sigma}_i = \tilde{\sigma}_i(p_a, \pi_z) := \frac{\sigma_k(p_a, \pi_z) + \sigma_M(p_a, \pi_z) - D_M(p_a) + \sum_{j \in N, j \neq k} \sigma_j(p_a, \pi_z)}{\sum_N D_j(p_a)} D_i(p_a). \quad (\text{A11})$$

deposits, where the numerator in (A11) is the 'number' of deposits contained in S . It is easy to see that (A11) yields $\tilde{\sigma}_i(p_a, \pi_z) = D_i(p_a)$ in the equilibrium of the game.

A4: Efficiency of outcome with deposit market II and price-taking coalition

Assume that the price function \hat{P} from (26) possesses a fixed point, i.e. set $p = p_a$, and denote the equilibrium price by $p = p^o$. Collecting the information from the preceding analysis, the fuel supplies and demands are

- $x_M^o = \hat{X}_M(p^o)$, $y_M^o = \hat{Y}_M(p^o)$ from (20) due to the deposit purchases A and B ;
- $y_i^o = D_i(p^o) \quad \forall i \in N$ from (6);
- $x_i^o = \hat{\sigma}_i(p^o) \quad \text{for } i \in N, i \neq k$, from (7) due to the deposit purchases A ;
- $x_k^o = \tilde{\sigma}_k(p^o) = \hat{\sigma}_k(p^o) + (\sigma_M(p^o) - D_M(p^o))$ due to the deposit purchases A and B .

Consider first the special case in which the coalition takes fuel prices as given – as do all countries – and suppose that $p^o = p^*$. We then have

$$\left. \begin{aligned} x_M^o &= \hat{\sigma}_M(p^*) - (\hat{\sigma}_M(p^*) - D_M(p^*)) = D_M(p^*) \quad \text{and} \quad y_M^o = D_M(p^*); \\ x_i^o &= \hat{\sigma}_i(p^*) \quad \forall i \in N, i \neq k \quad \text{and} \quad y_i^o = D_i(p^*) \quad \forall i \in N; \\ x_k^o &= \hat{\sigma}_k(p^*) + (\hat{\sigma}_M(p^*) - D_M(p^*)) \quad \text{for } k \in N. \end{aligned} \right\} \quad (\text{A12})$$

From (A12) follows $\sum_{\Omega} (x_j^o - y_j^o) = \sum_{\Omega} (x_j^* - y_j^*) = 0$, where $x_i^* = \hat{\sigma}_i(p^*)$ and $y_i^* = D_i(p^*) \quad \forall i \in \Omega$ characterize the fuel market equilibrium in the efficient equilibrium of the game with deposit market II when the coalition takes the fuel price as given.

A5: Welfare implications in the games

			Coalition	Group N
GAME $G(A_{ns})$ (Purchases A without strategic action)	Best for coalition	1	$U_M(A_{ns}) = \bar{U}_M^A$	$U_N(A_{ns}) = \underline{U}_N^A$
	Worst for coalition	2	$U_M(A_{ns}) = \underline{U}_M^A$	$U_N(A_{ns}) = \bar{U}_N^A$
	Intermediate	3	$U_M(A_{ns}) = U_M^A(\lambda) = \lambda\bar{U}_M^A + (1 - \lambda)\underline{U}_M^A$	$U_N(A_{ns}) = U_N^A(\lambda) = \lambda\underline{U}_N^A + (1 - \lambda)\bar{U}_N^A$
GAME $G(A_s)$ (Purchases A plus strategic action)	Best for coalition	4	$U_M(A_s) = \bar{U}_M^A + \Delta U_M^s$	$U_N(A_s) = \underline{U}_N^A - \Delta U_M^s - \Delta U_N^s$
	Worst for coalition	5	$U_M(A_s) = \underline{U}_M^A + \Delta U_M^s$	$U_N(A_s) = \bar{U}_N^A - \Delta U_M^s - \Delta U_N^s$
	Intermediate	6	$U_M(A_s) = U_M^A(\lambda) + \Delta U_M^s$	$U_N(A_s) = U_N^A(\lambda) - \Delta U_M^s - \Delta U_N^s$
GAME $G(AB)$ (Purchases $A + B$ no strategic distortion)	Best for coalition	7	$U_M(AB) = \bar{U}_M^A + \Delta U_M^s + \Delta U_N^s$	$U_N(AB) = \underline{U}_N^A - \Delta U_M^s - \Delta U_N^s$
	Worst for coalition	8	$U_M(AB) = \underline{U}_M^A + \Delta U_M^s$	$U_N(AB) = \bar{U}_N^A - \Delta U_M^s$
	Intermediate	9	$U_M(AB) = U_M(\lambda) + \Delta U_M^s + \mu\Delta U_N^s$	$U_N(AB) = U_N^A(\lambda) - \Delta U_M^s + (1 - \mu)\Delta U_N^s$

Table 1: Welfare implications of three different games in comparison (based on the Figures 3a and 3b)

$$(\bar{U}_M^A + \underline{U}_N^A = \underline{U}_M^A + \bar{U}_N^A = \text{first best}; \lambda \in [0, 1]; \mu \in [0, 1])$$