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Spatial Productivity Differences and the Optimal Tax Treatment of Commuting Expenses*

J. Malte Zoubek[a]

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Abstract

Spatial wage differences offer incentives to change the location of work either by commuting or by moving to the new work location. Combining an intensive labor supply margin with an extensive, productivity-enhancing margin of work place change due to commuting or moving, I study how spatial urban-fringe productivity differences and labor mobility shape optimal redistribution under tax deduction of commuting expenses. My study underlines the significance of the tax system for local labour market and settlement pattern.

JEL classification: H21, R12, R23, R51, C61, J61

Keywords: optimal taxation, urban wage premium, commuting, deduction, local labour markets, spatial taxation, regional inequality, multi-dimensional screening

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1 Introduction

The possibility to deduct specific expenses from the income earned or from the tax liability to be paid is a structural part of many tax systems in developed countries. Besides deduction possibilites for capital income, a substantial amount of deductions is related to the costs someone has due to her dependent employment. The rationale for deduction possibilites in both the capital and the income tax system is, basically, that costs that accrue from earning money shall be reckoned up with the money earned. From a normative perspective, reasons in favor of such deduction possibilites in the income tax system may be seen in the claim that such deduction possibilities directly lower the costs to work for an individual and, hence, indirectly rise individual's work supply, increase the size of the labour market and overall efficiency. Deduction possibilites are thought of as to compensate expenses paid to earn money and to generate dependent labour income, i. e. such expenses are seen as work-related expenses. Commuting cost, i.e. the costs that occur for traveling form home to work, account for a large part of work-related costs. In a lot of continental European countries such as Germany or France or Scandinavian countries such as Sweden or Denmark, commuting expenses are deductable whereas in several OECD countries such as the United States, the United Kingdom, Canada or Italy such costs are not deductable, see Harding (2014).

Besides this more policy related discussion (see next chapter), recently it emerges increasingly interest in the (empirical) taxation literature on aspects of commuting and taxation, see, among others, Dörrenbeg et al. (2017), Eeckhout and Guner (2017), and Wrede (2009).

Despite the discussion of deductions in the optimal capital taxation literature and despite the important role that commuting expenses play in everday life, there is, by now, only limited theoretical work dealing with the pros and cons of the deduction possibility of commuting expenses with respect to efficiency-equity considerations from the optimal income tax literature. Productivity differences between different cities and/or rural areas offer commuting and moving possibilites that lead to externalities. Living in one city or the urban fringe or the country side and commuting to a work place in another location creates externalities that should be taken into account when shaping the optimal income tax schedule. I study commuting cost deduction possibilities in an optimal taxation framework with spatial productivity differences. By this, I contribute to the literature in different fields of economics. First, I study optimal income taxation in an urban economics setting of a more productive city and a less productive fringe. Due to the commuting and moving possibilities between the fringe and the city, my study contributes to the urban economics literature by dealing with the bidirectional influence between, on the one hand, the tax system and, on the other hand, population and commuting pattern. I incorporate spatial productivity differences and the resulting commuting pattern which describe important aspects of local job markets in urban economics. Second, I contribute to the literature of optimal income taxation the possibility of deductions, in particular deductions for commuting expenses. Third, I show how the ambitious three-dimensional screening problem can, under plausibel assumptions, be reduced to an almost two-dimensional problem which is solved using the intuitive perturbation approach.

2 Actual tax treatment of commuting expenses

There is a wide spectrum without international standardization even within the industrialized countries whether and, if applicable, how costs for commuting to one's location of work is deductable from income or tax liabilities, respectively, see Harding (2014), Wrede (2001). According to a recent OECD study, however, there are two contrary opinions about commuting expenses within OECD member countries, see Harding (2014): For roughly half of the interviewed OECD countries, commuting expenses are seen to be fully under personal control and treated like all other personal expenses. It follows that commuting costs are by no means travel costs. On the other hand, the opinion of the other half of OECD countries is that commuting costs are not fully under personal control but necessary expenses for work. According to the second opinion, commuting expenses are like other travel expenses. The difference in opinion directly results in differences in the tax treatment of commuting expenses. Following the first opinion, commuting expenses, in general, do not have any relationship to personal taxation. According to the second opinion, a branch of deduction possibiltes of commuting costs is established, varying over countries.

In Germany, for example, one of the most important income deduction possibility out of several hundred are the expenses to commute from home to work. Regardless of the actual costs for traveling to work at another place, every worker is allowed to deduct a commuting allowance of € 0.3 per kilometer of her way from home to work (*Entfernungspauschale*) until a certain cap of deductions is reached (€ 4,500). This applies regardless of the actual means of commuting, e.g. it applies for commuting by car, train, bicycle or even walking, too. For 2011, the German Federal Statistical Office accounts that almost 12 million individuals applied for this commuting allowance which amounts to almost half of all individuals who filed an income tax declaration in that year, see Statistisches Bundesamt (2015).[1] Furthermore, the Federal Statistical Office accounts

[1]Note that there is a general fixed amount (€ $1,000$ for a single) that is deducted automatically from

that, in 2011, the average distance applied for deductions was 32.6 kilometres, in sum € 20 billion of commuting costs which is equivalent to more than 2% of all incomes from dependent work declared in that year.

In countries without such deduction possibilities, like the US, it is argued that, first, (almost) everyone has to travel to work and, second, the commuting decision is a soley private decision since nobody is forced to live close or far away from her working place. In addition, it is argued that there is no reason why someone shall be allowed to deduct her commuting expenses from, say, her home town San Fransisco to her working place New York City. On the other hand, in countries that allow for deduction of commuting expenses, the forenmentioned argument is taken into account using upper caps for commuting expenses. The idea behind the policy in deduction-friendly countries is that commuting expenses are, at least to some extent which is limited using a upper cap, not a solely private decision but necessary costs for generating income, hence, travel expenses. Even if (almost) everyone has to commute to work and has related commuting costs, it is additionally argued, that personal characteristics cause a difference in the effect commuting expenses have on the working decision, e.g. it makes a difference whether someone with high earning ability or with low earning abilities has to cover commuting expenses. In short, commuting expenses, first, distort individuals' working decision differently and, second, are seen as implicit redistribution. It is argued that, with a progessive tax system, deduction possibilities for commuting expenses take into account this distortion and redistribution effects. An addition rationale for the generous German deduction possibilities for commuting expenses may be seen in the German concept of equivalent living conditions all around the country.[2] The concept's first legal determination is given in the federal law on comprehensive regional planning in 1965 (*Raumordnungsgesetz*) which postulates a permanently balanced and comprehensive regional planning on all administrative levels with the aim of equivalent living conditions across all regions of Germany. One impact of this concept is the responsibility of the administration to counteract unbalanced urbanization. Subsidizing commuting counteracts urbanization. This rationale, again, emphasizes the important link between commuting, taxation, and urban economics, and points out the relevance of my study from another perspective. In addition, since

every person's income who files an income tax declaration. Only declared expenses that exceed this fixed rate are in the statistic mentioned. Hence, the given numbers with respect to commuting expenses are those persons who declare overall work-related deductions (including commuting costs) that exceed this fixed rate of € 1,000. To exceed this overall income deduction fixed rate of € 1,000, several other work-related deductable expenses are taken into account such as expenses for a working place at home, or accommodation expenses in the working place city, e.g. for a second flat, if the commuting individual is married and if she has her main place of living at her spouse's place.

[2] Note that the concept of *equivalent* living conditions in all German regions is not to be confused with a concept of *equal* living conditions.

individuals can commute using different means of travel, the consideration of commuting by tax regulations relates to aspects of public goods provision, such as the use of public transportation vs. private cars, as well as to aspects of environmental economics since commuting deduction possibilites forster incentives to commute, see e.g. Harding (2014).

3 Related literature

Deductions, in particular commuting deductions, are less discussed in optimal income tax frameworks. Nonethelesse, there is a discussion in the urban economic literature on several aspects of commuting deductions and taxation, see e.g. Borck and Wrede (2005), Wrede (2001), Hirte and Tscharaktschiew (2013). A seminal work on the topic of commuting, income taxes, and efficiency aspects in an urban framework is Wildasin (1985) who studies the optimal time allocation between work, commuting, and leisure in a monocentric city model. He finds that the income tax causes welfare losses due to two effects. Higher income taxes shift time to leisure and to travel, and, hence, lower the working time since the implicit value of time is lower by the tax. The effect of the income tax on the commuting time causes spatial dispersion. A workhorse in this literature is Fujitu (1989) who offers a monocentric city model with continuous space. This type of models is applied, among others, by Wrede (2009). Wrede (2009) studies the welfare effects of commuting subsidies in a wage tax framework and extends the two-region-home-attachment model of Wrede (2001) to a model with bidirectional commuting possibilities and two exogenous central business districts. Both studies support the welfare-improving claim in favor of commuting subsidies. However, a study of commuting deductions with key aspects of an optimal taxation framework seems not at hand in this branch of the literature.

Within the branch of the taxation literature, there is a broad discussion on optimal capital taxation that incorporates deductions, e. g. see Auerbach (1979) or Feldstein and Hartman (1979), but lacks a sufficient theoretical discussion on the optimal treatment of deductions in income taxation. One reason may be the belief that deductions only effect the tax base by reducing the income taxed. Hence, deductions directly affect the taxable income. There is a huge literature on taxable income, in particular on the elasticity of taxable income to be a sufficient statistic for the government to estimate the welfare cost of taxation, e. g. see Saez, Slemrod, and Gierts (2012), Chetty (2009), Gruber and Saez (2002), Slemrod and Kopczuk (2002)). However, recent empirical studies emphasize the role deductions have on responses to changes in the tax rate and, in addition, they address the externalities generated by deductions, see, among others, Dörrenbeg et al. (2017), in additon, Matikka (2014), Bastani and Selin (2014). These findings support the reasoning against the elasticity of taxable income to be a sufficient statistic. Using rich German panel data, Dörrenbeg et al. (2017) show that deductions respond to tax changes.

The study at hand corresponds to the class of two-dimensional screening models, see e.g. Lehmann et al. (2014) who combine the intensive labor supply margin with an extensive migration margin or Jacquet et al. (2013) who focus on participation. Kleven et al. (2009) present an optimal taxation paper that deals with participation, too, but with the taxation-participation problem of couples when there is the possibility to have a secondary earner. In their model, the secondary earner does not adjust to tax changes on the intensive margin but solely on the extensive margin. Structurally, to have such a costly and fixed secondary earning as presented in Kleven et al. (2009). In this regard, the structure of the study at hand has some aspects in common with their model. However, my model differs substantially in several aspects to Kleven et al. (2009). One important characteristic of their model is that the participation of secondary earners does not affect wages, neither wages of the primary earners nor of the secondary earners. With respect to the wage of primary earners one may justify missing wage effects by refering to an implicit assumption that the job market of secondary earners differ from the job market and the actual jobs of primary earners, especially if secondary earners work just for relatively smaller and fixed incomes. This rationale is not plausible for missing wage effects on the job market of secondary earners. However, in the urban fringe-city framework of my model, missing wage effects of changing the work place from the urban fringe to the city can be justified by the persistent urban wage premium (city premium) paid for workers who have their jobs in the city, see, among others, Glaeser and Maré (2001), Gould (2007), Yankow (2006).

To solve the optimization problem described by the study at hand, I give a heuristic proof of the optimal tax formulae as applied in other studies of the optimal tax problem, see, among others, Kleven et al. (2006, 2009) or Saez (2001). However, a rigorous solution can be obtained using delayed optimal control technique as has been analyzed by Göllmann et al. (2008) in its entire generality with the necessary conditions for optimal control in such a setting stated in Abdeljawad et al. (2009). In the model I present, the delay is a non-fixed lag, though, given that the productivity gain from changing the location of work is not restricted to be constant but I treat it as a function of the innate productivity.

Delayed optimal control methods have been first used in optimal taxation by Kessing et al. (2015) in a framework of regional productivity inequality and internal migration. However, Kessing et al. (2015) do not take deduction possibilites into account. The deduction possibility makes an additional policy instrument available to the government and can be interpreted as a (tax-free) subsidy. Including deduction in the model allows to contrast the pure tax rate instrument to additional policy instruments that, moreover, are common in real life. Thus, my model is a bit more in spirit of the Mirrleesian tradition of endogenous policy instrument restrictions in contrast to the Ramsey line of models that usually exhibit exogenous given restrictions. In addition, the focus of Kessing et al. (2015) is the moving decision whereas the complementary commuting decision is not studied. In their study, moving costs are ability dependent. Furthermore, Kessing et al. (2015) second focus is the study of regional differentiated income tax schedule, i.e. the regionally different income tax schedules are not set by the region but by the federal government. For the case of uniform tax schedules among regions, Kessing et al. (2015) finds that the moving possibility of individuals has a rather small but, nevetherless, not negligible effect on optimal income tax rates under plausible parameters settings.

My study also corresponds to aspects of federal city-fringe differentiated tax schedules with a growing literature on federal regionally (implicitly) differentiated taxation, see, among others, Albouy (2009), Eeckhout and Guner (2017), and Kessing et al. (2015). However, regional different federal income tax schemes necessarily need certain information for the government about the actual place of living. In a framework with moving and commuting decisions, the possibility for individuals to cheat is more pronounced. In a framework without commuting possibility as in Kessing et al. (2015) this information problem is negligible. However, in my framework this information problem would be present in case of spatial differentiated taxation. Hence, I focus on the uniform taxation case.

4 The framework

I consider three sources of heterogeneity across workers: *innate* productivity n, moving costs q, commuting costs c. These original individual characteristics are distributed over $[n_{\min}, n_{\max}] \times [0, +\infty) \times [0, +\infty)$, and the government can neither observe productivity nor moving costs nor commuting costs. Imagine, there is a homogeneous urban fringe region A and a homogeneous city region B, each of equal size with total population normalized to two. Half of the population resides originally in the urban fringe A, the other half of the population resides in the city B.

Individuals may relocate and move from the urban fringe to the city or vice versa. Alternatively, individuals may stay at the fringe (city) but commute to the city (fringe). In both cases, an individual changes her working place. The actual region of living of each individual is indicated by the superscript j, that is, $j = A, B$. Originally, each resident works according to his home region, but the endogenous decisions of individuals on moving or commuting change these work population shares. The actual region of working is indicated by the subscript i, that is, $i = A, B$.

My key assumption is that the city and the urban fringe differ in their productivity. This is in line with important empirical studies, see, among others, Glaeser and Maré (2001). An individual's *actual* or *realized* productivity n_i is a function of her innate productivity and her region of working $n_i = \omega(n, i) = \omega_i(n)$, where ω_i is strictly increasing in n. I normalize $n_A = \omega_A(n) = n$. Due to this normalization, the function $n_B = \omega_B(n) = \omega(n)$ assigns the actual productivity to all initial worker of city B, and, moreover, it also indicates the transformation of productivity for individuals who change her working place from the urban fringe A to the city B. It is assumed that the city B is more productive, hence, $\omega(n) > n$ for all ability levels n. This is in line with the empirical findings on the urban wage premium, see, among others, Glaeser and Maré (2001). I define a lag function $\kappa(n) \equiv w(n) - n$ that gives the increase in ability from chaging the working place by commuting or moving. Innate productivity is distributed in each region j according to the unconditional probability distribution $f(n)$ on $[n_{\min}, n_{\max}]$. The tax schedule $T^j(z)$ for the same innate ability worker but different realized income due to the working place may differ depending on the the location of work, i.e. $T_i^A(z) \neq T_i^B(z)$. However, spatially differentiated tax schedules are very uncommon. Hence, I focus on spatially uniform tax schedules in both the city and the urban fringe, i.e. $T_i^A(z) = T_i^B(z)$.

Moving costs are made up of two components, pecuniary costs q_p, like expenses for moving services or real estate broker services, and money-metric non-pecuniary costs q_{-p}, like giving up existing social networks or the strong family ties, such that $q = q_p + q_{-p}$. However, it is assmued that the pecuniary moving costs are negligible, so $q_p = 0$ and $q = q_{-p}$. As the already mentioned effect of moving possibilites on uniform regional optimal tax rates even with ability dependent moving costs is small (see Kessing et al. (2015)), and I focus just on spatially uniform tax schedules, it is reasonable to neglect pecuniary moving costs p_{-p}. Another rationale for this assumption is that, first, pecuniary moving expenses are typically relatively low compared to non-pecuniary moving costs, and, second, they arise only once but non-pecuniary moving costs and also commuting costs arise permanently. Even so it is not explicitly a dynamic model, it is straightforward to interpret my model in a quasi-dynamic way since, e.g. income or taxes make only sense in a quasi-dynamic way of permanent flows.

Like moving costs the costs for commuting are made up of two components, too, that is of pecuniary expenses c_p, like petrol or train tickes, and of money-metric non-pecuniary costs c_{-p}, like traveling time, such that $c = c_p + c_{-p}$. Pecuniary commuting costs c_p are deductible. Deductions are set off against the gross income, hence, leading to a lower taxable income. Furthermore, independent of the (innate) location j and innate productivity n, pecuniary commuting expenses c_p are assumed to be fixed. This is reasonable in an urban city framework since, for example, everyone keeps a car or buys a permanent ticket for public transportation. Furthermore, it is assumed that, given the non-pecuniary moving costs, q_{-p}, of an individual of productivity level n, the corresponding non-pecuniary commuting costs, c_{-p}, of that same individual are always smaller, i.e. $c\left(q|\, n\right)_{-p} < q\left(n\right)_{-p}$. As pecuniary commuting costs are fixed, pecuniary moving costs are negligible, and using the previous assumption on non-pecuniary costs, the originally three-dimensional problem can be treated like a two-dimensional problem since the third dimension only affects the split of all work location changer into commuters and movers. Hence, I will characterize each individual only two-dimensional to be of type (n, q).

In the optimal income taxation literature, there is increasing work on general equilibrium effects and on wage effects. However like in most of the optimal taxation literature, I treat wages as exogenous and independent of individual labor supply and aggregate work location decisions. Accordingly, the analysis applies to a situation where the effect on wages due to commuting or moving flows within an urban area is negligible. The empirical evidence on the persistance of the urban wage premium supports this assumption, see, among others, Glaeser and Maré (2001).

Following Diamond (1998), I use preferences that are separable in consumption and labor. The utility function of a worker of type (n, q) is similar to the formulation in Kleven et al. (2009), but depends on the region of working together with the commuting and moving decision, respectively,

$$u\left(c, z, l, m\right) = c^j - n_i h\left(\frac{z_i}{n_i}\right) - cl - qm, \tag{1}$$

where l (m) is an indicator variable that takes the value of 1 in case of commuting (moving). The function $h(\cdot)$ is increasing, convex and twice-differentiable. It is normalized such that $h'(1) = 1$ and $h(0) = 0$. The other variables have standard interpretations. In general, consumption c^j equals gross income z_i minus taxes T^j, which itself depend on gross income, $c^j = z_i - T^j\left(z_i\right)$.

Each individual chooses l and m to maximize her utility (1). By this, she decides in which region j she will live and in which region i she will work. If the decision leads to different regions for living and working the individual (implictly) decides to commute.

From 1, the first order condition for gross earnings are given by

$$h'\left(\frac{z_i}{n_i}\right) = 1 - \tau^j\left(z_i\right), \tag{2}$$

where τ_i is the marginal tax rate. In case of no taxation, $\tau^j = 0$, cross income equals realized productivity. Hence, n_i can be interpreted as potential income. In what follows, the elasticity of gross earnings with respect to net-of tax-rate as a function of gross earnings and the location of working is a useful concept. It is defined as

$$\varepsilon_i \equiv \frac{1-\tau^j}{z_i}\frac{\partial z_i}{\partial\left(1-\tau^j\right)} = \frac{n_i h'\left(\frac{z_i}{n_i}\right)}{z_i h''\left(\frac{z_i}{n_i}\right)}.$$

To focus on urban city-fringe productivity differences, I assume that $\varepsilon_i = \varepsilon$ for all individuals and independent of the place of living and working. This simple benchmark arises with an iso-elastic formulation, i.e. $h\left(\frac{z_i}{n_i}\right) = \frac{1}{1+\epsilon}\left(\frac{z_i}{n_i}\right)^{1+\epsilon}$, such that $\varepsilon_A = \varepsilon_B = 1/\epsilon$, for example. In addition, the following property is required.[3]

Assumption 1 The function $x \to \frac{1-h'(x)}{xh''(x)}$ is decreasing.

I now derive the case without any deduction possibilities. Later on, I allow for and introduce deductions. The proceeding is two-stage. First, individuals decide where to work. Second, individuals choose m and l given their realized working location on stage one, i.e. on stage two individuals decide by their decision whether to commute to work or to move to their working region if their place of living differ from the place of working differ.

Consider first the work place decision. Let us denote by $p\left(q|\, n\right)$ the density of q conditional on n, and by $P\left(q|\, n\right)$ the cumulated distribution of q conditional on n, and by $\zeta(n) \equiv qm + cl$ the choice-dependent joint expression of the moving and commuting costs for each innate productivity type. Conditional on working in place i, the individuals' choice of gross earnings is determined by (2), which allows to define indirect utility conditional on the place of working and net of the costs $\zeta(n)$, as

$$V_i\left(n_i\right) = z_i - T^j\left(z_i\right) - n_i h\left(\frac{z_i}{n_i}\right).$$

Individuals choose their working region i different from their (innate) living region j, i.e. $i \neq j$, whenever their costs $\zeta(n)$ are below the net gain from changing the working region,

[3] This property ensures that the solution optained is in fact an optimum and not a minimum, see Kleven et al. (2009) for further discussion.

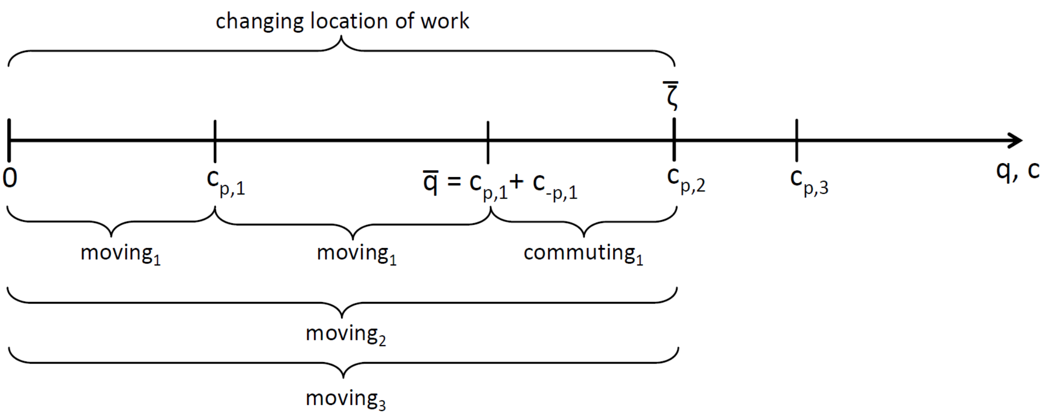

Figure 1: Fraction of individuals at innate ability level n that commute and move, respectively, relative to all individuals n that change their location of work ($\overline{\zeta}$), for different fixed commuting cost levels $c_1 < \overline{\zeta}$, $c_2 = \overline{\zeta}$, $c_3 > \overline{\zeta}$.

such that $\overline{\zeta}_i \equiv \max\{V_i(n_i) - V_{-i}(n_{-i}), 0\}$ is the critical level of costs that determines the actual number of individuals with $i \neq j$ (at the first stage) for any innate productivity level.

Now, turn to the decision to commute or tom move, respectively. This stage is relevant only for those individuals with $i \neq j$ (at the first stage) and, equivalent to $\overline{\zeta}_i > 0$. On the other hand, individuals who do decide to work in their innate living region, $i = j$, (at the first stage) choose implicitly $l = m = 0$.[4] For all other individuals, the decision is based on the maximization of their consumption to maximize their utility by choosing the minimum of the commuting and moving costs, respectively. This means on stage two, individuals choose l and m with $l \neq m$ according to $\zeta(n)$.

Individuals of innate productivity n decide to move (commute) to their new working place if $q(n) < c(n)$ $(q(n) > c(n))$. It is straightforward to define the critical level of costs that determine the actual number of moving (commuting) individuals to be $\overline{q}_i \equiv \min\{\overline{\zeta}_i, c\}$ ($\overline{c}_i \equiv \max\{\overline{\zeta}_i - c, 0\}$). This is obvious from Figure (1).

In Figure (1), three different levels of fixed pecuniary commuting costs are indicated relative to the critical cost level $\overline{\zeta}$, $c_{p,1}$, $c_{p,2}$, $c_{p,3}$. With fixed pecuniary commuting costs above or equal to $\overline{\zeta}$ like $c_{p,2}$ or $c_{p,3}$, all individuals will move to their new work location. However, with fixed pecuniary commuting costs below the critical level, like $c_{p,1} < \overline{\zeta}$, a fraction of individuals will move and another part will commute. The pecuniary commuting cost determine one part of the individuals that will move, see $c_{p,1}$ in Figure (1). However, the exact split of individuals that will move or commute, respectively, depends

[4]This is true since I abstract from location dependent living cost differences and direct utility effects of the location. Hence, there is no rationale for an individual to keep the innate place of working but relocate solely her place of living. It follows that $l = m = 1$ is not possible in the model.

on the the non-pecuniary commuting costs c_{-p} relative to the moving costs q. The split point is characterized by $q = c = c_p + c_{-p}$, for $c < \overline{\zeta}$, and denotey by $\overline{q}$. In Figure (1), this split point is depicted for the commuting costs $c_1 = c_{p,1} + c_{-p,1}$.[5]

4.1 The government's optimal tax problem

Given the focus on uniform tax-transfer system, that is $T_i^A(z) = T_i^B(z)$, and given that, for now, no deductions are allowed, the government wants to maximize the social welfare function

$$\sum_{i=A,B} \int_{n_{\min}}^{n_{\max}} [\int_{\overline{\zeta}_i}^{+\infty} \Psi\left(V_i\left(n_i\right)\right) p(q|n_i)dq + \int_{\overline{q}_i}^{\overline{\zeta}_i} \Psi\left(V_i\left(n_i\right) - c(n_i)\right) p(q|n_i)dq + \tag{3}$$

$$+ \int_0^{\overline{q}_{-i}} \Psi\left(V_i\left(n_i\right) - q(n_i)\right) p(q|n_i)dq]dn_i,$$

where $\Psi(.)$ is a concave and increasing transformation of individual utilities. Denoting by E the exogenous expenditure requirements, it needs to respect the budget constraint

$$\sum_i \int_{n_{\min}}^{n_{\max}} \int_0^{+\infty} T(z_i) p\left(q|n_i\right) f(n_i) dq dn_i \geq E. \tag{4}$$

Moreover, the government's tax schedule needs to be incentive compatible. This implies

$$\dot{V}\left(n\right) = \left[-h\left(\frac{z_i}{n_i}\right) + \frac{z_i}{n_i}h'\left(\frac{z_i}{n_i}\right)\right]\omega_i'\left(n\right) \geq 0, \tag{5}$$

where the derivative with respect to n is indicated by a dot above a variable. In the appendix, it is shown that a path for z_A and z_B can be truthfully implemented by the government using a non-linear tax schedule.

Let $\lambda > 0$ be the multiplier associated with the budget constraint (4). The government's redistributive tastes is dependent on the place of living. Hence a distinction is made between, after the work location decision is made, individuals that live in the city and individuals that live in the urban fringe. This redistribution taste may be represented by regional-dependent marginal social welfare weights. Marginal social welfare weights reflect society's regard for fairness, see Saez and Stantcheva (2016).[6] In terms of income,

[5] Without the assumpation earlier made on non-pecuniary commuting costs, c_{-p}, relative to the non-pecuniary moving cost for same individual, $c\left(q|\,n\right)_{-p} < q\left(n\right)_{-p}$, the fraction of commuters may be zero for a lot of cases.

[6] From this perspective, differentiated these weights according to the place of living, e.g. between city and urban fringe, society reveals taste for regional fairness. Using such marginal social welfare weights,

the welfare weights will take the form of

$$g_i(z) = \frac{\Psi'\left(V_i\left(z\right)\right)\overline{s}_i(z) + \int_0^{\bar{q}_{-i}} \Psi'\left(V_j\left(z\right) - q\right) p(q|z)dq}{\lambda s_i(z)}, \tag{6a}$$

with $\overline{s}_i(z) \equiv 1 - P(\overline{q}_i|z)$ which is the total innate population 1 reduced by outgoing movers, $P(\bar{q}_i|z)$. It is the residue of the *innate population* size of the urban fringe or the city, respectively.[7] The numerator gives the sum of marginal welfare of the residue of the innate population of i (first term) and the marginal welfare of the ingoing movers to i. This sum is weighted by the resulting, actual *total population* size of i that incorporates the net flow of movers, $s_i(z) = 1 - P(\overline{q}_i|z) + P(\overline{q}_{-i}|z)$, in the denominator.

The government maximizes (3) subject to (4) and (5) through its choice of $T(z)$. This problem formally amounts to a delayed optimal control problem. Using the delayed optimal control technique, as used and discussed in an optimal taxation framework by Kessing et al. (2015), a more formal mathematical derivation of the solution is possible. However, Piketty (1997) and Saez (2001) introduced to the literature an intuitive perturbation approach to derive optimal tax schemes in a way that allows for partioning the economic efffects that determine optimal marginal tax rates. Using this method, the effects generated by the possibility of productivity-enhancing commuting and deduction are also separable. I show the solution to this optimal taxation problem using the intuitive heuristic approach.

4.2 Heuristic derivation of optimal marginal tax rates

In what follows, the endogenously realized distribution of gross incomes, $v_i\left(z_i\right)$, in both the fringe and the city, respectively, are used. Denote by k the endogenously defined, strictly increasing function that maps gross income earned in the low productivity fringe to the gross income earned in the high productve city, $z_B = k(z_A)$, for any (innately) identically skilled worker if this worker decides to work in another region.[8] Consider an increase in taxes for all individuals above gross income z, implemented through an marginal tax rate increase $d\tau$ in the small band $(z, z + dz)$. As a result, all individuals

aspects like the aforementioned German concept of equivalent living conditions across all regions can be addressed. Of course, social marginal welfare weights might, in contrary, be dependent on the location of work. That seems to be more plausible in a framework without moving possibility where individuals can only commute. In such a framework, one can think of the tag be the region you work in.

[7]Note that $P(\overline{\zeta}_i|z)$ is the amount of individuals of region i that change their working place (sum of commuters and movers orginally innnated in i) and that $P(\bar{q}_i|z)$ is the amount of individuals that move from i, cf. 1).

[8]In terms of formulation previously used, an individual of innate ability n receives gross income $z_A = z(n)$ in the fringe A and gross income $z_B = z(\omega(n))$ in the city B. However, note that this notation do not indicate that the gross income also depends on the tax schedule.

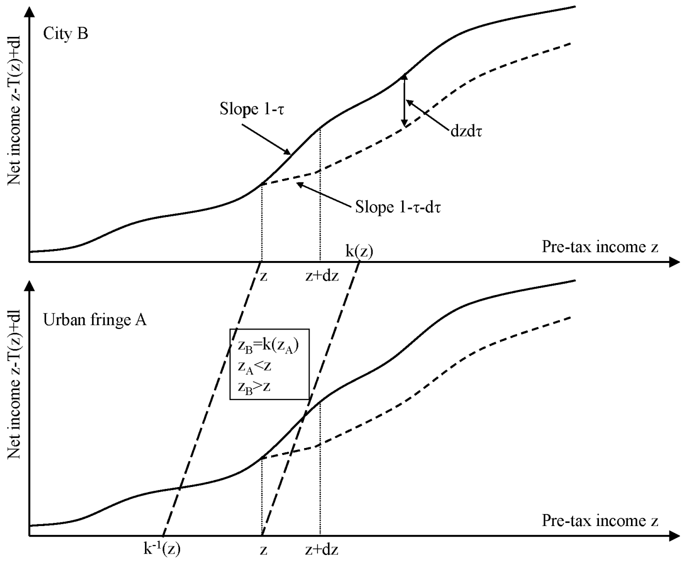

Figure 2: The effect of changing the location of work comes into play for individuals for which $z'_A < z$ and $z'_B \geq z$. (Modified version of Kessing et al. (2015).)

with gross earnings above z have to pay $dzd\tau$ higher tax payments. This tax increase causes three different effects.

Mechanical **Revenue effect** All taxpayers both in the fringe and in the city pay additional taxes of $dzd\tau$ if their gross incomes is higher than z . This tax payment gives rise to a net welfare effect $dzd\tau\,(1 - g_i\,(z'))$ for each affected individual in region i with gross earnings z', resulting in a total revenue effect

$$M = dzd\tau \int_z^\infty \{[1 - g_A\,(z')]\, v_A\,(z')\, s_A(z') + [1 - g_B\,(z')]\, v_B\,(z')\, s_B(z')\}\, dz'.$$

Behavioral **Substitution effect** In addition, those individuals with an income in the band $(z, z + dz)$ will adjust their labor supply to respond to the marginal tax rate increase. Each individual in the aforementined band will, described using $\varepsilon \equiv \frac{1-\tau}{z}\frac{dz}{d(1-\tau)}$, reduce its income by $-d\tau\varepsilon\frac{z}{1-\tau}$. There are around $dz\,[v_A(z)s_A(z) + v_B(z)s_B(z)]$ of affected individuals. The correction term according to Golosov et al. (2014) to account for the circularity due to the nonlinearity of the tax schedule takes the form $1 + \frac{\tau'}{1-\tau}z\varepsilon$. Overall, this behavioral adjustment results in a substitution effect on tax revenue of

$$S = -d\tau dz\frac{\tau z\varepsilon}{1-\tau}\,[v_A(z)s_A(z) + v_B(z)s_B(z)]\,(1 + \frac{\tau'}{1-\tau}z\varepsilon).$$

Job Location effect There are some individuals that will change their work place decision due to the tax increase. These individuals are characterized by the fact that their pre-tax income in the urban fringe is below the tax increase threshold but above this threshold in the city in case they change their working location. The tax increase for incomes above z does not affect the work location decision of individuals with gross income above in both regions, i.d. $z'_A \geq z$ and $z'_B > z$. In this case, for these individuals the tax increase has the same effect in both regions. Equivalently, for all individuals with pre-tax income below z in both the fringe and the city, that is $z'_B = k(z'_A) < z$ and accordingly $z'_A = k^{-1}(z'_B) < z$, the tax increase does not affect their job location choice. However, the job location choice is negatively affected for individuals with $z'_A < z$ and $z'_B \geq z$ as obvious from Figure 2. In this range, all individuals whose joint cost expression ζ is in the band $\overline{\zeta}$ and $\overline{\zeta}$ $-dzd\tau$ will now decide not to change their working place since the expected higher tax payment makes it unprofitable to change the job location. The number of individuals that now do not change their work location is $p\left(\overline{\zeta}\middle|\, z\right) v_A(z)dzd\tau$ at any applicable level of income z. The effect resulting on tax revenues for each individual is $T_A(z) - T_B(k(z))$. Hence, the overall job location effect is

$$J = d\tau dz \int_{\tilde{z}}^{z} \left[T(z') - T\left(k(z')\right)\right] p\left(\overline{\zeta}\middle|\, z'\right) v_A(z')dz',$$

where $\tilde{z} \equiv k^{-1}(z)$.[9] Since the effects are given in terms of the posterior distribution the endogenous effect of the location choice on the income distribution in each region does, here, not need to be expressed explicitly.

In the optimum, all three effects must balance out, that is: $M + S + J = 0$. It follows the first result:

Proposition 1 *The optimal tax schedule without deduction allowance is, under assumption 1, characterized by*

$$\frac{\tau}{1-\tau} = \mathfrak{A}(z)\mathfrak{B}(z)\left[\mathfrak{C}(z) + \mathfrak{D}(z)\right], \tag{7}$$

where

$$\begin{aligned}
\mathfrak{A}(z) &\equiv \frac{1}{\varepsilon}\frac{1-\tau}{1-\tau+\tau' z\varepsilon},\ \mathfrak{B}(z) \equiv \frac{1}{z\left(v_A(z)s_A(z) + v_B(z)s_B(z)\right)}, \\
\mathfrak{C}(z) &\equiv \int_{z}^{\infty} \left\{\left[1-g_A\left(z'\right)\right] v_A\left(z'\right) s_A(z') + \left[1-g_B\left(z'\right)\right] v_B\left(z'\right) s_B(z')\right\} dz', \\
\mathfrak{D}(z) &\equiv \int_{\tilde{z}}^{z} \left[T(z') - T\left(k(z')\right)\right] p\left(\overline{\zeta}\middle|\, z'\right) v_A(z')dz'.
\end{aligned}$$

[9]Note that there is no welfare adjustment to take into account in the job location effect since, in contrast to the *mechanical* revenue effect, the job location effect is a kind of a *behavioural* adjustment made by individual choice including individual utitliy considerations.

Proof. This follows from the heuristic proof above. ■

This result can act as a benchmark for comparison to the optimal income tax schedules if the government has additional policy instruments like the introduction of deduction allowance. Its main message, so far, is that it extends the standard outcome in the literature for the optimal tax rates by an addition term, $\mathfrak{D}(z)$, that catch the location change effect.

Interestingly, optimal tax rates may be lowered by this effect if the tax difference $T(z') - T(k(z'))$ for the same innate productivity type would be negative and this negative effect would not be offset by the endogenously adjusted location dependent effects $\mathfrak{B}(z)$ und $\mathfrak{D}(z)$. As long as an environment with the same posterior income distribution resulting from the work location decisions applies, a government neglecting the work location decisions would set higher optimal marginal tax rates compered to a government that takes these decisions into account, see Kessing et al. (2015) and their similar reasoning.

On a first sight, (7) may look very similar to the internal migration outcome of Kessing et al. (2015). However, in detail there is a substantial difference. In Kessing et al. (2015), the marginal social welfare weights applyed in term $\mathfrak{C}(z)$ and the additional effect of changing the place of working, $\mathfrak{D}(z)$, *both* are dependent on the same amount of people changing their place of *living*. This is different in (7) since the marginal social welfare weights applying in term $\mathfrak{C}(z)$ are, indeed, dependent on the amount of people changing their place of living. However, the additional effect of changing the place of working, $\mathfrak{D}(z)$, is dependent on both individuals changing their place of living and those individuals that do not change their place of living but decide to *commute* to the other place. This may give raise to a more complex trade-off concerning welfare and efficiency aspects.

5 Deduction of commuting expenses

5.1 Lump sum deduction framework

Let us now study the case of lump sum deduction of commuting expenses. This feature may capture cases like the general fixed amount (€ $1,000$ for a single) in Germany that is deducted automatically from every person's income. In this model, the lump sum deduction $\overline{\delta}$ is a fixed amount deducted from actual income for every individual that commutes. Hence, for individuals that change their place of working the deduction results in a taxable income that is lower than their gross income. However, since this fixed deduction cannot exceed the actual income, the lump sum deduction actually takes the form $\delta(n) \equiv \min\{\overline{\delta}, z\}$.Note that there is no need to study separately those individuals that have a gross income $z \leq \overline{\delta}$ since, by definition of δ, it is $z - \delta \geq 0$.Using this definition, it is $T = T(z, \delta, l) = T(z - \delta l)$ and $c^j = z_i - T(z_i - \delta l)$.

$$u\left(c, z, l, m\right) = c^j - n_i h\left(\frac{z_i}{n_i}\right) - cl - qm,$$

The function $h(\cdot)$ is again normalized such that $h'(1) = 1$ and $h(0) = 0$.

From (1), we have the first order condition for gross earnings given by

$$h'\left(\frac{z_i}{n_i}\right) = 1 - \tau\left(z_i - \delta l\right), \tag{8}$$

where τ is the marginal tax rate. In case of no taxation, $\tau = 0$, cross income equals realized productivity. Hence, n_i can still be interpreted as potential income. The elasticity of gross earnings with respect to net-of tax-rate, ε, does not change as it is assumed to be constant over all individuals.

Furthermore, $\zeta(n) \equiv ql + pm$ stays the choice-dependent joint expression of the commuting and moving costs for each innate productivity type. Conditional on working in region i, the individuals' choice of gross earnings is determined by (8), which allows to define indirect utility conditional on the place of working and net of the costs $\zeta(n)$, as

$$V_i\left(n_i\right) = z_i - T\left(z_i - \delta l\right) - n_i h\left(\frac{z_i}{n_i}\right).$$

Individuals choose their working region i different from their (innate) living region j, i.e. $i \neq j$, whenever their costs $\zeta(n)$ are below the net gain from changing the working region, such that $\bar{\zeta}_i \equiv \max\left\{V_i\left(n_i\right) - V_{-i}\left(n_{-i}\right), 0\right\}$ is the critical level that determins the number of work location changer. Note that the value of $\bar{\zeta}_i$ in this tax deduction case is, ceteris paribus, higher than the value of $\bar{\zeta}_i$ in the benchmark case (7) as long as $\delta > 0$. The government's maximization problem remains

$$\sum_{i=A,B} \int_{n_{\min}}^{n_{\max}} [\int_{\bar{\zeta}_i}^{+\infty} \Psi\left(V_i\left(n_i\right)\right) p(q|n_i) dq + \int_{\bar{q}_i}^{\bar{\zeta}_i} \Psi\left(V_i\left(n_i\right) - c(n_i)\right) p(q|n_i) dq +$$
$$+ \int_0^{\bar{q}_{-i}} \Psi\left(V_i\left(n_i\right) - q(n_i)\right) p(q|n_i) dq] dn_i.$$

However, the budget constraint needs to account for the deduction. It becomes

$$\sum_i \int_{n_{\min}}^{n_{\max}} \int_0^{+\infty} T(z_i - \delta l) p\left(q|n_i\right) f(n_i) dq dn_i \geq E.$$

Note that the marginal social welfare weights are expressed in gross incomes. They do not include the deduction possibility directly since the deduction possibility is accounted for indirectly via the indirect utility, $V_i(z)$. Moreover, the government's tax schedule needs to be incentive compatible. The incentive compatible constraint as well as the formulae of the marginal social welfare weights remain unchanged as

$$\dot{V}(n) = \left[-h\left(\frac{z_i}{n_i}\right) + \frac{z_i}{n_i}h'\left(\frac{z_i}{n_i}\right)\right]\omega_i'(n) \geq 0,$$

$$g_i(z) = \frac{\Psi'(V_i(z))\,\bar{s}_i(z) + \int_0^{\bar{q}_{-i}} \Psi'(V_j(z) - q)\,p(q|z)dq}{\lambda s_i(z)}.$$

5.2 Heuristic derivation of optimal marginal tax rates

Consider an increase in taxes for all individuals above gross income, z, implemented through an marginal tax rate increase $d\tau$ in the small band $(z, z+dz)$. I take into account the deduction possibility δl in separate steps later on. Note that, due to this procedure, an individual's gross income z' is not necessary equal to the taxable income finally filed by an individuals. For now, all individuals are treated as if they do not claim any deduction. As a result, all individuals with gross earnings above z have to pay $dzd\tau$ higher tax payments. This tax increase causes four effects.

Mechanical **Revenue effect** All taxpayers both in the fringe and in the city, independent about their moving or commuting decision, pay additional taxes of $dzd\tau$ if their gross incomes is higher than z. This tax payment give rises to a net welfare effect $dzd\tau\,(1 - g_i(z'))$ for each affected individual in region i with gross earnings z', resulting in a total revenue effect

$$M = dzd\tau \int_z^{\infty} \{[1 - g_A(z')]\,v_A(z')\,s_A(z') + [1 - g_B(z')]\,v_B(z')\,s_B(z')\}\,dz'.$$

Behavioral **Substitution effect** In addition, those individuals with an income in the band $(z, z+dz)$ will adjust their labor supply to respond to the marginal tax rate increase. Each individual in the aforementined band will reduce its income by $-d\tau\varepsilon\frac{z}{1-\tau}$. There are around $dz\,[v_A(z)s_A(z) + v_B(z)s_B(z)]$ of affected individuals. Including the correction term to account for the nonlinearity of the tax schedule this behavioral substitution effect keeps the already known form

$$S = -d\tau dz\frac{\tau z\varepsilon}{1-\tau}\,[v_A(z)s_A(z) + v_B(z)s_B(z)]\,(1 + \frac{\tau'}{1-\tau}z\varepsilon).$$

Job Location effect Again, there are some individuals that will change their work place decision due to the tax increase. As before, the job location choice is negatively affected for individuals with $z_A' < z$ and $z_B' \geq z$. In this range, all individuals whose joint costs ζ are in the band $\overline{\zeta}\ -dzd\tau$ to $\overline{\zeta}$ will now decide not to change their working place. A number of $p\left(\overline{\zeta}\middle|\, z'\right) v_A(z)dzd\tau$ individuals at any applicable level of income z change their decision. The overall job location effect stays the same as in the benchmark case,

$$J = d\tau dz \int_{\tilde{z}}^{z} \left[T(z') - T\left(k(z')\right)\right] p\left(\overline{\zeta}\middle|\, z'\right) v_A(z')dz'.$$

Tax **Deduction effects** Now, I take the deduction possibility into account. The three effects, the mechanical revenue effect, the behavioural substitution effect, and the job location effect, have to be corrected according to the facts, first, that some of the individuals who decide to commute may end up with an taxable income lower than z due to the deduction of δl of their gross income z', and, second, that those individuals who in spite of their deduction δl still end up with an *taxable income* above z has to be considered differently compared to those individuals that do not commute but have the same gross income z'.

Let us consider the correction due to the first reason. All commuting individuals with gross income z' in the band $[z, z+\delta]$ are not affected by the mechanical revenue effect. There are $\chi_i(z') = P(\overline{\zeta}_i|z') - P(\bar{q}_i|z')$ commuters at every appropriate level of gross income z' in each the urban fringe and the city. This leads so a necessary correction of the mechanical revenue effect of

$$D_1 = dzd\tau \int_{z}^{z+\delta} \left\{\left[1 - g_A\left(z'\right)\right] v_A\left(z'\right) \chi_A(z') + \left[1 - g_B\left(z'\right)\right] v_B\left(z'\right) \chi_B(z')\right\} dz'.$$

In addition, those individuals of the commuters that have an gross income in the band $(z, z+dz)$ will not reduce their labor supply by $-d\tau\varepsilon\frac{z}{1-\tau}$ since they are not affected by the tax increase. It is reasonable to assume $\delta > dz$ as dz is by definition very small. This assumption ensures that the correction has to account for all commuters in the band $(z, z+dz)$. There are approximately $dz\left[v_A(z)\chi_A(z') + v_B(z)\chi_B(z')\right]$ of individuals in this band that are actually not affected by the tax increase. Hence, the behavioural substitution effect has to be correct by

$$D_2 = -d\tau dz \frac{\tau z\varepsilon}{1-\tau}\left[v_A(z)\chi_A(z') + v_B(z)\chi_B(z')\right](1 + \frac{\tau'}{1-\tau} z\varepsilon).$$

Furthermore, there has to be made a correction of the job location effect. The negative effect of the tax increase on the job location decision of those individuals with $z'_A < z$ and $z'_B \geq z$ as well as $\zeta \in [\overline{\zeta}\ -dzd\tau, \overline{\zeta}]$ may not apply to all individuals who are qualified to the deduction. As there is no differential taxation between the urban fringe A and the city B, those individuals who are not affected due to the deduction are characterized by having $z'_A < z$ and $z'_B \in [z, z+\delta]$ with joint costs ζ in the band $[\overline{\zeta}\ -dzd\tau, \overline{\zeta}]$. Note that, since

$dzd\tau$ is very small, it is reasonable to assume $dzd\tau \in [\overline{q}, \overline{\zeta}]$, hence, only individuals that are potentially commuters are in the affected band $[\overline{\zeta} - dzd\tau, \overline{\zeta}]$. This assumption rely on sufficiently small pecuniary commuting costs compared to the critical level of joined costs, $c_p(n) < \overline{\zeta}(n)$. Thus, the job location effect has to be corrected by the term

$$d\tau dz \int_{\tilde{z}}^{\min\{z, \widetilde{(z+\delta)}\}} [T(z') - T(k(z') - \delta)]\, p\left(\overline{\zeta} \middle| z'\right) v_A(z') dz'.$$

Moreover, the job location effect has to be corrected a second time for the following reason. Individuals that, yet, did not change their work location but that have $z'_A > z$ and $z'_B \in [z, z + \delta]$ can now avoid the higher tax payment since their taxable income in the city, $z'_B - \delta$, would be lower than the threshold level z. Each individual with $z'_A > z$ and $z'_B \in [z, z + \delta]$ as well as $\zeta \in [\overline{\zeta} - dzd\tau, \overline{\zeta}]$ will now decide to quit her job in the urban fringe and will commute to the city in order to work there. Note that there are only such individuals if the tax allowance is higher than the gross income increase due to job location change. This second adjustment of the job location effect takes the form

$$d\tau dz \int_{z}^{\max\{z, \widetilde{(z+\delta)}\}} [T(z') - T(k(z') - \delta)]\, p\left(\overline{\zeta} \middle| z'\right) v_A(z') dz'.$$

Note that the case $\delta > k(z') - z'$ is plausibel for relatively small incomes. In the other case, with $\delta \leq k(z') - z'$, it is $z >, \widetilde{(z + \delta)}$ and the second correction effect vanishes. Both adjustment effects can be added to a combined correction term of the job location effect

$$D_3 = d\tau dz \int_{\tilde{z}}^{\widetilde{(z+\delta)}} [T(z') - T(k(z') - \delta)]\, p\left(\overline{\zeta} \middle| z'\right) v_A(z') dz'.$$

Let us now study the correction due to the second reason. Taxes are imposed on taxable income, not on gross income. The mechanical revenue effect derived so far does take into account that the taxable income of commuters is not z' but $z' - \delta$. Note that commuters with gross income $z' \in [z, z + \delta]$ are already accounted for in D_1. However, all commuters with $z' > z + \delta$ actually only pay $d(z - \delta)d\tau$ additional taxes with an actual net welfare effect of $(1 - g_i(z'))$. In the urban fringe and in the city, in each case there are $\chi_i(z')$ commuters. Hence, the mechanical revenue effect needs to be corrected an additional time by

$$D_4 = (d\tau dz + d\tau d(z - \delta)) \int_{z+\delta}^{\infty} \{[1 - g_A(z')]\, v_A(z')\, \chi_A(z') + [1 - g_B(z')]\, v_B(z')\, \chi_B(z')\}\, dz'.$$

In the optimum, all four effects must balance out, that is:

$$M + S + J - D_1 - D_2 - D_3 - D_4 = 0.$$

The following proposition is obtained.

Proposition 2 *The optimal tax schedule with lump sum deduction allowance is, under assumption 1, characterized by*

$$\frac{\tau}{1-\tau} = \mathfrak{A}(z)\mathfrak{B}(z)\left[\mathfrak{C}(z) + \mathfrak{D}(z) - \mathfrak{E}(z) - \mathfrak{F}(z)\right], \tag{10}$$

where

$$\begin{aligned}
\mathfrak{A}(z) &\equiv \frac{1}{\varepsilon}\frac{1-\tau}{1-\tau+\tau' z\varepsilon},\ \mathfrak{B}(z) \equiv \frac{1}{z\{v_A(z)[s_A(z)-\chi_A(z)] + v_B(z)[s_B(z)-\chi_B(z)]\}},\\
\mathfrak{C}(z) &\equiv \int_z^\infty \{[1-g_A(z')]\, v_A(z')\,[s_A(z')-\chi_A(z')] + [1-g_B(z')]\, v_B(z')\,[s_B(z')-\chi_B(z')]\}\, dz',\\
\mathfrak{D}(z) &\equiv \int_{\tilde{z}}^{z} [T(z') - T(k(z'))]\, p\left(\overline{\zeta}\right| z')\, v_A(z') dz',\\
\mathfrak{E}(z) &\equiv \int_{\tilde{z}}^{\widetilde{(z+\delta)}} [T(z') - T(k(z')-\delta)]\, p\left(\overline{\zeta}\right| z')\, v_A(z') dz',\\
\mathfrak{F}(z) &\equiv \frac{d\tau d(z-\delta)}{d\tau dz}\int_{z+\delta}^{\infty} \{[1-g_A(z')]\, v_A(z')\,\chi_A(z') + [1-g_B(z')]\, v_B(z')\,\chi_B(z')\}\, dz'.
\end{aligned}$$

In comparison to the benchmark case (7), now in (10) the difference of the realized total population size and the amount of commuters that reside in i, $s_i(z) - \chi_i(z)$, is considered in both terms $\mathfrak{B}(z)$ and $\mathfrak{C}(z)$. Due to this, commuters seem to have less direct influence on the optimal marginal tax rates according to (10). On the opposite, the number of non-changer and the movers in the fringe and in the city, respectively, largely determine the factors $\mathfrak{B}(z)$ and $\mathfrak{C}(z)$. Since their number is smaller than or equal to the respective realized total population size, i.e. $s_i(z) - \chi_i(z) \leq s_i(z)$, the overall factor $\mathfrak{B}(z)$ tend to be higher in the deduction case than in the benchmark case without deduction allowance in (7) which, under positive marginal tax rates, would increase optimal marginal tax rates. The opposite is true with respect to the term $\mathfrak{C}(z)$ which tend to be smaller than in the benchmark case.

However, the commuters become relevant for optimal marginal tax rates through all three remaing terms in the bracket, $\mathfrak{D}(z)$, $\mathfrak{E}(z)$, and $\mathfrak{F}(z)$. With sufficiently small pecuniary commuting costs compared to the critical level of joined costs, $c_p(n) < \overline{\zeta}(n)$, marginal individuals, $p\left(\overline{\zeta}\right| z')$, are only commuters, cf. Figure 1. This applies to both terms $\mathfrak{D}(z)$ and $\mathfrak{E}(z)$. Moreover, the correction term $\mathfrak{F}(z)$ incorporates the sum of the welfare effects due to commuting weighted by the deduction-induced ratio of an additional tax payment from taxable income, $d\tau d(z-\delta)$, to the corresponding tax payment from gross incomes, $d\tau dz$. The reduction of the term, $\mathfrak{C}(z)$, is reinforced by the term $\mathfrak{F}(z)$.

The term $\mathfrak{D}(z)$ catches the effect of the productivity change caused by commuting on optimal marginal tax rates. $\mathfrak{D}(z)$ may be negative. However, the term $\mathfrak{E}(z)$ adjusts this effect. For $\mathfrak{E}(z) < 0$, this adjustment would be overall positive and would reduce the negative effect of $\mathfrak{D}(z)$ on optimal marginal tax rates. However, the term $\mathfrak{E}(z)$ itself may become positive due to the effect of the deduction allowance, δ. Then, $\mathfrak{E}(z)$ would reinforce the negative effect of $\mathfrak{D}(z)$ on optimal marginal tax rates. The influence by the term $\mathfrak{E}(z)$ depends crucially on the relationship of the productivity increase, expressed in gross income by $k(z) - z$, relatively to the deduction allowance, δ. If δ is close to the productivity increase, that is $\delta \approx k(z) - z$, then the adjustment effect of $\mathfrak{E}(z)$ on $\mathfrak{D}(z)$ almost disappears because $T(k(z) - \delta) \approx T(z)$ and the tax difference within the integral of $\mathfrak{E}(z)$ almost vanishes. This is even so the two integrals of $\mathfrak{D}(z)$ and $\mathfrak{E}(z)$ would have almost the same support in this case. However, if the deduction allowance is higher than the increase in gross income due to commuting, $\delta > k(z) - z$, which is plausibel for relatively small incomes, then the tax difference, $T(z') - T(k(z') - \delta)$, becomes positive and, by this, $\mathfrak{E}(z)$ would reduce optimal marginal tax rates.

Overall, the sum of terms in the bracket of (10) tend to be smaller in the deduction case compared to the sum of terms in the bracket of (7). Nevertheless, the effect of $\mathfrak{B}(z)$ is not negligible, it may compensate the other effects or go beyond that and may lead to higher optimal marginal tax rates. To sum up, since several variables change endogenously one cannot directly obtain an overall assessment of the effect of a lump sum tax deduction allowance for commuting and moving costs on optimal marginal tax rates from (10).

6 Concluding remarks

Productivity differences between different cities and/or rural areas offer commuting and moving possibilites that lead to externalities. Those externatilies have to be taken into account when formulating the optimal income tax schedule. This inequality and the corresponding possibility of productivity-enhancing change of the work location can be an important determinant of the optimal redistributive tax-transfer scheme. A government that is constrained to use a unified redistribution policy faces an additional equity-efficiency trade-off beyond the intensive labor supply margin. Optimal income taxation needs to take the fiscal commuting and moving externalities into account. Moreover, deduction possibilites extend the available policy instruments. Fixed tax allowance for commuting expenses, as it is offered by the German tax system, results in additional channels determining the optimal income tax scheme and, at the same time, influencing commuting and moving flows. This underlines the significance of the tax system for local labour market and settlement pattern.

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