UNIVERSITÄT SIEGEN

Siegen, den 09.08.2010

Vorträge

SAMSON (Siegen Center of Analysis, Modelling, Simulation, Optimization and Numerics) und die **AG Geomathematik** laden hiermit alle Interessierten zu den folgenden beiden Vorträgen ein, die im Rahmen des Oberseminars Geomathematik

am Donnerstag, den 19. August 2010, 10:00 s.t.

in Raum ENC-B 205

stattfinden:

- **Prof. Dr. Frederik J. Simons (Princeton University):**

  „Spectral-domain maximum-likelihood methods for the estimation of the parameters of and the input to non-linear differential equations from noisy observations of their output made on irregular geographical domains. Or: How strong is the Earth's lithosphere really? A statistical perspective from gravity/topography data"

- **Dipl.-Math. Doreen Fischer (Universität Siegen):**

  „Recovering a Tomographic Model of the Earth by Sparse Regularization of Inverse Gravimetry"

für die AG Geomathematik:

Prof. Dr. V. Michel

Abstract

F.J. Simons: Spectral-domain maximum-likelihood methods for the estimation of the parameters of and the input to non-linear differential equations from noisy observations of their output made on irregular geographical domains. Or: How strong is the Earth's lithosphere really? A statistical perspective from gravity/topography data

The lithosphere is modeled using a differential equation characterized by a set of parameters, at least one of which, under the assumption of elastic behavior, is generally thought of as a proxy for its strength: the flexural rigidity (D), or, by extension, the elastic thickness. This lithospheric system then takes an input: topographic loading by mountain building and other processes, and maps it into an output: the gravity anomaly and the final, measurable, topography. The input is not measurable but some of its properties can be characterized. The outputs are measurable but the relation between them is obfuscated by their stochastic nature and the presence of unmodeled components. Estimating D, most usually in the spectral domain, generally involves constructing summaries of gravity and topography. Both admittance and coherence are popular; both are ratios of the cross-spectral density of gravity and topography to the power spectral densities of either, the whole sometimes squared. Despite the fact that neither admittance nor coherence are Gaussian, estimating D usually comes down to the least-squares fitting of a parameterized curve, where Gaussian behavior is tacitly assumed. In this two-step procedure, admittance or coherence are first estimated, and subsequently inverted for the strength parameters. Rarely, if ever, are lithospheric models found that satisfy both coherence and admittance to within their true error. Why don't they? Poorly characterized errors of admittance and coherence are not the only problems with this procedure. There is also the implicit annihilation of information during the construction of these statistics (coarsely sampled, sometimes squared, ratios, measures of the data as they are) themselves. Then there is the fact that we do not want to know coherence and admittance at all - we want to know properties of the lithosphere! In this presentation, we intend to abandon coherence and admittance studies for good, by proposing an entirely different method of estimating flexural rigidity, which returns it and its confidence interval, as well as a host of tests for the suitability of the assumptions made along the way, and the possible presence of correlated loads and anisotropy in the response. The crux of the method is that it employs a maximum-likelihood formulation that remains very grounded in the data themselves, and which is formulated in terms of variables that do have a Gaussian distribution.

Abstract

D. Fischer: Recovering a Tomographic Model of the Earth by Sparse Regularization of Inverse Gravimetry

To recover the density of the Earth we invert Newton's gravitational potential

$$V(x) = \gamma \int_{\mathcal{B}} \frac{\rho(y)}{|x-y|} \mathrm{d}y,$$

which corresponds to a Fredholm integral equation of the first kind, where γ is the gravitational constant, $\mathcal{B}$ is an approximation of the Earth and ρ is the density that we are looking for. It is a well-known fact that this problem is ill-posed. Moreover, it even becomes exponentially ill-posed if we use satellite data as input. Thus, we need to develop a regularization method to solve this problem.

In our research, we applied the idea of a matching pursuit to recover a solution stepwise. At step $n+1$ the basis function d_{n+1} and the weight α_{n+1} are selected to best match the data structure, i.e. to minimize

$$||R^n - \alpha \mathcal{F} d||^2_{\mathbb{R}^l} + \lambda ||F_n||^2_{\mathrm{L}^2(\mathcal{B})}$$

where $R^n = y - \mathcal{F} F_n$ is the residual, y is the given data, $\mathcal{F}$ is an operator corresponding to the problem, λ is the regularization parameter and $F_n = \sum_{k=1}^{n} \alpha_k d_k$ is the solution that was recovered in the preceding steps.

One big advantage of this method is that all kinds of different basis functions can be taken into account to improve the model stepwise and the sparsity of the solution can be controlled directly. Moreover, this new approach generates models with a resolution that is adapted to the data density and the detail density of the solution. Note that we were able to parallelize the implementation of our method to speed up the computations.

We applied our method to solve a variety of current problems, e.g. the reconstruction of the density distribution of the Earth or the seasonal changes in the area of the Amazon.

References:

P. Berkel, D. Fischer, V. Michel: Spline multiresolution and numerical results for joint gravitation and normal mode inversion with an outlook on sparse regularisation, International Journal of Geomathematics, accepted for publication, 2010.