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The Hamilton Regression in Comparison: Evidence from German Business Cycles Since 1950

Lars-H. R. Siemers†

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Abstract

Business-cycle adjustment is mostly determined via filter methods, especially the HP filter, or, e.g. within the EU fiscal rules, by a production function approach. James Hamilton put big doubt on the quality of the HP filter estimates, and proposed an alternative regression approach to decompose trend and cycle of time-series. We investigate how the new Hamilton filter compares to the common methods. We find that the Hamilton regression produces partly significantly different results. The average estimated output gap, and its variance, is significantly higher. As a consequence, the average identified cycle length is the shortest in comparison. By construction, the Hamilton regression produces odd results for periods of massive crisis, while it performs better in context of structural breaks. The highest correlation of the Hamilton gaps we find for the EU production-function approach. The identified business cycles, in contrast, do not differ in most cases since 1950. In an ex post evaluation, the HP filter with smoothing factor 20 performs well for Germany, and precisely fulfils the assumption of symmetry.

Keywords: business-cycle identification • Hamilton regression • EU production-function approach • HP filter • Swiss HP filter

JEL codes: C18; C22; E32; E62; H60; N14

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1 Introduction

The valuation of the position within a business cycle is crucial in many economic circumstances. The Keynesian idea of stabilising the economy via counter-cyclical economic policy, for instance, requires knowledge of the true position within the business cycle. Misjudgement regularly causes harmful cyclical interventions.[1] The important issue of reflecting and evaluating past policies in this context asks for an ex-post identification of the business cycles, to learn from past flawed estimations and the corresponding mistakes causing it. Based on Keynes' idea of deficit spending, debt brakes and regulations of public debt have been established in public finance, for, in practice, politics regularly increased expenditures in downturns without decreasing them in upswings[2] – with the consequence of accumulating public debt from business cycle to business cycle (for Germany, see e.g. GCEE 2007).[3] In this context, the correct assignment of particular years to the position of the business cycle is imperative for the determination of the structural financial position of a budget, mainly measured as cyclically-adjusted budget balance (CAB), for instance, in the context of the medium-term budget objectives (MTOs) in the EU or the German federal as well as states' debt regulation. The source of debt accumulation is also a suble expenditure and deficit bias due to a dynamic political economics common-pool problem (Persson and Tabellini 2000: Chapt. 13) that may play a role in form of many varieties.[4] And it turns out that this political bias towards more spending and deficits is more severe in boom times,

1 Galí and Perotti (2003) and Lane (2003) discuss cyclical properties of fiscal policy in OECD countries. Mohr (2001), e.g., states procyclical fiscal policy in Germany in the 1990s. In developing economies procyclical fiscal policy is widespread (Alesina et al. 2008; Gavin and Perotti 1997; Kaminski et al. 2004; Talvi and Vegh 2005).

2 Debt accumulation was partly caused by a widespread asymmetric "misuse of the idea of deficit spending." But in recent years, public debt also rose due to the 2007-2009 financial crisis and the COVID-19 pandemic (cf. for Germany, e.g., Gebhard and Siemers 2020a).

3 The trend of accumulating public debt started in the beginning 1970s in nearly all OECD countries (cf. Alesina and Perotti 1995; Mikosch and Überlmesser 2007).

4 Cf., e.g., Alesina and Perotti (1995), de Haan et al. (2013), Feld (2010, Sect. 2.2), Feld and Reuter (2017, 2019), Heinemann (2006), Mikosch and Übelmesser (2007), Perotti and Kontopoulos (2002), Velasco (2000), von Hagen and Harden (1995), or Woo (2003).

where political rents rise and voters want to participate via tax cuts and public goods as well as other spending (Alesina et al. 2008). This generates a procyclical fiscal-policy bias.[5]

But also policy and history analyses embed their respective arguments in business cycle evaluations, focusing on economic downturns (up to economic turmoil) and upturns. Even back a century it has been stated that there is a long tradition of a theory of economic interpretation of history, which discusses changes in the economy – most pronounced the 19th century Industrial Revolution – that are accompanied by profound social and historical changes (Ogburn and Thomas 1922) – and emphasized that “also brief swings in economic conditions through prosperity and depression” (p. 324) are correlated with the social conditions such as crime, marriage, divorces, deaths, or birth. In economics, historic reforms pushing an upswing may be considered as counter-productive, while in the opposite case they might be considered as having a clever additional counter-cyclical side effect. Historical developments are hence often discussed in the light of the respective business cycle situation. Negative turning points in history often come along with economic downturns and depressions; many historic revolutions, for instance the 1789 French Revolution, the 1848 German Revolution, or the fall of the Berlin Wall and Iron Certain rooted, beyond other aspects, in economic downturns that caused revolts. It is also argued that the coming-into-power of Adolf Hitler in 1933 was facilitated by the economic turmoil these days, including the Great depression and a massive bank crisis as well as a pro-cyclical economic policy of the Brüning government (Orlow 1984; Lutz 1941).[6] Especially in the U.S., the business cycle has a robust significant effect on election outcomes (Erikson 1989; Guntermann et al. 2021). A precise business-cycle identification may help to understand these issues. That is,

5 Cf. also the idea of a “voracity effect” in Lane (2003). Moreover, there arises a strategical political debt bias when there is a polarization of spending objectives and incumbent’s re-election is unlikely, so that there are alternating governments (Alesina and Tabellini 1990; Persson and Svensson 1989).

6 Borchardt (1979, 1980, 1990) considers Brüning’s economic policy as consequence of a fiscal dilemma and unavoidable: the challenge was a twin bank and currency crisis (Schnabel 2004) causing a sudden-stop credit crisis (Accominotti and Eichengreen 2016) in the context of the Dawes and Young plans (cf. Ritschl 2023).

identifying the business cycles, with the corresponding years of downturns and booms, is imperative.

We estimate the historic business cycles of the German economy since 1950 based on growth cycles. Given uncertainty about the most adequate method to identify business-cycles, we apply different methods, and especially investigate the Hamilton regression approach for separating cyclical and trend component of GDP (Hamilton 2018). We compare its results with the standard approaches of business cycle analysis used in practice: the EU production-function approach, the HP filter as well as the modified version of the HP filter used for the Swiss debt brake. This allows to figure out how the different methods compare to each other, to identify potential striking differences as well as where there are no significant deviations. We provide comprehensive results for the complete existence of the West-German Bonn Republic after WWII up to today's reunified Berlin Republic.

2 Relation to the Literature

In the tradition of Burns and Mitchell (Mitchell 1913; Burns and Mitchell 1936, 1946) and, e.g., Harding and Pagan (2002a) we identify cycles based on patterns of aggregate economic activity. We compare the output gaps and business-cycle identification estimated by the method of the European Union (EU) (Havik et al. 2014), the Hodrick and Prescott (1981, 1997) (HP) filter, the Swiss modified HP (mHP) filter (Bruchez 2003), and the new Hamilton approach (Hamilton 2018), to draw conclusions referring differences of the methods in practise.[7] Beyond identifying all business cycles since the start of the German "Bonn

7 Of course, there are many further methods to estimate potential output / trend and output gaps, e.g., Baxter-King filter (Baxter and King 1999), Beveridge-Nelson decomposition (Beveridge und Nelson 1981) or univariate time-series models of unobserved components (Harvey 1989). While the Beveridge-Nelson decomposition is criticized for producing implausible implications (Mc Morrow and Roeger 2001), the Baxter-King filter and the models of unobserved components, being more complex and easier to manipulate, generate quite similar results as the HP filter (Mc Morrow and Roeger 2001; Mohr 2001). Generally, potential output can be estimated using (i) smoothing and de-trending methods as the HP filter, the split-time trend method, the log-linear time trend method, or the moving-average method, (ii) econometric estimates such as the production function method or Hamilton's regression filter, or (iii) survey data on capacity utilisation (see, e.g., Fisher et al. 1997).

Republic"[8] in 1949 for each method, we provide details on approach-specific gaps and its standard deviation as well as on average duration in total as well as its troughs and booms.

Referring the HP filter, it is well-known that it suffers from an end-point bias: the trend estimates for the final (and starting) three years are markedly volatile.[9] Kaiser and Maravall (1999) hence suggest to include proper forecasts and backcasts, and Bruchez (2003) modifies the weights for the final three years, to reduce the bias; the latter is applied in the context of the Swiss debt brake. Applying the HP filter, the user has two degrees of freedom: the choice of the smoothing parameter, labelled λ, and the time frame included. There is especially a controversy about the correct value of λ. While e.g. Backus and Kehoe (1992) use the standard value of 100 for annual data, Cooley and Ohanian (1991) as well as Correia et al. (1992) apply a value of 400. Baxter and King (1999) argue, in contrast, the correct value was 10. Based on the value deduced in Hodrick and Prescott (1997) for quarterly data (1600), in turn, Ravn and Uhlig (2002) analytically derive a value of only 6.25 for annual data; Reeves et al. (2000), however, put doubt on the plausibility of the quarterly value of 1600 in practice, for it is only shown to be applicable for white-noise cyclical components, and the empirical sample estimate of the parameter deviates from their prior belief. Elaborating on the correct value for λ in Germany, a study of the German central bank arrives at a value of 20 (Mohr 2001), and in the harmonized approach of cyclical adjustment within the European System of Central Banks (ESCB) a value of 30 is used for price-adjusted data (Bouthevillain et al. 2001). Swiss law determines for the Swiss debt brake the modified HP filter with $\lambda =$ 100. The time horizon the estimates lean on are often simply determined by the time span

8 After WWII Germany was occupied and, in the aftermath, separated into several areas of occupation territories, majorly the British, French, Soviet, US, and Polish. While the latter (and smaller further territories) were given to other nations, the people of Western occupation zones were allowed to form a Western-German state, and the major Western Soviet zone became the East-German state (cf., e.g., Redding and Sturm 2008). The Bonn Republic was established by the (West-)German constitution declaration on 24 May 1949 and the first election on 14 August 1949; the first annual observation for West-Germany's GDP is for the year 1950. Data on the East-German counterpart involves massive difficulties (Becker et al. 2020: 144; Ritschl and Spoerer 1997), and could not be used.

9 In early years, the HP filter was criticised for generating artificial cycles (Cogley and Nason 1995; Harvey and Jaeger 1993; King and Rebelo 1993). These distortions, however, are unavoidable and a feature of even any optimal filter (Ehlgen 1998).

available, though this choice may also affect the result. Overall, therefore, the HP filter approach involves a high degree of haziness: there is a high degree of freedom in choosing the smoothing factor, the time frame included, and whether and how many future periods to include as forecasted values. We hence focus on a comparison of common practice and analyse three HP filter versions: we use the standard value of 100, the analytically deduced value of 6.25, and the value of 20, specifically deduced for Germany. Referring the time frame, we include all observations available, given we want to estimate trend, potential output, gaps, and business cycles for as many years as possible.

Hamilton (2018) puts considerable doubt on the reliability of the HP filter. He argues that the common practice of setting the smoothing parameter for the HP filter cannot obtain optimal decompositions into trend and cycle – even worse, the HP filter generally does not base on the respective true data-generating processes and hence generates spurious results for the statistical trend. Hamilton offers an alternative approach to deduce reliable estimates of the cyclical component. He demonstrates that a comparably simple regression of a variable's value vector in a period $t+h$, labelled y_{t+h}, on a constant and the four most recent realization vectors of that variable as of date t (so y_t, $y_{t\text{-}1}$, $y_{t\text{-}2}$, and $y_{t\text{-}3}$) allows to consistently estimate the cyclical component reasonably well for a wide range of non-stationary data-generating processes. We apply h=2 combined with p=1, as suggested in Hamilton (2018) and Schüler (2018) for annual data and business-cycle analysis.[10]

However, the Hamilton approach may also suffer from weaknesses. Following Quast and Wolters (2022) as well as Schüler (2018) it amplifies long, medium-term cycles, while muting short cycles of length of two years or below. Schüler (2018) finds that it also generates artificial cycles, suffers from a small-samples bias, smooths structural breaks (as the HP filter does), and the choice of the lag structure via parameters h and p is similarly ad hoc. Based on Monte-Carlo simulations, Jönsson (2020a) demonstrates that even if the data is generated

10 A longer lag h, for instance h=5 for annual data, is only adequate for analyses that are interested in identifying longer cycles than typical business cycles.

without any cyclical dynamics, the Hamilton regression produces similar cyclical component dynamics as the HP filter. While for the HP filter this is often labelled spurious dynamics, the Hamilton dynamics simply root in a specific definition of cycle and trend components, which can be misleading. That is, the Hamilton filter would modify the original cyclical structure of the data as well. In contrast to Schüler (2018), Quast and Wolters (2022) find that the trend estimates are not smooth, and thus hardly can be interpreted as a measure of potential output. The Hamilton regression approach is found to be weaker for business cycle analysis due to overemphasising cycles longer than typical business cycles.[11] In line, Hall and Thomson (2021) also did not find an advantage of using the Hamilton regression in comparison to HP or Baxter-King filters for New Zealand macro data. Schüler as well as Quast and Wolters, however, reveal that the Hamilton approach suffers less from a bias at the tails of the time-series (cf. also Jönsson 2020b), which is an important property for economic policy in practise, where data revision often changes recent gap estimates significantly.[12] In contrast to Schüler (2018), we find that the Hamilton regression does not smooth structural breaks, such as the HP filters, which is an advantage; this is in line with Quast and Wolters (2022), who find that Hamilton gaps are rather robust to controlling for structural breaks. In contrast to Schüler (2018) as well as Quast and Wolters (2022), we find that, with annual data, the Hamilton regression, on average, rather identifies shorter cycles in comparison. In line with Hall and Thomson (2021), we find that the Hamilton regression involves, in comparison, very volatile output gaps. While Quast and Wolters (2022), Schüler (2018), and Hall and Thomson (2021) alike compare Hamilton's regression only with the HP filter with the standard smoothing parameter, we include different values in our comparison. Moreover, we also compare it to the Swiss modified HP filter, and the official EU approach

11 Schüler (2018) states that the h=5 (annual data) Hamilton regression outperforms the Basel-III credit-to-GDP gap (based on HP filtering) in identifying imbalances in the financial markets, where longer cycles are to be considered. In contrast, Drehmann and Yetman (2018) find that the Hamilton filter can be outperformed by the HP filter referring the credit-to-GDP gap in predicting crises in financial markets.

12 Quast and Wolters (2022) suggest a modified Hamilton regression that may overcome all the mentioned weaknesses, preserving its advantages.

(see below), which, to best of our knowledge, has not been done so far. In contrast to many other studies, we use annual data.

The standard alternative to both approaches is decomposing trend and cycle via a production-function approach. While HP and Hamilton filter represent purely data-driven statistical methods, the production-function approach is more theory-driven. Based on macroeconomic theory, it estimates the unobservable theoretical variable of potential or natural output.[13] It assumes total output can be caught by the theoretical concept of a macroeconomic production function, with the inputs capital, labour, and technical progress (cf., e.g., Denis, McMurrow, and Röger 2002; Denis et al. 2006; D'Auria et al. 2010; Havik et al. 2014).[14] In a first step, the potential of capital, labour, and technical progress is, however, again estimated purely with statistical filter methods. Moreover, in practice, many forecasts explicitly assume that the output gap is zero in the final period forecasted; this is also the case in the used data on potential output of the Federal Government of Germany (FedGov 2021), which applies the EU cycle-adjustment approach. We directly use these official numbers without any adjustment, to compare the results with its alternatives.

The estimated gaps within the EU regulations are published as, and based on, annual data. Comparing these official EU gap estimates with those of the Hamilton regression or HP thus requires the use of annual data. The idea of structural debt rules in public finance is that debt accumulated due to cyclical downturns is symmetrically compensated for by surpluses accumulated in cyclical upswings. For the EU MTO and German debt brake the annual budget numbers are combined with these annual output-gap estimates. That is, in public finance as well as other contexts, annual business-cycle data are required. Gebhard and

13 The structural path is economic growth and measured via the unobservable potential output, so the situation when there is "full" employment of all production factors (capacity) – given all market frictions – and thus no inflation pressure (Phillips curve). If actual observed GDP is higher or below estimated potential output this relates to cyclical fluctuations and come along with positive or negative output gaps.

14 The idea of a production function to capture the input-output mechanism is much older. It was algebraically formulated, for instance, at least already by von Thünen (1826) and Wicksteed (1894). There was a debate criticising the use of production functions (e.g., Robinson 1971, Shaikh 1974). For a defence confer, e.g., Solow (1974).

Siemers (2020b) criticise the official EU approach with respect to using real-time interest rates, for abnormal levels of interest rates, e.g. in a low-interest-rate period, also bias the estimates of the structural budget position, in addition to business-cycle distortions. They hence propose an extension of these rules by substituting abnormal interest rates by non-distorted cyclically-adjusted interest rates. They suggest to use average interest rates over former business-cycles, which, however, requires identifying former completed business cycles of the economy. Overall, all these applications require a separation of structural and cyclical components as well as a categorisation of business cycles at annual basis.

Business-cycle researchers, interested in evaluating the position in the business cycle in progress, and recognizing turning points as soon as possible, typically identify business cycles on basis of multiple monthly or quarterly business-cycle indicators, so a system of indicators more sophisticated than the production function approach (e.g., Breuer et al. 2022; GCEE 2017, Chapt. 3: 133-34; Gehringer and Mayer 2021; RWI 2017). Gehringer and Mayer (2021), for instance, use 20 economic activity indicators at monthly basis. As a consequence, evidence on business cycles, and comparative methodological studies, at basis of annual data are rare. For the time frame back to 1950, investigated here, these sophisticated multidimensional approaches are, due to a lack of required data, not applicable. As stated by Schirwitz (2009), for ex post business-cycle analysis as well as comparison, which is our focus, aggregated GDP data is the adequate candidate, while the evaluation of the current cycle and forecasting requires multivariable approaches and cycle indicators published monthly or at least quarterly before GDP. The German Council of Economic Experts (Breuer et al. 2022) bases its dating on only five monthly and 11 quarterly indicators; the monthly sources are available back to 1948 to 1952, but the quarterly, with two exceptions, are available not before 1970.[15] Therefore, they also use annual data for all earlier years, and

15 Pioneering works in this field are, e.g., Hamilton (1989) as well as Stock and Watson (1989,1991). A further recent contribution is, e.g., Kyo et al. (2022). Krüger (2021) investigates the performance of leading indicators of the German business cycle, Pažický (2021) the yield curve in Germany and the US.

recently provided a business-cycle dating back to 1950, as we do. We use their dating as a benchmark, and hence are able to compare their approach with those analysed here.

As demonstrated, annual cyclically adjusted variables are, for instance, crucial in the context of public finances. Gebhardt and Siemers (2020b) argue that using annual data on output gaps produces very similar results referring the years of turning points and length of business cycles, compared to more sophisticated approaches based on monthly or quarterly data. This suggests that, focusing on annual-based issues as in public finance, one-dimensional annual estimates might be sufficient. We contribute to this literature by comprehensively identifying the chronology of German business cycles back to 1950 via many different approaches, including the new Hamilton method, exploiting annual GDP data. In doing so, we focus on those simpler methods that are widespread in practice. In contrast to Schirwitz (2009), who presents a German classical business-cycle dating for the period 1970 to 2006, we especially provide a growth-cycles dating in the period 1950 to 2021. In contrast to Breuer et al. (2022), we focus the standard GDP-based approaches used in practice at basis of annual data, including the new Hamilton regression. In line with Schirwitz (2009: 290), we believe that the big advantages of these approaches is higher transparency, long-term uniformity, and (international) comparability vis-á-vis expert committee approaches. We thus provide a comprehensive basis to calculate cyclically adjusted variables or parameters for the complete period of post-WWII German Federal Republic – for instance, cyclically adjusted public finance positions published annually, including cyclically-adjusted interest rates suggested for determining better MTOs within the EU, Euro area, or Germany.

3 Modus Operandi

We follow a two-step approach. We start with comparing the results of the new Hamilton regression approach (HR) with the official production function approach of the EU (EUPFA), the standard HP filter, and the modified Swiss HP filter (mHP); for the two HP filters we apply different values for the smoothing factor. We first estimate potential output or structural component (trend), and deduce the corresponding output gap, respectively. Based on these gaps we determine the business cycles of the German economy. The data basis is the spring projection 2021 of the German federal government (FedGov 2021), which provides price-adjusted (2015=100) GDP numbers for the years since 1960, and additionally involves forecasted GDP numbers for 2021 to 2025. In the context of the fiscal surveillance procedure of the EU, it also reports potential GDP estimated by EUPFA since 1980; given earlier official estimates are not available the comparative analysis is done for the years since 1980. In this part, we analyse and compare the methods in depth using several potential different parameters, and identify which methods can be considered as quasi-identical with respect to business-cycle identification. Doing so, we reveal pecularities of the HR technique, and other alternatives, but also their similarities. In a second step, we then deduce the complete business-cycle history of Germany since 1950 at basis of the HR in comparison to the deduced most relevant alternative method.

The identification of a business cycle requires the identification of the start and end point of the cycle. The cycle can either be identified from one minimal turning point (trough) to the next minimal turning point, or from a maximal turning point (peak) to the next maximal turning point. The adequate choice depends on the time period analysed and which result makes a better case. For our first comparative study we choose the peak-to-peak option, as we find, later in the second step, that the year 1979/1980 represents a cycle peak – this result is also confirmed by our benchmark studies. In the second part, the first year in the data is 1950, and it remains uncertain when exactly the pre-peak and pre-trough is located. Hence, we apply both options, for comparison. In our cycle identification we require a complete business cycle to involve a clear boom and a clear downturn phase, including positive and negative output gaps; whenever there is an up and down without a change of the sign of the gap, we consider it as an interim cycle only.[16] Having identified start and end point of a cycle, we determine its length in a second step.

To determine the business cycles we analyse the output gaps of the annual price-adjusted GDP (2015=100).[17] Given the uncertainty of the unobservable potential output, we compare four different techniques: EUPFA, HP, the Swiss mHP, and the new HR filter. For the two versions of HP filter we use the standard value of the smoothing factor λ for annual data of 100 (Hodrick and Prescott 1981, 1997), the value of 20 which was deduced specifically for Germany (Mohr 2001), and the value of 6.25 deduced analytically by Ravn and Uhlig (2002). For the EU approach we one-by-one adopt the official data on potential output and gap published by the federal government of Germany, rooting exactly in the official EU regulation (FedGov 2021). The data also covers price-adjusted GDP data (2015=100) for the period, which we use to apply the HP/mHP and HR filter techniques, so that all estimates root in identical data and are, in this respect, fully comparable. We include all forecasted GDP values up to 2025, also provided in the data, to mitigate the end-point problem of the HP filters (e.g. Kaiser and Maravall 1999; Mohr 2001).

Given the German Federal Statistical Office does not provide a time-series on (real) GDP back to the founding years of the post-war republic, we use, for the second part of our study,

16 The results provided, however, allow each reader to identify those interim cycles, if desired. Our approach is intensionally not applying strict identification rules, e.g., Markov switching models, which has been found to be too narrow to produce convincing results (Harding and Pagan 2002b; Schirwitz 2006). Identifying a particular period as turning point just because of a marginal change of a parameter appears kind of arbitrary. Additional inspection of each candidate is adequate, and, for instance, also the approach of the NBER, CEPR, or GCEE (Breuer et al. 2022). Finally, in the context of annual data, strict requirements for monthly (e.g. Bry and Boschan 1971) or quarterly (e.g. Harding and Pagan 2002a) data – representing minimum durations of less than a year – are not applicable. The same holds for the "newspaper-method" definition of a recession (two consecutive quarters of negative growth), or its modification by Boldin (1994).

17 It should be emphasized that national business cycles cannot be assumed to be uniform within an economy. Germany's regions reveal quite heterogenous economic structures, and thus regional-specific cyles (Lehmann and Wikman 2022); especially East-German states followed a catch-up effect after reunification (Ragnitz 2019) that resulted in a special East-German business cycle (Gießler et al. 2021). Similarly, Fortin et al. (2022) find that the cycle of Canada's region Quebec followed a region-specific cycle that were, prior to 1980, correlated more with the U.S. cycle than with the residual-Canada cycle.

a time series provided by the German Bundesbank,[18] which starts at 1950, the first complete year of the new West-German republic after WWII. Note that after the East-German Revolution and the emancipation of the East-European nations from the Soviet Union in 1989 the two West- and East-German states were reunified on 3 October 1990; statistically reunification was implemented for the year 1991. The Bundesbank price-adjusted GDP time-series, therefore, provides GDP only for West-Germany until 1990, and the complete economy of the West- and East-German states afterwards.[19] The Bundesbank chained the two time-series via the 1991 annual average.[20]

4 Results

4.1 The Hamilton Approach in Comparison

4.1.1 Output Gap, Dispersion, and Other Features

Comparing the different approaches to separate cycle and trend, respectively structure, in the years 1980 to 2020 reveals important differences and similarities. The analysed period involves a structural break (German reunification 1989 to 1991, cf. Appendix A.1) and the great global crisis in the years 2007-2009, which allows to investigate how the different methods are affected by these in comparison. Table 1 informs about important aggregated statistical numbers for our comparative analysis.[21] In the first two lines we state the mean output gap over the 40 years and the standard deviation (SD). While the mean informs us about how balanced the cycles are, the SD informs about how much the estimated gaps spread around the estimated trend, respectively potential output. We learn that all methods alike produce gaps that (at least nearly) are balanced over the 40 years, but that the estimated gaps fluctuate very heterogeneously: the SD varies from only 1.4 (HP6.25 and

18 The exact Bundesbank code of the time-series is BBNZ1.A.DE.N.H.0000.L.

19 A detailed discussion of (potential) structural breaks is provided in Appendix A.1.

20 The Federal Statistical Office nominal GDP data in 1991 for West-Germany and East-cum-West Germany reveal that the former contributed 89% to the latter. The estimates of Ritschl and Spoerer (1997) for the year 1989 referring the East-German real GDP suggest that, referring to the hypothetical common real GDP, the West-German GDP would have accounted for even about 92% of it.

21 The partly strikingly deviating output gaps are reported for all years in detail in Appendix A.2.

mHP6.25) to 2.9 (HR). Among the two HP filter methods the result is easily explained by the increasing smoothing factor from 6.25 to 100: the higher λ the more linear the trend and the more deviations from the trend can be found. Due to the increasing smoothing factor HP filters, therefore, estimate the cyclical part to be bigger, and the respective SD increases. We find that the HP100 SD (1.9) is most similar to the SD of the EUPFA (2.0); the Swiss mHP100 involves only a slightly higher SD (2.1). The most striking result is that the HR approach produces a markedly higher SD than all the other methods: the HR SD (2.9) is one SD unit higher than that of EUPFA, and still 0.9 units higher than that of the second highest SD (2.1, mHP100); that is, the HR SD is 53% higher than the EUPFA value and 38% higher than the second highest SD. To prevent a biased result due to incomplete business cycles in this context, we also restricted this comparison to the set of completed business cycles in the lines three and four of the table (cc).[22] Interestingly, we find means that are all positive and non-zero. While the means hereto varied in a range of 0.1 (EUPFA and HR) to 0.1 (mHP100 and mHP20), they now vary in a range of 0.1 (EUPFA, mHP6.25, and HR) to 0.3 (HP100 and mHP100). Based on our identification approach for cycles, this suggests that the methods alike involve a minor upwards bias in estimating output gaps. Our discussion of the SDs, however, remains qualitatively basically the same.

In a next step, we focused on the absolute values of the gaps; this allows to deduce the average size of the estimated gaps, irrespective of whether it is positive or negative. In line with our results so far, we find that the average size of gap is smallest for the HP filters (1.9) with smallest smoothing factor (HP6.25 and mHP6.25). The average gaps most similar to the official EU method (average of 1.4) are HP20 and the Swiss modification of it (mHP20) (average of 1.3). As suggested by our study so far, the HR value (2.3) is strikingly higher – the HR produces, by far, the biggest output gap estimates. This result is not changing for restricting analysis to only completed cycles. However, we find that the HP filters with

22 We used the partly heterogeneous identified cycles of the respective method, reported below.

smoothing factors 20 or 6.25, and the EUPFA value, remain basically unchanged, while the filters with factor 100, in contrast, decrease.

Table 1: Statistical Indicators in Comparison

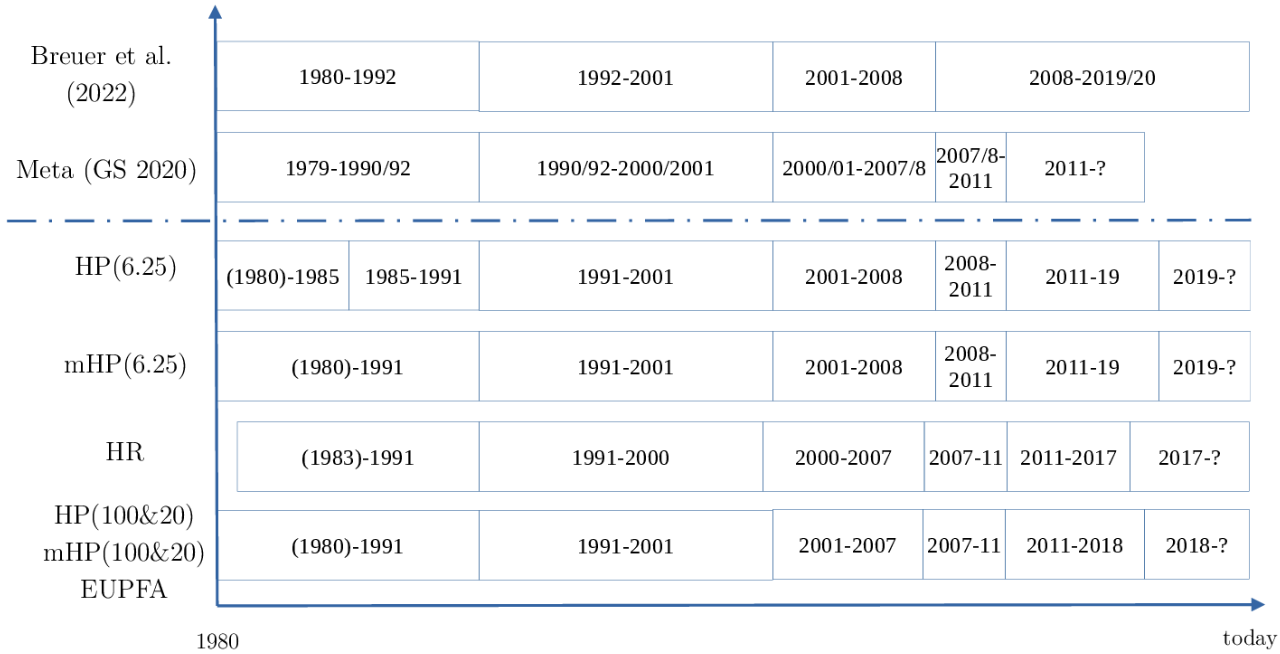
	EUPFA	HP100	HP20	HP6.25	mHP100	mHP20	mHP6.25	HR
mean	-0,1	0,0	0,0	0,0	0,1	0,1	0,0	-0,1
SD	1,9	2,0	1,6	1,4	2,1	1,7	1,4	2,9
mean(cc)	0,1	0,3	0,2	0,1	0,3	0,2	0,2	0,1
SD(cc)	1,9	1,8	1,6	1,4	1,8	1,6	1,4	2,8
mean (abs.)	1,4	1,6	1,3	1,1	1,6	1,3	1,1	2,3
SD (abs.)	1,2	1,2	1,0	0,9	1,3	1,1	0,9	1,8
mean(abs. cc)	1,4	1,4	1,3	1,1	1,3	1,3	1,1	2,2
SD(abs. cc)	1,2	1,2	1,0	0,8	1,2	1,0	0,9	1,7
				Correlation Matrix				
EUPFA	*1,00*	0,93	0,95	0,91	0,89	0,93	0,91	0,93
HP100	0,93	*1,00*	0,97	0,90	0,99	0,97	0,92	0,83
HP20	0,95	0,97	*1,00*	0,98	0,94	0,99	0,98	0,84
HP6.25	0,91	0,90	0,98	*1,00*	0,87	0,96	0,99	0,81
mHP100	0,89	0,99	0,94	0,87	*1,00*	0,97	0,90	0,82
mHP20	0,93	0,97	0,99	0,96	0,97	*1,00*	0,98	0,83
mHP6.25	0,91	0,92	0,98	0,99	0,90	0,98	*1,00*	0,81
HR	0,93	0,83	0,84	0,81	0,82	0,83	0,81	*1,00*

Source: Own calculations of the mean output gap, its standard deviation, and the linear correlation of the respectively estimated output gaps at basis of 1980-2020 period (correlation coefficient) and the FedGov (2021) data. – SD: standard deviation; cc: only completed business cycles (based on the respective method); abs.: all gaps in absolute values.

Finally, at the bottom of the table, we provide the correlation matrix of the compared approaches. We find that the output-gap estimates of the official EUPFA are at least linearly correlated by 0.91 (HP6.25) with the standard HP filter methods; the highest correlation exists with the HP20 filter (0.95). Comparing these numbers with the modified version, we see that this correlation decreases slightly. In contrast, the HR again stands out of the sample: its correlation coefficient varies from only 0.81 (HP6.25 and mHP6.25) to at most 0.84 (HP20) with the two HP filter methods. Comparing HR with the EUPFA approach, however, we find a correlation coefficient of 0.93.

Turning to the handling of structural breaks and economic crises, we use a graphical approach. In Appendix A.2, we provide figures of the development of all estimated potential outputs, respectively trends, in comparison to price-adjusted GDP (including the estimated gaps) (Figures A.2.1 to A.2.8). Comparing these, we again learn that the HR is standing out compared to the other approaches. While all other techniques smooth the ups and downs of GDP, the HR is rather shifting current developments to the future: the purple curve of estimated potential output is a kind of subdued right shift of the real GDP development. This is of no surprise, given it is a lagged dependent variable time-series regression. As a consequence, potential output is estimated to follow real GDP with some time lag, while all other techniques produce smoothing central-tendency curves through the ups and downs. As is well-known, we observe that the HP filter, modified or not, estimates trend curves following real GDP the more closer the lower smoothing factor λ . That is, for factor 6.25 potential output is de facto estimated to be quite elastic to current GDP movements, less elastic for 20, and least elastic for 100, with the consequence that in situations of structural breaks (reunification) and massive shocks (great financial and economic crisis in 2009) potential output is assumed to be quite stable in the short and medium run for $\lambda=100$, medium stable and more elastic for $\lambda=20$, and quite elastic for $\lambda=6.26$. The EU approach, in turn, is particular, as it combines filter techniques at the production factor level with the theoretical idea of a production function. The graphical comparison reveals that the estimated potential output is close to real GDP in normal times and relatively stable in times of turmoil. Considering German reunification, there is a slight increase of the slope of the EU potential output curve; the output gap starts to be positive in 1989 and is big in the years 1990 to 1992. Before and afterwards potential output is estimated to be close to real GDP. This is in line with a realistic description of the development. Comparing this with the HP and mHP filter, we see that HP100 produces similar results around the reunification but smoothing produces a doubtful upwards movement of the trend curve in the years 1986 to 1989. For lower smoothing factors this upwards movement in pre-years of structural breaks is weaker, and hence an advantage. Especially with a factor of only 6.25, however, this is involving the assumption that potential output would have increased quite elastically. Today's reading of reunification is that the stock of capital and production potential of the former GDR was much lower than expected; the increase of GDP was mainly driven by a massive increase of East-German demand satisfied by production in West-Germany, so by big positive output gaps, as estimated by EUPFA, (m)HP100, and still by (m)HP20. Turning to the big financial crisis in 2009, we had the special situation that Germany was hit by this crisis right in a moment where the economy experienced a boom phase. Hence, the pattern is much in line with a standard downswing from a boom, but, of course, a quite harsh one. As a consequence, all techniques produce similar patterns. The only special observation is for EUPFA because it estimates potential output in all the years since 2000 to be higher.

Figure 1: Business cycles in the period since 1980, identified via different cyclical adjustment methods



Source: own dating based on FedGov (2021), Breuer et al. (2022), and Gebhardt and Siemers (GS) (2020b).

In comparison to this the HR also produces big positive gaps from 1989 to 1992, and the slope of the potential-output curve increases starting in 1991 to 1993, so a bit delayed; there is no artificial upwards movement of the trend curve in advance. Hence, it is a reasonable description of the development. Turning to the big financial crisis, however, the lagged-dependent-variable regression produces an instable up and down of potential output in the years 2003 to 2012, and estimates that the slope of the potential-output curve even increased during the financial crisis, which is odd: potential output increased significantly just in the global crisis years 2007 to 2010. This roots in the fact that Germany experienced a boom just before the crisis, which, by construction of the HR, causes increasing potential output levels in the aftermath. Therefore, the HR may cause odd results when an economy is hit by a massive crisis during a period of boom, or when experiencing a significant ad-hoc boom start after a period of weak growth. After all, the HR may deal with structural breaks better than especially the standard HP100 approach, but has potential weaknesses, in comparison, when there are bigger fluctuations in estimating a plausible potential output development.

4.1.2 Business Cycles, Peaks, Troughs, and Cycle Length

The identified business cycles are summarised in Figure 1. We find striking similarities of the different approaches. The first identified business cycle covers at least the complete 1980s until 1991; this is identified by seven out of eight approaches. Only the HP6.25 separates this period into two different cycles: the first from at least 1980 to 1985, and the second from 1985 to 1991. The next cycle identified covers the complete 1990s from 1991 onwards to 2001; only the HR suggests a slight deviation and ends this cycle already in the year 2000. The following cycle is located in the years 2001 to 2008, where again the HR result deviates and locates it in the years 2000 to 2007. All approaches agree that the next cycle ended in 2011. With regard to the end of the last complete cycle in Germany the results are more heterogeneous: while using EUPFA as well as the HP and mHP filters with smoothing factors 100 and 20 result in ending this cycle in 2018, the HP and mHP filter with smoothing factor 6.25 suggest the year 2019; the HR, in contrast, ends the cycle already in 2017. Since then, all methods alike find decreasing gaps; the German economy started at the end of the last decade to slump, and entered the global crises of COVID-19 pandemic and Russian war of aggression against the Ukraine.

At the top of the figure we add the results of the recent business-cycle study of the German Council of Economic Experts (GCEE) (Breuer et al. 2022), and of a study of Gebhardt and Siemers (2020b) who evaluated several business-cycle datings. In contrast to our analysis, these results are (majorly) based on monthly and quarterly multidimensional business data;

we aggregated these results to comparable annual values. The comparison reveals that the methods analysed here, applied to more aggregated annual GDP data, produce more or less the same business cycles.[23] The only deviation is that the GCEE (Breuer et al. 2022) does not consider 2011/12 as an end of the business cycle starting in 2008; they also find a cycle in this period, but, based on other indicators, they draw the conclusion that this period represents only an interruption of the business cycle which continues afterwards.

Table 2: Length of business cycles

full cycles	EUPFA	HP100	HP20	HP6.25	mHP100	mHP20	mHP6.25	HR
1.	10	10	10	6	10	10	10	9
2.	6	6	6	10	6	6	7	7
3.	4	4	4	7	4	4	3	4
4.	7	7	7	3	7	7	8	6
5.				8				
Ø	6,75	6,75	6,75	6,80	6,75	6,75	7,00	6,50
SD	2,50	2,50	2,50	2,59	2,50	2,50	2,94	2,08

Source: Own calculations, based on FedGov (2021).

We summarise the found results on respective peaks and troughs as well as cycle lengths in Tables 2 and 3. We define peaks as upper tipping point of a cycle and troughs as lower tipping points. We do not observe the starting peak of the first cycle. However, the meta analysis of Gebhardt and Siemers (2020b) suggests that it is located in 1979, and the GCEE (Breuer et al. 2022) identify the start of the first cycle exactly at January 1980.[24] For the cycle length (Table 2) we find that the mHP6.25 and HR partly produce a bit specific results: while all other methods alike suggest an average cycle length of 6¾ years (the HP6.25 value is only slightly higher), the mHP6.25 value is a quarter longer, and the HR value a quarter shorter.[25] The HR, however, involves the lowest SD of cycle length, that is, the most homogenous duration of cycles, while the length of the mHP6.25 cycles are most heterogenous. With regard to length of down- and upswings (Table 3) we find more

23 In Gebhardt and Siemers (2020b) the end of the cycle starting in 2011 were not yet identified.

24 During cycles there can arise intermediate tipping points in the sense of some up and down; in Table A.2.2 (Appendix) we also add the location of these “mild” tipping points.

25 Our results are in line with those found for the OECD (Kufenko and Geiger 2017: 4.2 to 7.4 years), and typical cycle frequencies between 1.5 and 8 years stated by Burns and Mitchell (1946) or Stock and Watson (1999).

heterogenous results. While the estimated average duration of upswings (trend up) of both HP100 and HP20 version is 3.6 years (about 43 month), respectively 3.83 (46 month) for HP6.25, the other methods find a significantly longer mean upswing between 4.4 (53 month, EUPFA) and 4.8 years (57½ month, HR). For the downswings we find a more similar but still quite heterogenous result: while the five methods EUPFA, (m)HP100, and (m)HP20 all suggest a mean duration of 3¼ years, the others suggest significantly lower expected values between only 2¼ years (mHP6.25) and at most 2.8 years (HP6.26); the HR mean of 2½ years is in between those. Restricting the time slot to the reunified period since 1991, we obtain a more accentuated contrast: now the EUPFA is also in line with both (m)HP100/20 versions in suggesting an expected value of 3½ years of upswing and 3¼ years of downswing, so that downturns are only a quarter shorter in duration.

Table 3: Troughs, peaks, and their lengths

tp	EUPFA	HP100	HP20	HP6.25	mHP100	mHP20	mHP6.25	HR	GCEE
trough	1983	1987	1987	1982	1987	1987	1987	(1983)	1982
peak				1985					
trough				1987					
peak	1991	1991	1991	1991	1991	1991	1991	1991	1992
trough	1996	1996	1996	1996	1996	1996	1993	1994	1993
peak	2001	2001	2001	2001	2001	2001	2001	2000	2001
trough	2005	2005	2005	2005	2005	2005	2005	2003	2003
peak	2007	2007	2007	2008	2007	2007	2008	2007	2008
trough	2009	2009	2009	2009	2009	2009	2009	2009	2009
peak	2011	2011	2011	2011	2011	2011	2011	2011	
trough	2013	2013	2013	2013	2013	2013	2013	2013	
peak	2017/18	2018	2018	2019	2018	2018	2019	2017	2019
Ø years (all observed years)									
trend up	4,40	3,60	3,60	3,83	3,60	3,60	4,60	4,80	8,25
trend down	3,25	3,25	3,25	2,80	3,25	3,25	2,25	2,50	1,50
Ø years (since 1991)									
trend up	3,50	3,50	3,50	4,00	3,50	3,50	4,75	4,00	7,67
trend down	3,25	3,25	3,25	3,00	3,25	3,25	2,25	2,50	1,33

Source: Own calculations based on FedGov (2021); tp – turning point; GCEE – German Council of Economic Experts, based on Breuer et al. (2022), where we aggregated the monthly identification to years. For the HR value 4.8 (trend up) we used the first observed gap as first trough, as it fits the required characteristics. For the GCEE value on trend down we included the peak identified by GCEE in January 1980.

In contrast, HP6.25 and HR estimate a duration of 4 years of upturn, and mHP6.25 even of 4¾ years; with regard to the downturn value, the HP6.25 mean of 3 years is only a quarter shorter than the EUPFA/(m)HP100/20 counterpart, but the HP6.25 and HR means of 2¼ and 2½ years is significantly smaller. Therefore, while EUPFA and (m)HP100/20 rather suggest nearly balanced durations of up- and downswing, with the latter being only slightly shorter, the others suggest an asymmetry with downswings being significantly shorter in duration. The benchmark GCEE approach clearly supports the asymmetry hypothesis: it suggests 8¼ years (upswing) vis-á-vis only 1½ years downswing (since 1980), respectively 7.7 years vis-á-vis only $1^1/_3$; this, however, roots in the decision of the GCEE not to identify another independent cycle in the last decade. Overall, EUPFA, HP, mHP, and HR produce very similar cycle datings, unless the (m)HP smoothing factor is not 6.25; the HR sometimes involves turning points that are located one year in advance. The (m)HP6.25 filters produce more specific cycle datings, where HP6.25 even identifies an additional cycle.

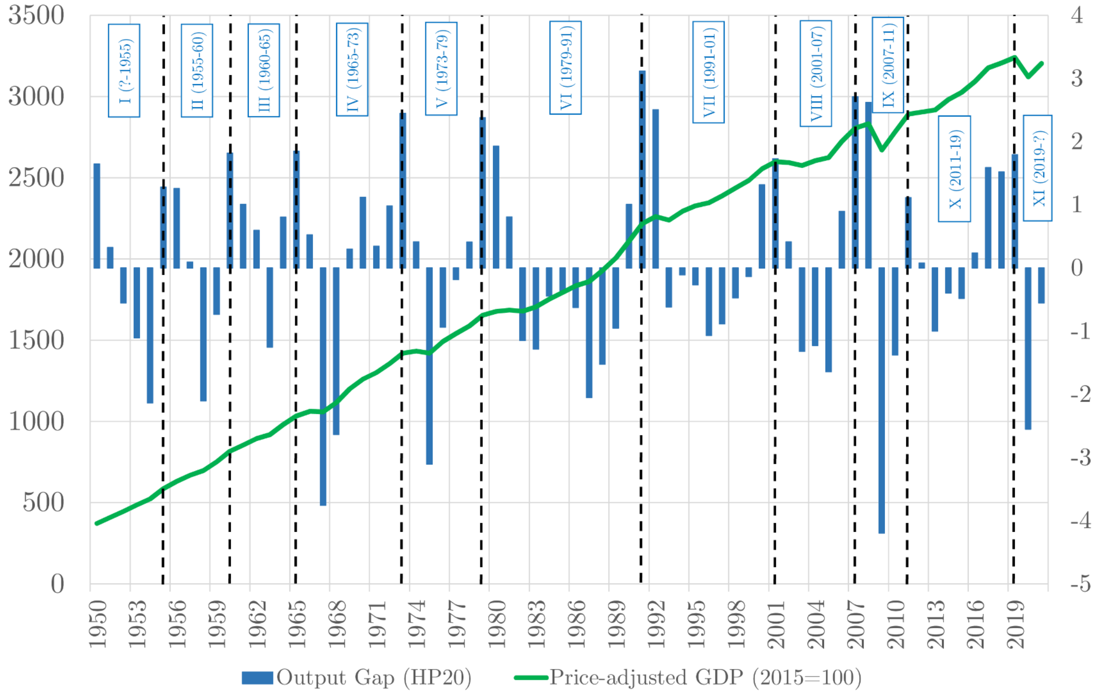
4.2 Business-Cycle History Since 1950

For the business-cycle history since 1950, therefore, we restrict ourself to the new HR approach and the HP20 filter, especially suggested for Germany, and use data of the German Central Bank (Bundesbank 2022). Figure 3 summarises the output gaps of the HP20 filter, where we again identify a cycle peak-to-peak. We find ten completed cycles, where the beginning of the first is probably not observed and unknown. The HR, in contrast, finds 11 completed cycles (Figure 4), so one extra cycle in comparison, namely, similar to our former HP6.25 result, in the period 1979 to 1991. Both methods (HR and HP20) suggest increasing gaps and a recovery after a trough in 1982/83 in pre-years of 1986. While the HR gaps turn to positive gaps in the years 1985 and 1986 – finalising the cycle from 1979 to 1986 – the HP20 gaps remain negative and decrease again afterwards, so that the cycle, that started in 1979, continue until 1991. The difference thus stems in relatively minor differences in the gaps: while the HR gaps in 1985 and 1986 are quite small (0.5 respectively), the negative HP20 gaps are relatively small in absolute terms as well (-0.4 and -0.6). Therefore, the 1979-

to-1986 cycle involves a quite underdeveloped boom period of only two years. The other remaining cycles are in most cases identical or very similar. Both suggest a first after-war peak in the year 1955/6, a second in 1960, and the third in 1965. A first significant deviation is found for the forth: while HP20 locates it at 1973, the HR suggests already 1970. Both again agree on the fifth at 1979. After the already discussed differences in the 1980s, both alike see 1991 as a peak year. The next three peaks afterwards are identical or similar: 2000/1, 2007, and 2011. While the last peak is located by the HP20 filter in 2019, the HR suggests 2017.

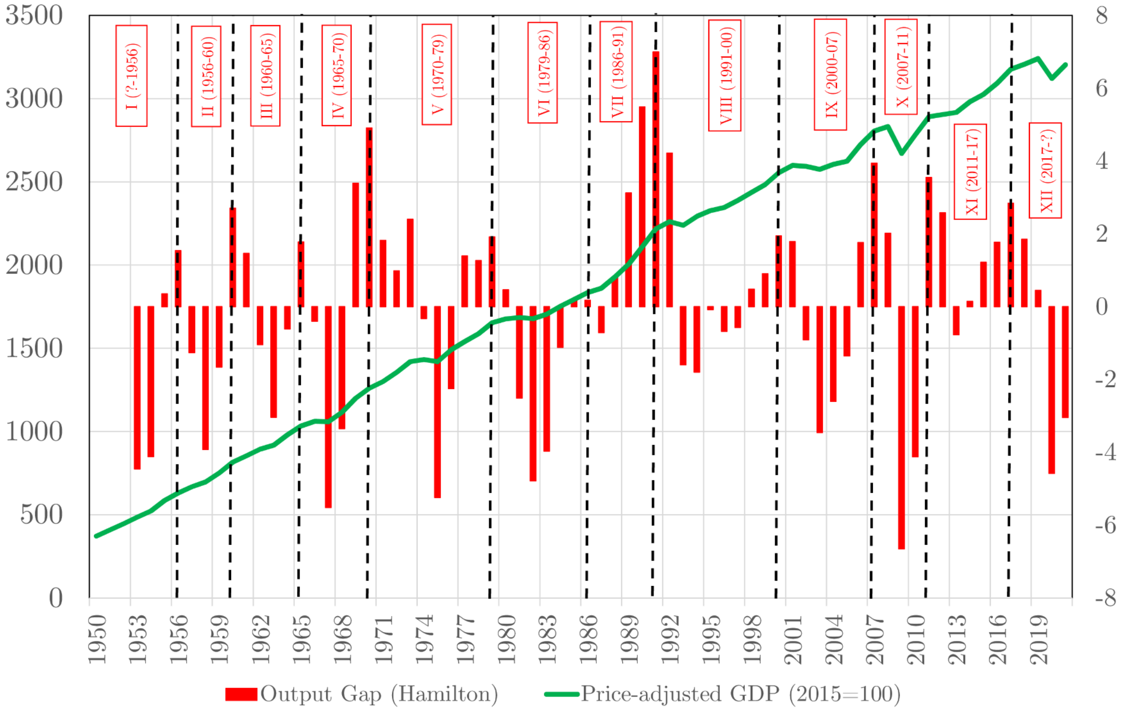
Utilising the historic GDP numbers in Ritschl and Spoerer (1997), and realising that 1945 is the most likely candidate for the pre-trough of the cycle running in 1950, we assume that the first after-war cycle started with a trough in 1945, which represents a year of total doom and collapse of the German economy. It followed a period of catch-up and absolute convergence towards steady state which presumably did not involve a second trough before 1950. Therefore, we also apply the alternative trough-to-trough identification, and use the premise that the first cycle started in 1945 and did not end before 1950. Figures 5 and 6 replicate the business cycles for this approach. Given the identified cycles start and end with a trough instead of peaks, we obtain different periods. The comparison of HP20 and HR, however, reveals very similar results. Until 1975 both approaches reveal identical or very similar cycles; the troughs are 1945, 1953/4, 1958, 1963, 1967, and 1975. Similar as in the former case, both agree in identifying the year 1987 as a trough, but the HR suggests that the period from 1975 to 1987 covers two cycles by locating another trough in 1982. The HP20 also finds an intermediate trough in 1983, but the following recovery does not turn the gap into a positive – and the cycle continues further until the deeper trough in 1987. The two trend-estimators agree afterwards in having troughs in the years 2009 (Great Financial Crisis), 2011, and 2020 (COVID-19 Crisis). For two troughs there is a deviation of two years, where, in both cases, the HR estimates it earlier in time: 1994 vs. 1996 and 2003 vs. 2005.

Figure 3: Price-adjusted GDP and business cycles of the German economy since 1950, based on HP20 (HP filter with $\lambda = 20$) and identifying cycles from peak to peak



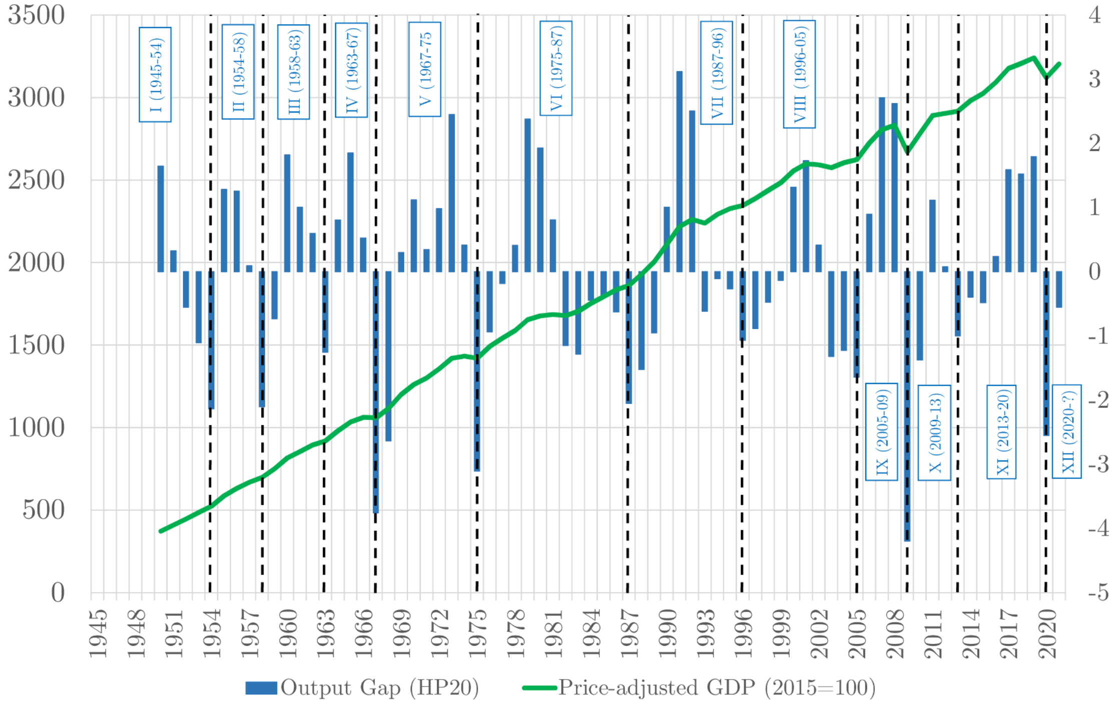
Source: Own estimations based on price-adjusted GDP (2015=100) from 1950 to 2021 (Bundesbank 2022)

Figure 4: Price-adjusted GDP and business cycles of the German economy since 1950, based on HR and identifying cycles from peak to peak



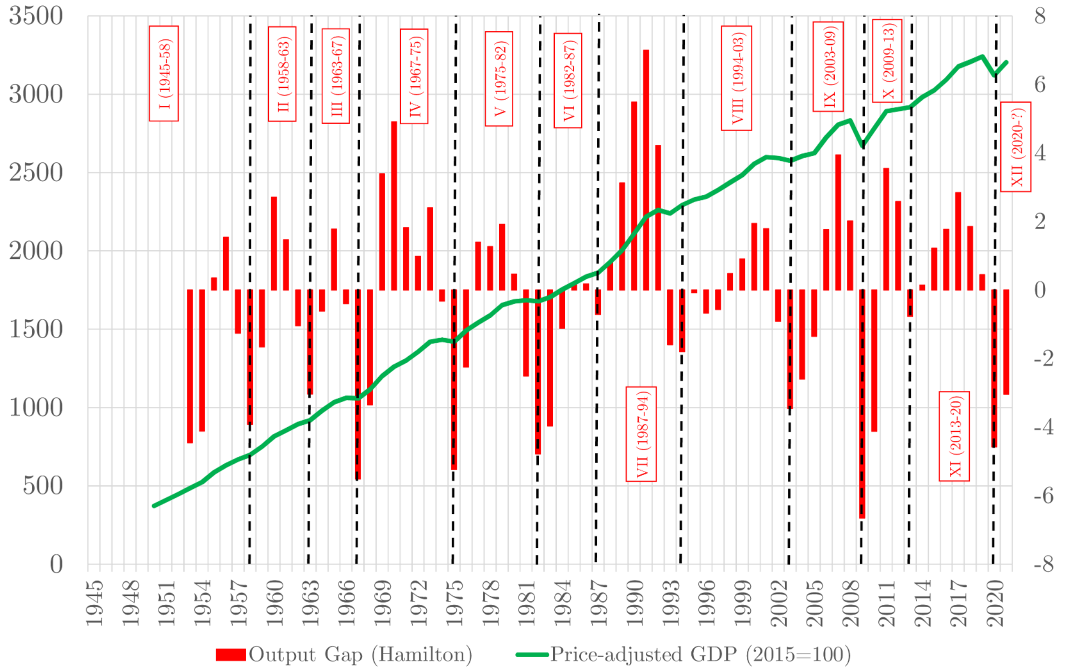
Source: Own estimations based on price-adjusted GDP (2015=100) from 1950 to 2021 (Bundesbank 2022)

Figure 5: Price-adjusted GDP and business cycles of the German economy since 1950, based on HP20 (HP filter with $\lambda = 20$) and identifying cycles from trough to trough



Source: Own estimations based on price-adjusted GDP (2015=100) from 1950 to 2021 (Bundesbank 2022)

Figure 6: Price-adjusted GDP and business cycles of the German economy since 1950, based on HR and identifying cycles from trough to trough



Source: Own estimations based on price-adjusted GDP (2015=100) from 1950 to 2021 (Bundesbank 2022)

Table 4: Statistical Analysis of Business Cycles since 1950, HR and HP20 filter

	mean abs. gap (cc) (SD)	mean gap (cc) (SD)	Ø length (cc) (SD)	Ø peak to trough (cc) (SD)	Ø trough to peak (cc) (SD)
			peak-to-peak identification		
HR	2,23	0,09	6,10	2,59	3,50
	(1,66)	(2,78)	(1,83)	(1,02)	(1,26)
HP20	1,25	0,07	7,06	3,10	3,50
	(0,91)	(1,55)	(2,58)	(1,98)	(1,64)
			trough-to-trough identification		
HR	2,26	-0,07	6,25	2,59	3,50
	(1,68)	(2,82)	(1,59)	(1,02)	(1,26)
HP20	1,28	0,00	6,82	3,10	3,50
	(0,92)	(1,58)	(2,66)	(1,98)	(1,64)

Source: Own calculations based on own estimations with the Bundesbank (2022) data; HR – Hamilton regression; HP20 – Hodrick-Prescott filter with smoothing factor equal to 20; cc – completed cycles; SD – standard deviations, in parenthesis below mean values; Ø – average (mean).

We report the respective means of gaps, cycle length, peak-to-trough length, and trough-to-peak length, including the respective standard deviations, in Table 4. With the longer time-series, we find that the mean absolute gaps of both approaches are nearly the same as in our 1980-to-2021 analysis, with a little bit lower standard deviations (peak-to-peak identification). The trough-to-trough identification produces very similar results, with only 0.03 points higher gaps, respectively. We deduce interesting results for the mean gaps. Compared to the former analysis, starting in 1980, these are higher for the longer time frame in this section (peak-to-peak identification), and thus farer away from the theoretical reference value of zero for symmetry. Turning to the alternative trough-to-trough identification, we find smaller deviations from symmetry, compared to both the shorter and longer time frame peak-to-peak identification. We also replicated the trough-to-trough identification for the former time frame starting in 1980, and find that this conclusion also holds for this shorter time frame.[26] Therefore, we provide empirical evidence that the

26 The respective means of all methods, with the exception of HR, are closer to the symmetry reference value of zero: -0.10 (EUPFA), 0.05 (HP100), 0.00 (HP20), -0.05 (HP6.25), 0.04 (mHP100), -0.01 (mHP20), -0.04 (mHP6.25), -0.33 (HR). The bigger deviation of the HR can be explained by the problem that the first trough of the HR is not before 1994, so that the first 14 years cannot be included.

symmetry characteristic is more in line with the trough-to-trough identification approach, at least for Germany in the post-war period. For the 1980-2021 as well as for the 1950-2021 time window the average HP20 gap over all completed cycles identified trough-to-trough is 0.00. That is, the HP20 generates gaps that perfectly fit the symmetry assumption.

Referring average cycle length we find that the HR method suggests 6.1 to 6.25 years, and HP20 6.8 to 7.1 years; the peak-to-peak identification generates for the HR a bit shorter cycle length vis-a-vis trough-to-trough identification, while it is vice versa for the HP20 method. The average years between a peak and trough on the one hand, and trough-to-peak on the other, is more balanced for the HR: while we found 2.5 and 4.8 in the former time frame, we now obtain 2.6 and 3.5. With HP20 we receive only slightly lower numbers. Overall, we hence provide empirical evidence that the downward-trend periods are shorter than the upwards-trend periods. This is in line with Breuer et al. (2022). Transformed to years, they suggest a mean peak-to-trough duration of 1.8-1.9 years, but 6-7 years for trough-to-peak periods. That is, with their method, they find even shorter mean downswing periods and longer upswing periods. In comparison, they identify longer cycles: their mean peak-to-peak length is 9 years, the trough-to-trough counterpart a bit less than 8½ years. As stated, this roots in their decision not to interpret mild cycles (e.g. due the peak in 2011) as own cycles but only as an interruption of running cycles. A striking difference to their result can be found in the 1950s and 1960s: while we identify peaks (troughs) in 1955/56 and 1960 (1954/1958 and 1963), they identify the first peak (trough) not before 1966 (1967). Finally, evaluating the SD of the gaps and durations (Table 4) we again find a higher variation of the HR gaps, but a lower of the durations, that is, cycles, downswings, and upswings are more homogenous in length when identified by HR.

5 Conclusion

Our comparative analysis revealed that the Hamilton regression (HR) approach involves, as an alternative to separate structural and cyclical parts of GDP development, several peculiarities in comparison; in many directions its results are standing out. Among the analysed methods it involves the highest output gaps and, by distant, the highest standard deviation (SD) of these: its SD is 53% higher than that of the official production-function approach of the EU, and still 38% higher than the alternative with the second highest SD (the Swiss modified HP filter). As a consequence, the HR method identifies, in comparison, the shortest cycles. This finding is in contrast to the conclusion in Quast and Wolters (2022) as well as Schüler (2018), who state that the HR amplifies longer and mutes short cycles. While the EU approach and (m)HP filter (with all applied smoothing factors) kind of smooth the GDP development, the HR rather shifts, by construction, current developments to the future, in order to identify the trend. Especially in periods of massive crisis this generates quite odd results. In times of structural breaks, however, the HR performs better – exactly for not smoothing the development. Finally, referring the statistical similarity of the estimated gaps, the HR gaps are mostly correlated with the results of the EU approach (correlation coefficient of 0.93), and least with the criticized HP filters (only 0.81 to 0.84). With only very rare single exceptions[27] all compared methods identify identical or very similar business cycles for Germany, and those are in line with more sophisticated business-cycle studies that exploit monthly and quarterly data as well as multidimensional business indicators. That is, being solely interested in annual developments and categorization the simple one-dimensional annual GDP data seem to work well, for instance, for fiscal analysis of structural budgets or calculation of business-cycle adjusted parameters, such as structural levels of interest rates.

We find that the average business cycle since 1950 lasted 6-7 years, the average upswing (trough to peak) 3½ years (42 months), and the average downturn (peak to trough) 2.6-3.1 years (31-37 months). It turns out that the identification of business cycles in post-war Germany is more adequate via trough-to-trough identification, for the economy started in

27 In contrast to all other methods, the Hamilton regression and the HP6.25 identify two peak-to-peak cycles located in the 1980s, instead of one. Considering weak boom or downswing phases as insufficient to end a cycle, this difference disappears.

1945 in a massive trough and then entered a period of super growth in the German "growth miracle."[28] The HP filter with smoothing factor 20, suggested for Germany by Mohr (2001), works well, and averaged over the trough-to-trough cycles identified, we find the symmetry assumption fully approved (average gap of zero); the average HR gap, in comparison, is slightly negative.

28 According to Manz (1968) and Abelshauser (1966) the period of super growth already started before the currency reform of 1948. Eichengreen and Ritschl (2009) argue that the reason for the growth miracle was mainly the postwar shock, and less neoclassical convergence, structural change, or an institutional shakeup.

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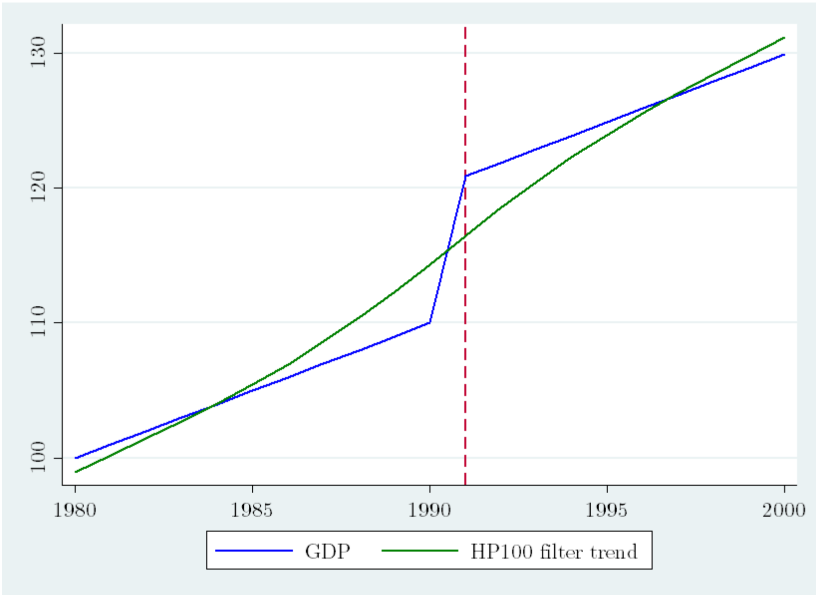
Appendix

Figure A.1.1: Illustration of smoothing structural breaks (HP100 filter)



Source: own illustration based on hypothetical GDP development with 1980=100 and a structural break as the reunification in the year 1991.

A.1 Structural Breaks in the German GDP Time-Series

It is often mentioned that a drawback of the HP filter (and all trend calculations based on two-sided moving averages) is that it cannot account for structural breaks (Bouthevillain et al. 2001; Brunila et al. 1999), and hence generates biased trend values in the neighbourhood of structural breaks, as the filter smooths the resulting kink in the trend as well (cf. Figure A.1.1); hence, the problem increases with the size of smoothing factor λ. This is exactly what we find in our study (cf. Figures A.2.3 to A.2.8). Following Schüler (2018) the same problem holds for the HR filter. Analysing Figure A.2.1, however, we do not find a structural-break smoothing of the HR. The EU approach, theoretically, faces the same problem, for it also uses statistical filters in identifying the structural part of the inputs that enter the production function. Inspecting Figure A.2.2, we find that EUPFA does not implausibly smooth the structural break, either. Therefore, we find that this is especially an issue of the HP filters. For our task of identifying business cycles structural breaks might bias the trends in its neighbourhood and cause the problem of distorting the tipping points that limit the business cycles in these years around structural breaks, especially using HP filters.

There were several potential structural breaks in the GDP time-series of Germany. First of all, the German Bundesbank provides own estimations for price-adjusted GDP in the years 1950 to 1959, while, starting in 1960, the data provide the data of the Federal Statistical Office, which involves a potential weak statistical break. Moreover, although the new West-German Federal Republic was founded in 1949, the region Saarland was still governed by a special French occupation regime until 1956, where a French-German treaty overcame this odd situation; and there was an economic transition process until 1959. However, the Bundesbank imputes in the years 1950 to 1956 own estimates for the Saarland, so that this potential structural break is healed. A similar statistical and territorial problem arose from the special status of Berlin: West-Berlin remained a special occupied city part ruled by a German city government below the almighty Western military occupation powers; according to international law, West-Berlin was not part of the West-German state, but the allied special status remained until 1990.[29] Again the Bundesbank imputes own estimations for West-Berlin in the West-German time-series, and the official statistics of the Federal Statistical Office later on included West-Berlin, too. Thus there is again no structural break in the data due to this issue.

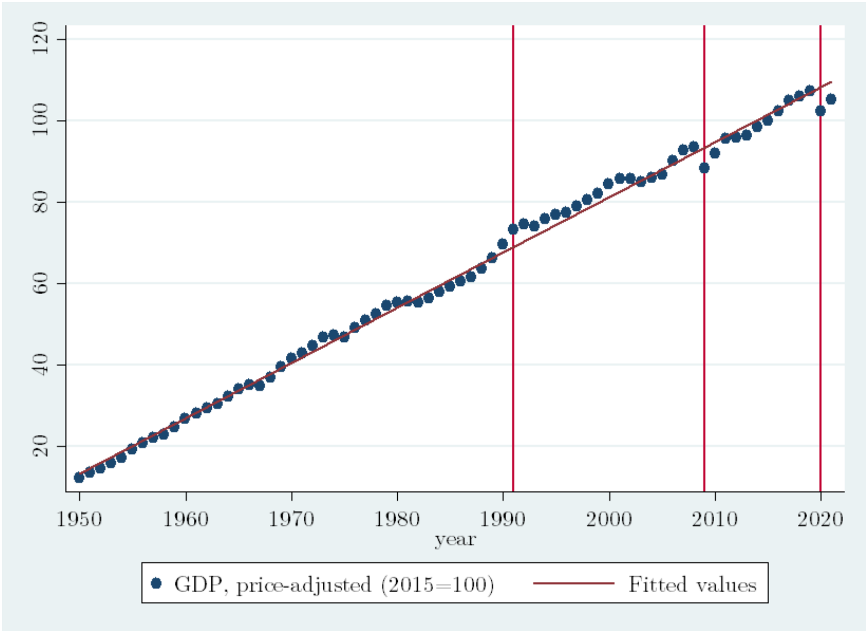
The major structural break issue is the reunification of the remaining two German states of East- and West-Germany at the end of the Cold War. Officially this happened on 3 October 1990, where the former East-German state entered into the Federal Republic of Germany (West-Germany). Therefore, the official statistics provide data on the reunified German states not before 1991. The structural-break issue is illustrated hypothetically in Figure A.1.1 for the HP100 filter: the HP filter smooths the break in 1991 over multiple pre- and post years, and is not able to identify the discontinuity in 1991. However, the reunification

29 The same held good for East-Berlin, though the Soviet occupation part of East-Berlin became even the East-German capital in 1949.

break was, in real world, much less significant than the considered hypothetical discontinuity of GDP. Firstly, it is important to note that the structural break in GDP by the reunification was in fact smaller than most expected: the East-German nominal GDP represented in 1991 only about 11% of the total, and in 1989 its real GDP even accounted for only about 8% of hypothetical common real GDP. Moreover, for the used German GDP time-series there are important special features to be accounted for, which caused the structural break to be smoothed over several years, instead of a sudden discontinuity. After the fall of the Berlin Wall and the inner German border on 9 November 1989, the reunification economically already started. Due to decades of renounce the East Germans generated massive catch-up effects on German aggregate demand. Given the East German economy had no supply of Western World goods, these catch-up effects were more or less entirely accommodated by the West-German economy, with the consequence that the total German aggregate demand to a huge extent already equalled the aggregate domestic demand of the West-German economy – thus there was no sudden increase of GDP in 1991. We observe above-average economic growth rates of nominal and price-adjusted GDP already in 1989 (7.0%, resp. 3.9%) and 1990 (9.1%, 5.3%). In 1991, these growth rates (8.4%, 5.1%) are even a bit lower than in 1990. This means that the data do not involve a harsh point of discontinuity in 1991. The Bundesbank chained the two time-series via the 1991 average, and in fact, the structural break of the reunification is reflected in the data as a relatively smoothing process over the years 1989 to 1991, with a downturn afterwards as the catch-up effects ended due to high inflation, and a European as well as global downturn, which were shifted in Germany due to the economic fireworks of the reunification by a few years. In 1992 the growth of price-adjusted GDP dropped to 1.9% and in 1993 to 1.0%.

Figure A.1.2 depicts the development of price-adjusted GDP with the overall linear OLS-trend-regression line; therein we mark the special years of reunification, the global financial crisis in 2009 (5.9%), and the Covid-19 Pandemic in 2020 (4.9%). We find that German GDP relatively straightly follows the linear trend without much deviation until 1988. In 1989, GDP begins to move upwards and remains above the linear trend until 2003. Afterwards the all-time linear trend seems to be adequate again. The massive downturns of the financial crisis and the COVID-19 pandemic involved similar changes in GDP as the reunification, but, in contrast, indeed with a massive discontinuity.

Figure A.1.2: Price-adjusted GDP and linear trend, 1950-2021



Source: own regression based on Bundesbank 2022

Therefore, we consider a smoothing increase of potential output or trend as adequate. The temporary boom starting in 1989 belongs to the cyclical part, and covers the catch-up effects in the years 1989 to 1992. However, the catch-up effects came along with a build-up of production capacity in the West, already starting in 1989; this part belongs to the structural part, and is indeed a smoothing trend increase.

A.2 More Detailed Results

Table A.2.1: Estimated output gaps in %, 1980-2025

year	EUPFA	HP100	HP20	HP6.25	mHP100	mHP20	mHP6.25	HR
1980	1,9	3,7	1,9	0,9	5,3	3,1	1,7	
1981	0,4	1,8	1,0	0,5	3,1	1,8	1,0	
1982	-1,8	-0,9	-0,9	-0,9	0,0	-0,4	-0,6	
1983	-2,0	-1,6	-1,1	-0,7	-1,0	-0,8	-0,6	-5,1
1984	-1,1	-1,3	-0,2	0,3	-0,9	-0,1	0,3	-2,3
1985	-0,8	-1,4	-0,2	0,4	-1,2	-0,2	0,4	-0,9
1986	-0,7	-1,8	-0,5	0,1	-1,7	-0,6	0,1	-0,8
1987	-1,6	-3,0	-2,0	-1,4	-3,0	-2,1	-1,4	-1,6
1988	-0,5	-2,1	-1,5	-1,1	-2,2	-1,6	-1,1	-0,1
1989	0,5	-1,1	-0,9	-1,0	-1,2	-1,0	-1,0	2,2
1990	2,6	1,3	1,0	0,5	1,3	1,0	0,5	4,8
1991	4,5	3,8	3,1	2,4	3,7	3,1	2,4	6,4
1992	3,2	3,3	2,5	1,8	3,2	2,5	1,8	3,8
1993	-0,5	0,1	-0,6	-1,0	0,0	-0,6	-1,0	-1,9
1994	-0,3	0,4	-0,1	-0,2	0,4	-0,1	-0,2	-2,3
1995	-0,6	0,0	-0,3	-0,1	0,0	-0,3	-0,1	-0,8
1996	-1,3	-0,9	-1,1	-0,8	-0,9	-1,1	-0,8	-1,1
1997	-0,8	-0,8	-0,9	-0,6	-0,8	-0,9	-0,6	-1,0
1998	-0,1	-0,4	-0,5	-0,3	-0,4	-0,5	-0,3	0,0
1999	0,3	0,0	-0,1	-0,2	0,0	-0,1	-0,2	0,5
2000	1,6	1,5	1,3	1,1	1,5	1,3	1,1	1,6
2001	1,8	1,9	1,7	1,5	1,9	1,7	1,5	1,5
2002	0,1	0,5	0,4	0,3	0,5	0,4	0,3	-1,1
2003	-1,8	-1,4	-1,3	-1,2	-1,4	-1,3	-1,2	-3,7
2004	-1,8	-1,4	-1,2	-1,0	-1,4	-1,2	-1,0	-3,0
2005	-2,2	-1,8	-1,6	-1,6	-1,8	-1,6	-1,6	-1,8
2006	0,3	0,8	0,9	0,7	0,8	0,9	0,7	1,5
2007	2,1	2,6	2,7	2,4	2,6	2,7	2,4	3,6
2008	1,8	2,4	2,6	2,5	2,4	2,6	2,5	1,9
2009	-4,8	-4,6	-4,2	-4,0	-4,6	-4,2	-4,0	-6,7
2010	-1,8	-1,8	-1,4	-1,1	-1,8	-1,4	-1,1	-4,3
2011	0,8	0,8	1,1	1,4	0,8	1,1	1,4	2,8
2012	-0,1	-0,1	0,1	0,3	-0,1	0,1	0,3	2,6
2013	-1,1	-1,0	-0,9	-0,8	-1,0	-0,9	-0,8	-0,7
2014	-0,2	-0,2	-0,3	-0,3	-0,2	-0,3	-0,3	0,0
2015	-0,3	0,0	-0,3	-0,5	0,0	-0,3	-0,5	1,0
2016	0,6	0,9	0,5	0,1	0,9	0,5	0,1	1,7
2017	1,9	2,2	1,8	1,3	2,2	1,8	1,3	2,7
2018	1,9	2,3	2,0	1,6	2,3	2,0	1,6	2,1
2019	1,3	1,8	1,7	1,6	1,8	1,7	1,6	0,3
2020	-4,7	-4,2	-4,1	-3,8	-4,2	-4,1	-3,8	-6,2
2021	*-2,6*	*-2,0*	*-1,8*	*-1,3*	*-2,0*	*-1,7*	*-1,3*	*-3,6*
2022	*-0,2*	*0,5*	*0,7*	*0,9*	*0,5*	*0,7*	*0,9*	*2,9*
2023	*-0,2*	*0,4*	*0,5*	*0,6*	*0,5*	*0,6*	*0,6*	*3,4*
2024	*-0,2*	*0,3*	*0,3*	*0,2*	*0,4*	*0,4*	*0,2*	*1,1*
2025	*0,0*	*0,3*	*0,2*	*-0,1*	*0,4*	*0,3*	*-0,1*	*0,3*
mean	-0,1	0,0	0,00	0,0	0,1	0,1	0,0	-0,12
SD	1,9	2,0	1,64	1,4	2,1	1,7	1,4	2,90
mean(cc)	0,11	0,33	0,21	0,09	0,31	0,21	0,19	0,11
SD(cc)	1,87	1,80	1,64	1,37	1,79	1,63	1,44	2,82

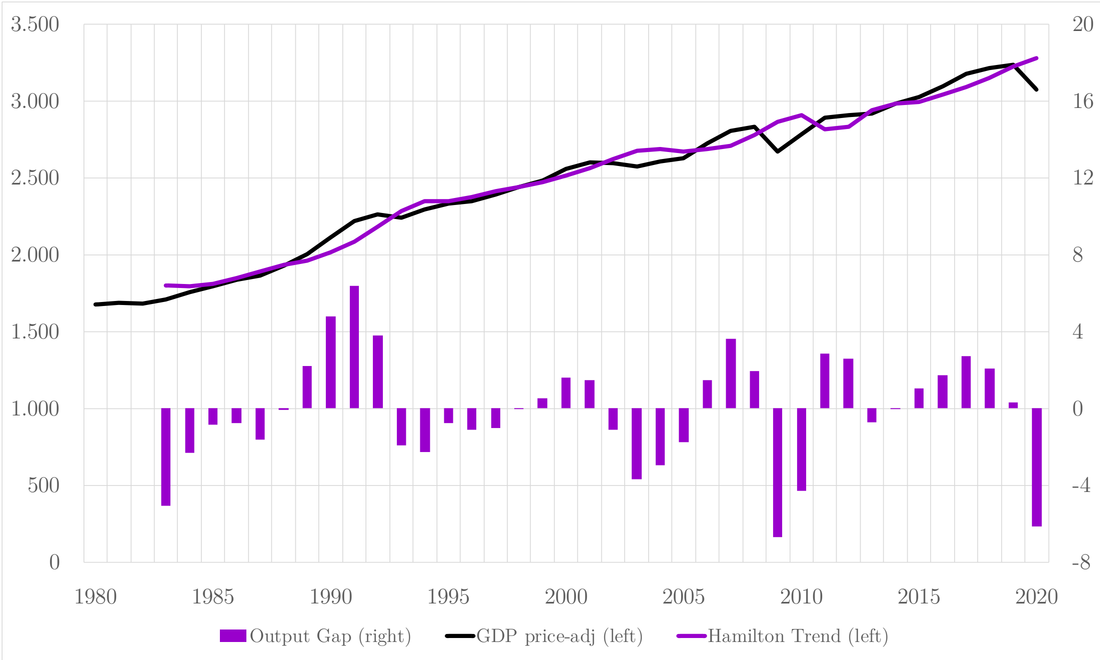
Source: own calculations of HP, mHP, and Hamilton filter based on price-adjusted GDP (FedGov 2021); EU approach numbers directly taken from FedGov (2021); numbers for 2021-2025 based on forecasted GDP only; SD – standard deviation; cc – completed cycles only.

Table A.2.2: Upper and lower turning points (peaks/troughs) and business cycles, 1980-2025

	EUPFA	HP100	HP20	HP6.25	mHP100	mHP20	mHP6.25	HR
1980	*upper tp*	*upper tp*	*upper tp*	*upper tp*	*upper tp*	*upper tp*	*upper tp*	
1981								
1982				lower tp			*lower tp*	
1983	lower tp		*lower tp*					*lower tp*
1984								
1985				**upper tp**			*upper tp*	
1986								
1987		lower tp	lower tp	lower tp	lower tp	lower tp	lower tp	
1988								
1989								
1990								
1991	**upper tp**	**upper tp**	**upper tp**	**upper tp**	**upper tp**	**upper tp**	**upper tp**	**upper tp**
1992								
1993				*lower tp*			lower tp	
1994								lower tp
1995								
1996	lower tp	lower tp	lower tp	lower tp	lower tp	lower tp		
1997								
1998								
1999								
2000								**upper tp**
2001	**upper tp**	**upper tp**	**upper tp**	**upper tp**	**upper tp**	**upper tp**	**upper tp**	
2002								
2003								lower tp
2004								
2005	lower tp	lower tp	lower tp	lower tp	lower tp	lower tp	lower tp	
2006								
2007	**upper tp**	**upper tp**	**upper tp**		**upper tp**	**upper tp**		**upper tp**
2008				**upper tp**			**upper tp**	
2009	lower tp	lower tp	lower tp	lower tp	lower tp	lower tp	lower tp	lower tp
2010								
2011	**upper tp**	**upper tp**	**upper tp**	**upper tp**	**upper tp**	**upper tp**	**upper tp**	**upper tp**
2012								
2013	lower tp	lower tp	lower tp	lower tp	lower tp	lower tp	lower tp	lower tp
2014								
2015								
2016								
2017	**upper tp**							**upper tp**
2018	**upper tp**	**upper tp**	**upper tp**		**upper tp**	**upper tp**		
2019				**upper tp**			**upper tp**	
2020	lower tp	lower tp	lower tp	lower tp	lower tp	lower tp	lower tp	lower tp
2021								
2022		*upper tp*	*upper tp*	*upper tp*	*upper tp*	*upper tp*	*upper tp*	
2023								*upper tp*
2024								
2025								

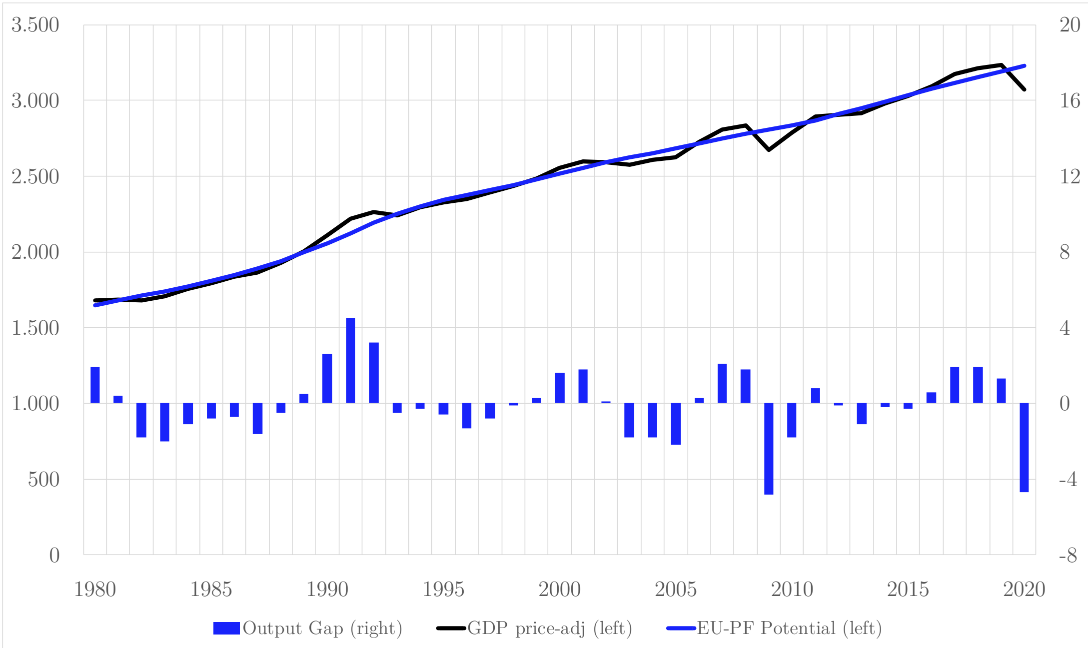
Source: based on Table A.2.1; cycles separated by red lines; tp – turning point; italic font mark upper and lower turning points of cycles that are considered as being only intermediate.

Figure A.2.1: Price-adjusted GDP and estimated potential GDP / trend (Hamilton)



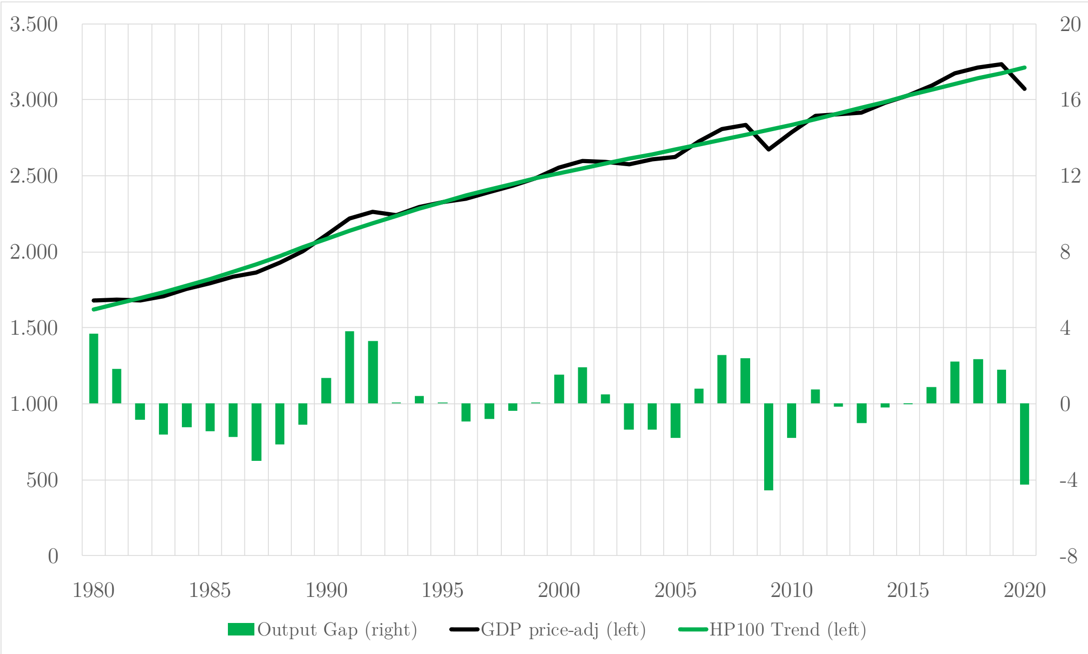
Source: own regression and calculations based on price-adjusted GDP (FedGov 2021)

Figure A.2.2: Price-adjusted GDP and estimated potential GDP / trend (EUPFA)



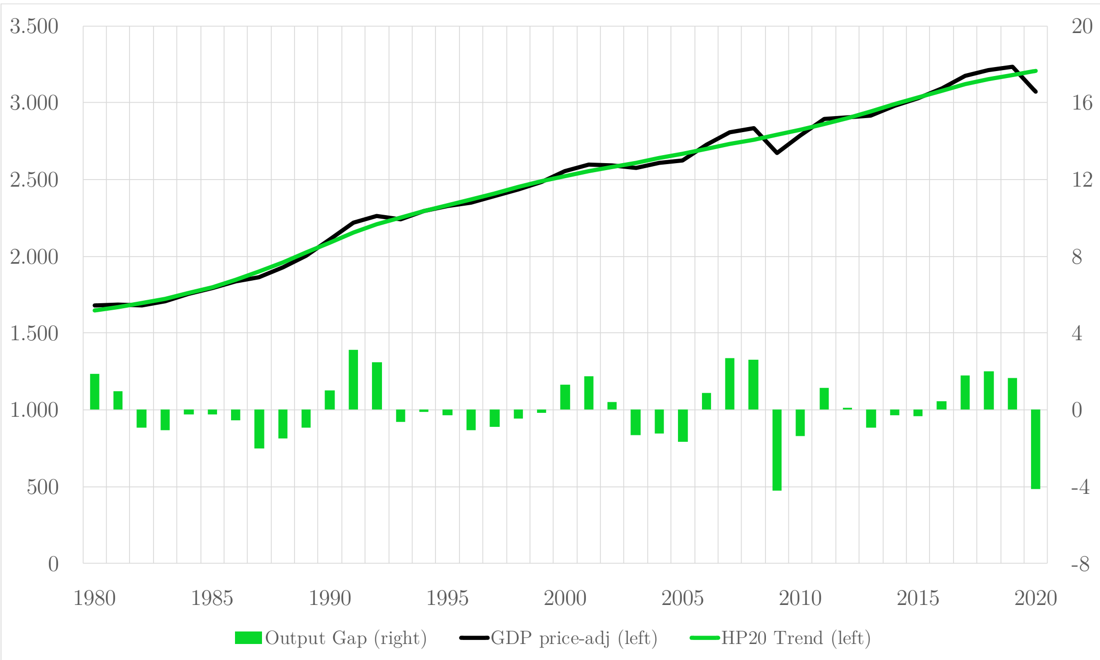
Source: own illustration based on FedGov (2021)

Figure A.2.3: Price-adjusted GDP and estimated potential GDP / trend (HP100)



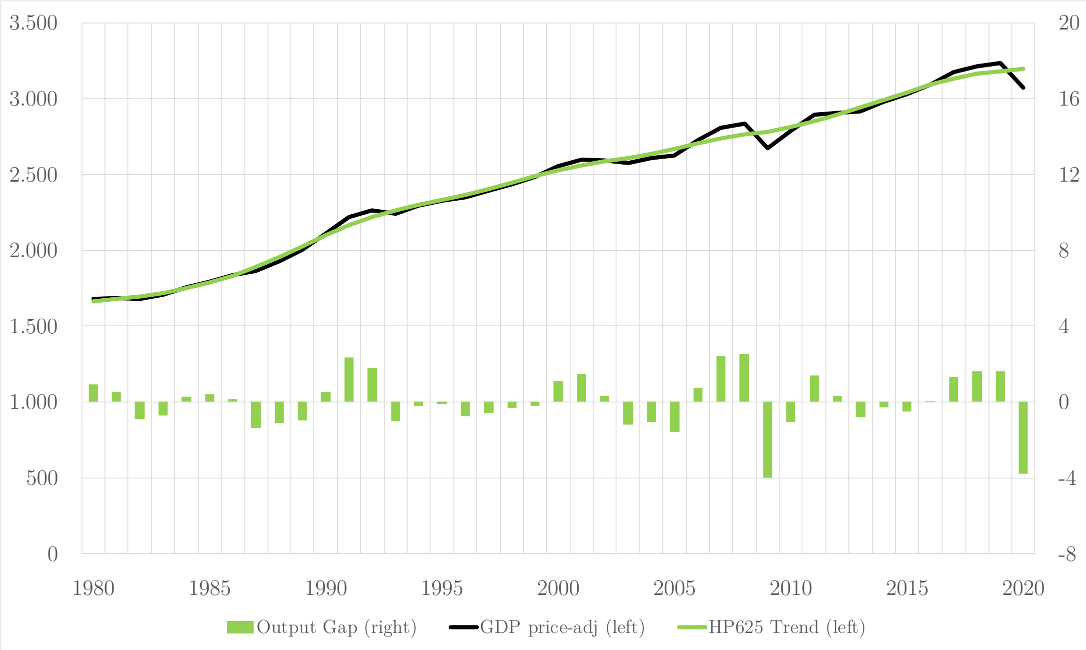
Source: own calculations based on price-adjusted GDP (FedGov 2021)

Figure A.2.4: Price-adjusted GDP and estimated potential GDP / trend (HP20)



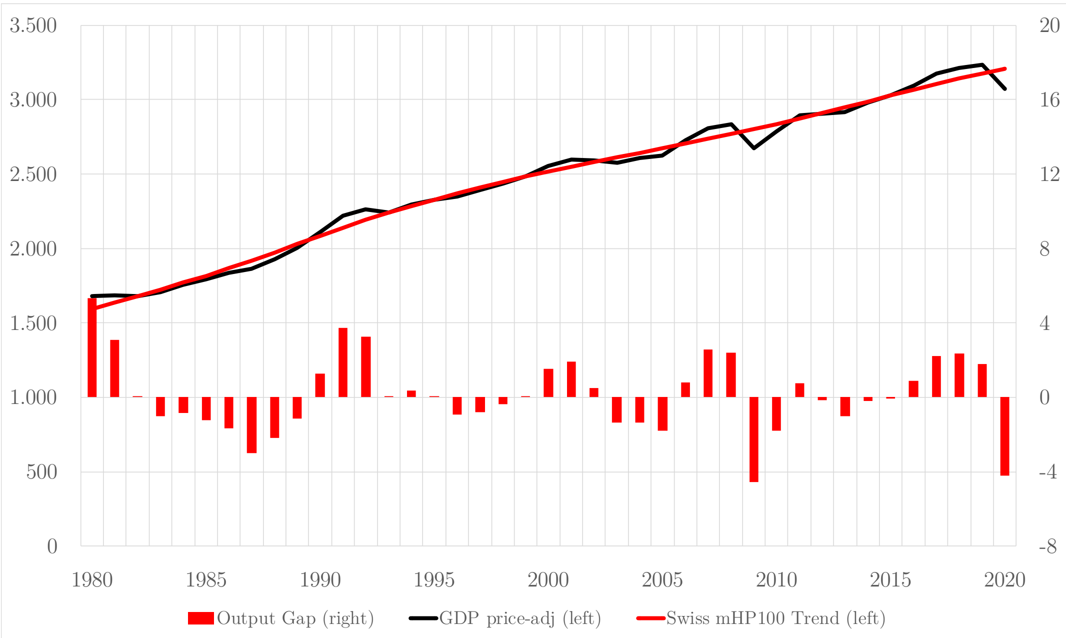
Source: own calculations based on price-adjusted GDP (FedGov 2021)

Figure A.2.5: Price-adjusted GDP and estimated potential GDP / trend (HP6.25)



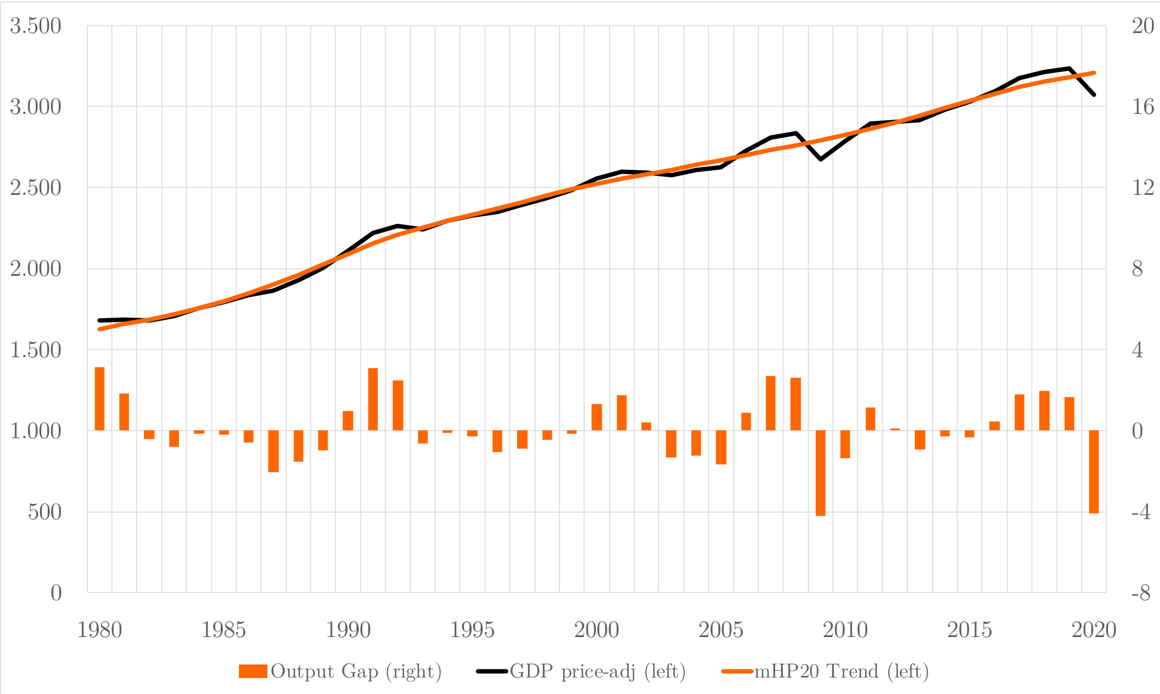
Source: own calculations based on price-adjusted GDP (FedGov 2021)

Figure A.2.6: Price-adjusted GDP and estimated potential GDP / trend (mHP100)



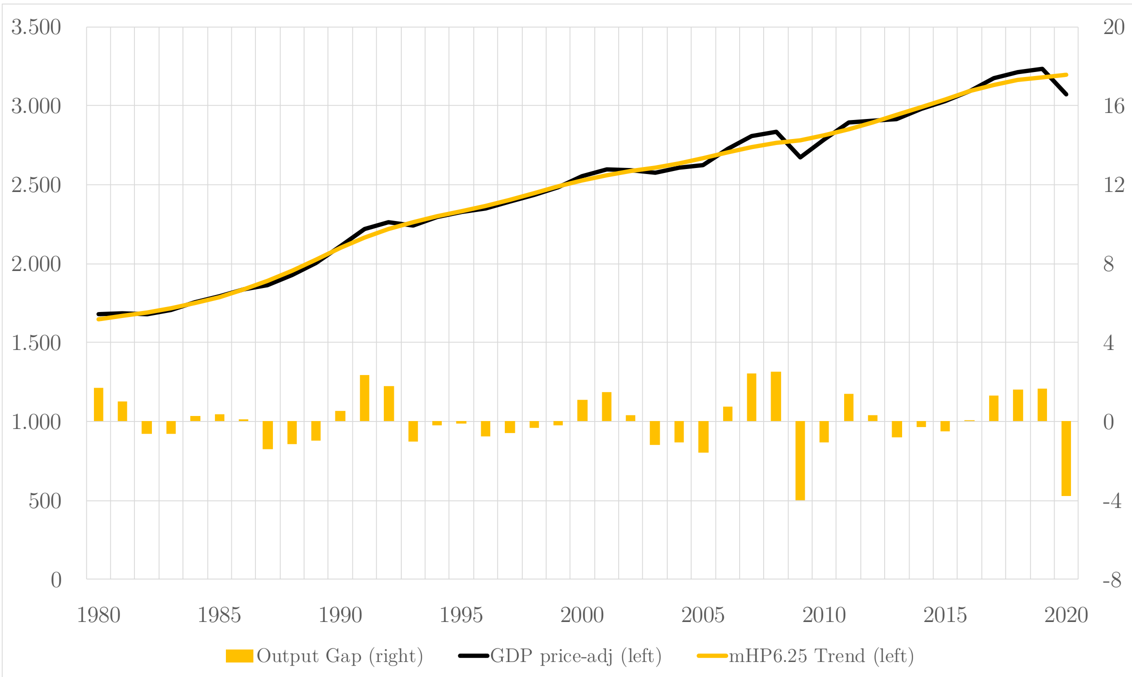
Source: own calculations based on price-adjusted GDP (FedGov 2021)

Figure A.2.7: Price-adjusted GDP and estimated potential GDP / trend (mHP20)



Source: own calculations based on price-adjusted GDP (FedGov 2021)

Figure A.2.8: Price-adjusted GDP and estimated potential GDP / trend (mHP6.25)

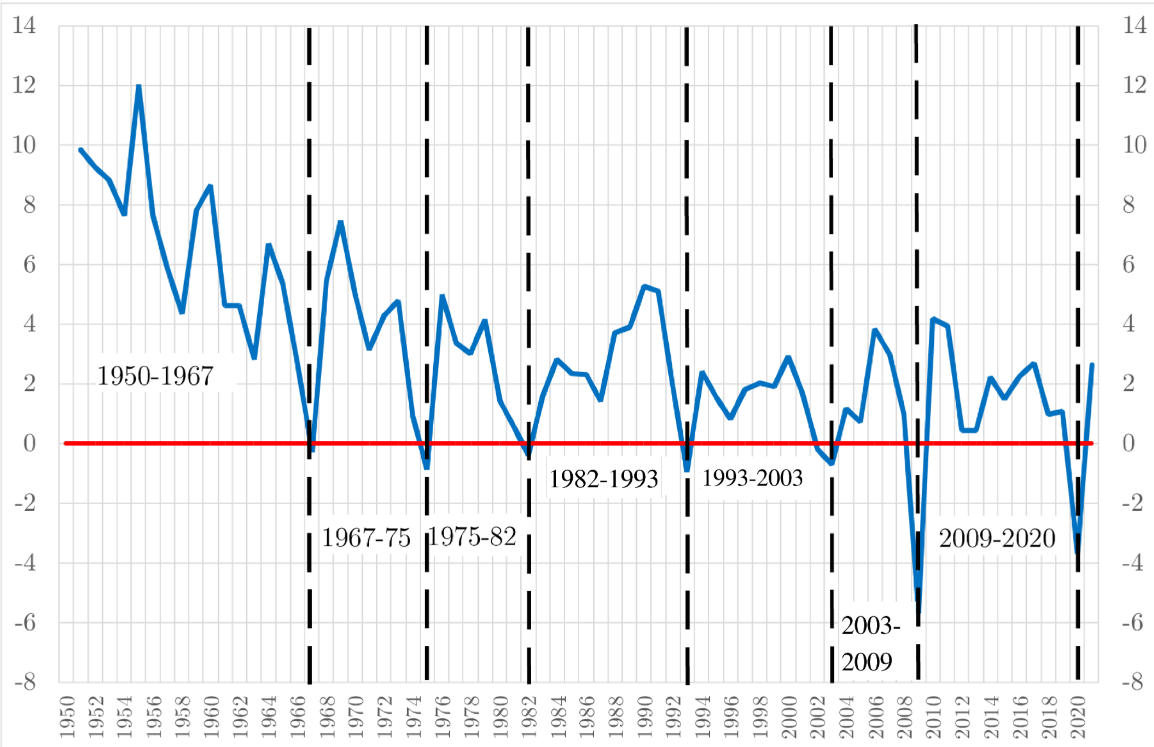


Source: own calculations based on price-adjusted GDP (FedGov 2021)

A.3 Classical Growth-based Identification

For comparison, we add to the so far discussed growth cycle approaches the classic descriptive identification approach, rooting in a GDP growth rate investigation: beginning and end of a business cycle is simply identified by years of negative GDP growth, that is, years where absolute GDP shrunk. The classical approach thus is closer to the trough-to-trough option, and directly applies the 2-phase scheme of Burns and Mitchell (1946).

Figure A.3.1: Business cycles classically identified by growth rate of price-adjusted GDP



Source: own calculations, based on Bundesbank (2022)

The result of the classical approach is illustrated by Figure A.3.1. We find seven completed business cycles: 1950 to 1967, 1967 to 1975, 1975 to 1982, 1982 to 1993, 1993 to 2003, 2003 to 2009, and 2009 to 2020. The first business cycle is obviously special, because it covers the recovery of the German economy after the massive destruction of Germany (and many areas of the World) in WWII; theoretically, it is a period of a catch-up effect and convergence back towards long-term equilibrium (Sala-i-Martin 1996; Solow 1956; Swan 1956).[30] The shortest

30 For Germany it has been shown that the log per-capita real GDP converged after WWI and WWII back to its 1900-to-1913 trend (Borchardt 1976, 1982). In the 1970s this process of convergence was finalised.

cycle (2002-2008), in turn, is special as well, for it stopped abruptly during a boom phase by the global financial crisis. The length of the cycles varies from six years (2003-2009) to 18 years (1950-1967), where the latter actually was even longer, since the first documented year back with negative growth rate is 1944, following Ritschl and Spoerer (1997), where the number of 1945 is not available; this would suggest a length of even 22 years, assuming that GDP growth was also negative in 1945. Excluding this unclear and after-WWII special cycle, the range is from six (2003-2009) to 11 (1982-1993 and 2009-2020); the average length thus is 8.8 years, and the typical cycle duration, described best by the median, is nine to ten years, depending on whether we exclude the special first cycle or not.

Comparing our results with those of Schirwitz's (2009) proposed "consensus turning points", rooting in several different classical cycle turning-point methods, we find that her troughs – identified based on quarterly data – are much in line with our results: the years 1975, 1982, and 1993 are identical, while she identifies 2004 and we 2003 as trough year; the latter minor deviation can presumably made plausible by the fact that her data were changed later on due to revisions that are covered in our data. The only real deviation is that she identifies additionally 1996 as a trough, which we do not – in our data there is also a "trough", but the growth rate is clearly positive. Turning to the peaks, however, there are marked deviations. While she identifies 1974, 1980, 1992, 1995, and 2002, we identify 1976, 1990, 2000, and 2006. Reducing our data to 1970-2006, for comparison, we also identify the year 1973 as a peak, because the true peak in 1969 is simply not observed; given that Schirwitz is identifying the peak in the first quarter of 1974, this is in line with 1973 in our annual data. All other deviations remain, and are potentially explained by the quarterly-vs.-annual data issue as well as data revisions after her study. Therefore, with regard to the trough-to-trough identification we find identically business cycles for the periods 1975-1982 and 1982-1993, but our business cycle 1993-2003 is separated into two (1993-1996 and 1996-2004), in contrast. The peak-to-peak cycles are significantly different: 1974-1980, 1980-1992, 1992-1995, and 1995-2000 (Schirwitz) versus (1973)-1976, 1976-1990, 1990-2000, and 2000-2006. For an ex post discussion of fiscal policy or other historical developments, or of the structural budget position it is doubtful whether relatively short monthly or quarterly developments are more informative than our focus on annual data. While a short downturn of just a few month is important for a business-cycle analysis of the present, it may not convince in explaining historical watershed moments or harmful pro-cyclically fiscal policy, as long as they are not involving a longer downturn period, as measured in annual data, or a strikingly massive short-term downturn.

While the classic approach is typically in the focus of the public, it is blurring the fact that "economic weakness does not necessarily involve a decrease" in GDP (Schirwitz 2009: 289). The classical identification via the growth rate is also comparatively arbitrary – which may also explain part of the revealed deviations. The first trough-to-trough cycle, for instance, ends in the year 1966 for the growth rate in 1967 is -0.3%, so is negative. This growth is close to zero, and if the growth rate would have been 0.001% instead the first cycle ended not until 1974. The classical identification via the growth rate also suffers from a lack of theoretical foundation, in contrast to capacity-based methods. On the other hand, economic as well as political actors and voters observe especially the GDP growth rate, as an important basis of their decisions.[31]

31 Voters determine their voting decision on recent macroeconomic variables, such as GDP, and governments use economic policy to affect the economic development, especially in advance of elections (e.g. Åkerman 1947; Nordhaus 1975; Bischoff and Siemers 2013), so that economic and political cycles are linked to each other.