Mathematik LK Q1 Junker

An Alternative Approach to Differential Calculus by Johann Bernoulli (1667 - 1748)

1. Introduction into the historical background

A few years before his death, Johann Bernoulli published a treatise entitled Lectiones mathematicae de methodo integralium. Although this did not appear until 1742, it is one of Bernoulli's first mathematical works, written in 1691. In a footnote in this work there is a reference to a previous work on differentials, the contents of which were never actually published under Bernoulli's name, but by Marquis de l'Hospital. Nevertheless, today we have access to Bernoulli's first thoughts on differential calculus. You will devote your attention to these in the following lessons. The mathematical approach to the beginnings of differential calculus by Bernoulli provides an alternative view of the origins of differential calculus, but is rather unwieldy and unfamiliar for students today, so it requires a little good will to engage with it. The following Poscast is intended to introduce you to the situation historically and to provide background knowledge in this regard.

***Private lessons with one of Europe's greatest minds - The creation of the first textbook on differential calculus.*[1]**

This story is about one of Europe's greatest mathematicians: Johann (I) Bernoulli. At his death in 1748, the Basel mathematics professor was a member of the most important European academies of science, such as the French academie de science, the Prussian Academy of Sciences in Berlin or the Royel Society in London. The Russian Academy of Sciences in St. Petersburg had even appointed him an honorary member. So he was well connected and highly respected – even without internet and airplanes - and used his contacts to make the brand new differential and integral calculus known among the mathematicians of Europe. In the dispute over who discovered calculus first - the Hanoverian Gottfried Wilhelm Leibniz or the Englishman Isaac Newton - he clearly took Leibniz's side and thus helped the Leibnizian version to achieve a breakthrough on the European continent. But that is another story ...

When Bernoulli was born in 1661 as the tenth child of a well-known Basel merchant family, his later fame and influence in the field of mathematics could not be foreseen

1 This text was made available to the students as a podcast and provides the historical framework for the source work.

at first. He was actually supposed to become a merchant at his father's request. However, he resisted this and first studied medicine at the University of Basel. Johann Bernoulli came to mathematics through his 12 years older brother Jakob, who was a professor of mathematics in Basel at that time. His talent was soon evident and Johann became Jakob's student. Together they worked on Leibniz's methods for the differential and integral calculus. Again and again there were disputes about the content, in which above all the younger brother wanted to show the older that he was the better mathematician.

If one wanted to get in touch with other scientists at that time, this could only be done in person or by letter. So in 1690, Johann Bernoulli went first to Geneva and then a year later to Paris to give a lecture on the new discipline of calculus. He did so in Paris in a circle of learned mathematicians that included the Marquis de L'Hospital.

L'Hospital, or more precisely Gillaume Francois Antoine Marquis de L'Hospital, belonged to a distinguished Parisian noble family. As befitted a son of this class, he was supposed to make a career in the military. Officially because of his shortsightedness, but unofficially because he much preferred to occupy himself with mathematics, he quit military service and devoted himself to mathematical studies. He was very interested in the new findings reported by Johann Bernoulli and so he invited Johann to his castle and took private lessons with him in 1691 and 1692. Even after Bernoulli's departure from France - he became a professor in Groningen (Netherlands) - the two exchanged letters about questions L'Hospital encountered while learning the calculus. The Marquis paid Johann for his services and they agreed in a secret contract that L`Hospital alone should have the right to publish what he had learned.

And while Bernoulli lectured on integral calculus in Groningen, De L'Hospital made use of his exclusive rights and, on the basis of Bernoulli's private lectures and letters, published the first textbook on differential calculus in 1696: "Analyse des infinement petits". The book was the basic work of analysis for many years and was printed in several editions until 1781. It helped the method of Leibniz and Bernoulli to achieve a breakthrough on the European continent and made De L'Hospital famous as a mathematician.

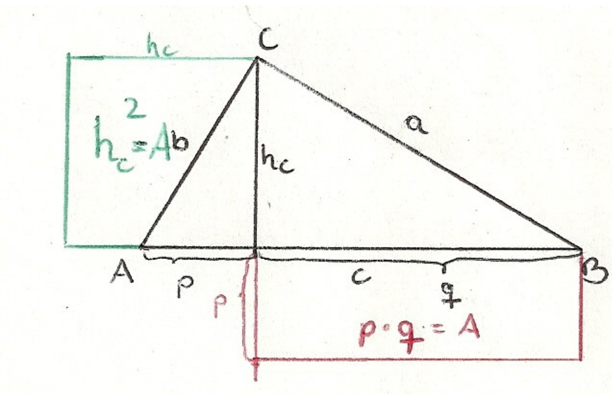
In the preface, he thanked Leibniz and the brothers Johann and Jakob Bernoulli for their preliminary work on the Calculus, but he did not name them as co-authors. This must have annoyed the already egotistical Johann, who in the meantime had followed his deceased brother to the chair of mathematics in Basel. But a contract is a contract.... And so it was only after L'Hospital's death that Johann criticised the fact that essential parts of the "analysis" went back to him. And indeed, the lecture notes from 1691/92 with which he introduced L'Hospital to calculus were found in Bernoulli's estate. Both were probably somewhat right: in terms of content, there are *many similarities, but L'Hospital put his own stamp on much of it and gave the first textbook of calculus its convincing systematic.*

2. Construction of curves by compass and ruler:

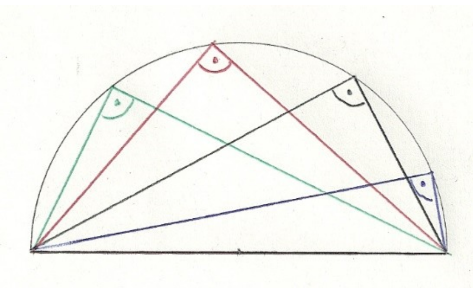
The podcast has given you a first insight into the historical situation during the lifetime of Johann Bernoulli and the Marquis de l'Hospital. Before you can get to the core of the task, you first have to realize that at that time it was not usual to view curves as graphs of functions in the coordinate system, but as geometric objects. Since this way of looking at things seems hardly conceivable nowadays and with today's knowledge, I would like to make it clear to you what this means with the following example. We will look at how Bernoulli was able to construct a parabola without using a coordinate system. For this we need paper, pencil, ruler, compasses, Euclid's height theorem and Thales' theorem.

Since most of you probably don't remember the height theorem and Thales' theorem, let's start with them:

The Altitude Theorem states that in a right triangle with hypotenuse c, the following equation holds: $h_c^2 = pq$. Geometrically, this means that the square with the side length h_c (green) has the same area as the rectangle with the side lengths p and q (red).



Thales' theorem says nothing other than that every triangle above the diameter of a circle is a right triangle:



The following two videos[2] show you how to construct the parabola for a given a. For the sake of simplicity, let's take $a = 1$, so we want to construct the parabola $y^2 = x$.

By the way: Strictly speaking, you don't even need a set square or a ruler with graduations for the construction, as the midpoints and perpendiculars can also be constructed with a compass and a ruler without graduations. Presumably, however, such a drawing would have led to even more confusion, so I have dispensed with it for now…;-)

Task:

Now construct the parabola for the equation $y^2 = 2x$ on your own with pencil, white paper, compass and set square. Please do not erase any auxiliary lines so that I can follow the drawing process. (Please upload the finished product under your name. Only then will the next worksheets be activated for you.)

2 Only accessible in German.

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Bernoulli begins his lectures on differential calculus with three postulates[1]. The first and the second are of interest to us in the following:

1. **a quantity which is diminished or increased by an infinitely smaller quantity is neither diminished nor increased.**
2. **every curved line consists of infinitely many straight lines, which are themselves infinitely small.**

After that the deduction of various derivation rules (sum rule, derivation of powers with natural exponents, product rule, derivation of root functions, etc.) follows. In particular, Bernoulli gives the derivative rule of the quadratic function:

"The differential of x^2 is 2x dx, which is proved thus: If one multiplies x + e by x + e, the product becomes $x^2 + 2ex + e^2$. Subtracting x^2 from this leaves $2ex + e^2$ and this is equal to 2ex = 2x dx because of the first postulate. Q.E.D."

(**Source:** Translation of Bernoulli, Johann: Die Differentialrechnung, Akademische Verlagsgesellschaft Leipzig 1924)

Working with Bernoulli‘s textbook 1:

1. What is Bernoulli's purpose when he multiplies x + e by x + e? Illustrate this geometrically.
2. Bernoulli then subtracts x^2. Why? What remains geometrically? Draw a sketch of the graph of $y = x^2$. Mark x, x+e and the difference $2ex + e^2$.
3. Why is e^2 now omitted? What remains arithmetically and what is the geometric meaning of the term 2x dx?
4. Bernoulli does not specify at any point what is meant by „differential“. This must be deduced from the text. Jean le Rond d'Alembert and Denis Diderot published their "Encyclopaedia of Knowledge" in 1751. Consider what could be meant by a differential and write a fictitious entry for Diderot and d'Alembert's "Encyclopaedia".

1 a requirement that seems indispensable

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Bernoulli used differential calculus to solve certain geometric problems. For example, to find the tangent to a parabola.

"Task 1.

To find the tangent of the parabola: According to the explanation of the parabola, $ax = y^2$, so also $a\,dx = 2y\,dy$ or $a : 2y = dy/dx$ and since according to postulate 2 it is assumed that every curve consists of infinitely many straight lines, so the tangent AD (Fig. 1) and the infinitely small piece DF of the parabola BDF will be a straight line. Therefore, if DG is drawn parallel to the diameter AE, DGF~ACD. Therefore FG : GD = CD : AC and s means the sub-tangent, so $dy/dx = y/s = a/2y$ (according to the preceding); therefore $s = 2y^2/a = 2ax/a = 2x$ (*).

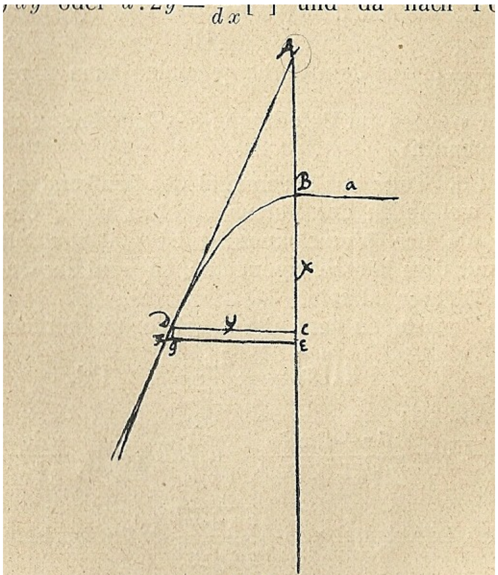

Fig. 1.

Therefore, if AC is taken twice as large as the abscissa BC of the curve point D and the straight line AD is drawn through A, it is the tangent that was to be found."

The differential of ax is a dx. Bernoulli explains why as following:

"The differential of ax is [a]dx. This is proved in this way: Multiply x + e, whereby e = dx, by a + 0, i.e. a and nothing, because a is a constant that has no differential. By this the product ax + ae results; if ax is subtracted from this, ae = a dx remains. Q.E.D."

(Source: Translation of Bernoulli, Johann: Die Differentialrechnung, Akademische Verlagsgesellschaft Leipzig 1924)

Working with Bernoulli's textbook 2:

1. The text says that the task is to find the tangent to the parabola. Try to make the task more concrete. At which point should the tangent be found? Mark in the drawing what exactly you are looking for.
2. The ratio equation FG : GD = CD : AC goes back to the intercept theorem. Mark the intercept-theorem-situation in the drawing (Fig. 1).
3. Mark the so called sub-tangent in the drawing?
4. Explain the chain of equations (*) in the original text.
5. How can Bernoulli construct the tangent line? Carry out the construction.

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Final task of the project:

Imagine you are l'Hospital and you have read Bernoulli's text on differentials. You have understood his procedure, but there is still a need for clarification and questions, especially because Bernoulli's procedure differs from the usual procedure you know for finding tangents. In this situation, you write the following letter to Bernoulli:

Dear Mr. Bernoulli,

I have read with great interest your remarks on the differential of the parabola. However, I still have some need for clarification. Until now, I always thought that the derivative of the parabola was...

Complete this letter in as much detail as possible.