Reflecting mathematics: an approach to achieve mathematical literacy

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Abstract: Mathematics plays a dominant role in today's world. Although not everyone will become a mathematical expert, from an educational point of view, it is key for everyone to acquire a certain level of mathematical literacy, which allows reflecting and assessing mathematical processes important in every day live. Therefore the goal has to be to open perspectives and experiences beyond a mechanical and tight appearance of the subject. In this article a framework for the integration of reflection and assessment in the teaching practice is developed. An illustration through concrete examples is given.

Kurzreferat: Mathematik ist in unserer Welt auf vielfältige Weise präsent. Was an dieser Mathematik müssen allgemeingebildete Lai/inn/en verstehen, um mathematisch mündig zu sein. Aus der Bildungsperspektive mathematischer Mündigkeit werden Reflektieren und Beurteilen von Mathematik als wichtige Tätigkeiten im Unterricht begründet. Es wird ein Rahmenkonzept für Reflexion im Unterricht ausgearbeitet und an Beispielen für den Unterricht konkretisiert. Mit dem Ansatz ist die Hoffnung verbunden, das oft starre und mechanistische Bild von Mathematik schrittweise in Richtung eines diskursiven Mathematikbildes wandeln zu können.

ZDM-Classification: D20, D30, C20, C30, C60.

1. Introduction

Very often people either blindly trust or strictly refuse to deal with mathematics. The core question of this article is how a mathematical education has to be designed in order to train pupils to become confident in handling and judging implications resulting from mathematical applications.

In the first part of this article the necessity to build mathematical literacy is explained, based on the relevance of mathematics in today's society. As literacy mainly concerns the handling of mathematics by human beings, reflecting and judging of mathematics are the most important activities in the process of learning.

Since reflecting can occur on various levels, the second part elaborates four key layers of reflection supporting the proper development of literacy.

In the third part, two concrete examples illustrate the meaning of reflection on the above mentioned layers, especially focusing on the concrete activities to be undertaken by the pupils.

In the conclusion it is discussed, how the subject of mathematics is changed through this approach and how it does effect the perception of mathematics for the pupils and the public area.

2. The social relevance of mathematics – what is mathematical literacy?

No doubt, mathematics does play a very important role in today's world. It is used as a tool to describe dependencies and provide solutions for real world problems. Moreover, embedded in technology, mathematics does shape our world; it has become both 'systemic' and part of the world we experience. Often, this is unconsciously influencing our human thinking and acting. (Fischer 1993). When analysing how mathematics is used to regulate the way we live together, a very important aspect of mathematics as a formatting power becomes obvious (Skovsmose 1998). Concrete examples amongst others are fair distribution, healthy weight and economic productivity. Skovsmose points out the importance to achieve critical faculty with respect to the 'formatting power' during the learning process:

'Does mathematics education produce critical readers of the formatting? Or does mathematics education prepare a general acceptance of the formatting, independent of the critical nature of the actual formatting?' (Skovsmose 1998, p. 197)

On the other hand, it is known from various sources (e.g. Heymann 1996) that there is a subtle discrepancy between the view of individuals on the social relevance of mathematics (rather described as important) as opposed to its personal relevance (often perceived as unimportant). This is caused by the fact, that laymen often trust the work of experts who are assumed to understand the subject in detail. It is felt to be enough to see and use the 'outcome' and not get involved in the process of deriving those results through a mathematical model. It is almost like driving a car: you do not need to understand how to build it in order to be able to drive it. However, a driving licence is needed, to ensure a proper usage. Similarly, any critical citizen has to integrate the mathematical perspective and its implications into a holistic understanding of democratic concerns.

In this sense mathematical literacy goes beyond pure mathematical content. This has been elaborated by Eva Jablonka, who worked out the various missions and directions of mathematical literacy (Jablonka 2003). Every definition of mathematical literacy has to reflect the abilities and possibilities of human beings to apply their knowledge. Reflecting and judging are two important activities in that context, which support and ensure a critical position during the process. Only through disconnecting from the pure mathematical objects it is possible to acquire a critical understanding of mathematical concepts, algorithms and patterns of application. Jablonka puts it this way:

„The ability to evaluate critically can neither be considered as mathematical, nor automatically follows from a high level of mathematical knowledge. Consciousness of the values and perceptions of mathematical knowledge associated with distinct mathematical practices and their history can compensate to a large extent for a lack of detailed expert knowledge. Introducing critical discussions as proposed here, means introducing a new discourse into school mathematics that will eventually establish a new practice of out of school mathematics of informed citizens." (Jablonka 2003, p. 98)

The goal of this paper is to call for a mathematical education, which does train the ability and attitude to act in a self-determined manner within social decision processes, especially if they involve mathematical methods and results. The person should reach the state of mathematical literacy, which is characterised through

- the increasing ability to work out opportunities as well as limitations of mathematics,
- the ability and desire to thoroughly judge the real-world usage of mathematics,
- the wish to intensify the continuous reflection of her/his own relation to mathematics,
- the capability to distance herself/himself from parts of mathematics and its application with a well-founded reasoning.

This characterisation shows that literacy is a continuous discourse between mathematics and human beings that in principle will never end (see Lengnink 2005). In this process reflecting should be stressed more than pure operating with mathematical tools. (For further discussion see Fischer w.y.)

3. Reflection - a view from the angle of mathematical literacy

In general, the concept of reflection is not very specific. It encompasses to contemplate an object as opposed to simply perceive it. This may explain why there is a whole range of approaches to reflection for mathematical education (see Lengnink/Siebel 2005, esp. sections of Lengnink, Prediger, Peschek, Siebel).

From the perspective of literacy the goal is to build a critical, self-determined and equal attitude towards mathematics. This does imply an understanding of reflection, for which the relationship between human being and mathematics on one hand and between mathematics and world on the other is a key consideration. Bauer (1990) puts the emphasis on the reflection of a person's relationship to mathematics. Skovsmose (1998) puts a stronger focus on the reflection of impacts of mathematics in our world. In this article, the synthesis of the two approaches is proposed, which is supplemented by the aspect of *reflection of situation*.

From the educational perspective of mathematical literacy for core layers have to be highlighted:

- *Reflection of situation* (ponder on the mathematical shape of a situation)
- *Reflection of sense* (ponder on the sense and meaning of fundamental concepts and approaches for their idealisation within pure mathematics on one hand and their connection to common thinking on the other)
- *Model-oriented and context-oriented reflection* (ponder on adequacy of a mathematical model for a certain purpose and its function for society)
- *Self-reflection* (ponder on the personal attitude towards mathematics and its applications)

Following those four aspects are elaborated in more detail:

Reflection of situation contributes to disclose the mathematical content of a situation, esp. around the question which aspect can (not!) be described mathematically. In that context purpose and moral concepts of the reflecting person play an important role to understand the sense of a mathematical perception after the reflection took place. In that respect, *reflection of situation* is closely linked to *self-reflection*.

Reflection of sense is understood as pondering over sense and meaning of fundamental mathematical concepts and approaches in their relationship to common thinking. In order to explore the momentousness of mathematical concepts, the question on how mathematical concepts and associations differ from common concepts and associations is very helpful. (What can (not!) be described with mathematical concepts? What can be gained from mathematical descriptions? How are the results to be interpreted?) Reflections of this kind do allow the pupils to connect their own thinking and acting with approaches and concepts within mathematics and in particular to consciously distinguish between the 'two worlds'. (Lengnink/Peschek 2001). This way, mathematics is embedded in a broader view of the world.

Model-oriented and context-oriented reflection touches the question if a specific mathematical modelling is appropriate for a given situation. (Which purpose will be served? Are there any alternative mathematical descriptions? Which conclusions can be drawn from the model and how do they relate to competing descriptions and their results?) The context-oriented reflection stresses the political and social context of the application of mathematics. (What is the purpose of using mathematics? Which function does mathematics play in social decision processes?)

Self-reflection has as a target to enable pupils to adopt an attitude towards mathematics which will affect their handling of mathematics. It is the goal to reflect the personal attitude towards mathematics and their applications especially in the affective and the cognitive layer. (Do you like a (specific) subject within mathematics? How to you feel about a (specific) mathematical subject? Does a (specific) mathematical modelling describe, what is important for you in the (specific) situation? How would you decide on this basis (argue!)? How comfortable do you feel with your judgement?) From the perspective of literacy, this is a core layer of reflection, because the holistic nature of human beings is activated and mathematics is embedded in this overall context. To do this in a reflected way, does support a critical attitude towards mathematics and their applications and can eventually build the courage to apply this attitude in practice.

It becomes obvious, that for the development of literacy it is not only necessary to change mathematical content taught. The shaping of attitude is also closely linked to the experienced culture during the classes.

4. The reflection way to mathematical literacy - concretions

The concept described in the previous section shall now be illustrated through two concrete examples. It is described, which activities will inspire reflection and which layers need to be touched in order to contribute to the process of mathematical literacy.

Dependency of values – the reflection way to learn the concept of functional dependency

Looking at *dependencies of values* does invoke associations with the real-world and raises questions, which can productively be linked with the mathematical concept of *functional dependency*. This approach was tested in a class study during 2001 with pupils aged 15 years (Lengnink 2002). The following real-world associations of the pupils were explored:

- Dependency as a mean to describe the cause-impact relationship
- Dependency as a mean to describe assignment
- Dependency as a mean to describe simultaneous change
- Dependency as a mean to describe 'the more …the…'
- Dependency as a mean to describe predictability

Those real-world associations were discuss during the classes and then compared with the mathematical concept of *functional dependency*. Commonalities and differences between real-world concepts and associations on one hand and those used in mathematics on the other were the focal point. Within the discourse, the following questions were raised for the respective layers of reflection:

Reflection of situation

- Which mathematical questions related to *dependencies of values* do raise your interest?
- Which questions on the subject chosen do raise your interest and can not be described in a mathematical fashion?

Reflection of sense

- For *dependencies of values* which are your associations and which of those can adequately be described through the concept of *functional dependency*?

Model-oriented and context-oriented reflection

- How is predictability dealt with in mathematics?
- Which are the mathematical representations of *dependencies of values* and for which situations are they advantageous?
- How are mathematical functions used in the real-world context?

Self-reflection

- Does the mathematical description of dependency make sense for yourself? Give examples and explain your judgement.

With this approach, pupils can connect to their own associations and reflect the describing character of mathematical concepts. They are empowered to build an emancipated and self-determined attitude towards mathematics, leading to increased literacy in using the respective mathematical concepts. Such attitude has to be supported during classes through a continuous discourse which allows a serious arguing about the value of mathematical descriptions.

Healthy weight – reflection of mathematical models

For pupils in classes 7 through to 10 (age 13-17) the own appearance is a crucial factor for their self-consciousness. This includes the perception of own weight as being too thin, just right or too fat. The so called 'Body Mass Index' (BMI) suggests a model, through which people can evaluate their own weight based on their body height.

$$BMI = \frac{\text{weight in kg}}{(\text{height in m})^2}$$

short weight	BMI <18.5
normal weight	BMI 18.5-25
over weight	BMI 25-30
adiposity	BMI >30

Characteristics of mathematical descriptions become very obvious with this measure for a healthy weight. The model does not deliver an impartial assessment; it is rather a setting based medical reasoning. This becomes even clearer, when the BMI figure is compared to other models that were frequently used in the past. The following calculation formula of the 'ideal weight' gives an illustration:

$$\text{ideal weight in kg} = \text{height in cm} - 100 - x\%$$

$$x=15 \text{ for women; } x=10 \text{ for men}$$

Also in this context it was possible and useful to apply the approach in the different layers of reflection. During another class study in 2003 with pupils aged 15, the following questions were raised:

Reflection of situation

- What does raise your interest around the topic of healthy weight?
- Which questions can be treated mathematically?

Reflection of sense

- How can the formula for BMI be interpreted? What are the components of the formula and why are they used? (Compute some examples and describe the way of calculation)
- Calculate the BMI for some persons you know. Have you awaited the respective results and their interpretation?

Model-oriented and context-oriented reflection

- What is the benefit to calculate healthy weight through a mathematical formula?
- How can this mathematical modelling of healthy weight be motivated? (Surf the internet to find places where the formula is applied and find reasons.)

- What are the limits of this mathematical model? For which group of people is it not applicable?
- Which approaches have been used in the past to measure healthy weight? What is the difference of those in comparison to BMI? (Draw a graph, to show the ranges for healthy weight in the past and today.)
- What is the function of those weight figures? In which context are they used and what are their limits?

Self-reflection

- What does the model mean for you? What do you conclude from your own BMI figure? What should you conclude in the opinion of doctors?
- Which are the possibilities beyond BMI to source information on healthy weight?

In the media there is currently a broad discussion on nutrition and healthy weight. Things are somewhat out of balance and there are good reasons to discuss the topic in schools. A lot of aspects have to be considered outside the mathematical context. However, the topic is rich seen from the educational perspective of mathematical literacy. A mathematical formula does fix what is seen to be healthy. To understand the social function of such a setting as well as to recognise its relativeness in comparison to past models becomes key to the competent dealing with such mathematical models in day-to-day life.

4. Conclusion

Mathematical education striving for literacy needs to include mathematics as an educational subject. During classes the debate goes beyond pure subject related topics. It is demonstrated to pupils that mathematical descriptions and knowledge is just one amongst multiple cultures of thinking which needs to be included in a broader understanding of our world.

It is necessary to run discourses showing the strengths as well as the limits of mathematics. The centre is about the meaning and the sense of mathematics for human beings; mathematics can and should be critically challenged.

Applying this approach in practise realises a piece of *Generalistic Mathematics* (Wille 1995) leading to a different understanding of mathematics. This includes reflection processes on

- Sense and meaning of mathematical concepts and algorithms, also with respect to their (historical and personal) evolution
- Purpose and appropriateness of mathematical models and their claims in the real world
- Impact and consequences of mathematics on society

This opens the opportunity for a critical discourse about background, meaning and conditions for the development of mathematics within the public area.

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