Geomathematics 2013

Workshop Honouring
Willi Freeden's 65th Birthday

15 April – 17 April 2013

in St. Martin/Palatinate, Germany

Organizing Committee: Geomathematics Group Siegen

www.geomathematics2013.de

Fachbereich Mathematik der
TU Kaiserslautern

Monday, 15 April 2013

Chair: V. Michel

- 13:00 Opening
- 13:10 Short Welcoming Speech by Prof. Dr.Dr.h.c. Helmut J. Schmidt, President of the University of Kaiserslautern
- 13:20 Short Welcoming Speech by Prof. Dr. Ralf Korn, on behalf of the Dean of the Department of Mathematics in Kaiserslautern
- 13:30 Volker Michel (Siegen): *The Reason why we are Here*
- 14:00 Willi Freeden (Kaiserslautern): *Essential Principles of Geomathematical Modeling*
- 14:45 Michael Schreiner (Buchs): *Sphere Oriented Approximation — Ideas and Techniques of Willi Freeden*
- **15:30 Coffee Break**

Chair: C. Gerhards

- 16:00 Frederik J. Simons (Princeton): *Maximum-likelihood Estimation of Lithospheric Flexural Rigidity, Initial-loading Fraction, and Load Correlation, Under Isotropy*
- 16:45 Henning Omre (Trondheim): *Generalized Gaussian Random Fields*
- 17:30 Ralf Korn (Kaiserslautern): *Aspects of Stochastic Modelling and Statistics in Applications With a View Towards Geothermics*
- 18:15 Matthias Augustin (Kaiserslautern): *Modeling the Stress Field in Geothermal Reservoirs*
- 18:35 Lutz Roese-Koerner (Bonn): *Convex Optimization Under Inequality Constraints in Rank-deficient Systems*
- **19:00 Dinner**

Tuesday, 16 April 2013

Chair: H. Nutz

- 8:00 Peter Maaß (Bremen): *Mathematical Formulation of Sparsity Models in Data Analysis*
- 8:45 Roger Telschow (Siegen): *Iterative Sparse Approximation of Extremely Scattered Data on the Sphere*
- 9:05 Volker Michel (Siegen): *How Mathematics can Help to Observe Climate Change — An Example*
- 9:25 Judith Schall (Bonn): *Optimisation of Point Grids in Regional Gravity Field Analysis*
- **9:45 Coffee Break**

- 10:15 Zuhair Nashed (Orlando): *Noise Models for Inverse Problems and Moment Discretization* — Chair: M. Schreiner
- 11:00 Helga Nutz (Kaiserslautern): *Multiscale Regularization of Tensorial Satellite Gravity Gradiometry*
- 11:20 Tim Masterlark (South Dakota): *Probing the Magma Chamber of an Active Volcano With InSAR, Seismic Tomography, and Finite Element Models*
- 11:40 Jürgen Kusche (Bonn): *Global Geodetic-geophysical Inverse Problems: Mass Redistribution and Sea Level*
- **12:30 Lunch**
- 13:30 Reiner Rummel (Munich): *GOCE Satellite Gradiometry and the Ocean* — Chair: S. Eberle
- 14:15 Sergei Pereverzyev (Linz): *Multi-penalty Regularization with Component-wise Penalization*
- 15:00 Valeriya Naumova (Linz): *Multi-penalty Regularization with a Possible Application in Geomathematics*
- 15:20 Otmar Scherzer (Vienna): *Optical Flow on Evolving Surfaces*
- **16:05 Coffee Break**
- 16:30 Erik Grafarend (Stuttgart): *The Runge-Walsh(-Krarup) Approximation Theorem in Critical Review and the Regularization of the Improperly Posed Problem of Geodetic Downward Continuation: Minimum Bias Versus Minimum of Mean Square Error* — Chair: M. Gutting
- 17:15 Peiliang Xu (Kyoto): *Regularization With Variance Components and Reduced Bias*
- 17:35 Mikhail Eremin (Tomsk): *Mathematical Model of Geomedium Failure*
- 17:55 Christian Blick (Kaiserslautern): *3D Wavelet Decomposition of Seismic Data*
- 18:15 Christophe Haynes (Paris): *A Sensitivity Analysis of Meteor-generated Infrasound*
- 18:35 Thomas Sonar (Braunschweig): *The Development of the Navigational Arts in Europe*
- 19:20 End of the Session
- **19:30 Banquet With Regional Specialties**

Wednesday, 17 April 2013

Chair: C. Blick

- 8:00 Nils Olsen (Lyngby): *The Magnetic Field — a Tool for Exploring the Earth's Interior*
- 8:45 Christian Gerhards (Sydney): *Multiscale Methods for Geomagnetic Modeling*
- 9:05 Alain Plattner (Princeton): *Source Field Estimation From Satellite Data Using Vectorial Spatiospectrally Concentrated Functions*
- 9:25 Wolfgang Erb (Lübeck): *An Alternative to Slepian Functions on the Unit Sphere — A Time-frequency Analysis Based on Localized Spherical Polynomials*
- **9:45 Coffee Break**

Chair: M. Augustin

- 10:15 Jürgen Prestin (Lübeck): *Quadrature Rules for Scattered Data on Spherical Triangles*
- 11:00 Daniel Potts (Chemnitz): *Fast Algorithms for the Computation of Optimal Quadrature Points on the Sphere*
- 11:20 Martin Gutting (Kaiserslautern): *Fast Multipole Accelerated Spline Approximation*
- 11:40 Stephan Dahlke (Marburg): *Recent Results on Shearlet Coorbit Spaces: An Overview*
- **12:30 Lunch**

Chair: R. Telschow

- 13:30 Andreas Meister (Kassel): *Thermal Fluid Structure Interaction*
- 14:15 Dmitri Kondrashov (UCLA): *Nonlinear Data-driven Reduction by Stochastic Energy-conserving Models With Memory Effects*
- 14:35 Sarah Eberle (Kaiserslautern): *Mathematical Modeling of Forest Fire Spreading*
- 14:55 Volker Michel (Siegen): *Conclusions*
- 15:00 Willi Freeden (Kaiserslautern): *The Final Say*

Abstracts

Monday, 15 April 2013

14:00 - 15:30

Essential Principles of Geomathematical Modeling

Willi Freeden,
Geomathematics Group, University of Kaiserslautern

In accordance with the Kaiserslautern philosophy the essential ingredients of geomathematical modeling can be roughly characterized by the following categorization:

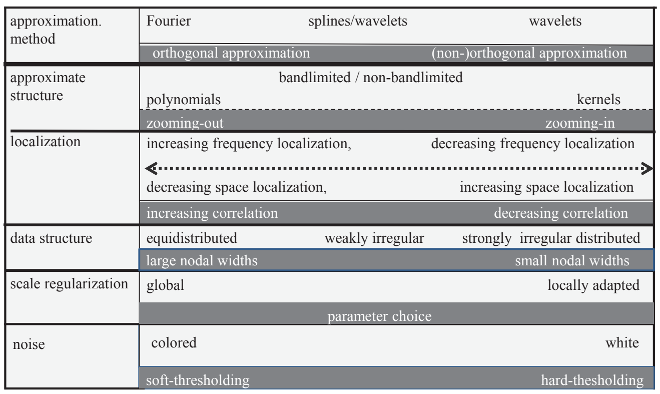

Figure 1: Illustration of geomathematical relationships.

The presentation aims at a more detailed mathematical explanation of the structural components of the scheme (Figure 1).

Sphere oriented Approximation — Ideas and Techniques of Willi Freeden

Michael Schreiner,
University of Applied Sciences Buchs NTB, Institute for Computational Engineering ICE

In honor of Willi Freeden's 65th Birthday, we try to give a summary of his most important ideas and techniques concerning sphere oriented approximation. The topices cover zonal functions on the sphere, spherical splines, pseudo differential operators, wavelets, inverse problems and more.

Monday, 15 April 2013, 16:00-18:55

Maximum-likelihood Estimation of Lithospheric Flexural Rigidity, Initial-loading Fraction, and Load Correlation, Under Isotropy

Frederik J. Simons[1] **and Sofia C. Olhede**[2],
[1] Department of Geosciences & Program in Applied and Computational Mathematics, Princeton University
[2] Department of Statistical Science, University College London

Topography and gravity are geophysical fields whose joint statistical structure derives from interface-loading processes modulated by the underlying mechanics of isostatic and flexural compensation in the shallow lithosphere. Under this dual statistical-mechanistic viewpoint an estimation problem can be formulated where the knowns are topography and gravity and the principal unknown the elastic flexural rigidity of the lithosphere. In the guise of an equivalent "effective elastic thickness", this important, geographically varying, structural parameter has been the subject of many interpretative studies, but precisely how well it is known or how best it can be found from the data, abundant nonetheless, has remained contentious and unresolved throughout the last few decades of dedicated study. The popular methods whereby admittance or coherence, both spectral measures of the relation between gravity and topography, are inverted for the flexural rigidity, have revealed themselves to have insufficient power to independently constrain both it and the additional unknown initial-loading fraction and load-correlation factors, respectively. Solving this extremely ill-posed inversion problem leads to non-uniqueness and is further complicated by practical considerations such as the choice of regularizing data tapers to render the analysis sufficiently selective both in the spatial and spectral domains. Here, we rewrite the problem in a form amenable to maximum-likelihood estimation theory, which we show yields unbiased, minimum-variance estimates of flexural rigidity, initial-loading fraction and load correlation, each of those separably resolved with little *a posteriori* correlation between their estimates. We are also able to separately characterize the isotropic spectral shape of the initial loading processes. Our procedure is well-posed and computationally tractable for the two-interface case. The resulting algorithm is validated by extensive simulations whose behavior is well matched by an analytical theory with numerous tests for its applicability to real-world data examples.

Generalized Gaussian Random Fields

Henning Omre and Kjartan Rimstad,
Department of Mathematical Sciences, Norwegian University of Science and Technology, Trondheim

We present a continuous non-Gaussian random field constructed by a selection mechanism on Gaussian random fields. The random field can capture skewness, multi-modality and to some extend heavy tails in the marginal distribution. The class of Generalized Gaussian random fields is closed under linear operations and conditioning and hence can be used for spatial interpolation. Moreover, the model parameters can be estimated by a maximum likelihood criterion. Although the random field is analytically tractable, numerical schemes must be used to sample from it, and an efficient McMC-based algorithm is defined. The properties of the random field and the simulation algorithm are demonstrated on both synthetic cases and real seismic data from a North Sea hydrocarbon reservoir. Considerable improvement in seismic inversion of the real data set is obtained.

Aspects of Stochastic Modelling and Statistics in Applications With a View towards Geothermics

Ralf Korn,
Dept. Mathematics, University of Kaiserslautern and Fraunhofer ITWM, Kaiserslautern

The use of stochastic models can either be motivated by both intrinsic uncertainty or mathematical convenience. The latter is often used to reduce a high-dimensional to a lower dimensional problem.
While deep knowledge of the physical behaviour of underlying phenomena and dynamics is indispensible when setting up a stochastic model, it is also important to stick to simple principles such as

- do not use too simple models, all available statistical information should be incorporated,
- do not introduce extra uncertainty via a too complex model,
- use only statistical models that tell you a story.

The use of these principles will be highlighted by some simple examples and some first experience from a geothermics project.

Modeling the Stress Field in Geothermal Reservoirs

Matthias Augustin,
Geomathematics Group, University of Kaiserslautern

With the rising demand for energy and the accompanying short running of resources, expanding the use of renewables is inevitable. A very promising source of energy is the heat stored in the earth's crust which is used by so-called geothermal facilities. Scientists from fields like geology, geo-engineering, geophysics and especially geomathematics are challenged to help making geothermics a reliable and safe energy production method. One of this challenges is modeling the mechanical stresses within a reservoir.

This talk will give an insight into stress field simulations. After introducing the basic equations and their relations to more familiar ones (heat equation, Stokes equations, Cauchy-Navier equation), we discuss the so-called method of fundamental solutions and how it can be used in our task. Based on the properties of the fundamental solutions, theoretical results will be established. The talk concludes with some numerical examples to inspire further investigations in the performance of the method and an outlook on further research goals.

Convex Optimization Under Inequality Constraints in Rank-deficient Systems

Lutz Roese-Koerner and Wolf-Dieter Schuh,
Theoretical Geodesy, University of Bonn

Many geodetic applications require the minimization of a convex objective function subject to some linear equality and/or inequality constraints. If we have a singular system (e.g. a geodetic network without a defined datum) there will be a manifold of solutions. Most state-of-the-art algorithms for inequality constrained optimization (e.g the Active-Set-Method or primal-dual Interior-Point-Methods) are either not able to deal with a rank-deficient objective function or yield only one of many particular solutions.

Therefore, in this contribution we develop a framework for the rigorous computation of the general solution of a rank-deficient problem with inequality constraints. We aim for the computation of a reproducible particular solution which fulfills predefined optimality criteria as well as for a representation of the homogenous solution. Additionally, the influence of the constraints on the solution will be quantified using a sensitivity analysis.

In a case study, our theoretical findings are applied to a real-life scenario to demonstrate the potential of the proposed framework.

Tuesday, 16 April 2013

8:00 - 9:45

Mathematical Foundation of Sparsity Models in Data Analysis

Peter Maaß,
University of Bremen

to be announced

Iterative Sparse Approximation of Extremely Scattered Data on the Sphere

Roger Telschow,
Geomathematics Group, University of Siegen

Recent applications often produce highly non-equidistributed data on the sphere. The distribution of such scattered data sets causes several problems which are hardly solvable with the established approximation methods. Whereas the expansion of the signal in a global basis fails for obvious reasons, the use of localized basis functions, such as spline bases, can also be connected to severe numerical drawbacks. In case of extremely scattered data, such bases most often yield highly ill-conditioned systems of equations. These systems demand a strong regularization which mostly results in a severe lack of accuracy. We present a novel algorithm based on an orthogonal matching pursuit which iteratively chooses the set of basis functions out of a large redundant dictionary to best match the signal. The outcome is a sparse and smooth solution whereas the expansion may combine different types of spherical trial functions. Particulary, we use spherical harmonics to reconstruct global trends as well as localized trial functions, such as the Abel-Poisson kernel, to represent more detailed structures of the signal. Moreover, the smoothness is controlled with a certain spherical Sobolev norm and the solution is adapted to the detail structure of the signal as well as to the data density. Numerical experiments are presented.

How Mathematics can Help to Observe Climate Change — An Example

Volker Michel,
Geomathematics Group, University of Siegen, Germany

The "Mathematics of Planet Earth 2013" initiative focusses on mathematical methods that enable us to solve the manifold scientific problems that are somehow linked to the Earth. One out of many problems of this kind is an accurate observation of climatic effects such as droughts, floods, El Niño effects, etc. As an example of benefits from mathematical innovations, we show that a novel mathematical method, the Regularized Functional Matching Pursuit (RFMP), yields a highly accurate reconstruction of water mass transports e.g. in the Amazon area from gravity data of the satellite mission GRACE. For instance, seasonal differences in the precipitation can be visualized and droughts respectively floods of the last decade can be resolved better.

The mathematical background is as follows: We have to solve an ill-posed inverse problem given by a Fredholm integral equation of the first kind, where large data sets are possible. A regularized version of a greedy algorithm is used to iteratively construct a solution of the inverse problem. This solution is combined from global trial functions (orthogonal polynomials) and localized trial functions (reproducing kernel based spline basis functions). Without the use of a priori information, the algorithm primarily uses localized basis functions in areas with a high detail structure. Hence, the obtained solution is sparse in the sense that essentially less trial functions than available are chosen by the algorithm for the computation of the solution.

References:

D. Fischer, V. Michel: *Sparse regularization of inverse gravimetry — case study: spatial and temporal mass variations in South America*, Inverse Problems, **28** (2012), 065012 (34pp).

D. Fischer, V. Michel: *Inverting GRACE gravity data for local climate effects*, Siegen Preprints on Geomathematics, **9**, 2012.

D. Fischer, V. Michel: *Automatic best-basis selection for geophysical tomographic inverse problems*, Geophysical Journal International, accepted for publication, 2013.

Optimisation of Point Grids in Regional Gravity Field Analysis

J. Schall, A. Eicker and J. Kusche,
Astronomical, Physical and Mathematical Geodesy, University of Bonn

In contrast to spherical harmonics, which realise global uniform resolution, regional basis functions have the advantage to be adaptable to the local data density and variability. But how to choose adequately their spatial configuration? Estimating the location of basis functions represents a non-linear problem, which is difficult to solve by ordinary statistical tools, especially when treating their number as an unknown too. For that reason, most of the regional approximation approaches define the point grid in advance.

Here, we tackle the question of how to globally optimise the number and location of regional basis functions jointly with their scaling coefficients. We propose a random sampling algorithm formulated in the context of Bayesian statistics as a practical solution for the problem. In contrast to the usual approach of defining a dense nodal point grid and using regularisation techniques to encourage simple models, we let the data themselves decide on the model resolution. Thus overparametrisation can be avoided and a more stable solution is achieved.

The method is analysed theoretically, followed by a demonstration of its applicability and performance by examples from the field of regional gravity field recovery from GOCE satellite observations.

Tuesday, 16 April 2013, 10:15 - 12:30

Noise Models for Inverse Problems and Moment Discretization

Zuhair Nashed,
Department of Mathematics, University of Central Florida, Orlando

The standard view of noise in ill-posed inverse problems is that it is either deterministic and small (strongly bounded noise) or random and large (not necessarily small). A new noise model was recently proposed and investigated by Eggermont et al. (see [1] and [2]), wherein the noise is weakly bounded. Roughly speaking, this means that the "local averages" of the noise are small. In this talk we describe the mathematical setting of this approach and give a precise formulation in a Hilbert space setting. We summarize an analysis of Tikhonov regularization of ill-posed problems with weakly bounded noise, and show that the noise model is particularly appropriate in the regularization of an operator equation or a variational inequality by perturbation (often called Lavrentiev regularization). The setting includes potential and quasi-potential operators, inverse monotone operators, and moment discretization problems, such as those that arise in geophysical and other inverse problemswhen the data are available at a discrete set of points.

[1] P. P. B. Eggermont, V. N. LaRiiccia and M. Z. Nashed, On weakly bounded noise in ill-posed problems, Inverse Problems 25 (2009): 115018 (14 pages).
[2] —, Noise Models for Ill-Posed Problems, in "Handbook of Geomathematics"

(W. Freeden, M. Z. Nashed and T. Sonar, Eds.", pages 741-762, Springer-Verlag, 2010.
[3] —, Moment discretization for ill-posed problems with discrete weakly bounded noise, GEM: International Journal on Geomathematics, 3 (2012), published on-line: 9 April 2012.

Multiscale Regularization of Tensorial Satellite Gravity Gradiometry

Helga Nutz,
Geomathematics Group, University of Kaiserslautern

In case of satellite gravity gradiometry (SGG) we have to deal mathematically with second order derivatives, preferably with the whole Hesse tensor, of the gravitational potential given at satellite's hight. In the context of inverse problems of mathematical geodesy, the calculation of the gravitational potential at the Earth's surface from its second order derivatives at satellite's height turns out to be exponentially ill–posed. A wavelet–based regularization method is proposed for tensorial data to overcome the calamities of the ill-posedness.

Probing the Magma Chamber of an Active Volcano With InSAR, Seismic Tomography, and Finite Element Models

Tim Masterlark,
South Dakota School of Mines

The migration of magma within an active volcano produces a deformation signature at the Earths surface, which can be mapped with interferometric synthetic aperture radar (InSAR). Quantifying magma migration in space and time is important, because the upward migration is a precursor to eruptive activity. Forward models predict the deformation of a volcano, based on known parameters that characterize the magmatic behavior within a given internal structure. Unfortunately, we face the far more challenging prospect of developing inverse models to estimate the controlling parameters that characterize the behavior of the magma at depth, based on InSAR-observed deformation. As such, models that simulate a volcano's response to magmatic loading provide the critical link between what we can directly observe (surface deformation) and what we want to know (magmatic migration at depth). I will present finite element models (FEMs) that integrate seismic tomography and inverse analyses of InSAR in an effort to probe the geometry of the magma chamber and magnitude of magma extraction during an eruption of Okmok volcano, Alaska. This integration of

multiple types of data into the FEMs provides a clearer image of magma migration, compared to customary interpretations that are based on a single type of data. For example, the estimated depth of the magma chamber for Okmok volcano, based on InSAR alone and standard half-space models, is about 3000 meters beneath the caldera floor. Alternatively, an FEM that accounts for the InSAR, as well as an internal distribution of material properties suggested by seismic tomography, indicates that the depth of the magma chamber is about 4000 meters beneath the caldera floor. These two estimates are very different, considering that the respective uncertainties are less than +/-100 meters. The deeper estimate, which accounts for both geodetic observations and seismic tomography, is consistent very long period tremor observations and lithostatic constraints that independently verify the deeper magma chamber configuration.

Global Geodetic-geophysical Inverse Problems: Mass Redistribution and Sea Level

Jürgen Kusche,
University of Bonn

The analysis of the Earth's time-variable gravity field, and its changing land and ocean surface geometry, plays an important role in Earth system research. Space-geodetic observables provide, for the first time, a measurement of the amount of mass that is redistributed at or near the surface of the Earth by oceanic and atmospheric circulation and through the hydrological cycle. In this contribution, we will revisit standard approaches and new avenues to solve the inverse problem of finding time-variable mass distributions consistent with the data and with the physics of a fluid redistributing on an elastic rotating Earth.

Tuesday, 16 April 2013, 13:30 - 16:05

GOCE Satellite Gradiometry and the Ocean

Reiner Rummel, Weiyong Yi and Alberta Albertella,
Astronomical and Physical Geodesy, Technische Universität München

The gravitational tensor consists of the nine second derivatives of the gravitational potential, usually expressed a local orthonormal coordinate system. GOCE is the first satellite equipped with a gravitational gradiometer; it measures gravitational acceleration differences in various combinations along the baselines of the three axes of the instrument. The objective of the mission is

the determination of the earth's gravitational field and geoid with great accuracy and spatial detail. The principle of gradiometry is applied, because of its spectral properties. It amplifies the signal content of the shorter spatial wavelengths relative to the long wavelength features. This is of particular importance at satellite altitude where due to the distance from the earth the short scale gravitational signal is highly damped. Unfortunately, because of the nature of the gradiometer instrument and due to the noise level of the various measured accelerations not all theoretical properties of the gravitational tensor can be exploited. Still, GOCE delivers the best and most detailed global gravity field and geoid. The geoid is used in oceanography for the determination of mean dynamic ocean topography and the global ocean geostrophic velocity field. In particular, the accuracy of geostrophic velocities benefit from the spectral properties of GOCE gravitational gradiometry.

Multi-penalty Regularization With a Component-wise Penalization

Valeriya Naumova and Sergei Pereverzyev,
Johann Radon Institute for Computational and Applied Mathematics (RICAM), Austrian Academy of Sciences

We are going to discuss a new regularization scheme for reconstructing the solution x_0 of a linear ill-posed operator equation $Ax = y$ from given noisy data y_δ in the Hilbert space setting. In this new scheme, the regularized approximation x_δ is decomposed into several components $u_i = u_{i,\delta},\ i = 1, 2, \ldots, m$, which are defined by minimizing a multi-penalty functional

$$\Phi_{\alpha_1,\alpha_2,\ldots,\alpha_m}(u_1, u_2, \ldots, u_m) := \left\| \sum_{i=1}^{m} Au_i - y_\delta \right\|^2 + \sum_{i=1}^{m} \alpha_i \|B_i u_i\|^2,$$

where B_i are some self-adjoint and positive definite operators. We show theoretically and numerically that under a proper choice of the regularization parameters $\alpha_1, \alpha_2, \ldots, \alpha_m$, the regularized approximation $x_\delta = \sum_{i=1}^{m} u_{i,\delta}$ exhibits the so-called compensatory property, in the sense that it performs similar to the best of the single-penalty regularization $x_{\alpha_i} = \arg\min\{\|Ax - y_\delta\|^2 + \alpha_i \|B_i x\|^2\}$.

Multi-penalty Regularization With a Possible Application in Geomathematics

Valeriya Naumova and Sergei Pereverzyev,
Johann Radon Institute for Computational and Applied Mathematics (RICAM), Austrian Academy of Sciences

It is well-known that a reconstruction of the gravitational potential from spaceborne data is an ill-posed problem and it should be treated by means of special regularization methods. Thus, in order to deal with this reconstruction problem, many schemes were proposed. Recently, the attention moved from the single-penalty regularization such as Tikhonov or Tikhonov-Phillips to the the multi-penalty regularization. A new generation of the multi-penalty regularization methods was discussed in [1, 2] for the determination of gravity potential from satellite tracking measurements.

In this talk we continue to consider the developments in this area, by introducing the new multi-penalty regularization scheme that exhibits the compensatory properties. In general, it means that the method performs similar to the best single-penalty regularization without requiring an additional knowledge such as a fulfillment of the so-called link conditions as it could be the case, for example, for the Tikhonov method.

We provide the theoretical justification of the proposed method, present the results of the numerical tests with synthetic data, and argue that the presented scheme could be of a potential interest for addressing the problems of satellite data processing.

1. Lu, S.; Pereverzev, S. V. Multiparameter Regularization in Downward Continuation of Satellite Data in *Handbook of Geomathematics*, ed. W. Freeden, M.Z. Nashed, T. Sonar, Springer-Verlag Berlin Heidelberg (2010), 814–831.
2. Xu, P.; Fukuda, Y.; Liu, Y. *Multiple parameter regularization: numerical solutions and applications to the determination of geopotential from precise satellite orbits*, J Geod 80 (2006), 17-27

Optical Flow on Evolving Surfaces

Otmar Scherzer,
Computational Science Center, University of Vienna

Optical flow is traditionally computed from a sequence of flat images. We extend the concept of optical flow to a dynamic non-Euclidean setting. It is the purpose of this talk to introduce variational motion estimation for images that are defined on an evolving surface. This is joint work with Lukas Lang and Celemens Kirisits (Univ. of Vienna)

Tuesday, 16 April 2013, 16:30 - 19:20

The Runge-Walsh(-Krarup) Approximation Theorem in Critical Review and the Regularization of the Improperly Posed Problem of Geodetic Downward Continuation: Minimum Bias Versus Minimum of Mean Square Error

Erik Grafarend,
University of Stuttgart

"The Runge-Walsh Approximation Theorem is applied in nearly all geodetic contributions. It has been brought up to the geodetic attention by Torben Krarup (Danish mathematician working in Geodesy) , Helmut Moritz (Austrian Geodesist: stochastic collocation) and Arne Bjerhammar (Swedish Geodesist, creator of the Bjerhammar Sphere). Willi Freeden very often refered to the Runge-Walsh Approximation Theorem , for instance in their book "Multiscale Potential Theory with Applications in Geoscience", W. Freeden and V. Michel, Birkhaeuser Verlag , Boston-Basel-Berlin 2004, in particular pages 127-155. In their contribution "Wavelet Approximation on Closed Surfaces ..." , Math.Meth.Appr.Sci. 21(1998) 129-163 by W.Freeden and F.Schneider, they list four conditions for a regular surface. Condition (iii) is the definition of a closed and compact surface "free of double points", a condition which does NOT apply to the REAL Earth Surface. An equivalent statement of the condition (iii) is : The mapping of a surface onto a sphere or onto an ellipsoid must be on-to-one. Here we show contrary examples and take reference to F.Sanso (1982) and O.Colombo(1983).

The second part is an introduction into the celebrated Slice Technique, here applied to the gravitational potential of three zones/ domains of type "external, topographic and internal". A series expansion of the topographic potential with respect to the ration h/R in terms of a multilayer representation (single, double layers etc) is presented.

Third, special attention is given to the estimation of the optimal Tykhonov-Phillips regularization parameter Lamda, namely treating the Improperly Posed Problem of Downward Continuation of Gravity Functionals , called Lamda-weighted BLE (Best Linear Estimation) . This is a special technique to balance BIAS and Minimum Mean Square Error in designing such an optimization problem.

Regularization With Variance Components and Reduced Bias

Yunzhong Shen[1] **and Peiliang Xu**[2],
[1] Tongji University, School of Geodesy and GeoInformation,
[2] Kyoto University, Disaster Prevention Research Institute

Ill-posed problems are solved by regularization with different types of data. In order to correctly determine the weighting factors for data from different sources, we have to simultaneously estimate the regularization parameter(s) and the unknown variance components. From the frequentist's point of view, a regularized solution is well-known to be biased. Although the biases of the estimated parameters can only be computed with the true values of parameters, we attempt to improve the regularized solution by using the regularized solution itself to replace the true (unknown) parameters for estimating the biases and then removing the computed biases from the regularized solution. We first analyze the theoretical relationship between the regularized solutions with and without the bias correction, derive the analytical conditions under which a bias-corrected regularized solution performs better than the ordinary regularized solution in terms of mean squared errors (MSE) and design the corresponding method to partially correct the biases.We then present two numerical examples to demonstrate the performance of our partially bias-corrected regularization method. The first example is mathematical with a Fredholm integral equation of the first kind. The simulated results show that the partially bias-corrected regularized solution can improve the MSE of the ordinary regularized function by 11%. In the second example, we recover gravity anomalies from simulated gravity gradient observations. In this case, our method produces the mean MSE of 3.71 mGal for the resolved mean gravity anomalies, which is better than that from the regularized solution without bias correction by 5%. The method is also shown to successfully reduce the absolute maximum bias from 13.6 to 6.8 mGal.

Mathematical Model of Geomedium Failure

P.V. Makarov and M.O. Eremin,
Institute of Strength Physics and Material Science Siberian branch of Russian Academy of Science

Studying the geological media with the purpose of its behavior prognosis is impossible without building the adequate mathematical models. In the present talk the problem of building the mathematical model is solved on the basis of mathematical theory of evolution of loaded solids and media [1]. Researches concerning the mechanical behavior of geomaterials and geomedia have been carried out for a long period of time as the physical modeling on uni-axial and multi-axial loading of geomaterials and field expeditions. The great bases of material properties are obtained during the physical modeling; however these results could not be applied to estimate for example strength of the geomedia because of the scale factor and appropriate accounting of gravitation. That is why the numerical simulation of mechanical behavior of loaded geomaterials and geomedia is very important for geosciences now. In the early 50s by Drucker and Prager it was shown [2] that it is very important to consider the processes of internal friction and dilatation in geomedia; however the associated flow-law proposed in this paper gave to high estimation of volumetric (dilatation) strain. That is why in paper [3] Nikolaevskiy V.N. and Garagash I.A. proposed a modification of Drucker-Prager constitutive model with a non-associated flow-law which became suitable for the description of volumetric (dilatation) strain as it gave the mean of dilatation close to the ones obtained in experiments. We use this modification as the basis for description of mechanical behavior of geomedia at its inelastic deforming. The classical approaches use the limit design which means that the geomedia failure occurs when the critical mean of loading force is reached. This approach gives good results for engineering however we cannot say anything about the failure process in geomedia especially about its prediction. That is why in the present talk the processes of inelastic deforming and failure are considered as the common evolutional process of geomedia destruction in the fields of operating forces. To descript this common process the approach of theory of damaged media is applied. The damage function depends on the mean of accumulated inelastic deformation, kind of the stress state calculated by the Lode-Nadai parameter and time. Introduction of the damage function into the constitutive model doesnt ruin the Adamar principle and there is no need to set the limit state of geomedia as it is formed during the loading and evolutional changes of geomedia in the field of operating forces. The system of equations of continuum mechanics in this approach represents the modified equations of elasto-plasticity which are solved with the second order accuracy scheme which is presented in details in paper [4]. As the main fundamental results of numerical simulation within the proposed model it is shown that the evolution of loaded geomaterials and geomedia corresponds to the fundamental laws of evolution of nonlinear dynamic systems (there is a quasi-stationary phase of evolution and a blow-up regime according to the terminology of Kurdyumov S.P. [5]). The frequency-dimensional distribution corresponds to the Guttenberg-Richter law. For the first time the migration of deformational activity is obtained in numerical simulation.

Literature:

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5. Kurdyumov S. P. Blow-up regimes. The evolution of the idea / editor G.G. Malineckiy. 2nd edition. M: FIZMATLIT, 2006. 312 p.

3D Wavelet Decomposition of Seismic Data

Christian Blick,
University of Kaiserslautern

In hydrothermal geothermic exploration, the search for water-bearing layers is realized by the interpretation of migration results of a seismic. The aim is to find a layer with an as high as possible water output and temperature. This data is, based on the nature of the signal and the faulting of the bedrock, difficult to interpret. In order to simplify the interpretation and hence lessening of the risk of imprecise or even wrong insights about the structural composition of the bedrock, we developed 3D-Wavelets, witch decorrelate the migration results. The method was already tested successfully in the 2D-case. Dependent on the choice of parameter, certain rock formations could be inhibited or pronounced, which simplifies the interpretation. First results regarding 3D-data shall be presented in this talk.

A Sensitivity Analysis of Meteor-generated Infrasound

C. P. Haynes[1] **and C. Millet**[2],
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In recent years, numerous bolide sources have been detected by the IMS infrasound arrays. Even though a variety of waveform data may be extracted from recorded signals, only a few parameters are used throughout infrasound research, the most common being the arrival time of signals.

A majority of results are obtained by employing energy estimates based on semi-empirical relations which possess errors. The justification of such estimates is questionable given that no analysis of the significant error has been performed. This is mainly due to the lack of models that are able to analyze the non-linear propagation of the waveform near the source right through to the infrasound station; given that an object entering the atmosphere generates a strong shockwave that decays to a weak shock front which propagates in this way until it arrives at the receiver, creating propagation models that accurately predict the waveform's characteristics are key to further understanding meteoric events.

In our goal of understanding how the dynamical and structural properties of a meteor (during atmospheric entry) affect the resulting emitted shockwave, we have developed a new propagation model and have performed a full sensitivity analysis of all unknown parameters. This model is applied to the controversial Carancas meteor event in 2007; a stony meteorite that formed an eleven-meter crater. Studies employing the standard energy estimate approach using recorded infrasound signals obtained at I08BO claim that the meteor fragmented mid-fall. We show that through variation of the input parameters it is possible to obtain alternative conclusions.

The core results of the Carancas meteor study demonstrate that the variation in the trajectory, entry angle, initial velocity, diameter of the meteor as well as the atmospheric environment completely govern the variation within the emitted waveform, whereas the variation of the density and drag coefficient of the meteor have very little effect on the waveform. Moreover, we show how the dominant input factors change over the parameter space, thereby leading to new advanced waveform data which consequently improves the characterization of the meteor.

The Development of the Navigational Arts in Europe

Thomas Sonar,
TU Braunschweig

We review the history of astronavigtion as it emerged from Portugal in the 15th century and then concentrate on the development of declination tables of the sun necessary to compute latitude. The art of the Portuguese was taken up by the Spanish and a tradition of 'almananacs' started to developed. We investigate in detail the tables given in Martin Cortés *Arte de navigar* which was translated into English soon after its first publication 1551. They can be traced back directly to the tables of Pedro Nunes. In only one generation after Cortés the English developed the tables into a form we know today. This is joint work with Matthieu Husson.

Wednesday, 17 April 2013

8:00 - 9:45

The Magnetic Field – a Tool for Exploring the Earth's Interior

Nils Olsen,
DTU Space, Kgs. Lyngby, Denmark

The Earth has a large and complicated magnetic field, the major part of which is produced by a self-sustaining dynamo operating in the fluid outer core. Magnetic field observations provide one of the few tools for remote sensing the Earth's deep interior, especially regarding the dynamics of the fluid flow at the top of the core. Also the electrical conductivity of the Earth's mantle, and geological processes in the Earth's crust, can be studied by means of the magnetic field.

However, what is measured at or near the surface of the Earth is the superposition of sources in the core, in the crust, by electric currents flowing in the ionosphere, magnetosphere and oceans, and by currents induced in the Earth by time-varying external fields. These sources have their specific characteristics in terms of spatial and temporal variations, and their proper separation, based on magnetic field observations, is a major challenge.

The talk will discuss approaches for separating the various sources based on magnetic field observations taken on ground and in space.

Multiscale Methods for Geomagnetic Modeling

Christian Gerhards,
University of New South Wales

In this talk we give a brief overview on the application of multiscale methods (as they have been developed at the Geomathematics Group in Kaiserslautern) to the modeling of the Earth's magnetic field. Particular focus is set on the crustal magnetic field. We indicate how scaling and wavelet kernels can be constructed to separate interior from exterior sources (including applications to real satellite data) and how they can be optimized in a certain sense to combine satellite and ground data.

Source Field Estimation from Satellite Data Using Vectorial Spatiospectrally Concentrated Functions

Alain Plattner[1] **and Frederik Simons**[1,2],
[1] Department of Geosciences, Princeton University, Princeton, NJ, USA
[2] Program in Applied and Computational Mathematics, Princeton University, Princeton, NJ, USA

In order to estimate the source magnetic field on Earth's surface from satellite magnetic field measurements at altitude we need to invert an exterior Dirichlet problem. This is classically done using a spherical-harmonic representation of the radial gradient component at altitude and then downward continuing the spherical-harmonic coefficients. The condition number of this downward continuation grows exponentially with the bandlimit of the representation of the field hence bandlimitation is of utmost importance. At the same time it would be beneficial to be able to estimate the field locally be it as a means of regularization in a specifically targeted investigation or in order to discern between areas of intrinsically different properties of the field, such as for example the crustal versus oceanic field. Unfortunately, a field can not be simultaneously band- and spacelimited. We can, however, construct functions that are bandlimited but spatially concentrated in an arbitrary geographical region by solving an energy concentration optimization problem. The resulting basis of so-called Slepian functions serves a dual purpose as the eigenfunctions of a bandlimited regional approximation problem. Scalar Slepian functions has proven to be useful in a wide range of fields including geodesy, gravimetry, geomagnetism, and geodynamics. In order to make use of all three components of the vectorial data set we recently developed vectorial Slepian functions. In this presentation we apply vectorial Slepian functions in an estimation scheme for the regional crustal magnetic field from vectorial data at satellite altitude. We propose three different ways of combining the spatiospectral concentration problem with downward continuation: Concentration at satellite altitude, concentration on Earth's surface, and estimation using Slepian functions that take the continuation into account. For the last estimation procedure the Slepian functions are constructed by finding the eigenfunctions of a bandlimited regional continuation and approximation problem.

An Alternative to Slepian Functions on the Unit Sphere - A Time-frequency Analysis Based on Localized Spherical Polynomials

Wolfgang Erb,
University of Lübeck

The aim of this talk is to present a time-frequency theory for spherical harmonics that runs parallel to the time-frequency analysis of bandlimited functions

developed by Landau, Pollak and Slepian. For this purpose, the spectral decomposition of a particular band-space-band limiting operator is studied. The spectral decomposition and the eigenvalues of this operator are closely linked to the theory of orthogonal polynomials on the one hand, and to the theory of Slepian functions on the unit sphere on the other. Results from both theories can be used to prove localization and approximation properties of the corresponding eigenfunctions. Moreover, particular weak limits related to the structure of the spherical harmonics provide information on the proportion of eigenfunctions needed to approximate localized functions on the unit sphere. Finally, a fast algorithm for the computation of the coefficients in the new localized basis is presented

Wednesday, 17 April 2013, 10:15 - 12:30

Quadrature Rules for Scattered Data on Spherical Triangles

Jürgen Prestin,
University of Lübeck

In this talk we present the construction of quadrature rules on arbitrary triangulations of the sphere which are exact for polynomials of some fixed degree. In the first part we study an explicit construction of quadrature rules exact for integrating multivariate trigonometric polynomials of a given coordinatewise degree on a spherical triangle. The theory is presented in the more general setting of quadrature formulas on a compact subset of the unit hypersphere, $\mathbb{S}^q$, embedded in the Euclidean space $\mathbb{R}^{q+1}$. The number of points at which the polynomials are sampled is commensurate with the dimension of the polynomial space. In a second part we apply Cholesky decomposition methods to obtain the weights for scattered data. For our numerical tests we use Mathematica where all calculations are carried out in high accuracy or even with exact numbers. Thus, we are able to overcome a lot of instability problems particularly for very small and thin triangles. Finally, we compare our local quadrature rules on triangulations and some small polynomial degree of exactness with global formulas on the whole sphere and a high degree of polynomial exactness. Particularly for clustered data the local methods seem to be better. This is joint work with Judith Beckmann (University of Lübeck) and Hrushikesh N. Mhaskar (California State University).

Fast Algorithms for the Computation of Optimal Quadrature Points on the Sphere

Daniel Potts,
Chemnitz University of Technology

Distributing points on the unit sphere $\mathbb{S}^2$ in the Euclidean space $\mathbb{R}^3$ in some optimal sense is a challenging problem. In this talk, we construct points $\mathcal{X}_M = \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_M\} \subset \mathbb{S}^2$ and weights $w_i \in \mathbb{R}$ such that

$$\int_{\mathbb{S}^2} p(\boldsymbol{x}) \mathrm{d}\mu_{\mathbb{S}^2}(\boldsymbol{x}) = \sum_{i=1}^{M} w_i \, p(\boldsymbol{x}_i), \qquad \text{for all } p \in \Pi^N(\mathbb{S}^2),$$

where $\mu_{\mathbb{S}^2}$ is the surface measure on $\mathbb{S}^2$ and $\Pi^N(\mathbb{S}^2)$ is the space of all spherical polynomials with degree at most N. Furthermore we discuss point sets with equal weights $w_i = \frac{4\pi}{M}$, $(i = 1, \ldots, M)$ called spherical designs. In the Hilbert space $\Pi^N(\mathbb{S}^2)$ with standard inner product the worst case quadrature error for the point set $\mathcal{X}_M$ and weights $\boldsymbol{w} = (w_1, \ldots, w_M) \in \mathbb{R}^M$ is defined by

$$E_N(\mathcal{X}_M, \boldsymbol{w}) := \sup_{\substack{p \in \Pi^N(\mathbb{S}^2) \\ \|p\|_2 \le 1}} \left| \int_{\mathbb{S}^2} p(\boldsymbol{x}) \mathrm{d}\mu_{\mathbb{S}^2}(\boldsymbol{x}) - \sum_{i=1}^{M} w_i \, p(\boldsymbol{x}_i) \right| .$$

We present optimization algorithms on the sphere for attacking this highly nonlinear and nonconvex minimization problem. The proposed methods make use of fast spherical Fourier transforms, which where already successfully applied in for solving high dimensional linear equation systems on the sphere.

This contribution is joint work with Manuel Gräf (Chemnitz University of Technology, Germany).

Fast Multipole Accelerated Spline Approximation

Martin Gutting,
Geomathematics Group, University of Kaiserslautern

The Runge-Walsh approximation, i.e. an approximation by trial functions that have a (slightly) larger domain of harmonicity, allows the construction of the solution in terms of harmonic splines. The approximation method requires a so-called Runge sphere to be situated inside the boundary surface. Harmonic splines which consist of reproducing kernels on the exterior of the Runge sphere also possess the Runge-Walsh approximation property.
Fast multipole methods are developed for some cases of the occurring kernels to obtain a fast matrix-vector multiplication. The main idea of the fast multipole algorithm consists of a hierarchical decomposition of the computational domain into cubes and a kernel approximation for the more distant points. The kernel

evaluation is performed directly only for points in neighbouring cubes on the finest level. Elsewhere the contributions of the points are transferred into a set of coefficients. The kernel approximation in terms of inner and outer harmonics is applied on the coarsest possible level using translations of these coefficients. This reduces the numerical effort of the matrix-vector multiplication from quadratic to linear in reference to the number of points for a prescribed accuracy of the kernel approximation.
The application of the fast multipole method to approximating splines that also allow the treatment of noisy data is presented with regard to regional modeling of the gravitational field of the Earth taking into account the Earth's topography. Finally, the solution of the oblique boundary value problem of potential theory is considered where the boundary is the known surface of the Earth itself.

Recent Results on Shearlet Coorbit Spaces: An Overview

Stephan Dahlke,
University of Marburg

In the context of directional signal analysis several approaches have been suggested such as ridgelets, curvelets, contourlets, shearlets and many others. Among all these approaches, the shearlet transform is outstanding because it is related to group theory, i.e., this transform can be derived from a square-integrable representation the so-called shearlet group. Therefore, in the context of the shearlet transform all the powerful tools of group representation theory can be exploited. Moreover, there is a very natural link to another useful concept, namely the coorbit space theory introduced by Feichtinger and Groechenig in a series of papers. By means of the coorbit space theory, it is possible to derive in a very natural way scales of smoothness spaces associated with the group representation. In this setting, the smoothness of functions is measured by the decay of the associated shearlet transform. Moreover, by a tricky discretization of the representation, it is possible to obtain Banach frames for these smoothness spaces. Once these new shearlet smoothness spaces are established some natural questions arise.

- How do these spaces really look like?
- Are there 'nice' sets of functions that are dense in these spaces?
- What are the relations to classical smoothness spaces such as Besov spaces?
- What are the associated trace spaces?

We show that for natural subclasses of shearlet coorbit spaces which correspond to 'shearlets on the cone', there exist embeddings into homogeneous Besov spaces and that for the same subclasses, the traces onto the coordinate axis can again be identified with homogeneous Besov spaces or shearlet coorbit spaces. To prove our results we use compactly supported shearlets and apply atomic characterizations of Besov spaces as well as molecular descriptions of coorbit spaces.

Wednesday, 17 April 2013, 13:30 - 14:35

Thermal Fluid Structure Interaction

Andreas Meister[1], **Philipp Birken**[1], **Tobias Gleim**[2] **and Detlef Kuhl**[2],
[1] University of Kassel, Institute of Mathematics,
[2] Institute of Mechanics and Dynamics

We consider thermal fluid structure interaction to model industrial gas quenching in steel forging, where hot steel is cooled using cold high pressured gas. This allows to define properties of the finished steel part, as for example yield strength, locally at low cost and without environmental problems.

For the numerical simulation, a partitioned approach via a Dirichlet-Neumann coupling and a fixed point iteration is employed. In time, previously developed efficient time adaptive higher order time integration schemes are used. The respective models are the compressible Navier-Stokes equations and the nonlinear heat equation, where the parameter functions are obtained from measurements on a specific steel. Whereas the discretization of the heat equation is enforced by a finite element method, the compressible Navier-Stokes equations are solved by means of a finite volume scheme which enable the simulation of both low and high Mach number flows.

Concerning the coupling, the use of different vector extrapolation methods for convergence acceleration techniques of the fixed point iteration is analyzed. In particular, Aitken relaxation, minimal polynomial extrapolation (MPE) and reduced rank extrapolation (RRE) are considered.

Nonlinear Data-driven Reduction by Stochastic Energy-conserving Models With Memory Effects

Dmitri Kondrashov,
University of California, Los Angeles

Comprehensive dynamical climate models aim at simulating past, present and future climate and, more recently, at predicting it. These models, commonly known as general circulation models or global climate models (GCMs) represent a broad range of time and space scales and use a state vector that has many millions of degrees of freedom. Considerable work, both theoretical and data-based, has shown that much of the observed climate variability can be represented with a substantially smaller number of degrees of freedom.

While detailed weather prediction out to a few days requires high numerical resolution, it is fairly clear that the dimension of the phase space in which a major fraction of climate variance can be predicted is likely to be much smaller. Low-dimensional models (LDMs) can simulate and predict that variability provided they are able to account for (i) linear and nonlinear interactions between the resolved high-variance climate components; and (ii) the interactions between the resolved components and the huge number of unresolved ones.

Here we will present applications of a particular data-driven LDM approach, namely energy-conserving formulation of empirical model reduction (EMR). As an operational methodology, EMR attempts to construct a low-order nonlinear system of multi-level prognostic equations driven by stochastic forcing, and to estimate both the dynamical operator and the properties of the driving noise directly from observations or from a high-order models simulation. The multi-level EMR structure for modeling the noise allows one to capture feedback between high- and low-frequency components of the variability, thus parameterizing the "fast" scales – often referred to as the "noise" – in terms of the memory of the "slow" scales, the "signal."

EMR already proved to be highly competitive for real-time ENSO prediction among state-of-the art dynamical and statistical models. New opportunities for EMR prediction will be illustrated in the framework of "Past Noise Forecasting", by utilizing on the one hand EMR-estimated history of the driving noise, and on the other hand the phase of low-frequency variability estimated by advanced time series analysis.

Mathematical Modeling of Forest Fire Spreading

Sarah Eberle,
Geomathematics Group, University of Kaiserslautern

Due to climate change, more and more woodlands will be endangered by forest fires in the future. Because of this, it is very important to get information about how forest fires expand to enhance prevention concerning forest protection and firefighting on a local level.

To model the complex processes of forest fire spreading, we will apply a physical model which considers heat transfer mechanisms (see e.g. Mandel et al., 2006). This leads us to a convection-diffusion-reaction-problem which is time-dependent and non-linear. Furthermore, we will have a look at the different parameters in the equations and their visualization in the form of maps. We will give a short introduction to the theoretical background of the numerical methods which are used for space and time discretization. Finally, we will present some simulations of forest fire spreading for different settings.