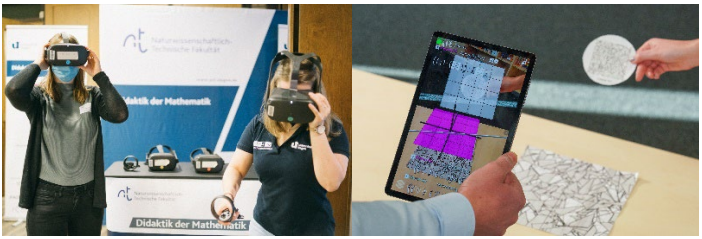
International Symposium on Augmented and Virtual Reality in Mathematics Education

October 6-7 2022



Augmented and virtual reality (AR/VR) are technologies of increasing importance in our society. In the field of mathematics education, these innovative technologies can offer a wide range of opportunities to support learning processes. Fascinating ideas have been developed and important scientific findings have been made during the last years. However, there is still a great need for concrete ideas and systematic scientific research. To tackle this desideratum, the collaboration of practice and research is essential. With this international symposium we want to contribute to the discourse by presenting existing insights and bringing parties from research and practice together.

Please send questions and comments to the following email address: dilling@mathematik.uni-siegen.de

The symposium will take place on Zoom. Please note the Dial-in data below.

Organizing Team: Dr. Frederik Dilling & Prof. Dr. Ingo Witzke

Universität Siegen

Guide for the International Symposium on AR / VR in Mathematics Education

Schedule

Time according to "Europe/Berlin" time zone.

Thursday October 6

9:00	Opening (Information on the conference)
9:15-10:15	Keynote (Prof. Dr. Andreas Dengel) *What is Immersive Education? - A Note on Its Terminology, Correlates, and Classroom Integration*
10:15-10:30	Break
10:30-12:00	1. Presentation block (Four presentations in two parallel sessions)
12:00-12:15	Break
12:15-13:45	2. Presentation block (Four presentations in two parallel sessions)
13:45-14:30	Lunch break
14:30-16:00	3. Presentation block (Four presentations in two parallel sessions)
16:00-16:15	Conclusion and outlook to the next day

Friday October 7

9:00	Open beginning/ Meet & Greet
9:30-11:00	4. Presentation block (Four presentations in two parallel sessions)
11:00-11:15	Break
11:15-12:45	5. Presentation block (Three presentations in two parallel sessions)
12:45-13:15	Conclusion (outlook, possibilities for publication)

The symposium will take place on the platform Zoom. You can find the dial-in data in the following:

AR / VR Symposium
Oct. 6 2022; 09:00 AM – 04:30 PM
7 2022; 09:00 AM – 01:30 PM

Calendar file: https://uni-siegen.zoom.us/meeting/u5Ulf-usrDojGNUP5sZ7PNyVC5r3kNsixL89/ics?icsToken=98tyKu-ppjMoHtaUuRuGR_McGoqga-jzpn5EjY1orEr2OXdjTBrAFbdQYblRA9HI

Dial-in data:
https://uni-siegen.zoom.us/j/61882421921?pwd=TmZTamxaYll6c2pMRGxiaTU2V2RwZz09

Meeting-ID: 618 8242 1921
ID-Code: 9KFZ%gU0

Overview: Presentations

Thursday October 6

1. Presentation block

10:30-11:15
Tim Läufer & Matthias Ludwig *Fostering Algorithmic Thinking in mathematics education with AR*
Adi Nur Cahyono, Muhammad Kharis & Muhammad Iqbal *VR STEM Trail: a math trail-based STEM Education using virtual reality technology for learning mathematical modelling*

11:15-12:00
Angelika Schmid & Lilla Korenova *Usage of the GeoGebra AR Application by the Teacher-training in the 3D Geometry courses*
Julian Sommer *(I)VR in mathematics education: perspectives from learning theory on learning in virtual environments*

2. Presentation block

12:15-13:00
Bilal Özçakir & Duygu Özdemir *Reliability and Validity Study of an Augmented Reality Supported Mathematics Education Attitude Scale*
Frederik Dilling & Jasmin Müller *Verbal and nonverbal communication during VR experiences - a case study on analytic geometry*

13:00-13:45

Roberto Capone, Eleonora Fagiano, Mario Lepore & Federica Mennuni
GeoGebra Augmented Reality with mathematics undergraduate students: a computational fuzzy semiotics analysis

Shereen El Bedewy
Mixed Reality Technologies use at the museum for applying STEAM practices

3. Presentation block

14:30-15:15

Gero Stoffels
GeoGebra AR in Cooperative Settings

Ben Haas, Yves Kreis, Zsolt Lavicza
Augmented Reality in primary education: New learning opportunities for students with learning difficulties in mathematics education

15:15-16:00

Eva Ulbrich, Shereen ElBedewy, Julia Handl, Branko Andjic, Yves Kreis & Zsolt Lavicza
3D Modelling for AR and 3D printing in Teacher Training

Qinqin Xiao
GEO2: An AR Geometrical Application with Cognition Awareness

Friday October 7

4. Presentation block

9:30-10:15

Janine Küng, Markus Vogel & Dorothee Brovelli
Teacher competencies in assessing augmented reality applications on spatial geometry

Bilal Özçakir & Erdinç Çakiroglu
An Augmented Reality Learning Toolkit for Fostering Spatial Ability in Mathematics Lesson: Design and Development

10:15-11:00

Astrid Beckmann, Julian Schendera & Irene Bouw
Specification of AR experiences in mathematics education - from interactive digital worksheets to the three-dimensional coordinate system to new perspectives in modeling

Frederik Dilling & Ingo Witzke
Enactive Learning through Marker-based AR

5. Presentation block

11:15-12:00

Neslihan Bulut, Mustafa Cevikbas & Gabriele Kaiser
Augmented Reality and Virtual Reality for Teaching and Learning Mathematics: A Systematic Literature Review

Mehmet Bulut & Rita Borromeo Ferri
Mathematical Modelling based STEAM-Approach in an Augmented Reality (AR) Environment

12:00-12:45

Kirstin Altmeyer, Lea Müller, Hamraz Javaheri, Melanie Platz, Paul Lukowicz & Roland Brünken
Evaluation of an AR-based learning environment on cube buildings to promote the use of spatial skills

Abstracts

Andreas Dengel
Goethe University Frankfurt, Germany

Keynote: What is Immersive Education? - A Note on Its Terminology, Correlates, and Classroom Integration

The technological advance of immersive media such as virtual reality and augmented reality has kicked off a new wave of research regarding their role for teaching and learning. Researchers in the emerging research area, often called „Immersive Education", „Immersive Learning", or „Immersive Teaching", come from different disciplines, such as the Educational Sciences, Human-Computer-Interaction, and Computer Science. Due to these very different backgrounds, discussions on terminologies lead to heavily differing definitions of the key concepts in this research area. This keynote presentation tries to give the audience a guide for three things:

- What are existing definitions in the research area and why are they so different?

- What are evidence-based findings on what factors influence learning in immersive experiences?

- How can immersive experiences be integrated in the everyday classroom and how does learning in the classroom differ from laboratory studies?

After the presentation, we will have an open discussion with the audience about what concepts they use and where their research is situated.

Kirstin Altmeyer, Lea Müller, Hamraz Javaheri, Melanie Platz, Paul Lukowicz & Roland Brünken
Saarland University, Germany

Evaluation of an AR-based learning environment on cube buildings to promote the use of spatial skills

Developing and using spatial skills is crucial in STEM education. They are addressed in elementary school math classes using cube buildings and corresponding blueprints. The current study with n = 16 elementary school students aimed at evaluating a paper-based augmented reality (AR) environment on cube buildings that was created based on theory-driven design principles. In contrast to traditional wooden cubes, the virtual and transparent cubes of the newly developed learning environment allow for a simultaneous presentation of cube buildings and blueprints, a systematic construction and deconstruction of cube towers and levels and provide insight into the inside of a building . The usability of the AR environment was rated as excellent and cognitive load was rated low. In a subsequent study, learning effects and students' observable learning strategies occurring in the AR-based learning environment will be further analyzed and compared to traditional non-AR teaching.

Astrid Beckmann, Julian Schendera & Irene Bouw
University of Education Schwäbisch Gmünd / Ulm University, Germany

Specification of AR experiences in mathematics education - from interactive digital worksheets to the three-dimensional coordinate system to new perspectives in modeling

In order to harness the opportunities of augmented reality (AR) for mathematics education, it is necessary to develop specific teaching materials for specific teaching scenarios and engage in critical reflection. Three examples drawn from the topic fields of data and

proportional relationships, prism properties, and the three-dimensional coordinate system demonstrate the wide-ranging potential of augmented reality. During subsequent reflection and detailed analysis from a mathematics didactics background, this paper also identifies key opportunities presented by AR in the topic field of modeling.

Mehmet Bulut & Rita Borromeo Ferri
University of Kassel, Germany

Mathematical Modelling based STEAM-Approach in an Augmented Reality (AR) Environment

Mathematical modelling is, briefly, the solution of real-world problems with the help of mathematical models (Borromeo Ferri, 2017). Furthermore, mathematical modelling is linked with students' development of twenty-first century skills and competencies emphasize the role of problems taken from the real world. Augmented Reality (AR) has shown promise as a resource, particularly for education in Science, Technology, Engineering, Arts, and Mathematics (STEAM) (Jesionkowska et al., 2020). Thus, the aim of this research is developing and evaluating a mathematical modelling based STEAM Approach in an AR Environment, using the dynamic geometry software for implementing it for mathematics learners.

- *Borromeo Ferri, R. (2017). Learning how to teach mathematical modeling in school and teacher education. Springer.*
- *Jesionkowska, J., Wild, F., & Deval, Y. (2020). Active learning augmented reality for STEAM education—A case study. Education Sciences, 10(8), 198.*

Neslihan Bulut, Mustafa Cevikbas & Gabriele Kaiser
University of Hamburg, Germany

Augmented Reality and Virtual Reality for Teaching and Learning Mathematics: A Systematic Literature Review

The current systematic review study explores the state-of-the-art research on the use of augmented reality (AR) and virtual reality (VR) in the field of mathematics education. In this systematic literature review, Preferred Reporting Items for Systematic reviews and Meta-Analysis (PRISMA) guidelines were used; and literature searches were conducted in two influential databases: Web of Science and SCOPUS. We analyzed peer-reviewed journal articles published in English, which substantially concentrated on AR or VR implementations in mathematics education. The qualitative content analysis of the included studies revealed that studies frequently used AR or/and VR to teach geometry, and several studies focused on employing these innovative technologies to assist students with learning disabilities such as dyscalculia. Our systematic review yielded promising results concerning the potential of AR/VR in mathematics education; however, some technological glitches and unfamiliarity with these emerging technologies may affect the success of the AR/VR applications in mathematics education.

Adi Nur Cahyono, Muhammad Kharis & Muhammad Iqbal
Universitas Negeri Semarang, Indonesia

VR STEM Trail: a math trail-based STEM Education using virtual reality technology for learning mathematical modelling

In this study, we propose VR STEM trail, a model of math trails-based STEM education that we developed using virtual reality (VR) technology. This concept is meant to be used to guide the implementation of STEM Education with math trails setting in a virtual environment. Examples of trail and tasks illustrate this

process. Results of an empirical investigation with teachers and students, which focusses on the teachers' math trail task design, how students utilize these VR tools when engaging with the developed virtual STEM trail, and how this model influence students' mathematical modelling ability, are presented. The results show that the developed model is a sound basis for initiating students' actions, to foster the development of mathematical modelling ability in a math trail-based STEM Education, and that virtual reality technology can support this process in a meaningful way.

Roberto Capone, Eleonora Fagiano, Mario Lepore & Federica Mennuni
University of Bari Aldo Moro, Italy

GeoGebra Augmented Reality with mathematics undergraduate students: a computational fuzzy semiotics analysis

This research focuses on designing and implementing teaching strategies using GeoGebra Augmented Reality to improve the conceptualization of specific mathematical objects. We describe a teaching sequence framed by Marton's Variation Theory, which involved thirty undergraduates' students in mathematics. The teaching sequence aims to foster students in characterizing paraboloids by easing the transition between different registers of semiotic representation, from the graphical to the analytical and vice versa. Our research hypothesis is that Augmented Reality facilitates students in the transition between different semiotic representations by boosting their motivation and engagement to improve their mathematical conceptualization process. The research methodology is based on mixed methods through a qualitative analysis of video-recorded activity and a quantitative analysis of the learner's status described through the parameters of conceptualization, inquiry and emotions, computed by using a Fuzzy Cognitive Map. The results are analyzed and discussed using Duval's theory of semiotic representations and Marton's Variation Theory.

Frederik Dilling & Jasmin Müller
University of Siegen, Germany

Verbal and nonverbal communication during VR experiences - a case study on analytic geometry

Virtual reality is generally recognized as a technology that allows people to have individual (virtual) experiences. By contrast, in classroom settings, the use of VR often does not take place in individual work, but collaboratively in groups, for reasons of learning theory but also for organizational reasons. Students who are wearing VR glasses are collaborating with other students who are not wearing VR glasses at the time. This imbalance of (mathematical) information available to the students leads to interesting negotiation processes, which consist of both verbal and non-verbal communication. This is theoretically explained in the presentation and explicated by means of a case study in the field of analytic geometry.

Frederik Dilling & Ingo Witzke
University of Siegen, Germany

Enactive Learning through Marker-based AR

The presentation discusses potentials and challenges of augmented reality technology for mathematics education using the example of an application programmed by the presenting authors. The application uses multi-marker tracking to provide students with an enactive approach to spatial geometry. Scenarios on geometric solids, the Pythagorean theorem in spatial contexts, congruence mappings, and analytic geometry are presented and analyzed from a didactic point of view. In an outlook, further developments and research potentials are discussed.

Shereen El Bedewy
Johannes Kepler University Linz, Austria / Egypt

Mixed Reality Technologies use at the museum for applying STEAM practices

Technology integration in educational practices can enhance learners' experiences. Augmented reality (AR) and virtual reality (VR) technologies extend learners' views of mathematical models providing opportunities for the learners to explore their digital models in real and virtual environments. We propose STEAM practices that integrate these technologies for the learners to examine their mathematical models created using GeoGebra. When using these practices in non-formal learning environments such as museums, learners could use museum collections as inspiration for their models. Implementing these practices with the teachers in various settings indicated that the study participants gained positive attitudes towards these STEAM practices in museum learning environments and in encapsulating these technologies. Moreover, the study showed that these practices supported integrating history, culture, arts, and architecture with mathematics while modelling the museum objects and visualizing them using AR tools.

Ben Haas, Yves Kreis, Zsolt Lavicza
Johannes Kepler University Linz, Austria / Luxemburg

Augmented Reality in primary education: New learning opportunities for students with learning difficulties in mathematics education

Although there are manifold connections between mathematics, foremost geometry, and the real-world (e.g., architecture, arts, functional objects), integration seldom happens in daily learning lessons in mathematics primary education. Learning three-dimensional geometric shapes, for example, is mainly done in a two-dimensional setting using textbooks instead of three-dimensional settings using technology or didactical material. This circumstance,

however, makes it far more difficult for students with learning difficulties in mathematics to understand mathematical properties, recognize shapes in the real world, and understand the possibilities of modulating shapes.
Students with learning difficulties learn efficient strategies to apply mathematics to their environment when shapes and connections are visualized with Augmented Reality within the real world. Based on several experiences and studies, we will present and discuss learning mathematics with Augmented Reality in primary education for students with learning difficulties.

Janine Küng, Markus Vogel & Dorothee Brovelli
University of Teacher Education Lucerne, Switzerland /
University of Education Heidelberg, Germany

Teacher competencies in assessing augmented reality applications on spatial geometry

Augmented reality (AR) can help learners understand complex relationships, models and structures. However, this requires not only augmented reality applications that are suitable for schools, but also teachers who can use them in lessons in a way that promotes learning. In this ongoing study, we assess and analyze the necessary teacher competencies in assessment of and planning with augmented reality applications in STEM teaching. Six AR applications on three different STEM topics, including two apps on spatial geometry, are assessed by experts and (prospective) teachers. The ability of (prospective) teachers to recognize potentials and challenges of the AR apps, to reflect on them, and to plan lessons with AR apps is evaluated. Teachers' characteristics, such as their attitudes or learning opportunities, are also considered in the evaluation. Preliminary results of the app selection and the pilot study will be presented.

Tim Läufer & Matthias Ludwig
Goethe University Frankfurt, Germany

Fostering Algorithmic Thinking in mathematics education with AR

Computational Thinking (CT), with its biggest part being Algorithmic Thinking (AT), is a still emerging topic present in virtually all modern sciences and needs to be addressed in modern education. Therefore, an integration into the corresponding subjects is appropriate. The <colette/>-project (Computational Thinking Learning Environment For Teachers in Europe, Erasmus+; KA201, 09/20 - 08/23) aims to provide a low-threshold approach to integrate CT and AT in mathematics and science classrooms, including high-quality educational resources and digital tools to create CT and AT Tasks, i.e. the “Building Cubes” and “Drone” tasks. There students use block-based programming to navigate a drone in a three-dimensional space with pre-existing shapes to achieve certain goals or construct their own three-dimensional shapes. With the AR-Viewer the students can see their solutions right in front of them (Building Cubes) or can follow the drone along the coded path (Drone).

Bilal Özçakir & Erdinç Çakiroglu
Alanya Alaaddin Keykubat University, Turkey

An Augmented Reality Learning Toolkit for Fostering Spatial Ability in Mathematics Lesson: Design and Development

Previous research claimed that integration of augmented reality on educational settings helps to improve academic achievement of students in collaborative learning environments, as well as to improve their retention and ability to translate this within other environments. Since augmented reality is still considered relatively novel technology in educational fields, there is an inherent need for research-based guides to design effective and feasible augmented

reality tools for school-based learning. The main aim of this study was therefore to design and develop an augmented reality learning toolkit to foster spatial ability in middle school students using mobile devices. The study was conducted in two parts, as preliminary research and a prototyping phase. The findings guided the characteristics for designing an augmented reality learning toolkit with a set of spatial tasks aimed at middle school students.

Bilal Özçakir & Duygu Özdemir
Alanya Alaaddin Keykubat University, Turkey

Reliability and Validity Study of an Augmented Reality Supported Mathematics Education Attitude Scale

Augmented reality provides an interactive learning medium for students to investigate abstract mathematical concepts as well as contributes positive results in understanding concepts and supports enjoyment in the learning environment. Due to providing unique and novel opportunities in the learning environment and the sparseness of related studies about determining students' attitudes regarding AR-based mathematics; in this study, an attitude scale was developed. Its validity and reliability studies were conducted. Six hundred and thirteen middle school students participated in the study. To ensure construct validity, exploratory factor analysis was conducted. To determine the representative power of the factors formed, confirmatory factor analysis was performed. It is seen that the scale developed according to the results of the study is a valid and reliable measurement tool.

Angelika Schmid & Lilla Korenova
University of Ostrava, Czech Republic & Comenius University Bratislava, Slovakia

Usage of the GeoGebra AR Application by the Teacher-training in the 3D Geometry courses

By teacher-training of future mathematics teachers, we target on increasing their motivation. We present them innovative teaching methods that they can use in their pedagogical practice. 3D geometry courses cover solid geometry and stereometry. We used the dynamic mathematics software GeoGebra 3D with its AR-application for learning 3D geometry to help students discover the properties of objects by visualizing virtual solids in real space.

As part of qualitative research, students worked in groups, solving tasks about the relative position of lines and planes in 3D. Task input was displayed on a cube, cuboid or pyramid and the solution had to be illustrated by using the AR app of GeoGebra 3D.

This learning model increased the students' understanding of the concept of the relative position of geometry objects.

In this contribution, we want to present some examples of our learning activities and report on the preliminary results of our qualitative survey.

Julian Sommer
University of Siegen, Germany

(I)VR in mathematics education: perspectives from learning theory on learning in virtual environments

Virtual reality HMDs can already be found in some schools. However, in order integrate these sustainably into mathematics teaching, basic research is needed. In the last decade, diverse studies have been conducted in learning theory research that have generated insights into learning in virtual environments. Without a closer look at the particular contexts and media used, the results seem contradictory - a plea to use the more precise term IVR (Immersive Virtual Reality). In

the talk, an app for mathematical education - the self-developed IVR learning game "Dreitafelprojektion VR" - and a related research project will also be presented.

Gero Stoffels
University of Siegen, Germany

GeoGebra AR in Cooperative Settings

AR and VR offer the possibility of new perspectives on geometric objects. At the same time, these perspectives are implemented with devices that often only allow one user to see them. This presentation will share results on two research projects that use this limitation in a constructive way in the context of the "builder and architect" method. One investigation focuses on comparing the identification of 2D figures and 3D bodies and associated modes of interaction with the device. In another study, the "builder and architect" method is implemented more conventionally with the help of GeoGebra AR, which is then reconstructed as an integrated empirical setting.

Eva Ulbrich, Shereen ElBedewy, Julia Handl, Branko Andjic, Yves Kreis & Zsolt Lavicza
Johannes Kepler University Linz, Austria

3D Modelling for AR and 3D printing in Teacher Training

School activities integrating students' environments into teaching aim to develop skills and strategies to solve problems in real-world situations and can be useful in hybrid teaching. Such activities can encourage and motivate exploring skills in Science, Technology, Engineering, Arts, and Mathematics (STEAM). Hybrid teaching usually uses technologies and connects virtual and physical worlds. We use technologies like 3D modelling for Augmented Reality (AR) or 3D printing with GeoGebra and created an exercise introducing them in a lecture for pre-service mathematics students. The exercise

combines the possibility to introduce these technologies, can be used in hybrid teaching and connects to the Austrian mathematics curriculum. The exercise consists of 3D modelling mathematical mazes that can be explored using AR on handheld devices and can also be 3D printed. We used it in online, offline and hybrid scenarios with pre- and in-service teachers and will show resulting presentations of teacher projects.

Qinqin Xiao
University of Rochester, USA

GEO2: An AR Geometrical Application with Cognition Awareness

The name of my Augmented Reality (AR) Geometrical Application is GEO2. Geometry is represented by the first O of O2, while cognition is represented by the second O of O2. The function of the GEO2 AR Geometrical Application is to divide the internal auxiliary lines of complicated aggregates in order to reinforce geometric space and distinct concepts, as well as to use wearing gloves to perceive and calculate the weight of geometric bodies with different material densities. The GEO2 AR Geometrical Application with Cognition Awareness was created to address the following key challenges in mathematics Education.

Information on the Book

After the symposium, a book of the contributions of the symposium will be published with Springer Publishing. All speakers (and participants) are warmly welcome to contribute with a paper.

A timeline can be seen below:

Submission of a paper	January 15, 2023
Submission of Reviews	February 15, 2023
Submission of revised papers	March 15, 2023
Publication of the book	Approx. July 2023

Details

Please use the template *Template International Symposium on AR/VR in Mathematics Education* for your contribution, which will be available for download on the homepage after the symposium: https://www.uni-siegen.de/fb6/didaktik/veranstaltungen/symposiumarvr.html

Your article should not exceed 20 pages within the template. All figures and diagrams will be printed as grey scale patterns (in color in the online version), please keep this in mind when creating your article. Please use the following information to submit your article:

Email address: dilling@mathematik.uni-siegen.de

Subject: ARVR_"Title"_"Author(s)"

Further information will be provided at the symposium.