Mini-Workshop on spectral analysis and curvature on fractals

Siegen, June 7, room EN-D 224 and June 8, room EN-D 308.

Program

Friday, 7th

09:15 – 10:15	Steffen Winter (Karlsruher Institut für Technologie) *Minkowski content and fractal curvatures of self-similar tilings*
10:30 – 11:30	Sabrina Kombrink (Universität Bremen) *Minkowski content and fractal curvatures of self-conformal sets and the Weyl-Berry conjecture*
11:30 – 12:00	Coffee break
12:00 – 13:00	Martina Zähle (Friedrich-Schiller-Universität Jena) *Stability properties of fractal curvatures for self-similar sets*
13:00 – 14:15	Lunch at the Mensa
14:15 – 14:45	Roland J. Etienne (Lycée Bel-Val, Luxemburg & Universität Siegen) *The moments of the eigenvalue distributions of fractal chains*
15:00 – 15:45	Tilman Bohl (Friedrich-Schiller-Universität Jena) *Curvature of self-conformal sets*
18:30	Workshop Dinner

Saturday, 8th

9:15 – 10:15	Ka-Sing Lau (Chinese University of Hong Kong) *Spectral problem of self-similar sets and measures*
10:30 – 11:30	Michael Hinz (Universität Bielefeld) 1-*forms, vector fields, and topologically* 1-*dimensional fractals*
11:30 – 12:00	Coffee break
12:00 – 12:30	Peter Arzt (Universität Siegen) *Spectral asymptotics on irregular and random Cantor-like fractals*
12:30 – 13:00	Raffaela Capitanelli ("Sapienza" Università di Roma) *Asymptotics for boundary value problems on domains with fractal boundary*

Abstracts

Minkowski content and fractal curvatures of self-similar tilings

Steffen Winter, Karlsruher Institut für Technologie

June 8th, 2013, 09:15–10:15.

We discuss the existence of the Minkowski content and of fractal curvatures of self-similar tilings and the general relations to the corresponding notions for the associated self-similar sets. Based on these relations, alternative formulas for the computation of the Minkowski content and of fractal curvatures of self-similar sets are obtained, which involve only data of the generator of the tiling. Also the regularity and curvature bound conditions needed to ensure the existence of fractal curvatures can be reformulated in terms of the generators.

If time allows, we will also discuss some recent joint work with Dusan Pokorny on the geometric meaning of curvature scaling exponents.

Minkowski content and fractal curvatures of self-conformal sets and the Weyl-Berry conjecture

Sabrina Kombrink, Universität Bremen

June 7th, 2013, 10:30–11:30.

Notions of fractal dimension such as Hausdorff, Minkowski, similarity and packing dimension are well-known and well-studied characteristics that describe the geometry of highly irregular sets. However, similarly to how differentiable manifolds of the same dimension can vary wildly in their geometric structure, sets of the same fractal dimension can exhibit very different geometric appearances. The Minkowski content and the fractal curvatures provide tools to characterise highly irregular sets beyond their dimension and can be viewed as substitutes of volume and curvature for fractal sets. In this talk we will discuss the existence of the Minkowski content and the fractal curvatures for self-conformal sets, i.e. invariant sets of iterated function systems consisting of conformal $\mathcal{C}^{1+\alpha}$ contractions and limit sets of conformal graph directed systems. We will see that the existence is highly related to the underlying function system being lattice or non-lattice. Furthermore, we will discuss what our results imply for the Weyl-Berry conjecture on the distribution of the eigenvalues of the Dirichlet-Laplacian on domains with a fractal boundary.

Joint works with U. Freiberg and M. Kesseböhmer.

Stability properties of fractal curvatures for self-similar sets

Martina Zähle, Friedrich-Schiller-Universität Jena

June 8th, 2013, 12:00–13:00.

Local Minkowski content and curvature measures for certain classes of fractal sets F have been introduced by means of approximation with tubular neighborhoods F_ε of small distances ε. It has been proved with different methods that the suitably rescaled volumes and Lipschitz-Killing curvature measures of F_ε converge (in the average) to fractal versions. We show that in the self-similar case one obtains the same limits if the parallell sets are replaced by neighborhoods admitting normal cycles in form of the pushforward of those of F_ε under asymptotically small perturbations. Consequences of this result are demonstrated on the example of the higher dimensional Sierpiński gasket.

The moments of the eigenvalue distributions of fractal chains

Roland J. Etienne, Lycée Bel-Val, Luxemburg & Universität Siegen

June 8th, 2013, 14:15–14:45.

Working on the vibrational spectrum of drums with fractal boundaries, C. Pomerance and M.L. Lapidus proved the modified Weyl-Berry conjecture for the asymptotics of the eigenvalues of the Laplace-operator for bounded open subsets of the real line (fractal strings), and in this context discovered new and intriguing relations between Minkowski-measurability and the Riemann Zeta function [1].

The work presented here proposes a slightly different view on the subject. Fractal strings are modelled by linear chains of masses coupled by harmonic springs, which in turn may be described by their dynamic matrices. This model, introduced as fractal chains by the author in a related context [2], provides a new approach to the study of oscillations in the spectrum of the eigenvalues.

In the case of self-similar chains, the spectral counting function does not asymptotically converge, a fact that can also be investigated through the moments of the eigenvalue distribution. Exact results as well as upper and lower bounds for the moments are given and some results on the relation between the moments of the eigenvalue distributions of fractal chains and the Minkowski-measurability of their boundaries are presented.

Bibliography

[1] M. L. Lapidus and C. Pomerance, Proc. London Math. Soc. (3) 66, pp.41-69 (1993).

[2] R. J. Etienne, 2nd Conference on Analysis and Probability on Fractals, Cornell University, Ithaca NY, 2005, (unpublished).

Curvature of self-conformal sets

Tilman Bohl, Friedrich-Schiller-Universität Jena

June 8th, 2013, 15:00–15:45.

What shape and how big is a fractal? Fractal curvature and the well-known Minkowski content give an answer. First the fractal is approximated with uniformly thin coatings (parallel sets). Then the rescaled Lebesgue measure of the approximation set converges in a Cesaro average sense. The same approach turns Lipschitz-Killing curvature-direction measures into fractal curvature-direction measures.

Self-conformal fractals generalize self-similar ones in $\mathbb{R}^d$: the generating system of contractions (IFS) need only locally preserve angles. We prove the open set condition is sufficient for the average Minkowski content (and its measure localization) to exist. Fractal curvatures need extra assumptions, weak enough for a partial converse. There is an integral formula.

The proof tracks the geometry inside a small ball, as it is preimaged and enlarged under the generating IFS. A multiplicative convergence theorem describes how the IFS's nonlinearities distort the ball. The measure version localizes curvature simultaneously to points in the fractal and to normals (in the sphere) onto them. A skew product group extension of the preimaging dynamical system rotates the normal directions correctly. The projection of fractal curvature-direction measures onto the sphere is (at least) rotation invariant under its ergodic fibres.

If time permits, direct limits (strengthening Cesaro average limits) will also be presented.

Spectral problem of self-similar sets and measures

Ka-Sing Lau, Chinese University of Hong Kong

June 8th, 2013, 09:15–10:15.

1-forms, vector fields, and topologically 1-dimensional fractals

Michael Hinz, Universität Bielefeld

June 8th, 2013, 10:30–11:30.

We consider compact connected topologically 1-dimensional metric spaces that carry a strongly local regular Dirichlet form. There is a natural definition of L_2-differential 1-forms associated with this Dirichlet form. As in the classical case the space of 1-forms decomposes into a subspace of exact and a subspace of harmonic forms. The space of harmonic forms is related to the first Čech cohomology of the metric space.

Spectral asymptotics on irregular and random Cantor-like fractals

Peter Arzt, Universität Siegen

June 8th, 2013, 12:00–12:30.

We study properties of the eigenvalues of the operator $\frac{d}{dm}\frac{d\mu}{dx}$ with homogeneous Neumann and Dirichlet boundary conditions. Our aim is to determine the asymptotic growth behaviour of the eigenvalue counting function if the measure μ has an irregular structure described by a so called environment sequence.

Asymptotics for boundary value problems on domains with fractal boundary

Raffaela Capitanelli, "Sapienza" Università di Roma

June 8th, 2013, 12:30–13:00.

Fractals provide an interesting setting for studying those phenomena that take place in bodies with small bulk and large surfaces. In this talk we present two ways of facing boundary value problems on domains with fractal boundary.

The first approach is the approximation of boundary value problems on domains with fractal boundary with the analogue boundary value problems on domains with the corresponding pre-fractal boundary.

The second approach is the study of boundary value problems on domains with fractal boundary from the point of view of the homogenization theory.