Reflected Acting in Mathematical Learning Processes

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Abstract: *Acting and thinking are strongly interconnected activities. This paper proposes an approach to mathematical concepts from the angle of hands-on acting. In the process of learning, special emphasis is put on the reflection of the own actions, enabling learners to act consciously. An illustration is presented in the area number representation and extensions of number fields. Using didactical materials, processes of mathematical acting are stimulated and reflected. Mathematical concepts are jointly developed with the learners, trying to address shortcomings from own experiences. This is accompanied by reflection processes that make conscious to learners the rationale of mathematical approaches and the creation of mathematical concepts. Teaching mathematics following this approach does intent to contribute to the development of decision-making and responsibility capabilities of learners.*

ZDM-Classification: B23, C33, D33, D83

1. Mathematical acting

Acting and thinking are strongly interconnected. Piaget (1971) considers for example thinking as internalized acting being a prerequisite for conscious acting. Also in mathematical didactics emphasis is given to the fact that acting is key in learning mathematics. A number of didactical approaches to mathematics have been created based on action oriented arguments.

- From the angle of the *theory of learning*, it is Bruner who highlights the layers 'enactive', 'iconic' and 'symbolic' (Bruner 1960). A variety of concrete examples for the day-to-day work in schools were developed subsequently.
- *Ethnomathematics* carves out 'universal' activities which lead to the development of mathematics using them as a foundation to build culturally guided curricula (cf. Bishop 1991).
- Typical mathematical procedures are interlinked with day-to-day patterns of acting within the *theory of fundamental ideas*. For example Schreiber (1983) tries to connect mathematical procedures with lifeword activities. The aim of fundamental ideas in general is to support learning through networking inner and outer aspects of mathematics (for an overview see Schweiger 1992, recent see Vohns 2005).

Since the importance of acting is common sense, it is key to also reflect actions during the process of learning mathematics itself. The learners will be empowered to discover new opportunities for acting through reflection of the goals, the motivation, the course, the tools and the results of their own actions. In addition this allows to include a process to develop consciousness about the role mathematics can claim in the course of human action.

This paper is built on the idea to base processes of concept formation on actions, where opportunities and limits of those concepts are jointly reflected with the learners. This follows the principle of operative concept formation (cf. Aebli 1985) which intents to stimulate thinking in the framework of acting, constructing a system of operations that will finally serve the acting itself. This principle has been implemented in mathematical didactics by Bender and Schreiber for the teaching of geometry (cf. Bender & Schreiber 1980, 1985). Geometric objects are considered as answering practical needs and idealisations of acting experiences. This way, mathematics comes to know in its modus operandi, its determination of aims, its abstractions and its idealisations.

In this paper the terms action and acting are understood in a broad sense, including not only material tasks but also mental processes (cf. Mittelstraß 2004). An action is a process within which 'tools' are used to reach a certain 'goal' wanted as the achievement by the 'actor' through the 'result'. Mathematical acting is hereby seen as acting to reach goals within the field of mathematical knowledge.

2. Number representation and extension of number domains: a learning environment

During the school year 2004/2005 the topic of number representation and extension was introduced to a group of 30 pupils aged between 13 and 16 through the provision of a variety of visualising material for calculations:

- peas
- stones (in different sizes)
- clay tablets
- laces of the Inca
- finger bargaining
- Roman numbers
- abacus

Two teachers advised the learners in the selection process of the materials, ensuring items were picked appropriate for their individual profiles. The pupils are used to learn in a group of mixed ages. Where necessary, materials were separately explained in reference guides. Pupils were asked to work pair wise on the following task.

> *Task:*
> Please perform the following calculations using your material. For which aspects is your material helpful, less helpful or even useless?
>
> 573 + 56 =
>
> 327 – 48 =
>
> 18 : 5 =
>
> For each calculation, please describe your approach.

The result of the exercise was then presented by the pair to the rest of the group, especially focusing on the own approach taken. This forced them to reflect their proceeding and make it explicit.

The aim of this introduction to the topic was twofold. First, learners needed to think about the materials used along with their strengths, weaknesses and limits for the purpose of number representation. Second, supported through targeted intervention by the teachers, the own mathematical acting had to be reflected.

A little episode around calculating with stones

Antonia and Valentin, aged 14 and both slow learners in mathematics, received 35 pebbles varying in size. They approach me after a couple of minutes: ‘We don’t have enough stones’. Antonia says they had counted the stones, but those were only sufficient to carry out the division. She points at the table showing a distribution of 18 stones to 5 clusters (18 : 5 = 3 remainder 3). ‘But those are not enough stones for the other tasks’ Antonia says ‘the group working with peas have much more in numbers, more than enough.’ I suggest thinking about what makes the pebbles different from the peas. They have a second look at the pebbles and discover the different looks and sizes. Valentin immediately starts to sort them.

In order to get to the mathematical breakthrough, the view needs to be broadened through a comparison with the other groups. I raise the following questions: ‘What does a pea represent in the calculation of the other group? Why does this not work with stones? How can you make use of your own sorting to progress?’ Finally, after this strenuous process of thinking about their own actions and other opportunities, one of the two starts to label the little stones with 1, the medium sized with 10 and the large ones with 100. Compared to this step of abstraction, it is now rather easy to solve the two remaining problems. But even there non elementary actions are required which can be mastered through the materialisation of the stones. During the summation the pupils are not yet coming across the idea to exchange ten middle sized stones through one large. However, in the subtraction such an exchange is unavoidable. Through that, Antonia even develops an idea how to deal with the unsatisfactory solution of the division (remainder 3). She gets 30 peas from the other group and distributes newly to represent the result accurate to a tenth.

Actions around calculating with stones

The above episode shows, how multifaceted the actions of the learners are, which is summarised in the following overview:

Actions to represent the calculation with stones were:

- count
- distribute
- recognise and compare
- sort and classify
- represent
- abstract and substantiate

Additional actions during the calculation with stones were:

- append
- remove

- decompose
- replace and exchange

3. Reflecting Mathematics

As pointed out by Jablonka (2003) in the discussion on mathematical literacy, reflections are of key importance during the learning process in order to achieve an appropriate handling of the subject in question. She argues that the claim of literacy always reaches beyond the pure knowledge about the subject itself, since one core aspect of the concept of literacy is the way how human beings handle knowledge. When developing mathematical literacy she considers indispensable a critical analysis of the subject which makes reflections about mathematics necessary. Those reflection processes always refer to the relationships between subject, person and the world.

In the mathematical literature different levels of reflections are considered (cf. e.g. Neubrand 1990, Bauer 1990, Kaune 2001, Skovsmose 1998, Fischer o. J., Lengnink 2005). The diagram shows how those level can be incorporated in the network of relations mentioned above.

Through the distinctive levels different meanings of reflections in the learning process are highlighted. Content reflection deals with inner questions of mathematics like correctness, clarity and sophistication. Reflections around conscious working and metacognition mainly support effective learning of mathematics. Reflections of meaning and sense as well as model and context oriented reflections underline mainly the reasonable dealing with and a critical judgment of mathematics including its applications. Self reflection emphasises the personal positioning vis-à-vis mathematics. Lifeworld reflection suggests to connect the own relation to the world with a reflection of benefits and limits of mathematics in describing real world issues as well as its standardising function within the real world. (What is important to me in a specific situation? Where is mathematics helpful in describing? Where are the limits? How does mathematics take effect in my lifeworld? What is my position in that respect?) The level of lifeword reflection is especially important for the process of developing mathematical literacy (mathematische Mündigkeit) (cf. e.g. Lengnink 2005a und 2005b).

Reflecting whilst calculating with stones

The learning experience described in section 2 stimulates reflection processes on different levels, which can contribute to a sensible usage of mathematics and the competence of conscious acting. This was suggested through the chosen setting of the situation and in addition explicitly requested by the teachers during the different learning phases.

Working in pairs opens possibilities to act at leisure and enables discussions in a sheltered environment. Reflections were initiated to make progress in acting, mainly activating the levels of content reflection and metacognition. Whilst calculating with stones, content reflection was kicked off through the restriction in the number

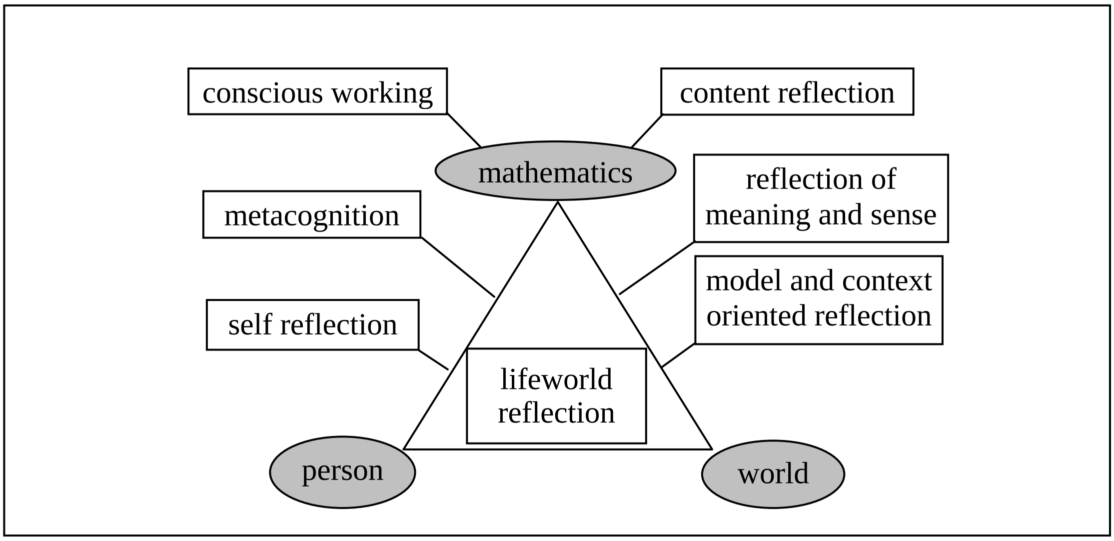

Figure 1: Levels of reflection within the area of mathematics, world and person

of stones. The guideline for thinking was: It has to be possible with a little number of stones. Metacognitive processes were started through the comparison of acting opportunities with those of the other groups. The leading principle was: We need another idea beyond the peas. We want to use one stone for the number 1, but that does not work. During this phase it is essential to give room for self reflection. The two pupils express themselves: ‘We are annoyed that you gave us only a small number of stones’. This level of reflection is already important during the process of acting as behind the scenes affects of the learners will very often impact their attitude and position towards the subject. The nuisance, that can unconsciously cause a slow down, injects in case of becoming conscious a positive energy into the learning process.

After their mathematical discovery, the learners were asked to *prepare a presentation* to the rest of the group. The requirement was to describe for each task the approach the group has taken, which addresses the level of conscious working. We as teachers suggested the following questions: How have we dealt with the task? Which actions were mathematically sustainable and why was that? Which ones were not? In this context, metacognitive processes were initiated retrospectively. An important question the learners asked themselves was: ‘What were our thinking barriers and how did we overcome them?’ This step delivered mixed results during the preparation but was subsequently offset in the joint group discussion.

The presentation of the results in front of the whole group deepened the level of conscious working and in addition did put focus on the reflection of meaning. The initiation was given through the question in the initial task setting: ‘For which aspect is your material helpful, less helpful or even useless?’ In that context, the trick to label differently sized stones with different powers of ten was recognised. The economical characteristics of the place value system vis-à-vis the counting of peas became very clear. The learners say: ‘Less stones work better than many peas’. Also in comparison with the Roman numbers the meaning of the place value system became apparent. Nanne says: ‘It is dam complicated to calculate with Roman numbers. Principally, I have calculated as usual in order to determine the result.’ This insight in the advantages to formalise and automate calculations in the place value system was further deepened through the presentation of the Abacus. It materially and directly demonstrated the idea of the place value system, which needed still cumbersome interpretation in the case of stones.

The performance a number representation through given material can deliver is not only determined by its mathematical usefulness but also by the function it is supposed to fulfil in a specific context. This context oriented reflection played an important role during the discussion on the materials in the group: The clay tables were identified as a means to store calculations due to the long-living material. The same held true for the laces of the Inca, which were used to archive inventory. Finger bargaining served the purpose of secret trading in the market place; it was not supposed to support calculations. However, the pupils did invent calculation symbols which they presented to the group. This let to a discussion of useful characteristics of those symbols. The group identified they need to be tangible and distinguishable. Interested readers can find an historical overview of number representations in the book of Ifrah (1991).

4. Elements of learning environments for reflected acting

Number representation and extensions of number domains

It is valuable to carve out the elements of the earlier presented learning environment, which was designed for reflected action and enabled a fruitful course of events during the class. They can be reused for the creation of other learning environments.

Request for own action

Through the distribution of materials and the given tasks learners were pushed into action. Goals and means were given, but the course of the process and the result of action were not determined although restricted through the presented materials. The choice for partner work was deliberate to generate a high degree of self activity at the same time proved possibilities to exchange views on their actions.

Leisure to act

It was key for the learning environment to give learners room to work at leisure on their own actions. This enabled reflection processes in the first place. It was only possible through slowing down the class process to build self experience during acting and to think about the own actions in comparison to the actions of the other groups. The partner work including preparations for the presentation therefore lasted about 90 minutes.

Limited scope for acting: Enable comparative reflection

Through the task setting the learners were given goals (representation of calculations through materials) and means (the materials). This consciously limited the scope for acting for each pair of pupils. This framework forces on a certain spectrum of courses as well as a profile of results.

The reflection potential in the learning environment was enabled through the distribution of different materials within the whole group of learners. The reflection of the course and results within the own group was inspired through the comparison with available scope for acting in the other groups.

Presenting: The task to structure retrospectively

Learners were asked to talk about their own actions and the results from a bird's eye view through the task to give a presentation. During the preparations important processes of cogitation took place through making explicit strategies, successes and failures. This contributed to the building of competencies in decision-making and responsibility.

Animate comparisons in class discussions

The presentations provoked class discussions within the whole group of learners within which opportunities and limits of the different materials could be reflected. In that context, materials were compared with respect to their ability to represent numbers as well as the quality of this representation. Also the issue of number representation was broached.

Creation of mathematical concepts as answers to overcome bottlenecks

Pending the respective purpose, the given materials proved to be more or less suitable. Those purposes became transparent and communicable through the different materials. However, some materials were transformed during the course of actions. The 35 stones for example were transformed into a different material through the labelling with numbers[1], which opened new options. (Presumably, this transformation would have not happened in case Valentin and Antonia would have been put in a gravel bed with lots of stones). Therefore, number representation and extensions of number fields were understandable as answers to bottlenecks in the presented learning environment, which lead to the alteration of the material itself satisfying the new prevailing circumstances.

Differentiation according to capabilities

Through the differently aged pupils it was necessary to differentiate the work within the class. This was expressed through the selection of the different materials and the advices given to learners when choosing the appropriate material. Peas and stones for example allow for manifold experiences also for pupils with less skill. In the presented example, they were guided to re-discover the place value system, which should have been already known to them at that stage. For the more skilled pupils, Roman numbers and the Abacus were quite challenging. On one hand they could learn to deal with this way of representation, on the other the limitations of those number representations became apparent. This was a good hook for the topic that followed next in the class: the extensions of number fields. Subsequently, negative numbers for the younger pupils and irrational numbers for the older pupils were introduced.

Culture of teaching and learning

In the described learning environment pupils did mainly work unassisted. They were given a high degree of autonomy personal responsibility for their own learning. On the teacher's side, this assumes a high level of confidence in the capabilities of the group. Also the responsibility has to be handed over step by step, stand-alone learning is part of the learning process as well as of learning goal. Teachers turn into moderators who listen, perceive thoughts, remind, summarise and stimulate comparisons. They support the learners in the shaping and reflection of their actions. Depending on the level of self-reliance of the pupils more or less instructions will be required of the teachers. The episode of Valentin and Antonia shows how inspiring and sustainable such an environment can be also for less

[1] This idea I owe to Lucas Amiras.

skilled pupils, provided sufficient leisure is given during the actions, accompanied by stimulating them to also reflect about those.

Attributes of learning environments for reflected acting

In summary, reflected acting can be successful, if:

- Situations of action demand active acting;
- Different scopes of acting are opened up;
- Clear goals are agreed;
- Leisure for active doing is given;
- Comparisons are stimulated;
- Based on own experience class discussion are held about both, the own actions and the actions of others;
- Reflections about purposes, goals and success of actions from a mathematical point of view are stimulated to illustrate specific mathematical approaches and claims;
- The process is accompanied through the establishment of a culture of learning, characterised through a high degree of autonomy of the learners as well as a high degree of confidence of the teachers in the capabilities of the learners going along with the courage and patience to listen.

5. Outlook and conclusion

In the learning environment described above, the reflections about actions were mostly restricted to mathematics. In the diagram, the main reflection processes are place in the upper part which is related to mathematics. Even reflection of meaning and sense underlines inner mathematical aspects, even though they are important within mathematics itself. It is the aspiration for 'good' representations which are easy to understand, prove to be economic, can be formalised, are transferable and carry the possibility to offload connected actions to machines. Only context oriented reflection and self reflection went beyond the inner mathematical framework; however they were not given a lot of time during the class.

Moreover, the frame of action was very specific in the presented learning environment as it was centred on acting with didactical materials. The far-reaching meaning of the term of acting, introduced in section 1, also allows for other acting opportunities.

Additional learning environment for reflected acting: an outlook

In the following, other class examples of reflected acting are briefly outlined. The first two remain mainly inner mathematical, whereas those described further below open possibilities for actions (in the broad sense) which can and should be reflected beyond.

Reflection of mathematical proofs

During the winter term 2005/2006 the approach to mathematical proofs was worked out with teacher trainees for Secondary School in the first semester at the TU Darmstadt. They were split into groups of four, whereby two students were responsible for the proof, the third was asked to raise doubts and the last one was supposed to put down notes on the observed situation, esp. to minute the task, the approach, auxiliary means, strategies and the roll of the doubter during the proof. The doubter was supposed to critically follow the process of the proof, provoking reasoning through questions. In this setting, the learners were supposed to be enabled to deliberately take on the above roles in order to later on take them into consideration for their own acting process.

The students were given a task for proving in the area of linear algebra and asked to fix the process visible for all on a poster. After the work in the groups a presentation about the results and observations was given to the remaining students. In the joint discussion, the focus was on conscious working. The learners gave answers to the following questions: How have we tackled the subject? Which heuristic strategies and means were applied? The next questions raised by the students show aspects of metacognition and content reflection: Where did we get into trouble with the proof? Which reasoning did the doubter request from us? When is a mathematical proof valid?

As a result of this learning environment some strategies were extracted supporting processes of proving.

Action oriented and reflected solving of equations

An action oriented approach to solve equations was chosen in the eighth and ninth class of a comprehensive school, where the ‘enactiv’ and the ‘symbolic’ level (cf. Bruner 1972) was developed in parallel during the learning process. The learners were asked to pack arbitrary numbers for their fellow pupils through applying mathematical operations. Those numbers could later on be unpacked.[2] Through the reflection of the analogy between the lifeworld processes of packing and unpacking on one hand and the mathematical matches it was possible to make the counter-operation subsumable and represent them symbolically.

This model of packing and unpacking of arbitrary numbers was then used action oriented access to solve equations of a specific type. Since the scope of this analogy is limited to a certain type of equations virtual scales were used as a model for equations later on in the process. Equations needed to be actively arranged with platelets on the scale, asking the learners to actively perform subtractions, additions, divisions and multiplications which at the same time needed to be noted down in a formal mathematical way. Mathematical and lifeworld actions were connected. Reflection of content, sense and meaning of mathematical operations and their effect were necessary to stimulate further mathematical understanding (cf. Lengnink, in preparation).

Reflecting mathematical acting and its connection to lifeworld thinking

During the beginners course in linear algebra in the first two semesters of students in the subjects mathematics and computer science the following guiding principle was unfolded: the meaning of linear algebra lays in its ability to provide a language that can give ‘good descriptions’ for our technological and scientific world (cf. Wille 1981, page 108). This motto implemented in the course on the level of concept formation and application but also on building up competences in acting mathematically. Thus a variety of occasions for acting were provided during exercise courses and tutorials within which the learners could intensively study the concept of ‘good description’. This model oriented reflection was enhanced through the aspect of lifeworld reflection through which the learners were intensively asked to think about the comparison of mathematical and lifeworld approaches with respect to acting. This way the course contributed to the formation of ‘acting competences’ in the area of linear algebra as well as to the understanding the specific mathematical point of view and its contribution to describing reality (details see Lengnink & Prediger 2000).

Reflected decision making using mathematics: What is behind the data?

Decisions using mathematics are often based on mathematical models that have been specifically built for this situation. Because models never embrace the whole situation, there are a variety of descriptions that are more or less favourable which pick up and follow up different aspects. Those models can be reflected with respect to there appropriateness to describe the subject and their function in a wider social context. Such approaches for model and context oriented reflection have been worked out by Ole Skovsmose, for example in the project ‘Citizenship’ (cf. Skosmose 1994).

Also in the research-field of the Generalistic Mathematics at the TU Darmstadt learning environments for model and context oriented reflections were developed that include lifeworld reflections. The claim for reflected action can be illustrated through the project ‘What is behind the data?’ that was run by Sybille Thamm. Jointly with pupils in classes ten through to twelve it was worked out, to which extent mathematical representations of data are useful to support the decision making process to select a profession. The learners were confronted with software for selecting a profession and were asked to apply it to their own situation. They were put in the ‘acting situation’ to inform themselves about professions. Their own criteria towards selecting a profession could be made explicit reflectively and mathematical methods could be reinvented, that gave a decent support for the matter. It was thereby possible for the learners to open up new mathematical approaches which fit better for the acting situation. A high level description of the project can be found in (Lengnink & Prediger 2001).

Conclusion

In all of the above mentioned approaches to a reflected acting in mathematics classes, aims of mathematical description mode and means of representation can be declared making conscious

[2] This idea is found in mathbu.ch 8 (cf. Affolter et al 2003).

perspectives. For this it is important to develop learning environment and appropriate materials which can be integrated in current class settings without deep structural adaptations. This way changes on the level of content as well as on the level of the learning culture can grow slowly with all parties involved in the class. In the long run such processes of concept formation can be connected with the competence in acting, which enables the learner to a literate handling of mathematics.

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