A Dynamic Model of Conflict and Cooperation*

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Abstract

We introduce a common-pool contest into a continuous-time, differential game setting to model the dynamic behavior of agents facing a trade-off between socially productive activities and appropriation. We are able to identify a unique linear Markov perfect equilibrium strategy which leads the economy to a state where 'partial cooperation' occurs. We show that such cooperation can be seen as a response to conflict. We also discuss the consequences of changes in the effectiveness of appropriation, the number of contenders, and the rate of time preferences on contest equilibria.

Keywords: Conflict, Cooperation, Differential Game, Markov Perfect Equilibrium, Nonlinear Markov strategy

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1 Introduction

This study is devoted to the analysis of the tension between individual and collective rationality in a dynamic environment where economic transactions are insecure. The more general intellectual challenge to identify the mechanisms relevant for the evolution of property rights in society apart, there are many reasons why we believe this analysis to be of interest. Specifically, the struggle of pharmaceutical, software and music enterprises exert great efforts to protect their intellectual property rights and their investments.[1] Business organizations of all kinds attempt to encourage autonomous individuals to sacrifice for the overall benefit of the firm, yet rewards and performance measures are structured around individual contributions. Many of the global frictions we witness today can be traced to large scale divergences between the self interests of political elites and the well being of the global community[2] – most conflicts have their origin in a quasi-Darwinian fight for existence between religious and racial groups.[3] Influential literature argues that resource abundance increases rent seeking activities.[4]

To endogenize the emergence of spontaneous order, more specifically, of property rights requires the dynamic analysis of behavior that predicts a trajectory of property rights. Building on the static models of Grossman (2001), Skaperdas (1992) and Hirshleifer (1991), we develop an agent-based infinite horizon general-equilibrium model to study the dynamic evolution of self-enforcing property rights. Such an analysis is potentially complicated because of the complex intertemporal interdependence between the desire to obtain the best possible outcome for oneself, i.e., aggression or greed, and production. The relevant

[1] According to the *Pew Internet & American Life Project*, 29% of internet users have indicated usage of file sharing computer applications.

[2] Collier and Hoeffler (2004) cite the Stockholm International Peace Research Institute, according to which only two of the 27 major armed conflicts listed in 1999 were internal.

[3] There are many real world economic amalgamations in which conflict can take place (between the states in a federation, between or within nations, ethnic groups, religious groups, etc.). For concreteness the argumentation here is primarily based on conflicts within a nation.

[4] Sachs and Warner (1999) estimate that an increase in natural resource intensity on GDP leads to a reduction in economic growth. Torvik (2002) shows that resource abundance increases the payoff from conflict and lowers growth of the economy.

state variable in the model we use is a durable (capital) stock which is produced using collective efforts of all involved in the production process. This durable stock is exhaustible or rival in the sense that one agent's use of the stock *does* diminish its availability to other agents. Hence each of the agents is tempted by the immediate benefit attainable from capturing the stock.

This model structure allows to explicitly consider insecure market transactions. They enter through the assumption that property rights are determined by the equilibrium level of aggression or greed. The basic mechanism is the following. We model the incentives of agents to exert effort in an attempt to defend their claims on the stock and challenge the claims of others. All agents who succumb to the temptation reduce their help in production of the common-pool stock to increase their efforts to convert claims on the common stock into effective property rights. More specifically, agents derive utility (or a payoff) from owning capital stock and, at every instant in time, choose how to allocate an endowment between appropriation of the common-pool stock (creating property rights) and participating in the production process to increase level of the capital stock in the economy. The investment decisions made independently and noncooperatively by each of the contenders jointly determine the evolution of the commonly accessible stock (the state variable).

We present a tractable version of a differential game formulation of this model of conflict between several agents who attempt to appropriate a common-pool durable stock over an infinite horizon. The solution concept employed is Markov perfect equilibrium, restricting strategies to be functions of the current payoff-relevant state variable. Not all the strategies that describe a solution of the intertemporal optimizing problem of an agent are Markov perfect equilibria. The key to determining which describe equilibrium outcomes is subgame perfection over the entire domain of a state variable. In our model, this requirement produces a unique linear strategy. This linear strategy has the following characteristics and implications. First, the solution suggests that initially poor countries will exhibit an increase in appropriation as the aggregate stock of the capital stock gets larger until a steady state is reached. Second, on the other hand, in economies with an affluent endowment of natural resources the 'marginal gain' of appropriation is high and agents substitute appropriation for production for a while until the state variable reaches a threshold level. From that threshold onwards, agents choose to invest in production to some extent until a steady state is reached where the output of production is only just sufficient to replace the capital stock. This result relates to the observation that rent-seeking activities in rich countries may result in deindustrialization as suggested by the literature on the resource curse (e.g., Sachs and Warner, 1999; Auty, 2001).[5] Third, our results also suggest that regardless of the difference in the initial levels of common-pool endowment economies converge to the same steady state. There, property rights are 'partially' enforced in the sense that appropriation and productive activities coexist, so that neither a totally peaceful (disarmed) equilibrium nor a full-fighting equilibrium emerges as a long run outcome.

Fourth we identify the possibility that countries may reach different long run equilibria in a case where each of the contenders anticipates that the domain of the prize in the contest is limited. We present the exact domain restriction required to obtain Markov perfect equilibrium strategies leading to a multiplicity of equilibria. As a result, the model predicts that some countries converge to a low-income steady state with more unstable property rights (which is socially less desirable), and some converge to a high income equilibrium with more stable property rights (which is socially more desirable).[6] Potentially, at least, a humped-shaped relation between appropriation and wealth can constitute a long run outcome.

There are a few papers which examine a dynamic variation of the one-shot conflicting game analyzed by Grossman (2001), Hirshleifer (1991, 1995) and

[5] There is evidence that resource abundance in the definition used by Sachs and Warner (1999) is associated with civil war (e.g., Collier and Hoeffler, 1998; Hodler, 2005).

[6] Auty (2001) argues that experiences in different countries are complex and diverse. Some countries like Malaysia, Australia, Norway, Botswana and Canada appear to have used their resources judiciously, whereas countries like Nigeria, Mexico and Venezuela seem to have squandered their oil windfalls. According to Acemoglu *et. al.* (2001) the limiting force of conflict is institutional quality as a key driver for economic growth and prosperity.

Skaperdas (1992).[7] Garfinkel (1990) examines a dynamic model in which agents make choices between productive and fighting activities. She uses a repeated game setting where threats and punishments are available. Existence of cooperative (or disarmament) equilibria can be established using Folk Theorem argements. Skaperdas and Syropoulos (1996) discuss a two-period model of conflict in which time-dependence is introduced by the assumption that second period resources of each agent are increasing in first-period's payoff. As a result, 'the shadow of the future' may impede the possibilities for cooperation. In other words, competing agents engage more in appropriation in order to capture a bigger share of today's pie. The equilibrium solution concept we employ in this paper allows us to identify possible cooperative outcomes as a result of decentralized decision-making by agents, without having to rely on the Folk Theorem of repeated games or enforceable commitments.

The organization of the paper is as follows. The next section describes the basic model. Section 3 derives an efficient solution (i.e., cooperative solution) as a reference path. Section 4 conducts comparative static analysis with respect to several principle structural parameters. Section 5 concludes the paper.

2 The Model

Consider an infinite horizon economy populated by $n \geq 2$ agents who strategically interact. Each of the agents derives utility from the consumption (or services) of a common-pool asset (such as land territories and natural resources)

[7] More recently, there is another class of dynamic conflicting models that include, e.g., Gradstein (2003) and Gonsalez (2005). There are several important differences between the models in these papers and the one in ours. First, in their models a flow of the output produced each period is subject to predation, while in our model a stock variable is subject to predation. Secondary and more importantly, those papers investigate the relationship between conflict and economic growth in the standard growth model based explicitly on the investment and saving decisions of a large number of economic agents. Hence, their models are mostly concerned with the macroeconomic consequences, such as growth effects, of insecure property rights. Since our model is a straightforward dynamic extension of Grossman, Hershleifer and Skaperdas which allows for static interaction among a few economic agents, it enables one to directly compare our results with those in static conflicting models and thus to highlight the strategic role of appropriation among those few agents in the intertemporal context.

or (tangible and intangible) capital stocks. We want our model to capture the role of productive and aggressive activities with the understanding that aggressive investment causes an inward shift of the aggregate production possibility frontier. Accordingly, we use a setup where appropriation and production are two substitutable investment choices. Specifically, let an individual decide at each point in time how much resources to devote for appropriation $a_i \geq 0$ and production $l_i \geq 0$. The individual resource (e.g. time) constraint is:

$$a_i + l_i = e_i, \tag{1}$$

where e_i is the endowment of a fill-in activity that is not subject to appropriation.[8] We will set $e_i = 1$ for ease of exposition. The time arguments have been suppressed in this and all subsequent equations.

The common-pool stock is subject to appropriation. The stock is generated by accumulation of output. Output is produced with a linear production technology:

$$Y(l_1, \ldots, l_n) = \sum_{j=1}^{n} l_j, \tag{2}$$

which captures the idea that higher productive efforts by agents cause an outward shift of the production possibility frontier for the economy as a whole. The output of production can be stored to augment the common-pool stock. However, storage entails costs such that the stock Z evolves according to

$$\dot{Z} = Y(l_1, \ldots, l_n) - \delta Z, \tag{3}$$

where $\delta \in (0, 1)$ is the rate at which output will depreciate if stored for future consumption, $\dot{Z}$ denotes the change of Z over time and $Z(0) \geq 0$ is the initial stock.

[8]The standard assumption that each agent has some essential property rights is implicit in this formulation. In the standard rent-seeking contests this activity is, for example, the investment in a perfectly secure project with a return that is normalized to be equal to one.

A main ingredient of the model is the conflict technology which, for any given values of $a_1, \ldots, a_n$, determines each agent's probability of winning sole possession in obtaining the stock Z in a given period. To model this probability for agent i, a natural assumption is that the probability is increasing in aggressive investment of agent i, the fraction of time player i devotes to aggression, and decreasing in the sum of aggressive investment of all agents. A plausible form of the conflict technology is the Tullock contest success function (Tullock, 1980). In its standard formulation this function reads:

$$p_i\left(a_1, \ldots, a_n\right) = \begin{cases} a_i^r \Big/ \left(a_i^r + \sum_{j \neq i}^n a_j^r\right) & \text{for } a_i > 0 \\ 1/n & \text{for } a_i = 0 \ \forall i \end{cases} \tag{4}$$

where the parameter r captures the effectiveness of aggression. From the contest success function (4) we obtain the relative success of contender i in the contest. Alternatively, the contest success function (4) may be interpreted as a sharing rule, or ownership of assets that depends on the respective efforts of aggression. It is natural to assume in the analysis that each agent has an equal access to the prize when agents do not engage in aggressive behavior; hence the assumption that $p_i\left(0, \ldots, 0\right) = 1/n$ will be in force throughout the analysis.

The instantaneous expected payoff to each agent is given by $p_i\left(a_1, \ldots, a_n\right) Z$.[9] Each of the agents chooses the streams of a_i and l_i to maximize the discounted value of total expected payoffs subject to the feasibility conditions introduced in (1)-(4):

$$\begin{aligned} &\max_{a_i} \int_0^\infty p_i\left(a_1, \ldots, a_n\right) Z e^{-\rho t} dt \quad \text{subject to} \\ &\dot{Z} = \sum_{j=1}^n \left(1 - a_j\right) - \delta Z,\ Z\left(0\right) = Z_0 \geq 0, \\ &0 \leq a_i\left(t\right) \leq 1 \quad \text{for all} \quad t \in [0, \infty), \end{aligned}$$

[9]Alternatively, one may view the prize as flow services, such as output or utility from the stock variable Z rather than Z itself. In this case we have to introduce a concave function, say $u\left(Z\right)$ instead of Z . This complication does not affect our results at all.

where $\rho > 0$ is the rate of time preference.

2.1 Solution Concept

We solve the differential game using the notion of a (stationary) Markov perfect Nash equilibrium, because we think that this equilibrium concept captures the essential strategic interactions over time. Markov prefect strategies are decision rules such that each agent's decision is the best response to those of the other players, conditional on the current payoff-relevant state variable Z (see, e.g., Chapter 4 in Docker et al., 2001). Markovian strategies rule out path dependence in the sense that they depend only on the current values of the state variables rather than strategy choices in history. As a result, it does not matter how one gets to a particular point, only that one gets there.

Markov perfect equilibrium strategies must satisfy the Hamiltonian-Jacobi-Bellman equation given by:

$$\rho V_i(Z) = \max_{a_i \in [0,1]} \left[p_i(a_1, \ldots, a_n) Z + V_i'(Z) \left\{ \sum_{j=1}^{n} (1 - a_j) - \delta Z \right\} \right], \tag{5}$$

where V_i denotes the maximum value agent i attributes to the game that starts at Z. Notice that

$$\frac{\partial^2 p_i}{\partial a_i^2} Z = r(n-1) \frac{n(r-1) - 2r}{n^3 a_i^2} Z < 0 \quad \text{for} \quad \begin{cases} n = 2 \wedge r > 0, \\ n > 2 \wedge 0 < r < n/(n-2), \end{cases} \tag{6}$$

implying that the r.h.s. of (5) is concave in $a_i \in [0, 1]$. We assume that $r < 1/(n-1)$ in what follows, anticipating that the linear strategy of each agent, which plays an important role in the later analysis, is a nonnegative value. The function that maximizes (5) can then be derived from

$$\frac{\partial p_i}{\partial a_i} Z - V_i'(Z) \begin{cases} = 0 \implies a_i \in [0, 1], \\ > 0 \implies a_i = 1, \\ < 0 \implies a_i = 0, \end{cases} \tag{7}$$

the l.h.s. of which is evaluated for all $a_i \in [0, 1]$. According to (7) each agent, when choosing a_i, trades the marginal increase in expected payoff from an increase in appropriation against the marginal loss in the discounted value of the future stream of payoffs which results from a reduction of productive effort. If the payoff gain from an increase in a_i is larger than the payoff loss implied by the decrease in l_i for all levels of a_i, then agent i will rationally devote all resources to appropriation. In contrast, the agent chooses $a_i = 0$ in cases where the discounted marginal gain from productive investment exceeds the instantaneous marginal gain from aggressive behavior for all levels of a_i.

2.2 Equilibrium

We can then make use of (7) to characterize subgame perfect equilibria of the differential game. Since we have started our analysis assuming identical agents, a natural focus is on symmetric equilibria. The symmetry assumption allows us to drop the subscript i in the subsequent discussion, and we will suppress this index unless strictly necessary for expositional clarity.

Let us first analyze interior solutions of a_i. Differentiation of the interior first-order condition in (7) gives

$$V''(Z) = \sum_{j=1}^{n} \frac{\partial^2 p_i}{\partial a_i \partial a_j} a_j'(Z) Z + \frac{\partial p_i}{\partial a_i} = -\frac{r(n-1)}{n^2 a^2} a'(Z) Z + \frac{r(n-1)}{n^2 a}. \tag{8}$$

At an interior solution of $a(Z)$ we may apply the envelope theorem to characterize $a'(Z)$. Using the symmetry assumption we obtain

$$a'(Z) = -\frac{\dfrac{1}{n} + \dfrac{r(n-1)}{n^2 a(Z)}\left[(1-a(Z))\,n - (\rho+2\delta)\,Z\right]}{\dfrac{r(n-1)}{n^2 a(Z)^2} Z\,(n-\delta Z)}. \tag{9}$$

We will employ phase-plane methods to characterize the qualitative solution of the nonlinear differential equation (9) and the associated Markov strategies. For this purpose we have to identify the steady state locus where $\dot{Z} = 0$, called C_1 in the following. Let us denote by C_2 the loci where $a'(Z)$ approaches infinity, and by C_3 the loci where $a'(Z)$ equals zero in the (Z, a) space:

$$\begin{aligned} C_1 &:= \{(Z,a) : \dot{Z} = (1-a(Z))\,n - \delta Z = 0\}, \\ C_2 &:= \{(Z,a) : a'(Z) \to \pm\infty\}, \\ C_3 &:= \{(Z,a) : a'(Z) = 0\}. \end{aligned} \tag{10}$$

The steady-state line C_1 is a downward-sloping, straight line in the (Z, a) space. It intersects the vertical axis at the point $(0, 1)$ and the horizontal axis at $(n/\delta, 0)$. Turn to C_2. Setting the denominator in (9) equal to zero, we obtain a vertical line at $(n/\delta, 0)$. The locus C_3 is obtained by setting the numerator in (9) equal to zero. Solving for a gives the following locus:

$$a = -\frac{r(n-1)}{1-r(n-1)} + \frac{r(n-1)}{1-r(n-1)}\frac{\rho+2\delta}{n}Z. \tag{11}$$

Using $1 - r(n-1) > 0$, which ensures the second-order condition (6) holds, (11) shows that the straight line C_3 has a positive slope and a negative intercept on the vertical axis, as shown in Figure 1. Moreover, the point of intersection

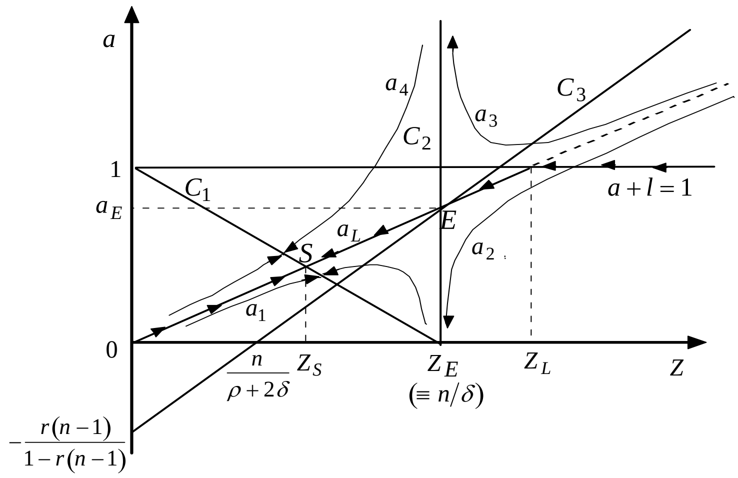

Figure 1: Phase diagram when $a_E < 1$.

between the straight lines C_2 and C_3, labelled E, is situated in the nonnegative region of the (Z, a) plane:

$$(Z_E, a_E) = \left(\frac{n}{\delta}, \frac{r(\rho + \delta)(n-1)}{[1 - r(n-1)]\,\delta} \right). \tag{12}$$

Note, however, that since point E may be located below or above the resource constraint (1), the value of a_E may or may not be less than 1. Depending on this value, we can draw two diagrams such as in Figs. 1 and 2.

It follows from (3) that any strategy $a(Z)$ above line C_1 implies that Z declines in time, while any strategy $a(Z)$ below line C_1 entails an increase of Z over time.

Collecting the arguments we can illustrate an uncountable number of the hyperbolic curves corresponding to the solutions satisfying the HJB equation (5) in Figs. 1 and 2. These figures display representatives of those integral curves that are divided into five types of the families of strategies. Arrows on the *families* of integral curves a_j, $j = 1, \ldots, 4$, and a_L illustrate the evolution

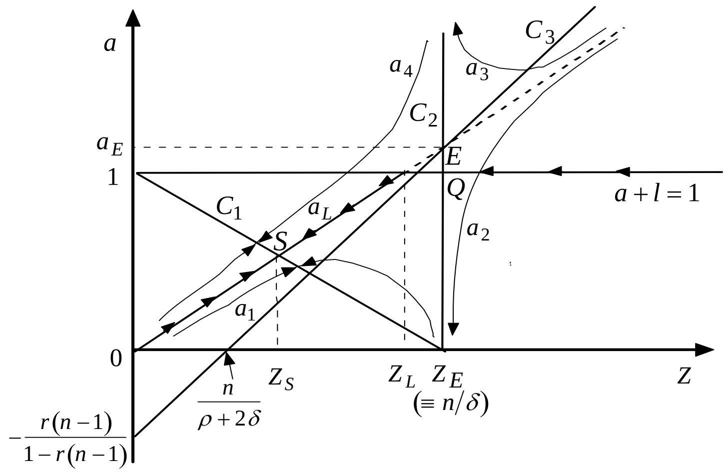

Figure 2: Phase diagram when $a_E > 1$.

of Z over time. Strategy a_L stands for the linear strategy which is obtained from the solution to (9):

$$a_L(Z) = \frac{r(n-1)(\rho+\delta)}{[1-r(n-1)]\,n}Z. \tag{13}$$

This strategy a_L can also be obtained from the well-established guessing method for a value function (see Appendix C). The left branch of the linear strategy a_L to the left of the steady state line C_1 starts from the origin, and then reaches point S on the steady state line C_1, while its right branch starts from any initial value $Z_0 > Z_S$ (we do not here take into account the resource constraint (1)), then reaching point S also. Moreover, it can be verified by substitution that this linear strategy also goes through the singular point E.

The left branch of the a_1-*family* of strategies starts from the origin and reaches a point on the steady state line C_1, while its right branch starts from point $(n/\delta, 0)$ and reaches the same point on line C_1; therefore, those strategies never hit the horizontal axis.[10] The left-branch of the a_4-*family* of strategies also starts from the origin, while its right-branch starts from any initial value $Z_0 < Z_E$, both of which reach the same point on line C_1. On the other hand, when the a_2- and a_3-families of strategies start from any initial value $Z_0 > Z_E$, the a_2-*family* of strategies approaches point $(n/\delta, 0)$, while the a_3-*family* of strategies goes to plus infinity, as illustrated in Figs. 1 and 2.

However, not all integral curves in Figs.1 and 2 are Markov perfect equilibrium strategies. There are three additional requirements which have to be met. The first prerequisite is that strategies should not violate the resource constraint (1). This implies that $a(Z)$ should be bounded to the nonnegative region below a horizontal line with intercept 1 in Figs. 1 and 2.

The second requirement is that strategies should cover the entire domain in a continuous way.[11] At first glance this requirement seems to eliminate all strategies a_j, $j = 1, \ldots, 4$, and a_L. But strategies can potentially be continuously extended either by the upper bound $a = 1$ given by the resource constraint (1), or by the non-aggressive strategy $a = 0$ on the horizontal axis. Both potential extensions are triggered by the corner solutions where the equality in (7) does not apply. In light of these observations, some strategies of the a_3-family (that do not reach the resource constraint (1)) are immediately eliminated because they can neither cover the entire domain by themselves nor can they be extended by any strategy in a continuous way.

[10] To see this note that there exists the self-evident solution $a(Z) = 0$ to the non-linear differential equation (9) for any $Z \geq 0$. Because the Lipschitz condition, which is satisfied for any $Z \geq 0$ except for the origin and point $(n/\delta, 0)$, requires the uniqueness of a solution from the Cauchy-Peano theorem, any other solution curves in the a_1-*family* of strategies cannot hit the horizontal axis. Note, moreover, that the Lipschitz condition is not satisfied at both the origin and point $(n/\delta, 0)$, since the denominator of (9) is equal to zero at either point. These facts together imply that there are many (a continuum of) solution curves emitting from these two points. However, it is easy to show that $a(Z) = 0$ cannot be consistent with the equality in the first-order condition (7), implying that it does not constitute a solution to (5).

[11] Tsutsui and Mino (1990), and Itaya and Shimomura (2001) restrict the state space in order to generate multiple equilibrium strategies. In particular, Tsutsui and Mino treat the domain of a state variable as endogenous to get different stable Markov perfect strategies associated with different steady states. Unfortunately, this approach prevents comparison of payoffs between strategies.

We then examine which of the other families of strategies may be continuously extended by the patching strategy $a = 1$ or $a = 0$. Extending the a_1- and a_2-families of strategies by the patching strategy $a = 0$ is not possible since both extended strategies are discontinuous at point $(n/\delta, 0)$. Furthermore, the a_3-family and the a_4-family of strategies cannot be continuously extended by the patching strategy $a = 1$. The reason is that both are also discontinuous at $Z_E = n/\delta$.

Turn to the linear strategy $\widehat{a}_L$, where the hat indicates those strategies extended by the patching strategy $a = 1$. Since strategy $\widehat{a}_L$ can continuously pass through point E in Fig. 1, the coordinates of which are given by (12), strategy $\widehat{a}_L$ is continuos over the entire domain $[0, \infty)$. This property is also obtained in the case illustrated in Fig. 2 where strategy $\widehat{a}_L$ does not go through point E. Here, the patching strategy $a = 1$ instead of the interior strategy a_L will cross locus C_2 and thus strategy $\widehat{a}_L$ is again continuos over the entire domain of Z. As a result, only the extended linear strategy $\widehat{a}_L$ survives as a candidate for a subgame perfect strategy.

The third and final requirement is subgame perfection. We have to show that there do not exist profitable deviations from strategy $\widehat{a}_L$. Strategy $\widehat{a}_L$ is stable in the sense that from an arbitrary initial value of Z strategy $\widehat{a}_L$ can reach the steady state point S in the long run. As a result, the convergence towards the bounded steady state point S ensures that the value function associated with strategy $\widehat{a}_L$ is bounded. Armed with this fact, we outline in Appendix B that the *extended* strategy $\widehat{a}_L$ can meet the requirement of subgame perfection over the global domain $[0, \infty)$. We summarize with the following theorem:

Proposition 1 *Assume that the domain of the state variable Z is given by $[0, \infty)$ and*

$$Z_0 - (\rho + \delta)^{-1} - \delta^{-1} G(Z_0) > 0, \tag{14}$$

where Z_0 is an arbitrary initial value of Z,

$$G(Z_0) \equiv \left(\frac{\rho}{\rho + \delta} \frac{Z_L}{Z_0} - \frac{Z_S}{Z_0} \right) \left(\frac{Z_0}{Z_L} \right)^{-\frac{\rho}{\delta}} - \left(\frac{Z_L}{Z_0} - \frac{Z_S}{Z_0} \right) \left(\frac{Z_0}{Z_L} \right)^{-\frac{\rho+\delta}{\delta[1-r(n-1)]}},$$

and $Z_L \equiv n\left[1-r\left(n-1\right)\right]/\left(n-1\right)r\left(\delta+\rho\right)$*. Then there exists a unique linear Markov perfect equilibrium strategy that supports the steady state equilibrium*

$$(Z_S, a_S) \equiv \left(\frac{n\left[1-r\left(n-1\right)\right]}{\left(n-1\right)r\rho+\delta}, \frac{r\left(n-1\right)\left(\delta+\rho\right)}{\left(n-1\right)r\rho+\rho}\right). \tag{15}$$

Proof. See the Appendix B. ■

Proposition 1 implies that a unique linear MP strategy exists, even in the case where the domain is globally defined. As a result, given any initial stock of Z, the economy approaches the steady state point S where the common-pool stock takes a positive value and individual aggressiveness takes an intermediate value between zero and one. In this sense, (implicit) 'partial cooperation' can be seen as a best response to the risk of appropriation.

Although the sufficient condition for subgame perfection (14) appears to be complicated, we can easily check that (14) holds in cases where Z_0 is sufficiently close to Z_L and where Z_0 is sufficiently large (see (B8) in Appendix B). To gain further economic insights, we can give a more restrictive but simpler sufficient condition than (14) (to be derived in Appendix B):

$$Z_L \geq \frac{\rho+2\delta}{\left(\rho+\delta\right)\delta} \iff \frac{n\delta}{n\delta+\rho+\delta} - r\left(n-1\right) > 0, \tag{16}$$

which also ensures that condition $1-r\left(n-1\right)>0$. When $Z_L \geq Z_E$ $(= n/\delta)$, which is illustrated in Fig. 1, it is easy to see that condition (16) is satisfied.

The intuition for Proposition 1 is best understood from the observation that each contender will have a stronger incentive to engage in appropriation if the prize $Z(t)$ is large. Consequently, strategy $a=1$ is more likely to be subgame perfect at larger values of Z. Moreover, in view of (16) either the larger is the rate of time preference ρ or the larger is the rate of depreciation δ, the more likely the first inequality in (16) is to hold and thus the lower is the incentive to deviate from strategy $a=1$. These results are clearly consistent with the intuition according to which higher values of those parameters make the accumulation of the stock Z less attractive, reducing the incentives to engage in production of the common-pool stock. The effect of the effectiveness parameter r on appropriation is subtle in the following sense. A higher r makes the contenders more aggressive (i.e., strategy a_L becomes steeper), so that appropriation can reach an upper bound on the resource constraint (1) at lower levels of Z_L. Since the resulting smaller Z_L in turn makes fighting activity less attractive when $Z(t)$ is close to Z_L, the subgame perfection of strategy $a = 1$ is less likely to hold.

We are now in a position to discuss the time profile of $\widehat{a}_L$ associated with the evolution of Z. In affluent economies where the level of the stock variable is sufficiently large, investment in aggression reaches the maximum possible level in finite time. It then is decreasing until the steady state S is reached. Put differently, in affluent societies where there is a large amount of the common pool stock, a full fighting strategy (i.e., $a = 1$) will be rationally and inevitably chosen during the transition to the steady state. On the other hand, if the level of the initial stock level is relatively low at the start of the game investment in aggressive behavior is monotonically increasing toward the steady state S over time. That is, as the common-pool stock Z gets larger over time, the contenders will become greedier, because the marginal gain of appropriation will be higher. In the long run (i.e., the steady state point S) the economy will reach a situation where 'partial cooperation' prevails in the sense that every agent chooses to contribute to the production of the common-pool stock Z to some extent.

3 Comparative Static Analysis

In this section we discuss the effects of a change in the model parameters on the transition path of the linear strategy a_L as well as on the associated long-run equilibrium point S. Consider first the effects of a change in the productivity (or effectiveness) of conflict technology. The recent developments in computer networks and their applications mentioned in the introduction are an example of technological change that potentially puts at risk the intellectual property rights of the software and music industry. In the model, a change in the productivity of the conflict technology is captured by a change in r. The shift of point S

can be calculated by differentiating (15) with respect to the parameter r. This yields:

$$\begin{aligned}\frac{da_S}{dr} &= \delta\left(\delta+\rho\right)\left(n-1\right)\Delta^{-2}>0,\\ \frac{dZ_S}{dr} &= -n\left(n-1\right)\left(\delta+\rho\right)\Delta^{-2}<0,\end{aligned}$$

where $\Delta \equiv (n-1)\,r\rho+\delta>0$. Although an increase in r does not affect the line C_1, this increase strengthens the intensity of appropriation associated with every level of the common-pool stock Z during the transition path, thus making the linear strategy (13) steeper. Since the productivity of appropriation becomes more effective with higher r, all competing agents engage in more aggressive behavior in the hope of capturing more resources. This finding is quite intuitive, and is also consistent with the static conflict models of Hirshleifer (1991, 1995).

An increase in the number of agents augments the aggregate endowment in proportion to n, since each entrant provides one additional unit of the endowment. The larger aggregate endowment will increase the payoff each agent can expect to obtain from a given investment in aggression, thereby intensifying each agent's aggressive behavior and thus making the linear strategy a_L steeper. At the same time the amount of aggregate endowment devoted to productive as well as to appropriation will be larger, which corresponds to an outward shift of the aggregate resource constraint C_1 (i.e., scale effect). Although these two effects together intensify individual appropriation, the long run effect on the common-pool stock Z is ambiguous:[12]

$$\begin{aligned}\frac{da_S}{dn} &= r\left(\delta+\rho\right)\delta\Delta^{-2}>0,\\ \frac{dZ_S}{dn} &= \left[\left\{1-r\left(n-1\right)-nr\right\}\left(-r\rho+\delta\right)-\rho n^2r^2\right]\Delta^{-2}\gtreqless 0.\end{aligned}$$

A higher depreciation rate causes a reduction in the level of the common-pool stock Z available to contenders, thereby discouraging appropriation. This negative effect on the prize causes a clockwise turn of line C_1 around point $(0,1)$

[12]This effect has been also found in Result 4B of Hirshleifer (1995).

(i.e., the aggregate resource constraint C_1 moves inward toward the origin). At the same time, a higher δ implies that the cost of maintaining the common-pool stock is increased relative to the cost of aggressive behavior, which in turn strengthens an incentive for investment in aggressive behavior, thus making the linear strategy a_L steeper. Although these two effects on appropriation operate in opposite directions, the following result indicates that the former effect will outweigh the latter effect in the long run:

$$\begin{aligned}\frac{da_S}{d\delta} &= -\rho r(n-1)\left[1-r(n-1)\right]\Delta^{-2} < 0,\\ \frac{dZ_S}{d\delta} &= -n\left[1-r(n-1)\right]\Delta^{-2} < 0.\end{aligned}$$

A decrease of the subjective rate of time preference makes the linear strategy a_L steeper, but it has no effect on line C_1. Hence we obtain the following long run effects:

$$\begin{aligned}\frac{da_S}{d\rho} &= r(n-1)\left[1-r(n-1)\right]\delta\Delta^{-2} > 0,\\ \frac{dZ_S}{d\rho} &= -n(n-1)\left[1-r(n-1)\right]r\Delta^{-2} < 0.\end{aligned}$$

The economic explanation is that there is a tendency to spend less resources on aggressive investment when contenders become more far sighted (i.e., smaller ρ). This result has apparently not been addressed by Hirshleifer (1991, 1995) and Skaperdas (1992), who use the static conflict models. It stands in contrast to Skaperdas and Syropoulos's (1996) result in which the higher is the valuation of the future (i.e., smaller ρ), the stronger is the intensity of fighting. The reason for this difference is that in their two-period's model agent's first-period expenditure on appropriation increases agent's second-period payoff. Rather, our result is similar to Garfinkel's (1990) Folk Theorem type result in repeated games where higher discount factors (i.e., smaller ρ) make it easier to sustain cooperative outcomes. An interpretation of our result is that long-sighted agents become less aggressive because they are more concerned about the future. We may then summarize the discussion in the following proposition:

Proposition 2
(i) An increase in the effectiveness of aggression leads to a higher level of aggression and to a lower level of the common-pool stock;
(ii) an increase in the number of agents leads to a higher level of aggression, but the effect on the common-pool stock is ambiguous;
(iii) an increase in the depreciation rate leads to a lower level of aggression and of the common-pool stock; and
(iv) a decrease in the subjective rate of time preference leads to a lower level of aggression and to a higher level of the common-pool stock.

4 The Cooperative Solution

We will characterize the explicit cooperative solution as a benchmark steady state in the following. Assume an outside enforcer or centralized agency has the power to induce every contender to execute its command. The cooperative strategy is one for which a centralized agency chooses the infinite-horizon planning profile of strategy $a \in R_+^n$ at the outset of the game so as to maximize $\int_0^\infty Ze^{-\rho t}dt$ subject to $\dot{Z} = n - \sum_{j=1}^n a_j - \delta Z$ where $a_j \in [0, 1]$ for all j. Clearly this optimization yields a totally peaceful solution, that is, $a_j(t) = 0$ for $t \in [0, \infty)$ for all j. The result is understood by noting that expenditure on appropriation is socially wasteful in the sense that it causes a deadweight loss because of the non-productive use of resources. This deadweight loss should be zero in the hypothetical case where a central agency can directly control the allocation between productive and appropriation. As a result, the superior authority should establish point $(n/\delta, 0)$ in the long run. Agents would benefit from an enforced peaceful resolution because Pareto-inefficient aggressive activity is completely eliminated.

Combined with the comparative static results in the previous section, we obtain the following results:

Proposition 3 *Assume that a centralized agency chooses an allocation between aggressive and productive investment so as to maximize aggregate payoff. The* *resulting allocation dictates that agents devote all resources to the socially productive activity to obtain the Pareto efficient point* $(n/\delta, 0)$ *in the long run. Moreover, a decrease in either the productivity of aggressiveness, the depreciation rate, or the subjective rate of time preference moves the resulting long run equilibrium closer to a Pareto efficient one.*

The nuclear nonproliferation treaty which deters the development of nuclear weapons (i.e., aggressive technology) would be socially desirable in a way that makes the long run outcome closer to the peaceful and efficient one. Another example is patent law, which aims at enforcing property rights on investment return and thus limits socially wasteful activities. Patent law potentially prevents a rapid fall in the expected return from new innovation, which would be a consequence of imitation by rivals. The increase in return on investment caused by secure property rights is approximately captured by the effect of a lower depreciation rate in our model.

The problem with using the cooperative solution as a benchmark is that the socially attractive steady state is not self-enforcing because it does not constitute a subgame perfect (Nash) equilibrium. There is a strong argument that a necessary condition for self enforcing agreements at every moment in time is that they can be established as a subgame perfect equilibrium. There must exist other ways to realize socially desirable outcomes (or making the long run equilibrium closer to the Pareto efficient point $(n/\delta, 0)$) without the need to assume outside enforcement or compulsion imposed by a strong central agency. For several reasons (see, e.g., Itaya and Shimomura, 2001), the domain of the state variable could be restricted, which may generate a multiplicity of equilibria. This is a defining characteristic of dynamic games which has never been captured by the one-shot conflict models. In the present model, if the domain of Z is restricted over the interval $[0, \bar{Z}]$, where $\bar{Z}$ $(< Z_E)$ represents the upper bound on the domain, we obtain an uncountable number of the a_1- and a_4-families of strategies which are subgame perfect over that domain. These strategies lead to a continuum of steady state equilibria. Figs. 3 and 4 draw a continuum of

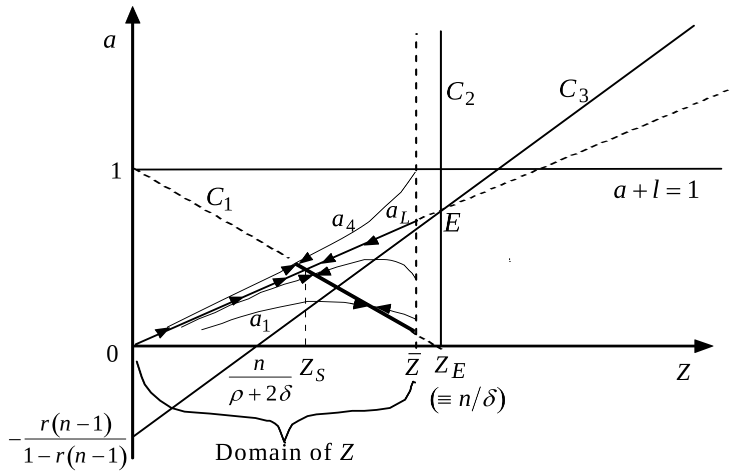

Figure 3: Multiplicity of equilibria in the restricted domain of Z when $a_E < 1$.

steady state equilibria as the bold line segments.[13] More importantly, it follows from Figs. 3 and 4 that more peaceful (efficient) transition paths and long run equilibria, as compared to the transition path of the linear strategy $\widehat{a}_L$ and the associated long run equilibrium point S, could be possible:

Proposition 4 *Assume* (14) *holds and that the domain of the state variable* Z *is restricted over the interval* $[0, \bar{Z}]$ *with* $0 < \bar{Z} < Z_E$. *There exist uncountable many nonlinear Markov perfect equilibrium strategies that constitute a continuum of steady states. When the domain of the state variable* Z *is restricted over the interval* $[0, \bar{Z}]$ *with* $\bar{Z} > Z_E$, *there exists only the linear Markov perfect equilibrium strategy that supports a unique steady state point* S.

When the domain of the state variable is restricted on the prescribed range stated above, agents in (initially) poor economies may have an infinite number of choices to increase the stock of the common-accessible stock since there exist an uncountable number of Markov perfect equilibrium strategies emitting from

[13] Although the upper bound on the state variable domain, $\bar{Z}$, can be closer to $Z_E(\equiv n/\delta)$ as much as possible, this upper bound cannot coincide with Z_E. This is because at Z_E the a_1-family of strategies are discontinuous.

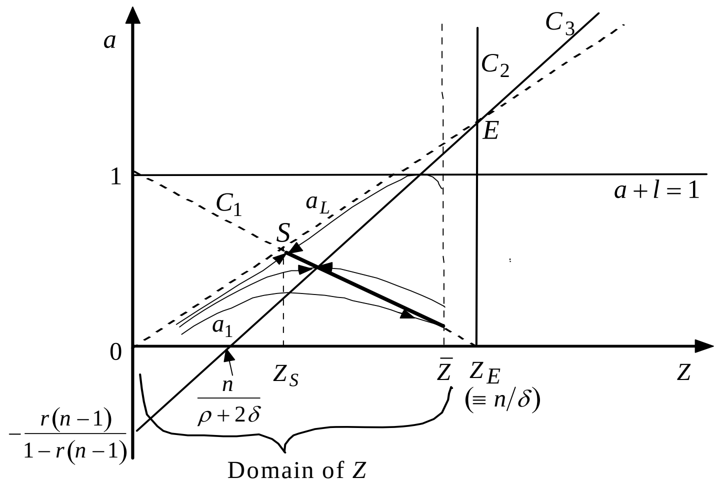

Figure 4: Multiplicity of equilibria in the restricted domain of Z when $a_E > 1$.

the origin. One of those strategies (i.e., strategy $\widehat{a}_L$) would lead the economy to point S. But other options exist. Following any of the strategies in the a_1-family of strategies implies that the level of aggression increases (and then may decrease) towards a continuum of steady states ranging over a proper set of the interval $[0, \bar{Z}]$. It is socially desirable to follow one of the less aggressive strategies and thus potentially realize socially better outcomes in the long run compared to the steady state point S supported by the linear Markov perfect strategy. As a result, agents have to be confronted with a coordination problem in order to select the most efficient strategy.

Accordingly, governments or central agencies can play dual roles in restricting the domain of the state variable as well as in solving the coordination problem stated above. Multiplicity entails that a window of opportunity is available for an anarchic society in the sense that the equilibrium is not only predetermined by the capital stock Z_0. A coincidental start abstracts from all means of communication and trust between agents. But agents have an interest in avoiding the loss caused by aggression and may transcend their lack of confidence to achieve the Pareto-dominant equilibrium. Benevolent governmental institutions might be important devices for solving the coordination problem, such that the society in the best circumstance is able to coordinate on the preferred among all feasible equilibria. Nevertheless, it should be stressed here that the multiplicity of equilibria we derive as a special case is vulnerable on theoretical grounds.

5 Conclusions

The first conclusion of this paper is that completely aggressive behavior is not necessarily a rational strategy for an agent in anarchic situations. Rather, every agent will voluntarily and uniquely choose 'partial cooperation', in which each agent devotes his individual resource both to productive and appropriation at the same time, even though agents act fully rational and are guided by their self-interest. The primary driving force is the durability of the common-pool stock in conjunction with the forward looking behavior of agents. These intrinsically dynamic ingredients induce each contender to behave 'partially cooperatively', even without punishments and threats, unlike Garfinkel (1990). In other words, either if the stock depreciates completely each period or if contenders have myopic foresight, they are less motivated to follow a cooperative behavior in producing a commonly-accessible good.

The second conclusion is that the domain of a state variable also plays a critical role in determining the nature of the equilibrium in addition to the equilibrium concept, which has not been addressed by the static conflict models. Nevertheless, one may cast doubt on the possibility of restricting the domain in reality. We may provide one possible justification for this argument. In reality agents cannot have the sophisticated ability to perfectly foresee and calculate any possible strategic interaction over an infinite horizon. More realistic agents do have a limited ability of calculating, collecting information and forecasting future events. Restricting the domain of a state variable may provide a good substitute mechanism for describing the behavior of such bounded rational agents. Properly speaking, *economic intuition* suggests that those agents could voluntarily narrow the domain of a state variable or truncate part of the domain. With such a voluntary restriction the possibility of multiple equilibria would become more likely. The multiplicity of equilibria then may be more causal than predicted by this theory. In this case it may be one of the primary functions of centralized institutions to play a critical role to resolve such a coordination problem.[14] It is a lesson from history that centralized institutions need to be an equilibrium outcome. It is this constructive aspect of anarchism that can explain the evolution of nation states. Therefore, bounded rationality paired with limited cognitive ability of the agents and the necessity for confidence in others may be the primary rational for the existence of a centralized institution or a constitution.

In addition to the above-mentioned role of the government, the results of the paper suggest that the government (or central agency) should also play the following roles in order to achieve a socially desirable outcome, and thus move the long run outcome closer to the more efficient ones. First, governments should attempt to contain the development of the conflict technology, reduce the depreciation rate of common-pool assets or induce people to have longer sight. Such structural or institutional reforms, including laws or institutional schemes, could reduce the likelihood of aggression, and thus lead to peaceful and more efficient outcomes in the long run. Second, if intervention by a centralized agency to directly control fighting expenditures is impossible, it would be socially desirable to introduce the domain restriction through some kinds of indirect regulation such as an imposition of a ceiling or floor imposed on the state variable.

The model presented in this paper should be developed further in several directions. In particular, introducing asymmetry among agents would enable us to compare the results of the present model with those static models which do incorporate asymmetric agents. The 'paradox of power' (Hirshleifer, 1991) may be generated in such an asymmetric dynamic conflicting model.

[14] In evolutionary games bounded players are usually assumed to be myopic. This assumption may stand in the spirit similar to our voluntary restriction of the domain in the present differential game.

Appendix A: Derivation on the HJB equation

In this appendix we show how to derive (9) in the text. Assuming an interior solution and solving (7) for each agent yields the optimal strategy $a_i = a_i(Z)$. By substituting this optimal strategy into (5), the HJB equation (5) associated with agent i is transformed into

$$\rho V_i(Z) = p_i(a_1(Z), ..., a_n(Z))Z + V_i'(Z)\left[\sum_{j=1}^{n}(1 - a_j(Z)) - \delta Z\right]. \tag{A1}$$

By differentiating (A1) with respect to Z and applying the envelope theorem to the resulting expression, we obtain

$$\begin{aligned}\rho V_i'(Z) &= \sum_{j=1}^{n}\frac{\partial p_i}{\partial a_j}a_j'(Z)Z + p_i(.) + V_i''(Z)\left[\sum_{j=1}^{n}(1 - a_j(Z)) - \delta Z\right] \\ &\quad + V_i'(Z)\left[-\sum_{j=1}^{n}a_j'(Z) - \delta\right].\end{aligned} \tag{A2}$$

Substituting (7) and (8) into $V_i'(Z)$ and $V_i''(Z)$ in (A2), respectively, and exploiting symmetry yields

$$\begin{gathered}0 = (n-1)\left[\frac{\partial p_i}{\partial a_k}Z - \frac{\partial p_i}{\partial a_i}Z\right]a'(Z) + p(.) + \\ \left[\frac{\partial^2 p_i}{\partial a_i^2}a_i'(Z) + (n-1)\frac{\partial^2 p_i}{\partial a_k \partial a_i}a_k'(Z)\right]Z\left[n(1 - a(Z)) - \delta Z\right] \\ + \frac{\partial p_i}{\partial a_i}\left[n(1 - a(Z)) - \delta Z\right] - (\delta + \rho)\frac{\partial p_i}{\partial a_i}Z, \ k \neq i.\end{gathered} \tag{A3}$$

Since the assumption of symmetry further allows us to make use of the following simple expressions:

$$
\begin{aligned}
p_i &= \frac{1}{n},\ \frac{\partial p_i}{\partial a_i} = \frac{r(n-1)}{n^2 a},\ \frac{\partial p_i}{\partial a_k} = -\frac{r}{an^2}, \\
\frac{\partial^2 p_i}{\partial a_i^2} &= r(n-1)\frac{n(r-1)-2r}{n^3 a^2},\ \frac{\partial^2 p_i}{\partial a_k \partial a_i} = \frac{r^2(-n+2)}{n^3 a^2},
\end{aligned}
$$

we substitute those expressions into (A3) yielding

$$
\begin{gathered}
0 = \frac{n-1}{an^2}\left[-r - r(n-1)\right] Za'(Z) + \frac{1}{n} + \\
\frac{r(n-1)}{n^3 a^2}\left[r(-n+2) + n(r-1) - 2r\right] Z \left[n(1-a(Z)) - \delta Z\right] a'(Z) \\
+ \frac{r(n-1)}{n^2 a}\left[n(1-a(Z)) - \delta Z\right] - (\delta+\rho)\frac{r(n-1)}{n^2 a} Z. \qquad \text{(A4)}
\end{gathered}
$$

Further rearranging (A4) gives rise to (9) in the text.

Appendix B: Subgame Perfection

In this appendix we shall check whether the extended linear strategy $\widehat{a}_L$ is subgame perfect over the global domain $[0, \infty)$. To do so, we divide the domain $[0, \infty)$ into the subintervals $[0, Z_S]$ and $[Z_S, \infty)$, and then investigate whether or not there is an incentive for each player to deviate from strategy $\widehat{a}_L$ when the initial stock of the common-pool asset Z_0 lies in the respective intervals. So we first consider the case where the initial stock Z_0 lies in the interval $[0, Z_S]$. Since in this case strategy $\widehat{a}_L$ satisfies the equality in (7) and thus the HJB equation (5), it is subgame perfect over the domain $[0, Z_S]$. In addition, the boundedness of the value function associated with this strategy clearly follows from the bounded instantaneous utility function, pZ, resulting from the bounded value of Z in the steady state point S and the convergence of strategy $\widehat{a}_L$ towards this steady state.

Next, we investigate whether strategy $\widehat{a}_L$ is subgame perfect when it starts from any initial stock $Z_0 \in [Z_S, \infty)$. We further subdivide this domain into two intervals $[Z_S, Z_L]$ and $[Z_L, \infty)$, where

$$Z_L \equiv \frac{n\left[1 - r\left(n-1\right)\right]}{\left(n-1\right) r \left(\delta + \rho\right)}.$$

is given by the intersection point of the linear strategy a_L, (13), with line $a = 1$, provided $1 - r\left(n-1\right) > 0$.

Without loss of generality, we suppose that strategy $a = 1$ is played from time 0 to t_1 and after time t_1 the interior linear strategy $a_L\left(Z\right)$, (13), will be taken, thus leading to the steady state point S. As a result, during $t \in [0, t_1]$ the state variable Z evolves according to $\dot{Z} = -\delta Z$, so that we have $Z\left(t\right) = Z_0 e^{-\delta t}$.

After time t_1, the linear strategy $a_L\left(Z\right)$ will be played, so that the evolution of $Z\left(t\right)$ follows

$$\dot{Z} = n\left[1 - \frac{\left(n-1\right) r \left(\delta + \rho\right)}{n\left[1 - r\left(n-1\right)\right]} Z\right] - \delta Z = n - \frac{\left(n-1\right) r\rho + \delta}{1 - r\left(n-1\right)} Z,$$

the solution of which is given by

$$Z\left(t\right) = Z_S + \left(Z_L - Z_S\right) e^{-\frac{(n-1)r\rho+\delta}{1-r(n-1)}t}, \tag{B1}$$

noting that $Z\left(t\right) \to Z_S$ as $t \to \infty$. Substituting the values of Z_L and Z_S into (B1) gives

$$Z\left(t\right) = \frac{n\left[1 - r\left(n-1\right)\right]}{\left(n-1\right) r\rho + \delta}\left[1 + \frac{\delta\left[1 - \left(n-1\right) r\right]}{\left(n-1\right) r \left(\delta + \rho\right)} e^{-\frac{(n-1)r\rho+\delta}{1-r(n-1)}t}\right].$$

Taken together, the resulting value function which starts from $Z_0 \geq Z_L$ is given by

$$V_i(Z_0) = \int_0^{t_1(Z_0)} \frac{1}{n} Z_0 e^{-\delta t} e^{-\rho t} dt +$$
$$e^{-\rho t_1(Z_0)} \int_{t_1(Z_0)}^{\infty} \frac{1}{n} \left[Z_S + (Z_L - Z_S) e^{-\frac{(n-1)r\rho+\delta}{1-r(n-1)} t} \right] e^{-\rho(t - t_1(Z_0))} dt. \tag{B2}$$

Note that since $Z(t)$ reaches Z_L at time $t = t_1$, we can express $Z_L = Z_0 e^{-t_1 \delta}$, which implies that $t_1 = \delta^{-1} \log(Z_0/Z_L)$ $(\equiv t_1(Z_0))$. Integration and rearrangement gives

$$\begin{aligned} V_i(Z_0) &= \frac{1}{n(\rho+\delta)} \left[Z_0 - e^{-\rho t_1(Z_0)} Z_0 \right] + \frac{Z_S}{n\rho} e^{-\rho t_1(Z_0)} \\ &\quad + \frac{1 - r(n-1)}{n(\rho+\delta)} (Z_L - Z_S) e^{-\frac{\rho+\delta}{1-r(n-1)} t_1(Z_0)}, \end{aligned}$$

the differentiation of which with respect to Z_0 yields

$$V_i'(Z_0) = \frac{1}{n(\rho+\delta)} + \frac{1}{n\delta Z_0} \left[\left(\frac{\rho}{\rho+\delta} Z_L - Z_S \right) e^{-\rho t_1(Z_0)} - (Z_L - Z_S) e^{-\frac{\rho+\delta}{1-r(n-1)} t_1(Z_0)} \right], \tag{B3}$$

noting that $t_1'(Z_0) \equiv 1/(\delta Z_0) > 0$.

Finally, by making use of the value function (B2) (and (B3)) we have to prove that $a_i(Z(t)) = 1$ is a best-response to $a_{-i} = (1, \ldots, 1) \in R_+^{n-1}$ for every player i at any point in time. In what follows, we suppose that only deviator i can adopt any strategy $a_i \in [0,1]$, while other players continue to play the $(n-1)$-tuple of strategies $a_{-i} = (1, \ldots, 1)$. Following Theorem 16 in Rowat (2002), we have to show that the following inequality holds for any strategy played by deviator i; i.e., for $a_i \in [0,1]$

$$\begin{aligned} \rho V_i(Z(t)) &= \frac{1}{n} Z(t) + V_i'(Z(t)) [-\delta Z(t)], \\ &\geq \frac{a^r}{n-1+a^r} Z(t) + V_i'(Z(t)) [1 - a - \delta Z(t)], \end{aligned} \tag{B4}$$

where noting that by symmetry the same inequality (B4) holds for every player.[15] By subtracting the l.h.s. of (B4) from its r.h.s., we define

$$F(a; Z(t)) \equiv \frac{(1-a^r)(n-1)}{n(n-1+a^r)} Z(t) - V'(Z(t))(1-a), \tag{B5}$$

We want to show that $F(a; Z(t)) \geq 0$ for any deviator's strategy $a_i \in [0,1]$. Without loss of generality, we set $Z(t) = Z_0$.

When deviator i plays an interior strategy $a \in (0,1)$ (we drop subscript i on strategy a from now on), the first-order condition (7) should hold with equality. Thus we substitute this equality into (B5) and rearrange to get

$$F(a; Z_0) \equiv \frac{a^r Z_0}{n-1+a^r} \frac{(1-a^r)(a^r+n-1) - (1-a)nra^{r-1}}{n(a^r+n-1)} > 0, \tag{B6}$$

because $1-a^r > 1-a$ and $a^r+n-1 > nra^{r-1}$.

Finally, when deviator i plays $a=0$, we substitute (B3) and $a=0$ into (B4) to get

$$F(0; Z_0) \equiv \frac{1}{n} Z_0 - V'(Z_0) = \frac{1}{n} Z_0 - \frac{1}{n(\rho+\delta)} -$$
$$\frac{1}{n\delta Z_0}\left[\left(\frac{\rho}{\rho+\delta} Z_L - Z_S\right) e^{-\rho t_1(Z_0)} - (Z_L - Z_S)\, e^{-\frac{\rho+\delta}{\delta[1-r(n-1)]} t_1(Z_0)}\right]. \tag{B7}$$

[15] In order to prove that the extended strategies, which are patched by either $a=1$ or $a=0$, or both strategies are subgame perfect over the domain $[0,\infty)$, we shall apply Theorem 16 in Rowat (2002) with slight modifications as follows: Given the instantaneous objective function $u_i(a_i(Z(t)), a_{-i}(Z), Z(t))$ and equation of motion (3), if $a_i^*(Z)$ satisfies

$$\begin{aligned}\rho V_i(Z(t)) &= u_i\left(a_i^*(Z(t)), a_{-i}^*(Z), Z(t)\right) + V_i'(Z(t))\dot{Z}(t)\\ &\geq u_i\left(a_i(Z), a_{-i}^*(Z), Z(t)\right) + V_i'(Z(t))\dot{Z}(t)\end{aligned}$$

for all admissible strategies $a_{-i}^*(Z) \equiv \left(a_1^*(Z), \ldots, a_{i-1}^*(Z), a_{i+1}^*(Z), \ldots, a_n^*(Z)\right) \in R_+^{n-1}$ and all i, then a sufficient condition for $a_i^*(Z)$ to be a best response to $a_{-i}^*(Z)$ is that $\lim_{T\to\infty} e^{-\rho T} V_i(Z(T)) \geq 0$, where $Z(T)$ is the result of any play, $a_i(Z(t))$ against $a_{-i}^*(Z(t))$ over $t \in [0,T]$ from the initial state Z_0.

It is easily verified that

$$\lim_{t_1(Z_0)\longrightarrow 0} F(0; Z_0) = \frac{1}{n} Z_L > 0 \text{ and} \lim_{t_1(Z_0)\longrightarrow \infty} F(0; Z_0) = \frac{1}{n} Z_0 - \frac{1}{n(\rho+\delta)} \longrightarrow \infty, \tag{B8}$$

since $Z_0 \longrightarrow Z_L$ as $t_1(Z_0) \longrightarrow 0$, while $Z_0 \longrightarrow \infty$ as $t_1(Z_0) \longrightarrow \infty$.

Using the following relations:

$$e^{-\rho t_1(Z_0)} \equiv \left(\frac{Z_0}{Z_L}\right)^{-\frac{\rho}{\delta}} \text{ and } e^{-\frac{\rho+\delta}{1-r(n-1)} t_1(Z_0)} = \left(\frac{Z_0}{Z_L}\right)^{-\frac{\rho+\delta}{\delta[1-r(n-1)]}},$$

(B7) can be rewritten as

$$nF(0; Z_0) \equiv Z_0 - \frac{1}{\rho+\delta} - \frac{1}{\delta}\left[\left(\frac{\rho}{\rho+\delta}\frac{Z_L}{Z_0} - \frac{Z_S}{Z_0}\right)\left(\frac{Z_0}{Z_L}\right)^{-\frac{\rho}{\delta}} - \left(\frac{Z_L}{Z_0} - \frac{Z_S}{Z_0}\right)\left(\frac{Z_0}{Z_L}\right)^{-\frac{\rho+\delta}{\delta[1-r(n-1)]}}\right]. \tag{B9}$$

Although the third term on the r.h.s. of (B9) may change its sign depending on values of Z_0, it is easy to show that $nF(0; Z_0) > 0$ under condition (14).

Moreover, it is easy to confirm that

$$G(Z_0) \equiv \left(\frac{\rho}{\rho+\delta}\frac{Z_L}{Z_0} - \frac{Z_S}{Z_0}\right)\left(\frac{Z_0}{Z_L}\right)^{-\frac{\rho}{\delta}} - \left(\frac{Z_L}{Z_0} - \frac{Z_S}{Z_0}\right)\left(\frac{Z_0}{Z_L}\right)^{-\frac{\rho+\delta}{\delta[1-r(n-1)]}} < 1, \tag{B10}$$

since $Z_0 \geq Z_L$ and $Z_S > Z_L$. It follows from (B9) and (B10) that

$$nF(0; Z_0) \equiv Z_0 - \frac{1}{\rho+\delta} - \frac{1}{\delta} G(Z_0) > Z_0 - \frac{1}{\rho+\delta} - \frac{1}{\delta} \geq 0,$$

as long as $Z_0 \geq Z_L \geq \dfrac{\rho+2\delta}{(\rho+\delta)\delta}$.

Appendix C: Linear Strategy

Under symmetry, rewrite the HJB equation (5) as follows:

$$\rho V(Z) = \max_{a_i \in [0,1]} \left[p\left(a_1, a_2, ..., a_n\right) Z + V'(Z) \left\{ n\left(1-a\right) - \delta Z \right\} \right]. \tag{C1}$$

Suppose that the value function is linear, that is, $V(Z) = A + BZ$, where A and B are unknown constants. Substitute this hypothetical value function into the above HJB equation to get

$$\rho\left[A + BZ\right] = \max \left[\frac{1}{n} Z + B\left\{ n\left(1-a\right) - \delta Z \right\} \right]. \tag{C2}$$

Substituting further the (interior) first-order condition (7), that is, $a = r\left(n-1\right) Z / Bn^2$ into a in (C2), we obtain

$$\rho A + \rho BZ = \frac{1}{n} Z + B\left\{ n\left(1 - \frac{r\left(n-1\right)}{Bn^2 a} Z\right) - \delta Z \right\}.$$

Further rearrangement gives

$$\rho A - Bn + \left[\rho B - \frac{1}{n} + \frac{r\left(n-1\right)}{na} + B\delta \right] Z = 0,$$

which is equivalent to

$$\rho A - Bn = 0 \text{ and } \rho B - \frac{1}{n} + \frac{r\left(n-1\right)}{na} + B\delta = 0. \tag{C3}$$

Solving the above simultaneous system of equations in terms of A and B to yields

$$B = \frac{1 - r\left(n-1\right)}{\left(\rho + \delta\right) n}.$$

Further substitution of this expression into the second equality in (C3) yields (13).

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