A SEQUENTIAL CONJUGATE GRADIENT METHOD FOR THE STABLE NUMERICAL SOLUTION TO INVERSE HEAT CONDUCTION PROBLEMS $^{\rm 1}$

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Abstract

In this paper we propose a new numerical method for solving noncharacteristic Cauchy problems (NCCPs) for parabolic equations, namely a time-stepping sequential variant of the conjugate gradient method (CGM). In contrast to other related papers, we do not assume that the initial condition is given. Moreover, we allow that the coefficients in the parabolic equations are nonsmooth and may depend on time.

The purpose of this paper is to present the method and to show a few numerical results. A detailed analysis of the method will be carried out in a forthcoming paper. The CGM itself applied to linear ill-posed problems has optimal regularization properties as it is shown by Nemirovskii. Our sequential variant of the CGM proceeds similarly to the well-known Beck method. It computes the solution of the associated direct problems only in a short time interval of - say - r time steps but takes, as a result, only the solution at the next (discrete) time. The latter is then used as the starting value for the following (sequential) step. Several advantages of the sequential version of the CGM will be explained in detail in the paper.

¹Inverse Problems in Engineering 2(1996), 263-272.

²Supported by the Deutsche Forschungsgemeinschaft (DFG)