REGULARIZATION OF A NONCHARACTERISTIC CAUCHY PROBLEM FOR A PARABOLIC EQUATION [1]

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Abstract

In this paper the non-characteristic Cauchy problem

$$\left.\begin{array}{rcll} u_t - a(x)u_{xx} - b(x)u_x - c(x)u & = & 0 & , x \in (0,l), t \in \mathbb{R}, \\ u(0,t) & = & \varphi(t) & , t \in \mathbb{R}, \\ u_x(0,t) & = & 0 & , t \in \mathbb{R} \end{array}\right\} \quad (*)$$

is considered. The problem $(*)$ is well known to be severely ill-posed: a small perturbation in the Cauchy data may cause a dramatically large error in the solution. In this paper the following mollification method is suggested for this problem: if the Cauchy data are given inexactly then we mollify them by elements of well-posedness classes of the problem, namely by elements of an appropriate $\infty-$regular multiresolution approximation $\{V_j\}_{j\in\mathbb{Z}}$ of $L^2(\mathbb{R})$ which is generated by the father wavelet of Meyer. Within V_J the problem (*) is well posed, and we can find a mollification parameter J depending of the noise level ε in the Cauchy data such that the error estimation between the exact solution and the mollified solution is of Hölder type. The method can be numerically implemented using fundamental results by Beylkin, Coifman and Rokhlin [1] on representing (pseudo)differential operators in wavelet bases. A stable marching difference scheme based on this method is suggested. Several numerical examples are given.

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