STABLE NUMERICAL FRACTIONAL DIFFERENTIATION BY MOLLIFICATION [1]

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Abstract

The problem of stable calculation of the fractional derivatives of a function f given in $L_p()$, $1 \leq p \leq \infty$ is considered by a mollification method. We have obtained stability estimates of Hölder type in L_p-norm for all $p \in [1, \infty]$ for the problem of numerical differentiation and $p = 2$ for the problem of numerical fractional differentiation. In contrast to other related papers, we do not assume the existence of the second order derivative for the problem of numerical differentiation, or the existence of the first order derivative for the problem of numerical fractional differentiation, as in other related papers, *but only that the desired (fractional) derivatives belong to L_p!* Note that the methods for stable numerical fractional differentiation for such functions up to now did not appear in the literature. Furthermore, we can use fast Fourier transforms to implement our method numerically. We consider also the problem of approximating the derivatives of very rough functions (that have no derivative in the classical sense) in a stable way. Finally, we describe several numerical experiments.

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