Theoretical and Phenomenological Interest in Rare B Decays

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Interest in Rare B Decays

- Rare B Decays $(b \to s\gamma, b \to s\ell^+\ell^-, ...)$ are Flavour-Changing-Neutral-Current (FCNC) processes $(|\Delta B| = 1, |\Delta Q| = 0)$; not allowed at the Tree level in the SM

- FCNC processes are governed by the GIM mechanism, which imparts them sensitivity to higher scales (m_t, m_W) and the CKM matrix elements, in particular, $V_{ti}; i = d, s, b$

- FCNC processes are sensitive to physics beyond the SM, such as supersymmetry, and the BSM amplitudes can be comparable to the (tW)-part of the GIM amplitudes

- Last, but not least, Rare B-decays enjoy great interest in the ongoing and planned experimental programme in heavy quark physics (CLEO, BABAR, BELLE, CDF, D0, LHC, Super-B factory)

Content

- Standard Model, Quark Flavour Mixing & the CKM Matrix
- The Standard Candle in Rare B-Decays: $\mathbf{B} \to X_s \gamma$
- Exclusive Radiative Decays $\mathbf{B} \to K^* \gamma$ & $\mathbf{B} \to (\rho, \omega) \gamma$
- Electroweak Penguins: $\mathbf{B} \to X_s \ell^+ \ell^-$
- Exclusive Decays $\mathbf{B} \to (K, K^*, \pi) \ell^+ \ell^-$
- Current Frontier of Rare B Decays: $\mathbf{B}_s \to \mu^+ \mu^-$ & $\mathbf{B}_d \to \mu^+ \mu^-$
- Outlook & Summary

Standard Model Lagrangian

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm GSW} + \mathcal{L}_{\rm QCD}$$

QCD [SU(3)]

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F^{(a)}_{\mu\nu} F^{(a)\mu\nu} + i \sum_q \bar{\psi}^\alpha_q \gamma^\mu (D_\mu)_{\alpha\beta} \psi^\beta_q$$

with $F^{(a)}_{\mu\nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu - g_s f_{abc} A^{(b)}_\mu A^{(c)}_\nu; \quad a, b, c = 1, ..., 8$

and $(D_\mu)_{\alpha\beta} = \delta_{\alpha\beta}\partial_\mu + i g_s \sum_a \frac{1}{2} \lambda^{(a)}_{\alpha\beta} A^{(a)}_\mu$

Electroweak $[SU(2)_I \times U(1)_Y]$

$$\mathcal{L}_{\rm GSW} = \mathcal{L}_{\rm gauge}(W_i, B, \psi_j) + \mathcal{L}_{\rm Higgs}(\phi_k, W_i, B, \psi_j)$$

$$\mathcal{L}_{\rm gauge}(W_i, B, \psi_j) = -\frac{1}{4} F^i_{\mu\nu} F^{\mu\nu}_i - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \sum_{\psi_L} \overline{\psi_L} i D_\mu \gamma^\mu \psi_L + \sum_{\psi_R} \overline{\psi_R} i D_\mu \gamma^\mu \psi_R$$

$$\mathcal{L}_{\rm Higgs}(\phi_k, W_i, B, \psi_j) = \mathcal{L}_{\rm Higgs}(\text{gauge}) + \mathcal{L}_{\rm Higgs}(\text{fermions})$$

$$\mathcal{L}_{\rm Higgs}(\text{gauge}) = (D_\mu \Phi)^* (D^\mu \Phi) - V(\Phi)$$

$$D_\mu \Phi = (\mathrm{I}(\partial_\mu + i\frac{g_1}{2}B_\mu) + ig_2\frac{\tau}{2}\cdot \mathrm{W}_\mu)\Phi;\ V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

$$\mathcal{L}_{\text{Higgs}}(\text{fermions}) = Y_u^{ij}\bar{Q}_{L,i}\tilde{\Phi}u_{R,j} + Y_d^{ij}\bar{Q}_{L,i}\Phi d_{R,j} + \text{h.c.} + \ldots$$

- 3 Quark families: $Q_{L_j} = (u_L,\ d_L); (c_L,\ s_L); (t_L;\ b_L); \bar{u}_R,\ \bar{d}_R; \ldots$
- Flavour mixings in the SM reside in the Higgs-Yukawa sector of the theory
- Flavour symmetry broken by Yukawa interactions

$$Q_i Y_d^{ij} d_j \phi \ \longrightarrow\ Q_i M_d^{ij} d_j$$

$$Q_i Y_u^{ij} u_j \phi^c \ \longrightarrow\ Q_i M_u^{ij} u_j$$

$$M_d = \text{diag}(m_d,\ m_s,\ m_b);\ \ M_u^\dagger = \text{diag}(m_u,\ m_c,\ m_t) \times V_{\text{CKM}}$$

- V_{CKM} a (3×3) unitary matrix is the only source of Flavour Violation, as all gauge interactions (involving $\gamma,\ Z^0,\ g$) are Flavour diagonal
- All observed phenomena involving flavour changes in the hadrons are consistently described by the CKM framework; i.e., in terms of 10 fundamental parameters: 6 quark masses, 3 mixing angles and 1 phase

The Cabibbo-Kobayashi-Maskawa Matrix

$$V_{\rm CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

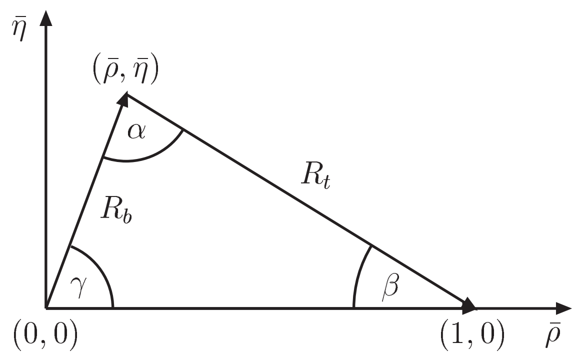
- Customary to use the handy Wolfenstein parametrization

$$V_{\rm CKM} \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3\left(\rho - i\eta\right) \\ -\lambda(1+iA^2\lambda^4\eta) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3\left(1-\rho-i\eta\right) & -A\lambda^2\left(1+i\lambda^2\eta\right) & 1 \end{pmatrix}$$

- Four parameters: $A, \ \lambda, \ \ \rho, \ \ \eta$
- Perturbatively improved version of this parametrization

$$\bar{\rho} = \rho(1-\lambda^2/2), \ \ \bar{\eta} = \eta(1-\lambda^2/2)$$

- The CKM-Unitarity triangle $[\phi_1 = \beta; \ \ \phi_2 = \alpha; \ \ \phi_3 = \gamma]$



Phases and sides of the UT

$$\alpha \equiv \arg\left(-\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}}\right), \qquad \beta \equiv \arg\left(-\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\right), \qquad \gamma \equiv \arg\left(-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right)$$

- β and γ have simple interpretation

$$V_{td} = |V_{td}| e^{-i\beta}, \qquad V_{ub} = |V_{ub}| e^{-i\gamma}$$

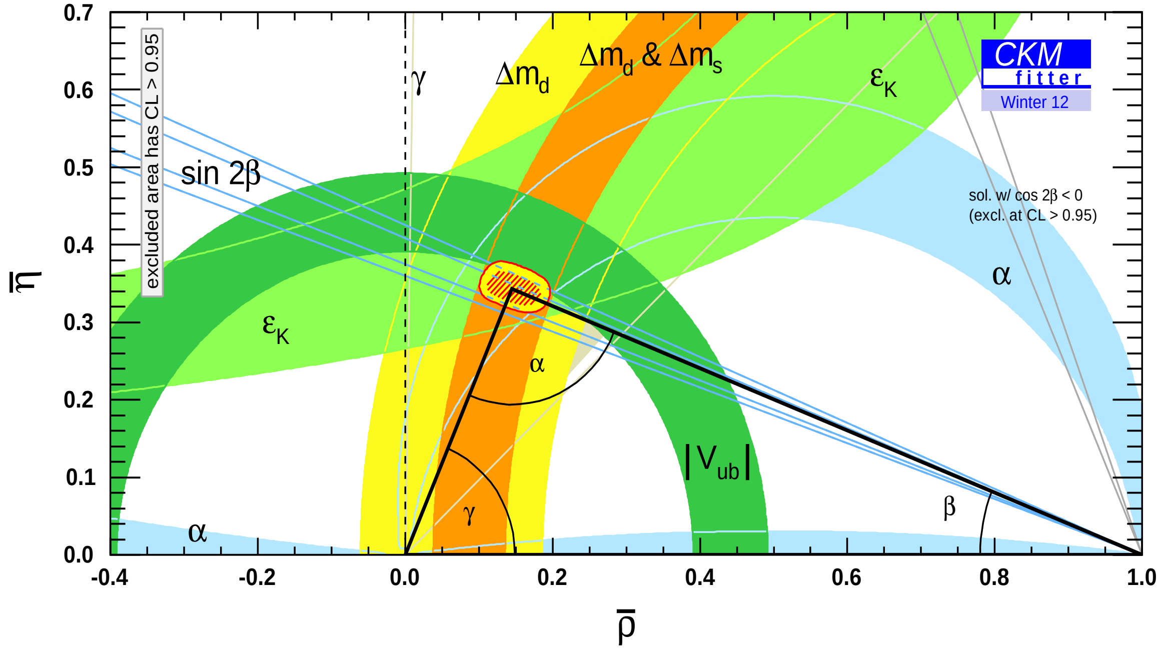
- α defined by the relation: $\alpha = \pi - \beta - \gamma$
- The Unitarity Triangle (UT) is defined by:

$$R_b \mathrm{e}^{i\gamma} + R_t \mathrm{e}^{-i\beta} = 1$$

$$R_b \equiv \frac{|V_{ub}^* V_{ud}|}{|V_{cb}^* V_{cd}|} = \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left|\frac{V_{ub}}{V_{cb}}\right|$$

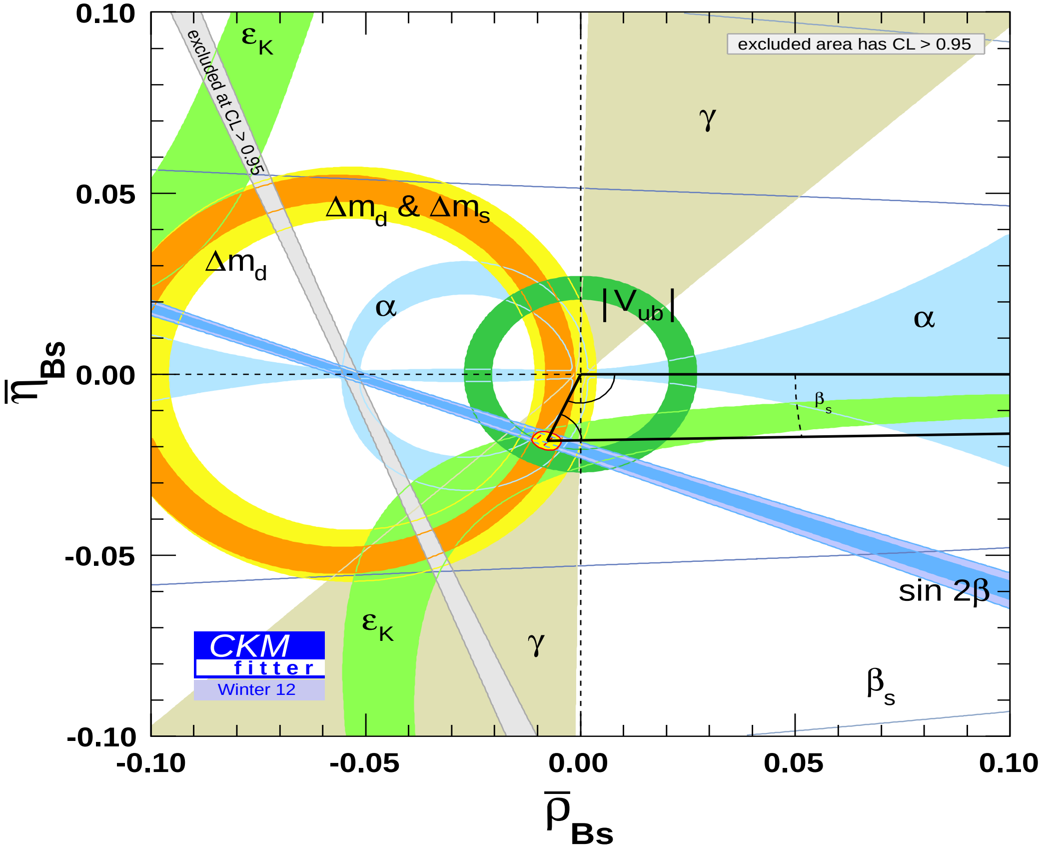
$$R_t \equiv \frac{|V_{tb}^* V_{td}|}{|V_{cb}^* V_{cd}|} = \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left|\frac{V_{td}}{V_{cb}}\right|$$

Current Status of the CKM-Unitarity Triangle [CKMfitter]



- $\sin 2\beta = 0.820^{+0.024}_{-0.028}$ [Fit-value]
  ($= 0.691 \pm 0.020$ [Direct Measurement]
- $\alpha = [95.9^{+2.2}_{-5.6}]^\circ$ [Fit-value]
  $\alpha = [88.7^{+4.6}_{-4.2}]^\circ$ [Direct Measurement]
- $\gamma = [67.1 \pm 4.3]^\circ$[Fit-value]
  $\gamma = [66 \pm 12]^\circ$ [Direct Measurement]
- Direct and indirect measurements of angles agree well; largest Pull is on $\sin 2\beta$ $(= 2.6\ \sigma)$

Current Status of the Squashed UT_s Triangle [CKMfitter]



- $\bar{\rho}_{B_s} = -0.0078 \pm 0.0015$ [Fit-value]
- $\bar{\eta}_{B_s} = -0.01837^{+0.00080}_{-0.00082}$ [Fit-value]
- $\sin 2\beta_s = 0.0364 \pm 0.0016$[Fit-value]
  where $\beta_s = -\arg(-V_{cs}V_{cb}^*/V_{ts}V_{tb}^*)$]

The Standard Candle: $B \to X_s\gamma$

Interest in the rare decay $B \to X_s\gamma$ transcends B Physics!

- First measurements by CLEO (1995); well measured at the B-factories by Belle and BaBar; more precise measurements anticipated at SuperB-factories

Theoretical Interest:

- A monumental theoretical effort has gone in improving the perturbative precision; $B \to X_s\gamma$ in NNLO completed in 2006

  - First estimate of $\mathcal{B}(B \to X_s\gamma)$: M. Misiak et al., Phys. Rev. Lett. 98:022002 (2007)

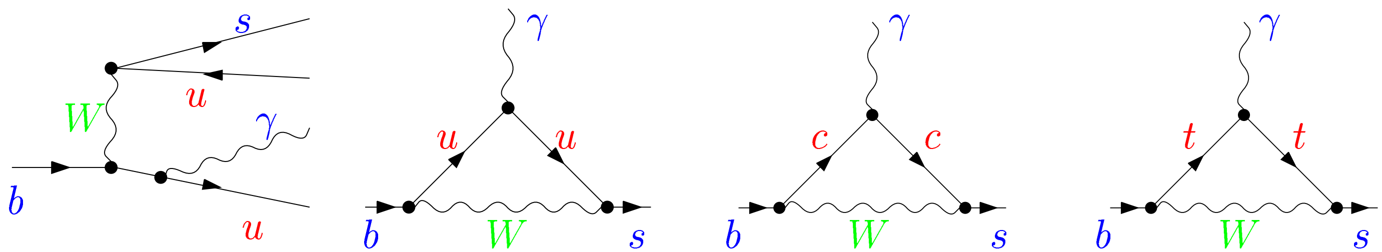
  - Analysis of $\mathcal{B}(B \to X_s\gamma)$ at NNLO with a cut on the Photon energy, T. Becher and M. Neubert, Phys. Rev. Lett. 98:022003 (2007)

- Non-perturbative effects under control thanks to HQET

- Sensitivity to new physics; hence constrains parameters of the BSM models such as the 2HDMs and Supersymmetry

- A crucial input in a large number of precision tests of the SM in $b \to s$ processes, such as $B \to X_s\ell^+\ell^-$

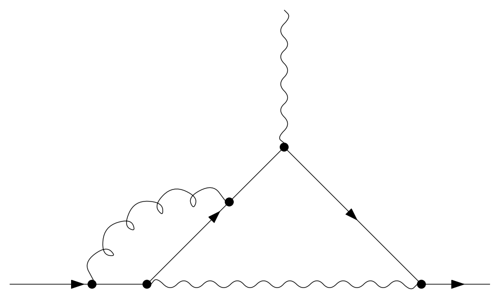
Examples of the leading electroweak diagrams for $B \to X_s \gamma$



$$\left|\frac{V_{ub}V_{us}}{V_{cb}}\right| \simeq \left|\frac{V_{ub}V_{us}}{V_{ts}}\right| \simeq 2\%$$

$\sim +200\%$ $\sim -100\%$

In the amplitude, after including LO QCD effects.



- QCD logarithms $\alpha_s \ln \frac{M_W^2}{m_b^2}$ enhance $\mathrm{BR}(B \to X_s\gamma)$ more than twice
- Effective field theory (obtained by integrating out heavy fields) used for resummation of such large logarithms

The effective Lagrangian for $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

$$\mathcal{L} \;=\; \mathcal{L}_{QCD\times QED}(q,l) \;+\; \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i$$

$(q = u, d, s, c, b, \;\; l = e, \mu, \tau)$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b), & i = 1,2, & |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma_i' q), & i = 3,4,5,6, & |C_i(m_b)| < 0.07 \\ \frac{em_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, & C_7(m_b) \sim -0.3 \\ \frac{gm_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8, & C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2}(\bar{s}_L\gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), & i = 9, 10 & |C_i(m_b)| \sim 4 \end{cases}$$

Three steps of the calculation:

Matching: Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and the effective theory Green functions

Mixing: Deriving the effective theory RGE and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$

Matrix elements: Evaluating the on-shell amplitudes at $\mu_b \sim m_b$

Structure of the SM calculations for $\bar{B} \to X_s \gamma$

$$\mathcal{H}_{\rm eff} \sim \sum_{i=1}^{10} C_i(\mu) O_i$$

• $\mathcal{H}_{\rm eff}$ independent of the scale μ, while $C_i(\mu)$ and $O_i(\mu)$ depend on μ
$\Longrightarrow$ Renormalization Group Equation (RGE) for $C_i(\mu)$:

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ij}^{\rm T} C_j(\mu)$$

• γ_{ij}: anomalous dimension matrix

• Matching usually done at high scale $(\mu_0 \sim M_W, m_t)$

• Full theory and the matrix elements of the effective operators have the same large logarithms

$\mu_0 \sim O(M_W)$
$\downarrow$ RGE
$\mu_b \sim O(m_b)$: matrix elements of the operators at this scale don't have large logs; they are contained in the $C_i(\mu_b)$

• Evaluation of the on-shell amplitudes at $\mu_b \sim m_b$

Wilson Coefficients in the SM

Wilson Coefficients of Four-Quark Operators

	$C_1(\mu_b)$	$C_2(\mu_b)$	$C_3(\mu_b)$	$C_4(\mu_b)$	$C_5(\mu_b)$	$C_6(\mu_b)$
LL	-0.257	1.112	0.012	-0.026	0.008	-0.033
NLL	-0.151	1.059	0.012	-0.034	0.010	-0.040

Wilson Coefficients of Other Operators

	$C_7^{\text{eff}}(\mu_b)$	$C_8^{\text{eff}}(\mu_b)$	$C_9(\mu_b)$	$C_{10}(\mu_b)$
LL	-0.314	-0.149	2.007	0
NLL	-0.308	-0.169	4.154	-4.261
NNLL	-0.290		4.214	-4.312

- Obtained for the following input:

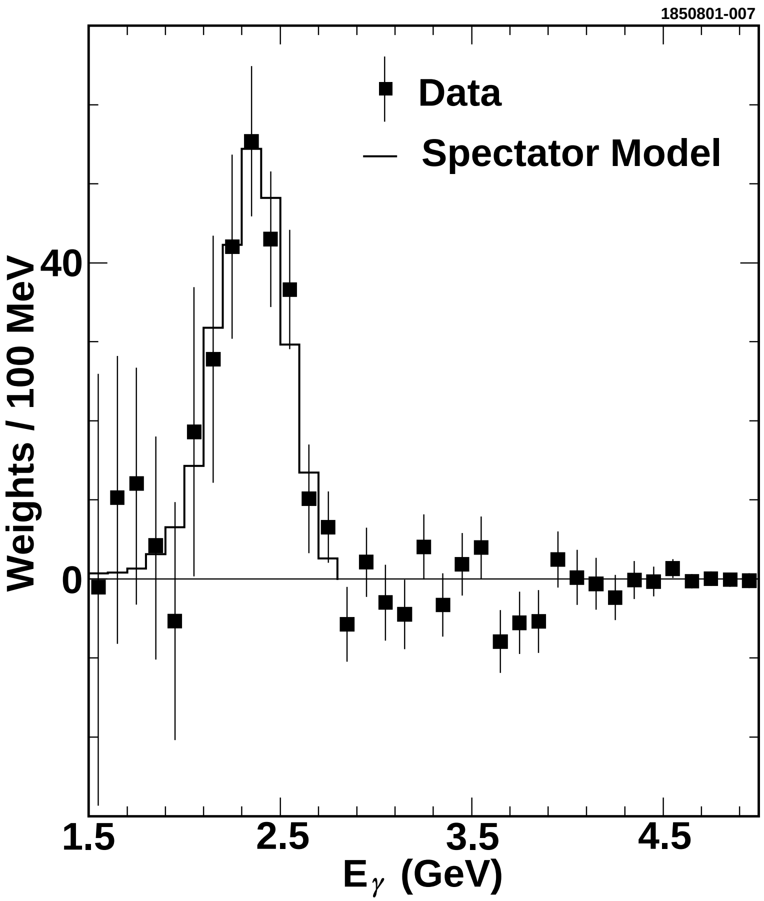
$$\mu_b = 4.6 \text{ GeV} \qquad \bar{m}_t(\bar{m}_t) = 167 \text{ GeV}$$

$$M_W = 80.4 \text{ GeV} \qquad \sin^2\theta_W = 0.23$$

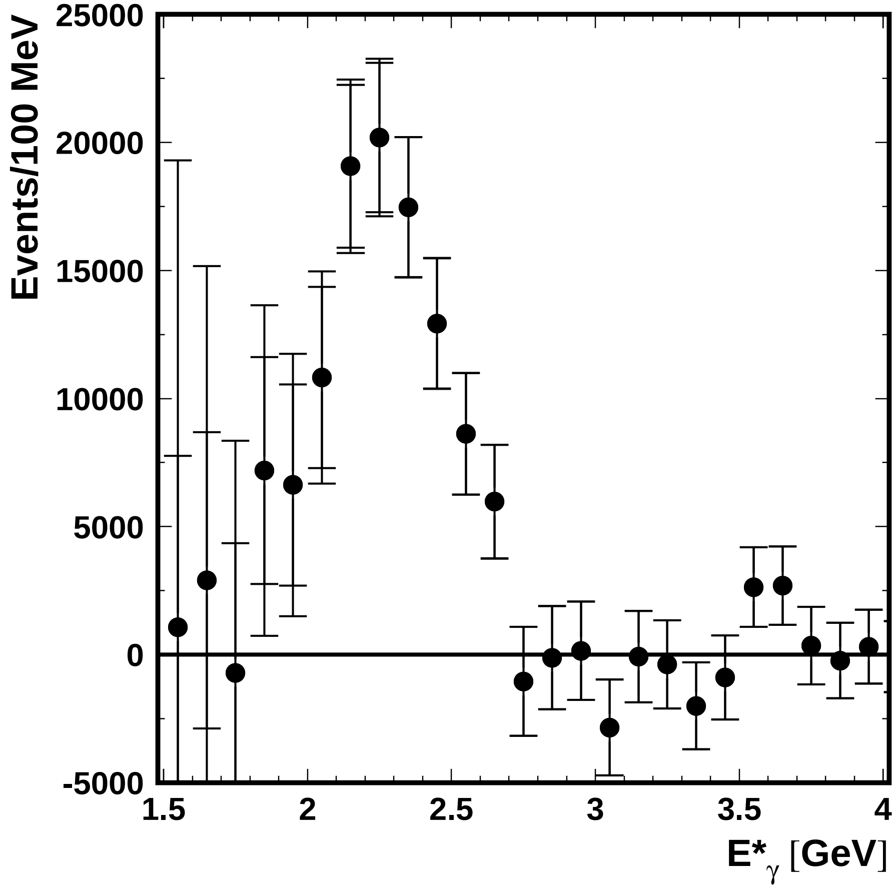
- Three-loop running is used for α_s coupling with $\Lambda_{\overline{\text{MS}}}^{(5)} = 220$ MeV

Measurement of $\bar{B} \to X_s \gamma$

Spectator Model: Greub, AA; PLB 259, 182 (1991)



CLEO
hep-ex/0108032
PRL 87 (2001) 251807



BELLE
hep-ex/0403004
PRL 93 (2004) 061803

Experimental data

Experimental Data on $B \to V\gamma$ Decays

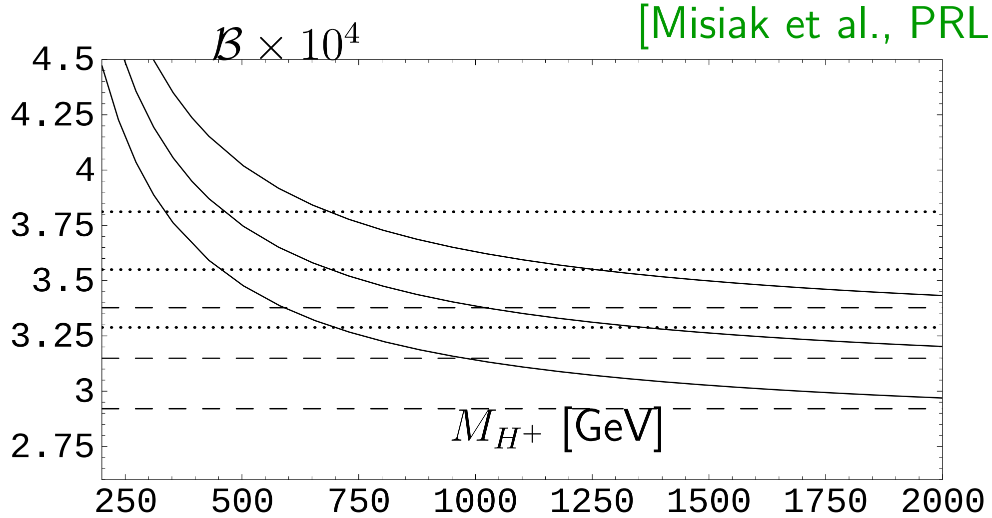
Branching ratios (in units of 10^{-6}) [HFAG, Summer 2012]

Mode	BABAR	BELLE	CLEO	Average [HFAG]
$B \to X_s\gamma$	$331 \pm 35 \pm 34$	$347 \pm 15 \pm 40$	$327 \pm 44 \pm 28$	337 ± 23 ‡
$B^+ \to K^*(892)^+\gamma$	$42.1 \pm 1.4 \pm 1.6$	$42.5 \pm 3.1 \pm 2.4$	$37.6^{+8.9}_{-8.3} \pm 2.8$	42.1 ± 1.8
$B^0 \to K^*(892)^0\gamma$	$44.7 \pm 1.0 \pm 1.6$	$40.1 \pm 2.1 \pm 1.7$	$45.5^{+7.2}_{-6.8} \pm 3.4$	43.3 ± 1.5
$B^+ \to K_1(1270)^+\gamma$		$43 \pm 9 \pm 9$		43 ± 12
$B^+ \to K_2^*(1430)^+\gamma$	$14.5 \pm 4.0 \pm 1.5$			14.5 ± 4.3
$B^0 \to K_2^*(1430)^0\gamma$	$12.2 \pm 2.5 \pm 1.0$	$13.0 \pm 5.0 \pm 1.0$		12.4 ± 2.4
$B^+ \to \rho^+\gamma$	$1.20^{+0.42}_{-0.37} \pm 0.20$	$0.87^{+0.29+0.09}_{-0.27-0.11}$	< 13.0	$0.98^{+0.25}_{-0.24}$
$B^0 \to \rho^0\gamma$	$0.97^{+0.24}_{-0.22} \pm 0.06$	$0.78 \pm 0.17 \pm 0.09$	< 17.0	0.86 ± 0.14
$B^0 \to \omega\gamma$	$0.50^{+0.27}_{-0.23} \pm 0.09$	$0.40^{+0.19}_{-0.17} \pm 0.11$	< 9.2	$0.44^{+0.18}_{-0.16}$
$B \to (\rho,\omega)\gamma$	$1.63 \pm 0.29 \pm 0.16$	$1.14 \pm 0.20 \pm 0.11$	< 14.0	1.30 ± 0.18
$B^0 \to \phi\gamma$	< 0.85		< 3.3	< 0.85
$B^0 \to J/\psi\,\gamma$	< 1.6			< 1.6

‡ Calculated for the photon energy range $E_\gamma > 1.6$ GeV

$\mathcal{B}(\bar{B} \to X_s \gamma)$: Experiment vs. SM & 2HDM

[Misiak et al., PRL 98:022002 (2007)]



[$\cdots$ (exp); $- - - --$ (SM); solid (2HDM)]

- Expt. [ICHEP 2012]: $(E_\gamma > 1.6$ GeV$)$: $\mathcal{B}(\bar{B} \to X_s \gamma) = (3.37 \pm 0.23) \times 10^{-4}$
- NNLO SM: $\mathcal{B}(\bar{B} \to X_s \gamma) = (3.15 \pm 0.23) \times 10^{-4}$
- Ratio=Expt./SM $= 1.07 \pm 0.10$, Limits most NP models
- In 2HDM, $\mathcal{B}(B \to X_s \gamma)$ bounds M_{H^+}

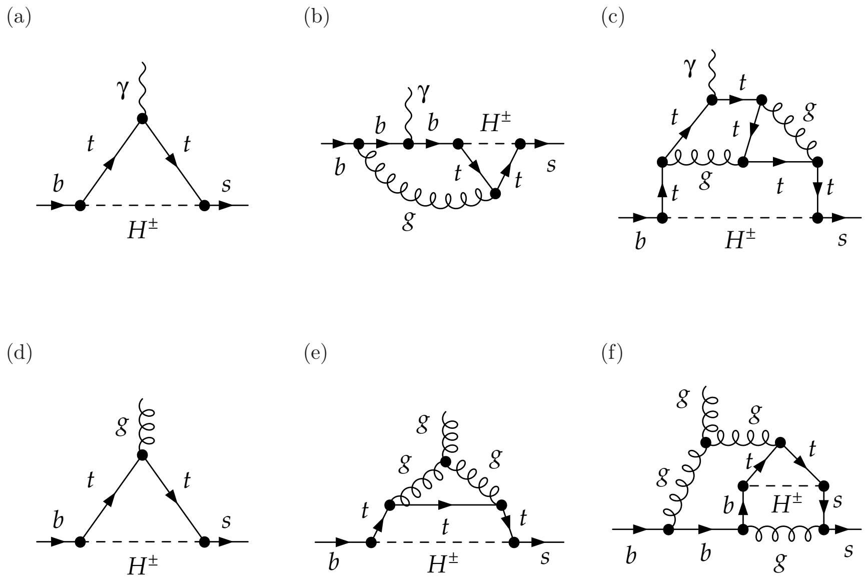
$B \to X_s\gamma$ in 2HDM

- NNLO in 2HDM calculated recently [Hermann, Misiak, Steinhauser; arxiv:1208.2788]

$$\mathcal{L}_{H^+} = (2\sqrt{2}G_F)^{1/2}\Sigma_{i,j=1}^{3}\bar{u}_i(A_u m_{u_i} V_{ij} P_L - A_d m_{d_j} V_{ij} P_R) d_j H^* + h.c.$$

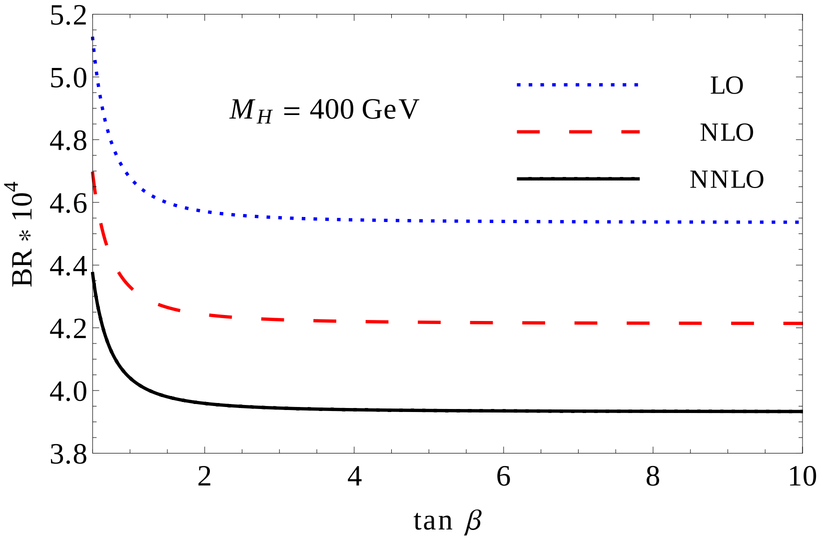
with $P_{L/R} = (1 \mp \gamma_5)/2$

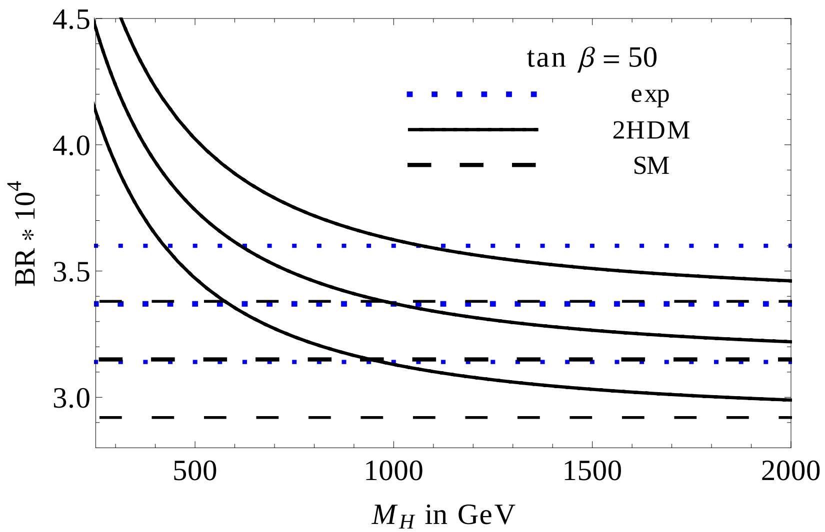
- 2HDM contributions to the Wilson coefficients are proportional to $A_i A_j^*$
  - 2HDM of type-I: $A_u = A_d = \frac{1}{\tan\beta}$
  - 2HDM of type-II: $A_u = -1/A_d = \frac{1}{\tan\beta}$



$\bar{B} \to X_s \gamma$ in Type-II 2HDM

[Hermann, Misiak, Steinhauser; arxiv:1208.2788]





- $M_{H^+} > 380$ GeV (at 95% C.L.)
- $M_{H^+} > 289$ GeV (at 99% C.L.)

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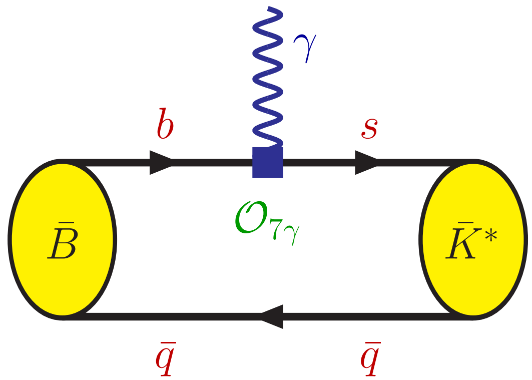
$B \to K^*\gamma$ Branching Fraction in LO

- In LO, only the electromagnetic penguin operator $\mathcal{O}_{7\gamma}$ contributes to the $B \to K^*\gamma$ amplitude; involves the form factor $T_1^{(K^*)}(0)$



$$\mathcal{M}^{\rm LO} = -\frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* C_7^{(0)\rm eff} \frac{e\bar{m}_b}{4\pi^2} T_1^{(K^*)}(0) \left[(Pq)(e^*\varepsilon^*) - (e^*P)(\varepsilon^* q) + i\,{\rm eps}(e^*, \varepsilon^*, P, q)\right]$$

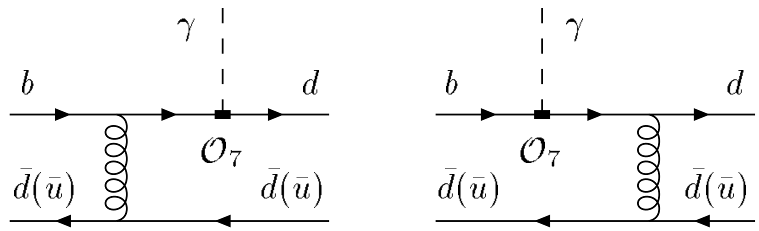
Here, $P^\mu = p_B^\mu + p_K^\mu$; $q^\mu = p_B^\mu - p_K^\mu$ is the photon four-momentum; e^μ is its polarization vector; ε^μ is the K^*-meson polarization vector

- Branching ratio:

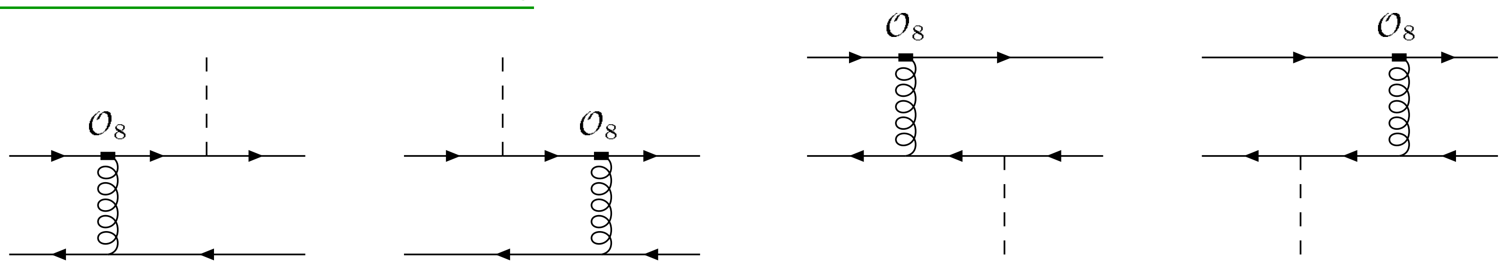
$$\boldsymbol{\mathcal{B}^{\rm LO}(B \to K^*\gamma) = \tau_B \frac{G_F^2 |V_{tb}V_{ts}^*|^2 \alpha M^3}{32\pi^4} \bar{m}_b^2(\mu_b) \, |C_7^{(0)\rm eff}(\mu_b)|^2 \, |T_1^{(K^*)}(0, \mu_b)|^2}$$

Hard spectator contributions in $B \to (K^*, \rho)\,\gamma$

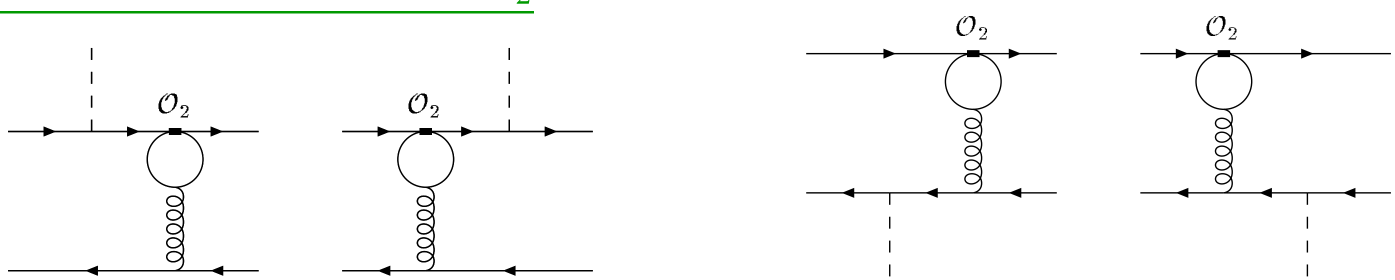
Spectator corrections due to $\mathcal{O}_7$



Spectator corrections due to $\mathcal{O}_8$



Spectator corrections due to $\mathcal{O}_2$



$B \to K^* \gamma$ decay rates in NLO

- Perturbative improvements undertaken in three approaches (QCD-F; PQCD; SCET)

Factorization Ansatz (QCDF):

[Beneke, Buchalla, Neubert, Sachrajda; Beneke & Feldmann]

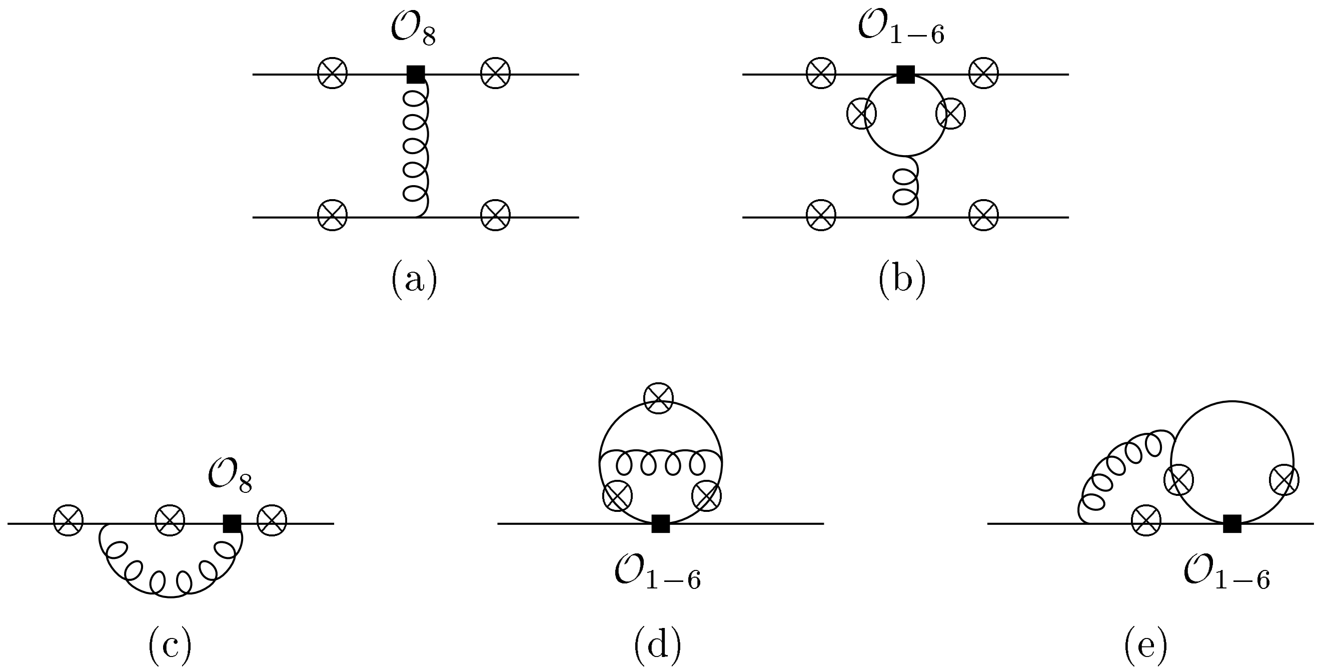
$$\langle V\gamma|Q_i|\bar{B}\rangle = t_i^I \zeta_{V_\perp} + t_i^{II} \otimes \phi_+^B \otimes \phi_\perp^V + \mathcal{O}(\frac{\Lambda_{\rm QCD}}{m_b})$$

- $\zeta_{V_\perp}$ (form factor) and $\phi^{B,V}$ (LCDAs) are non-perturbative functions
- t^I and t^{II} are perturbative hard-scattering kernels

$$t^I = \mathcal{O}(1) + \mathcal{O}(\alpha_s) + ..., \quad t^{II} = \mathcal{O}(\alpha_s) + ...$$

- The kernels t^I and t^{II} are known at $\mathcal{O}(\alpha_s)$ for some time; include Hard-scattering and Vertex corrections [Parkhomenko, AA; Bosch, Buchalla; Beneke, Feldmann, Seidel 2001]

Nonfactorizable α_s Corrections



- First line: hard-spectator corrections
- Second line: $b \to s\gamma$ vertex corrections

SCET factorization formula for $B \to K^* \gamma$

[Chay, Kim '03; Grinstein, Grossman, Ligeti '04; Becher, Hill, Neubert '05]

$$\langle V\gamma|Q_i|\bar{B}\rangle = \Delta_i C^A \zeta_{V_\perp} + (\Delta_i C^{B1} \otimes j_\perp) \otimes \phi_\perp^V \otimes \phi_+^B$$

- $\zeta_{V_\perp}, \phi_\perp^V, \phi_+^B$ are matrix elements of SCET operators
- Hard-scattering kernels t^I, t^{II} = SCET matching coefficients

$$t_i^I = \Delta_i C^A(m_b); \quad t_i^{II} = \Delta_i C^{B1}(m_b) \otimes j_\perp(\sqrt{m_b \Lambda}) \quad \text{(subfactorization)}$$

- Derivation of factorization in SCET

1) QCD $\to$ SCET$_I$: Integrate out m_b; Defines vertex corrections $\Delta_i C^A = t_i^I$

$$Q_i \to \Delta_i C^A(m_b) J^A + \Delta_i C^{B1}(m_b) \otimes J^{B1} + ...$$

2) SCET$_I$ $\to$ SCET$_{II}$: Integrate out $\sqrt{m_b \Lambda_{\rm QCD}}$; Defines spectator corrections

$$J^{B1} \to j_\perp(\sqrt{m_b \Lambda_{\rm QCD}}) \otimes O^{B1,\text{SCET}_{II}}(\Lambda_{\rm QCD})$$

3) Large logs in t_i^{II} resummed by solving RG equations

$$[\Delta_i C^{B1} \otimes j_\perp] \to [\Delta_i C^{B1}(\mu_h) \otimes U(\mu_h, \mu_{hc}) \otimes j_\perp(\mu_{hc})]$$

$B \to K^* \gamma$ in SCET at NNLO

[Pecjak, Greub, AA '07]

Vertex Corrections

$$\Delta_i C^A = \Delta_7 C^{A(0)} \left[\Delta_{i7} + \frac{\alpha_s(\mu)}{4\pi} \Delta_i C^{A(1)} + \frac{\alpha_s^2(\mu)}{(4\pi)^2} \Delta_i C^{A(2)} \right]$$

- Contributions from O_7 and O_8 exact to NNLO $O(\alpha_s^2)$
- Contribution from O_2 exact at NLO $O(\alpha_s)$ but only large-β_0 limit at $O(\alpha_s^2)$

Spectator Corrections at $O(\alpha_s^2)$

$$t_i^{II(1)}(u, \omega) = \Delta_i C^{B1(1)} \otimes j_\perp^{(0)} + \Delta_i C^{B1(0)} \otimes j_\perp^{(1)}$$

- Status of $O(\alpha_s^2)$ Calculations
  - The one-loop jet-function $j_\perp^{(1)}$ known
    [Becher and Hill '04; Beneke and Yang '05]
  - The one-loop hard coefficient $\Delta_7 C^{B1(1)}$ known
    [Beneke, Kiyo, Yang '04; Becher and Hill '04]
  - The one-loop hard coefficient $\Delta_8 C^{B1(1)}$ known
    [Pecjak, Greub, AA '07]
  - $\Delta_i C^{B1(1)}$ $(i = 1, ..., 6)$ remain unknown (require two loops)

Estimates of BR($B \to K^* \gamma$) in SCET at NNLO

[Pecjak, Greub, AA; EPJ C55: 577 (2008)]

Estimates at NNLO in units of 10^{-5}

$$\mathcal{B}(B^+ \to K^{*+}\gamma) = 4.6 \pm 1.2[\zeta_{K^*}] \pm 0.4[m_c] \pm 0.2[\lambda_B] \pm 0.1[\mu]$$

[Expt. 4.2 ± 0.18 (HFAG 2012)];

$$\mathcal{B}(B^0 \to K^{*0}\gamma) = 4.3 \pm 1.1[\zeta_{K^*}] \pm 0.4[m_c] \pm 0.2[\lambda_B] \pm 0.1[\mu]$$

[Expt.: 4.33 ± 0.15 (HFAG 2012)];

$$\mathcal{B}(B_s \to \phi\gamma) = 4.3 \pm 1.1[\zeta_{\phi}] \pm 0.3[m_c] \pm 0.3[\lambda_B] \pm 0.1[\mu]$$

[Expt.: $5.7^{+2.1}_{-1.8}$ (BELLE); 3.9 ± 0.5 (LHCb)]

Comparison with current experiments

- $\frac{\mathcal{B}(B^+ \to K^{*+}\gamma)_{\rm NNLO}}{\mathcal{B}(B^+ \to K^{*+}\gamma)_{\rm exp}} = 1.10 \pm 0.35[\text{theory}] \pm 0.04[\text{exp}]$
- $\frac{\mathcal{B}(B^0 \to K^{*0}\gamma)_{\rm NNLO}}{\mathcal{B}(B^+ \to K^{*0}\gamma)_{\rm exp}} = 1.00 \pm 0.32[\text{theory}] \pm 0.04[\text{exp}]$
- $\frac{\mathcal{B}(B_s \to \phi\gamma)_{\rm NNLO}}{\mathcal{B}(B_s \to \phi\gamma)_{\rm exp}} = 1.1 \pm 0.3[\text{theory}] \pm 0.1[\text{exp}]$
- Theory error is about 30%; dominantly from $\zeta_{V_\perp}$, m_c and λ_B; SM decay rates in good agreement with the data

$B \to \rho\gamma$ decay

Penguin amplitude $\mathcal{M}_{\rm P}(B \to \rho\gamma)$

$$-\frac{G_F}{\sqrt{2}} V_{tb}V_{td}^* C_7 \frac{e m_b}{4\pi^2} \epsilon^{(\gamma)\mu}\epsilon^{(\rho)\nu} \left(\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta - i\left[g^{\mu\nu}(q.p) - p^\mu q^\nu\right]\right) T_1^{(\rho)}(0)$$

Annihilation amplitude $\mathcal{M}_{\rm A}(B^\pm \to \rho^\pm\gamma)$

$$e\frac{G_F}{\sqrt{2}} V_{ub}V_{ud}^* a_1 m_\rho \epsilon^{(\gamma)\mu}\epsilon^{(\rho)\nu} \left(\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta F_A^{(\rho);\rm p.v.} - i\left[g^{\mu\nu}(q.p) - p^\mu q^\nu\right] F_A^{(\rho);\rm p.c.}\right)$$

- $F_A^{(\rho);\rm p.v.}(0) \simeq F_A^{(\rho);\rm p.c.}(0) = F_A^{(\rho)}(0)$ [e.g., Byer, Melikhov, Stech]

$$\epsilon_{\rm A}(\rho^\pm\gamma) = \frac{4\pi^2 m_\rho a_1}{m_b C_7^{eff}} \frac{F_A^{(\rho)}(0)}{T_1^{(\rho)}} = 0.30 \pm 0.07$$

- Holds in factorization approximation
- $O(\alpha_s)$ corrections to annihilation amplitude $\mathcal{M}_{\rm A}(B^\pm \to \rho^\pm\gamma)$: Leading-twist contribution vanishes in the chiral limit [Grinstein, Pirjol]; non-factorizing annihilation contribution likely small; testable in $B^\pm \to \ell^\pm \nu_\ell \gamma$

Annihilation amplitude $\mathcal{M}_{\rm A}(B^0 \to \rho^0\gamma)$

- Suppressed due to the electric charges ($Q_d/Q_u = -1/2$) and colour factors (BSW Parameters: $a_2/a_1 \simeq 0.25$)

$$\Longrightarrow \epsilon_{\rm A}(\rho^0\gamma) \simeq 0.05$$

$B \to (\rho,\ \omega)\gamma$ decay rates

[Parkhomenko, A.A.; Bosch, Buchalla; Lunghi, Parkhomenko, AA; Beneke, Feldmann, Seidel]

$$R(\rho\gamma) \equiv \frac{\overline{\mathcal{B}}(B \to \rho\gamma)}{\overline{\mathcal{B}}(B \to K^*\gamma)} = S_\rho \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(1 - m_\rho^2/M^2)^3}{(1 - m_{K^*}^2/M^2)^3}\,\zeta^2\ [1 + \Delta R(\rho/K^*)]$$

$$R(\omega\gamma) \equiv \frac{\overline{\mathcal{B}}(B \to \omega\gamma)}{\overline{\mathcal{B}}(B \to K^*\gamma)} = 1/2 \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(1 - m_\omega^2/M^2)^3}{(1 - m_{K^*}^2/M^2)^3}\,\zeta^2\ [1 + \Delta R(\omega/K^*)]$$

- $S_\rho = 1$ for $B^\pm \to \rho^\pm\gamma$; $= 1/2$ for $B^0 \to \rho^0\gamma$
- $\zeta = \frac{T_1^{(\rho)}(0)}{T_1^{(K^*)}(0)} \simeq 0.85 \pm 0.10$; $T_1^\omega(0) \simeq T_1^{(\rho)}(0)$ [SRs, Lattice Average]

  $\zeta = \simeq 0.85 \pm 0.06$; $T_1^\omega(0) \simeq T_1^{(\rho)}(0)$ [Ball, Zwicky, 2006]
- $\Delta R(\rho^\pm/K^{*\pm}) = 0.12 \pm 0.10$
- $\Delta R(\rho^0/K^{*0}) \simeq \Delta R(\omega/K^{*0}) = 0.1 \pm 0.07$

Branching Ratios (SM) vs. Expt.

$\mathrm{BR}(B^\pm \to \rho^\pm\gamma) = (1.35 \pm 0.4) \times 10^{-6}$ (SM)$= (0.98 \pm 0.24) \times 10^{-6}$ (Expt.)
$\mathrm{BR}(B^0 \to \rho^0\gamma) \simeq \mathrm{BR}(B^0 \to \omega\gamma) = (0.65 \pm 0.2) \times 10^{-6}$
$\mathrm{BR}(B^0 \to \rho^0\gamma)$ (Expt.) $= (0.86 \pm 0.14) \times 10^{-6}$
$\mathrm{BR}(B^0 \to \omega\gamma)$ (Expt.) $= (1.30 \pm 0.18) \times 10^{-6}$

Experiment vs. SM $(b \to d\gamma)$

SM Estimates [Lunghi, Parkhomenko, AA; PLB 595 (2004) 323]

$$\bar{\mathcal{B}}[B \to (\rho,\omega)\,\gamma] \equiv \frac{1}{2}\left\{\mathcal{B}(B^+ \to \rho^+\gamma) + \frac{\tau_{B^+}}{\tau_{B^0}}\left[\mathcal{B}(B_d^0 \to \rho^0\gamma) + \mathcal{B}(B_d^0 \to \omega\gamma)\right]\right\}$$
$$= (1.38 \pm 0.42) \times 10^{-6}$$

$$R[(\rho,\omega)/K^*] \equiv \frac{\bar{\mathcal{B}}[B \to (\rho,\omega)\,\gamma]}{\bar{\mathcal{B}}[B \to K^*\,\gamma]} = 0.033 \pm 0.010$$

Expt. HFAG-2012

$$\bar{\mathcal{B}}_{\rm exp}[B \to (\rho,\omega)\,\gamma] = (1.30^{+0.18}_{-0.19}) \times 10^{-6}$$

$$R[(\rho,\omega)/K^*] = 0.030 \pm 0.005\ ({\rm stat})^{+0.003}_{-0.002}\ ({\rm syst})$$

$$|V_{td}/V_{ts}| = 0.20 \pm 0.02\ ({\rm exp}) \pm 0.04\ ({\rm theo})$$

- In good agreement with the determination from the ratio $\Delta M_s/\Delta M_d \Longrightarrow$ $|V_{td}|/|V_{ts}| = 0.211 \pm 0.001({\rm exp}) \pm 0.006\ ({\rm theo})$ in the SM, but less precise
- A correlated study of $R[(\rho,\omega)/K^*]$ and $\Delta M_s/\Delta M_d$ provides valuable constraints on the parameters of the underlying theory

D. Mohapatra (BELLE)[EPS 2005]

Extraction of $|V_{td}/V_{ts}|$

$$\frac{B(\overline{B}\to(\rho,\omega)\gamma)}{B(B\to K^*\gamma)}=\left|\frac{V_{td}}{V_{ts}}\right|^2\left(\frac{1-M_\rho^2/M_B^2}{1-M_{K^*}^2/M_B^2}\right)\zeta^2[1+\Delta R]$$

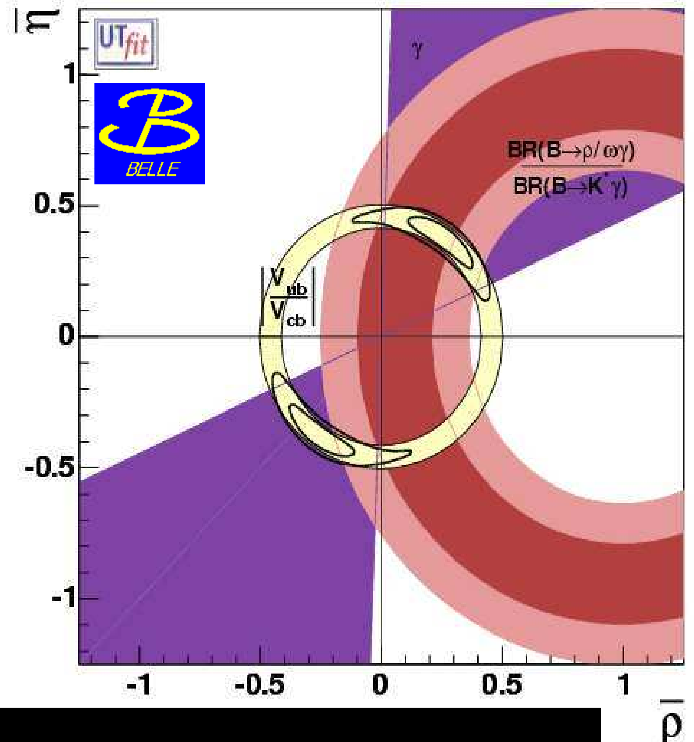
Form factor ratio $\zeta=0.85\pm0.10$

SU(3)-breaking effect $\Delta R=0.1\pm0.1$

$$\frac{B(B\to(\rho,\omega)\gamma)}{B(B\to K^*\gamma)}=0.032\pm0.008^{+0.003}_{-0.002}$$

$$0.143<\left|\frac{V_{td}}{V_{ts}}\right|<0.260$$

(95 % C.L. interval)



$$\left|\frac{V_{td}}{V_{ts}}\right|=0.200^{+0.026}_{-0.025}(\text{expt.})^{+0.038}_{-0.029}(\text{theo.})$$

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Isospin violation in $B \to \rho\gamma$ decays

$$\Delta = \frac{1}{2}\left[\Delta^{+0} + \Delta^{-0}\right], \qquad \Delta^{\pm 0} \equiv \frac{\Gamma(B^\pm \to \rho^\pm \gamma)}{2\Gamma(B^0(\bar{B}^0) \to \rho^0 \gamma)} - 1$$

$$\Delta_{\rm LO} = 2\epsilon_A \left[F_1 + \frac{\epsilon_A}{2}\left(F_1^2 + F_2^2\right)\right] = 2\epsilon_A F \cos\alpha + O(\epsilon_A^2)$$

$$\Delta_{\rm NLO} \simeq \Delta_{\rm LO} - \frac{2\epsilon_A}{C_7^{(0)\rm eff}} F\cos\alpha \left[A_R^{(1)t} + A_R^u F \cos 2\alpha\right] + O(\epsilon_A^2)$$

$$F_1 = F\cos\alpha; \quad F_2 = F\sin\alpha; \quad F = \frac{R_b}{R_t} \simeq 0.5$$

$$\Delta^{\rm SM}(\rho\gamma) = (1.1 \pm 3.9)\% \;\; \text{for} \;\; \alpha = (92 \pm 11)^\circ; \quad \Delta^{\rm expt}(\rho\gamma) = -0.46^{+0.17}_{-0.16}$$

$$\Delta^{(\rho/\omega)} \equiv \frac{1}{2}\left[\Delta_B^{(\rho/\omega)} + \Delta_{\bar{B}}^{(\rho/\omega)}\right]$$

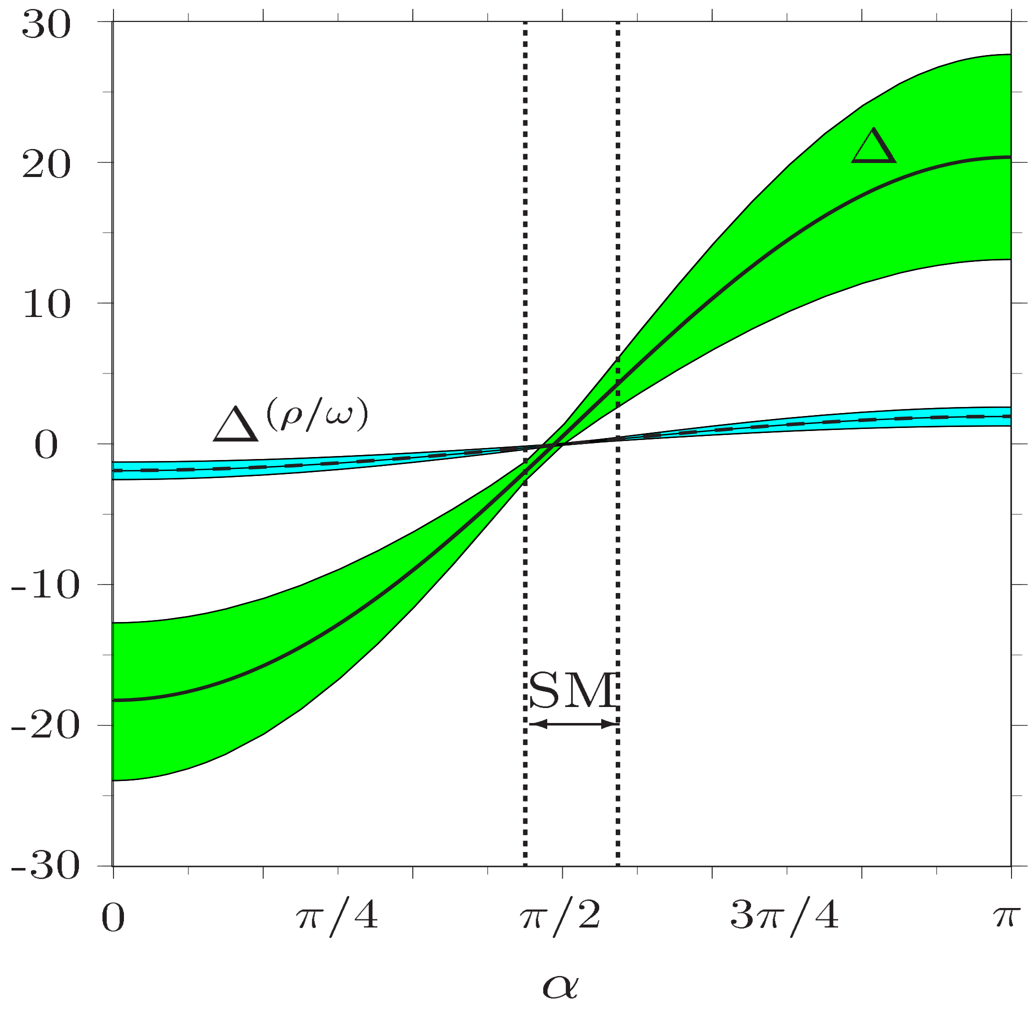
$$\Delta_B^{(\rho/\omega)} \equiv \frac{(M_B^2 - m_\omega^2)^3 \mathcal{B}(B^0 \to \rho\gamma) - (M_B^2 - m_\rho^2)^3 \mathcal{B}(B^0 \to \omega\gamma)}{(M_B^2 - m_\omega^2)^3 \mathcal{B}(B^0 \to \rho\gamma) + (M_B^2 - m_\rho^2)^3 \mathcal{B}(B^0 \to \omega\gamma)}$$

with $\Delta_{\bar{B}}^{(\rho/\omega)} = \Delta_B^{(\rho/\omega)}(B^0 \to \bar{B}^0)$

$$\Delta_B^{(\rho/\omega)} = (0.3 \pm 3.9) \times 10^{-3} \;\; \text{for} \;\; \alpha = (92 \pm 11)^\circ$$

Isospin-violating ratio Δ in $B \to \rho\gamma$ decays

[AA, Lunghi, Parkhomenko; PLB 595 (2004) 323]



$\bar{B} \to X_s l^+ l^-$

- The NNLO calculation of $\bar{B} \to X_s l^+ l^-$ corresponds to the NLO calculation of $\bar{B} \to X_s \gamma$, as far as the number of loops in the diagrams is concerned.

- Wilson Coefficients of the two additional operators

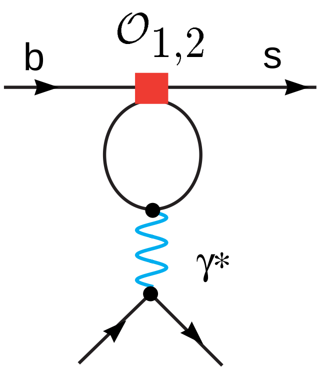
$$O_i = \frac{e^2}{16\pi^2}(\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), \qquad i = 9, 10$$

have the following perturbative expansion:

$$C_9(\mu) = \frac{4\pi}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_9^{(1)}(\mu) + \ldots$$
$$C_{10} = C_{10}^{(0)} + \frac{\alpha_s(M_W)}{4\pi} C_{10}^{(1)} + \ldots$$

- After an expansion in α_s, the term $C_9^{(-1)}(\mu)$ reproduces (the dominant part of) the electroweak logarithm that originates from photonic penguins with charm quark loops:

$$\frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) = \frac{4}{9} \ln \frac{M_W^2}{m_b^2} + \mathcal{O}(\alpha_s)$$



$C_9^{(-1)}(m_b) \simeq 0.033 \ll 1 \quad \Longrightarrow \quad \frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) \simeq 2$

On the other hand: $C_9^{(0)}(m_b) \simeq 2.2$; need to calculate NNLO

NNLO Calculations of $\mathrm{BR}(\bar{B} \to X_s \ell^+ \ell^-)$

- Two-loop matching, three-loop mixing and two-loop matrix elements have been completed
  - Matching: [Bobeth, Misiak, Urban]
  - Mixing: [Gambino, Gorbahn, Haisch]
  - Matrix elements:
    [Asatryan, Asatrian, Greub, Walker;
    Asatrian, Bieri, Greub, Hovhannissyan;
    Ghinculov, Hurth, Isidori, Yao;
    Bobeth, Gambino, Gorbahn, Haisch]
- Power corrections in $B \to X_s \ell^+ \ell^-$ decays
  - $1/m_b$ corrections [A. Falk et al.; AA, Handoko, Morozumi,Hiller; Buchalla, Isidori]
  - $1/m_c$ corrections [Buchalla, Isidori, Rey]
- NNLO Phenomenological analysis of $B \to X_s \ell^+ \ell^-$ decays [AA, Greub, Hiller, Lunghi]
  - $\mathbf{BR}(\bar{B} \to X_s \mu^+ \mu^-); \quad q^2 > 4m_\mu^2 = (4.2 \pm 1.0) \times 10^{-6}$
  - $\mathbf{BR}(\bar{B} \to X_s e^+ e^-) = (6.9 \pm 0.7) \times 10^{-6}$

Inclusive $B \to X_s \ell^+ \ell^-$ in NNLO in SM

Dilepton Invariant Mass

$$\frac{d\Gamma(B \to X_s \ell^+ \ell^-)}{d\hat{s}} = \left(\frac{\alpha_{em}}{4\pi}\right)^2 \frac{G_F^2 m_{b,pole}^5 \left|V_{ts}^* V_{tb}\right|^2}{48\pi^3} (1-\hat{s})^2$$

$$\times \left((1+2\hat{s}) \left(\left|\tilde{C}_9^{\text{eff}}\right|^2 + \left|\tilde{C}_{10}^{\text{eff}}\right|^2 \right) + 4\,(1+2/\hat{s}) \left|\tilde{C}_7^{\text{eff}}\right|^2 + 12\text{Re}\left(\tilde{C}_7^{\text{eff}} \tilde{C}_9^{\text{eff}*}\right) \right)$$

$$\begin{aligned}
\tilde{C}_7^{\text{eff}} &= \left(1 + \frac{\alpha_s(\mu)}{\pi}\omega_7(\hat{s})\right) A_7 \\
&\quad - \frac{\alpha_s(\mu)}{4\pi} \left(C_1^{(0)} F_1^{(7)}(\hat{s}) + C_2^{(0)} F_2^{(7)}(\hat{s}) + A_8^{(0)} F_8^{(7)}(\hat{s}) \right) \\
\tilde{C}_9^{\text{eff}} &= \left(1 + \frac{\alpha_s(\mu)}{\pi}\omega_9(\hat{s})\right) \left(A_9 + T_9\, h(\hat{m}_c^2, \hat{s}) + U_9\, h(1, \hat{s}) + W_9\, h(0, \hat{s}) \right) \\
&\quad - \frac{\alpha_s(\mu)}{4\pi} \left(C_1^{(0)} F_1^{(9)}(\hat{s}) + C_2^{(0)} F_2^{(9)}(\hat{s}) + A_8^{(0)} F_8^{(9)}(\hat{s}) \right) \\
\tilde{C}_{10}^{\text{eff}} &= \left(1 + \frac{\alpha_s(\mu)}{\pi}\omega_9(\hat{s})\right) A_{10}
\end{aligned}$$

- A_7, A_8, A_9, A_{10}, T_9, U_9, W_9 are linear combinations of the Wilson coefficients

Comparison of $B \to X_s \ell^+ \ell^-$ with Data

[AA,Greub, Hiller, Lunghi 2001 (AGHL); Ghinculov, Hurth, Isidori, Yao 2004 (GHIY); Huber, Lunghi, Misiak, Wyler 2005 (HLMW); Bobeth, Gambino, Gorbahn, Haisch 2003]

- Inclusive $B \to X_s \ell^+ \ell^-$ BRs

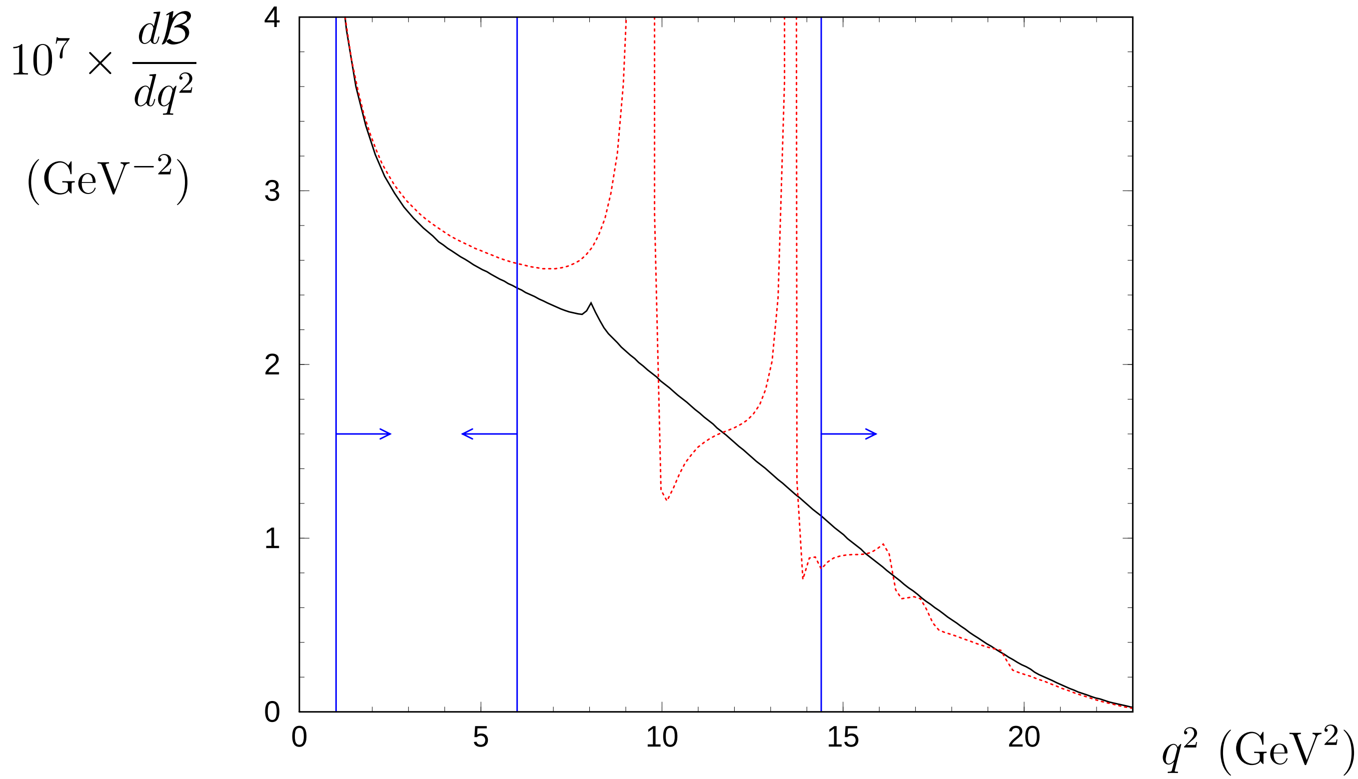
$$\mathcal{B}(B \to X_s \ell^+ \ell^-)(M_{\ell\ell} > 0.2 \text{ GeV}) = (3.66^{+0.76}_{-0.77}) \times 10^{-6} \quad [\text{HFAG}'12]$$

$$SM: (4.2 \pm 0.7) \times 10^{-6} \quad [\text{AGHL}]; \quad (4.6 \pm 0.8) \times 10^{-6} \quad [\text{GHIY}]$$

- Partial BRs (integrated over lower range of q^2)
  - $\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-); \quad q^2 \in [1,6] \text{ GeV}^2 = (1.63 \pm 0.20) \times 10^{-6}$ [GHIY]
  - $\mathcal{B}(\bar{B} \to X_s \mu^+ \mu^-); \quad q^2 \in [1,6] \text{ GeV}^2 = (1.59 \pm 0.11) \times 10^{-6}$ [HLMW]
  - $\mathcal{B}(\bar{B} \to X_s e^+ e^-); \quad q^2 \in [1,6] \text{ GeV}^2 = (1.63 \pm 0.11) \times 10^{-6}$ [HLMW]
  - Experiment: $\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-) \quad q^2 \in [1,6] \text{ GeV}^2 = (1.60 \pm 0.51) \times 10^{-6}$
- Partial BRs (integrated over higher range of q^2)
  - $\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-); \quad q^2 > 14 \text{ GeV}^2 = (4.04 \pm 0.78) \times 10^{-7}$ [GHIY]
  - $\mathcal{B}(\bar{B} \to X_s \mu^+ \mu^-); \quad q^2 > 14.4 \text{ GeV}^2 = 2.40(1^{+0.29}_{-0.26}) \times 10^{-7}$ [HLMW]
  - $\mathcal{B}(\bar{B} \to X_s e^+ e^-); \quad q^2 > 14.4 \text{ GeV}^2 = 2.09(1^{+0.32}_{-0.30}) \times 10^{-7}$ [HLMW]
  - Experiment: $\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-) \quad q^2 > 14.4 \text{ GeV}^2 = (4.4 \pm 1.2) \times 10^{-7}$

Dilepton invariant mass distribution in $\bar{B} \to X_s \ell^+ \ell^-$:

[Ghinculov, Hurth, Isidori, Yao 2004]

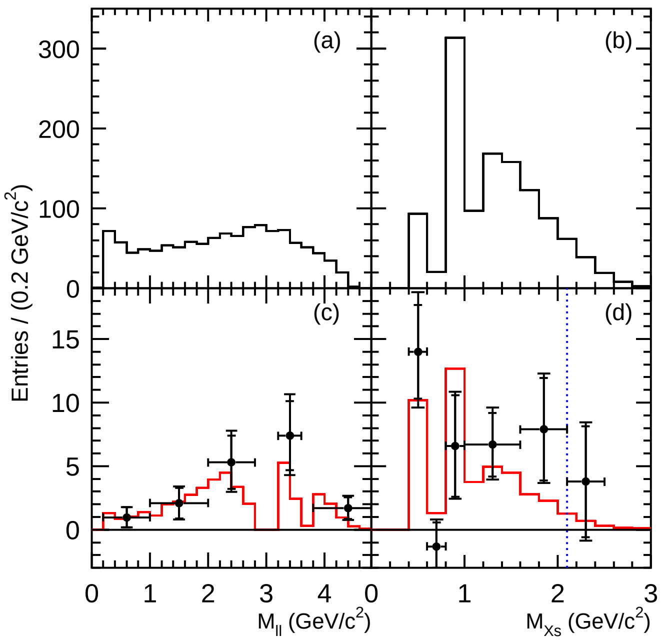


- $\mathbf{BR}(\bar{B} \to X_s \ell^+ \ell^-);\quad q^2 \in [1,6]\ \mathrm{GeV}^2 = (1.63 \pm 0.20) \times 10^{-6}$
- $\mathbf{BR}(\bar{B} \to X_s \ell^+ \ell^-);\quad q^2 > 14\ \mathrm{GeV}^2 = (4.04 \pm 0.78) \times 10^{-7}$
- $\mathbf{BR}(\bar{B} \to X_s \ell^+ \ell^-);\quad q^2 > 4m_\mu^2 = (4.6 \pm 0.8) \times 10^{-6}$,

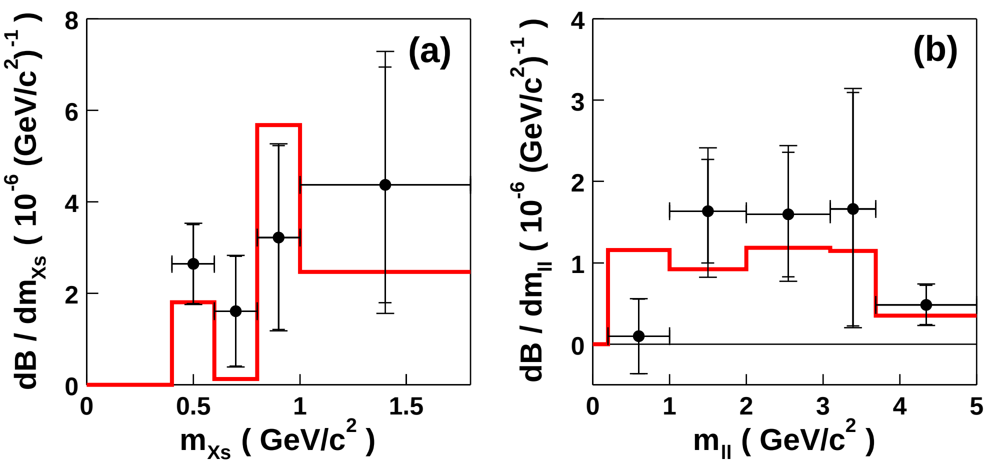
Decay distributions in $\bar{B} \to X_s \ell^+ \ell^-$

$M_{\ell\ell}$ and M_{X_s} Spectra

[BELLE]



[BABAR]



- In agreement with the NNLO SM calculations

Forward-Backward Asymmetry in $B \to X_s \ell^+ \ell^-$

[Proposed in AA, Mannel, Morozumi, PLB 273, 505 (1991)]
[NNLL: Asatrian, Bieri, Greub, Hovhannisyan; Ghinculov, Hurth, Isidori, Yao]

Normalized FB Asymmetry

$$\overline{A}_{\rm FB}(\hat{s}) = \frac{\int_{-1}^{1} \frac{d^2\Gamma(b \to X_s \ell^+\ell^-)}{d\hat{s}\, dz} \, \mathrm{sgn}(z)\, dz}{\int_{-1}^{1} \frac{d^2\Gamma(b \to X_s \ell^+\ell^-)}{d\hat{s}\, dz}\, dz}$$

Unnormalized FB Asymmetry

$$A_{\rm FB}(\hat{s}) = \frac{\int_{-1}^{1} \frac{d^2\Gamma(b \to X_s \ell^+\ell^-)}{d\hat{s}\, dz} \, \mathrm{sgn}(z)\, dz}{\Gamma(B \to X_c e \bar{\nu}_e)} \mathrm{BR}_{\rm sl}$$

$$\int_{-1}^{1} \frac{d^2\Gamma(b \to X_s \ell^+\ell^-)}{d\hat{s}\, dz} \, \mathrm{sgn}(z)\, dz = \left(\frac{\alpha_{\rm em}}{4\pi}\right)^2 \frac{G_F^2\, m_{b,\rm pole}^5 \left|V_{ts}^* V_{tb}\right|^2}{48\, \pi^3} (1-\hat{s})^2$$
$$\times \left[-3\, \hat{s}\, \mathrm{Re}(\widetilde{C}_9^{\rm eff} \widetilde{C}_{10}^{\rm eff*}) \left(1 + \frac{2\alpha_s}{\pi} f_{910}(\hat{s})\right) - 6\, \mathrm{Re}(\widetilde{C}_7^{\rm eff} \widetilde{C}_{10}^{\rm eff*}) \left(1 + \frac{2\alpha_s}{\pi} f_{710}(\hat{s})\right)\right]$$

- NNLL Contributions stabilize the scale ($= \mu$) dependence of the FB Asymmetry

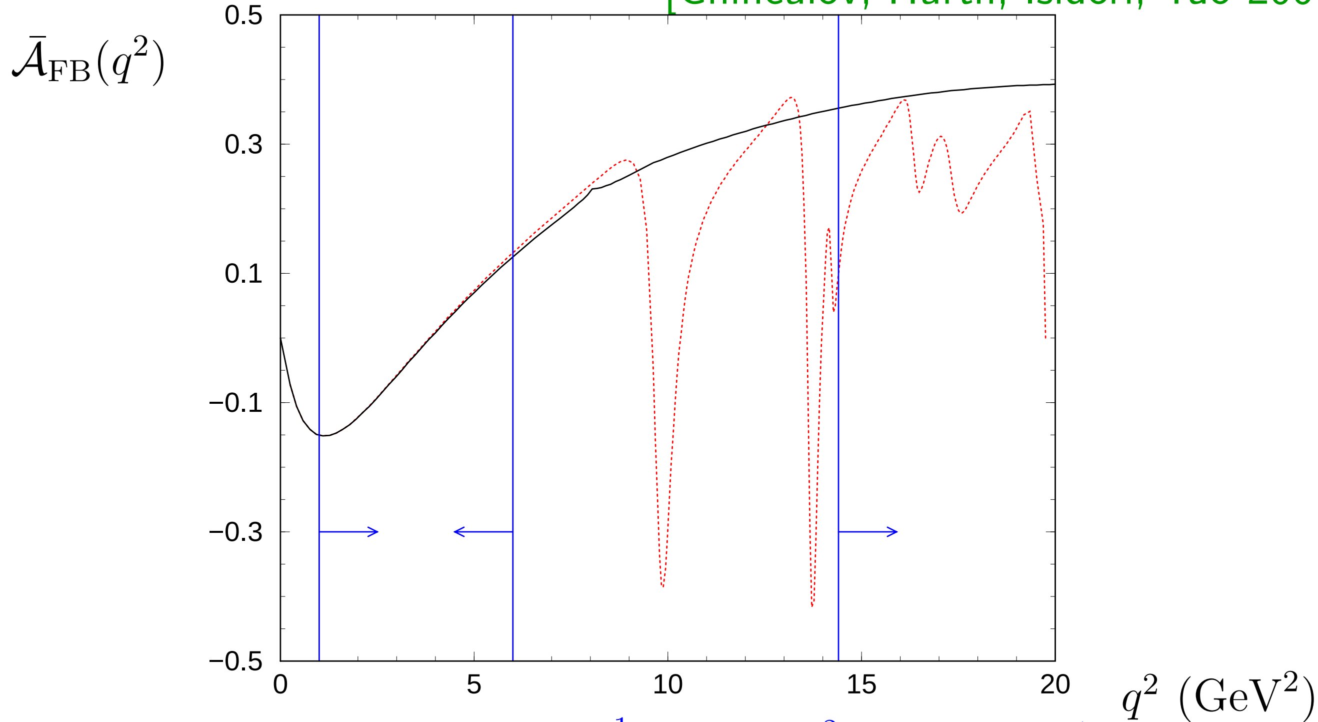
$$A_{\rm FB}^{\rm NLL}(0) = -(2.51 \pm 0.28) \times 10^{-6}; \quad A_{\rm FB}^{\rm NNLL}(0) = -(2.30 \pm 0.10) \times 10^{-6}$$

- Zero of the FB Asymmetry is a precise test of the SM, correlating $\widetilde{C}_7^{\rm eff}$ and $\widetilde{C}_9^{\rm eff}$

$$\hat{s}_0^{\rm NLL} = 0.144 \pm 0.020; \quad \hat{s}_0^{\rm NNLL} = 0.162 \pm 0.008$$

Normalized FB-Asymmetry in $\bar{B} \to X_s \ell^+ \ell^-$:

[Ghinculov, Hurth, Isidori, Yao 2004]



$$\bar{\mathcal{A}}_{\rm FB}(q^2) = \frac{1}{d\mathcal{B}(B \to X_s \ell^+ \ell^-)/dq^2} \int_{-1}^{1} d\cos\theta_\ell \, \frac{d^2\mathcal{B}(B \to X_s \ell^+ \ell^-)}{dq^2 \, d\cos\theta_\ell} \mathrm{sgn}(\cos\theta_\ell)$$

- Zero of the FB-Asymmetry is a precision test of the SM

$$q_0^2 = (3.90 \pm 0.25)\ \mathrm{GeV}^2$$ [Ghinculov, Hurth, Isidori, Yao 2004]

$$q_0^2 = (3.76 \pm 0.22_{\rm theory} \pm 0.24_{m_b})\ \mathrm{GeV}^2$$ [Bobeth, Gambino, Gorbahn, Haisch 2003]

Exclusive Decays $B \to (K, K^*)\ell^+\ell^-$

- $B \to K$ (pseudoscalar P); $B \to K^*$ (Vector V) Transitions involve the currents:

$$\Gamma^1_\mu = \bar{s}\gamma_\mu(1-\gamma_5)b, \quad \Gamma^2_\mu = \bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b$$

$$\langle P|\Gamma^1_\mu|B\rangle \supset f_+(q^2), f_-(q^2)$$

$$\langle P|\Gamma^2_\mu|B\rangle \supset f_T(q^2)$$

$$\langle V|\Gamma^1_\mu|B\rangle \supset V(q^2), A_1(q^2), A_2(q^2), A_3(q^2)$$

$$\langle V|\Gamma^2_\mu|B\rangle \supset T_1(q^2), T_2(q^2), T_3(q^2)$$

- 10 non-perturbative q^2-dependent functions (Form factors) $\Longrightarrow$ model-dependence
- Data on $B \to K^*\gamma$ provides normalization of $T_1(0) = T_2(0) \simeq 0.28$
- HQET/SCET-Approach allows to reduce the number of independent form factors from 10 to 3; perturbative symmetry-breaking corrections [Beneke, Feldmann, Seidel; Beneke, Feldmann]
- HQET & $SU(3)_F$ relate $B \to (\pi, \rho)\ell\nu_\ell$ and $B \to (K, K^*)\ell^+\ell^-$ to determine the remaining FF's

Experimental data vs. SM in $B \to (X_s, K, K^*)\ell^+\ell^-$ Decays

Branching ratios (in units of 10^{-6}) [HFAG: 2012]

SM: [A.A., Greub, Hiller, Lunghi PR D66 (2002) 034002]

Decay Mode	Expt. (BELLE & BABAR)	Theory (SM)
$B \to K\ell^+\ell^-$	0.45 ± 0.04	0.35 ± 0.12
$B \to K^* e^+ e^-$	$1.19^{+0.17}_{-0.16}$	1.58 ± 0.49
$B \to K^* \mu^+ \mu^-$	$1.15^{+0.16}_{-0.15}$	1.19 ± 0.39
$B \to X_s \mu^+ \mu^-$	$2.23^{+0.97}_{-0.98}$	4.2 ± 0.7
$B \to X_s e^+ e^-$	$4.91^{+1.04}_{-1.06}$	4.2 ± 0.7
$B \to X_s \ell^+ \ell^-$	$3.66^{+0.76}_{-0.77}$	4.2 ± 0.7

- Inclusive measurements and the SM rates include the cut $M_{\ell^+\ell^-} > 0.2$ GeV
- SM & Data agree within $\mathbf{25\%}$

Forward-Backward Asymmetry in $B \to K^*\ell^+\ell^-$

$$\frac{dA_{FB}}{d\hat{s}} = -\int_0^{\hat{u}(\hat{s})} d\hat{u} \frac{d\Gamma}{d\hat{u}d\hat{s}} + \int_{-\hat{u}(\hat{s})}^{0} d\hat{u} \frac{d\Gamma}{d\hat{u}d\hat{s}}$$

$$\sim C_{10}[\mathrm{Re}(C_9^{eff})VA_1 + \frac{\hat{m}_b}{\hat{s}} C_7^{eff}(VT_2(1-\hat{m}_V) + A_1T_1(1+\hat{m}_V))]$$

- $\boldsymbol{T_1, T_2, V, A_1}$ form factors
  - Probes different combinations of WC's than dilepton mass spectrum; has a characteristic zero in the SM $(\hat{s}_0)$ below $m^2_{J/\psi}$

Position of the $A_{FB}(\hat{s})$ zero $(\hat{s}_0)$ in $B \to K^*\ell^+\ell^-$

$$\mathrm{Re(C_9^{eff}(\hat{s}_0)) = -\frac{\hat{m}_b}{\hat{s}_0} C_7^{eff}(\frac{T_2(\hat{s}_0)}{A_1(\hat{s}_0)}(1-\hat{m}_V) + \frac{T_1(\hat{s}_0)}{V(\hat{s}_0)}(1+\hat{m}_V))}$$

  - Model-dependent studies $\Longrightarrow$ small FF-related uncertainties in $\hat{s}_0$ [Burdman '98]
  - HQET provides a symmetry argument why the uncertainty in $\hat{s}_0$ is small. In leading order in $1/m_B$, $1/E$ $(E = \frac{m_B^2 + m_{K^*}^2 - q^2}{2m_B})$ and $O(\alpha_s)$:

$$\frac{T_2}{A_1} = \frac{1+\hat{m}_V}{1+\hat{m}_V^2 - \hat{s}}(1 - \frac{\hat{s}}{1-\hat{m}_V^2}); \quad \frac{T_1}{V} = \frac{1}{1+\hat{m}_V}$$

  - No hadronic uncertainty in $\hat{s}_0$ [AA, Ball, Handoko, Hiller '99]:
$C_9^{eff}(\hat{s}_0) = -\frac{2m_b M_B}{s_0} C_7^{eff}$

$B \to K^* \ell^+ \ell^-$ decay in SCET

[AA, Gustav Kramer, Guohuai Zhu; hep-ph/0601034 (EPJC (2006))]

- Soft Collinear Effective Theory (SCET): Applicable to any QCD processes which contain collinear meson or jet, i.e. $P^2 \ll Q^2$, in the final states
- The idea is borrowed from HQET and NRQCD, but technically SCET is more involved than HQET because of the collinear degrees of freedom
- For $B \to K^* \ell^+ \ell^-$ decay, in the region 1 GeV$^2 \leq q^2 \leq 8$ GeV^2

$$P^\mu_{K^*} = (2.34, 0, 0, 2.16) \text{ GeV} \qquad [q^2 = 4 \text{ GeV}^2]$$

- Light-cone vectors $n^\mu = (1, 0, 0, 1)$, $\bar{n}^\mu = (1, 0, 0, -1)$, satisfying $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 2$

$$P^\mu = n \cdot P \frac{\bar{n}^\mu}{2} + \bar{n} \cdot P \frac{n^\mu}{2} + P^\mu_\perp = (P_+, P_-, P_\perp) \sim E(\lambda^2, 1, \lambda)$$

$$[P_+ = 0.18 \text{ GeV}, \; P_- = 4.5 \text{ GeV}, \; \lambda \sim 0.2]$$

- Power counting and expansion in λ, $\lambda \sim \frac{\Lambda_{QCD}}{E}$

Leading order in $1/m_b$ and all orders in α_s

[AA, Kramer, Zhu; EPJ (2006) 625]

The factorization formula in SCET

$$
\begin{aligned}
\langle K_a^* \ell^+ \ell^- | H_{eff} | B \rangle &= T_a^I(q^2)\, \xi_a(q^2) + \\
&+ \sum_{\pm} \int_0^\infty \frac{d\omega}{\omega}\, \phi_\pm^B(\omega) \int_0^1 du\, \phi_{K^*}^B(u)\, T_{a,\pm}^{II}(\omega, u, q^2)
\end{aligned}
$$

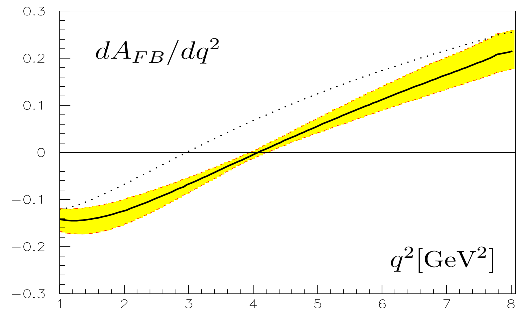
where $a = \|, \perp$ denotes the polarization of the K^* meson.

- formally coincides with the formula in QCD Factorization [Beneke/Feldmann/Seidel 2001], but valid to all orders of α_S,
- for T^{II}, the logarithms are summed from $\mu = m_b$ to $\sqrt{m_b \Lambda_h}$,
- compared with BFS, the definition of $\xi_{\|,\perp}$ is also different here.

Reduction of Scale Uncertainty in SCET

Introduction $B \to K^* \ell^+ \ell^-$ decay Summary SCET formulae Phenomenological discussion

Forward-backward asymmetry



$A_{FB}(q_0^2) = 0$ free of hadronic uncertainties [Burdman1998, Ali et al., 2000]

$q_0^2 = (4.07^{+0.16}_{-0.13})\ \mathrm{GeV}^2$ with $\Delta(q_0^2)_{\mathrm{scale}} = {}^{+0.08}_{-0.05}\ \mathrm{GeV}^2$

QCD-F [Beneke/Feldmann/Seidel 2001]

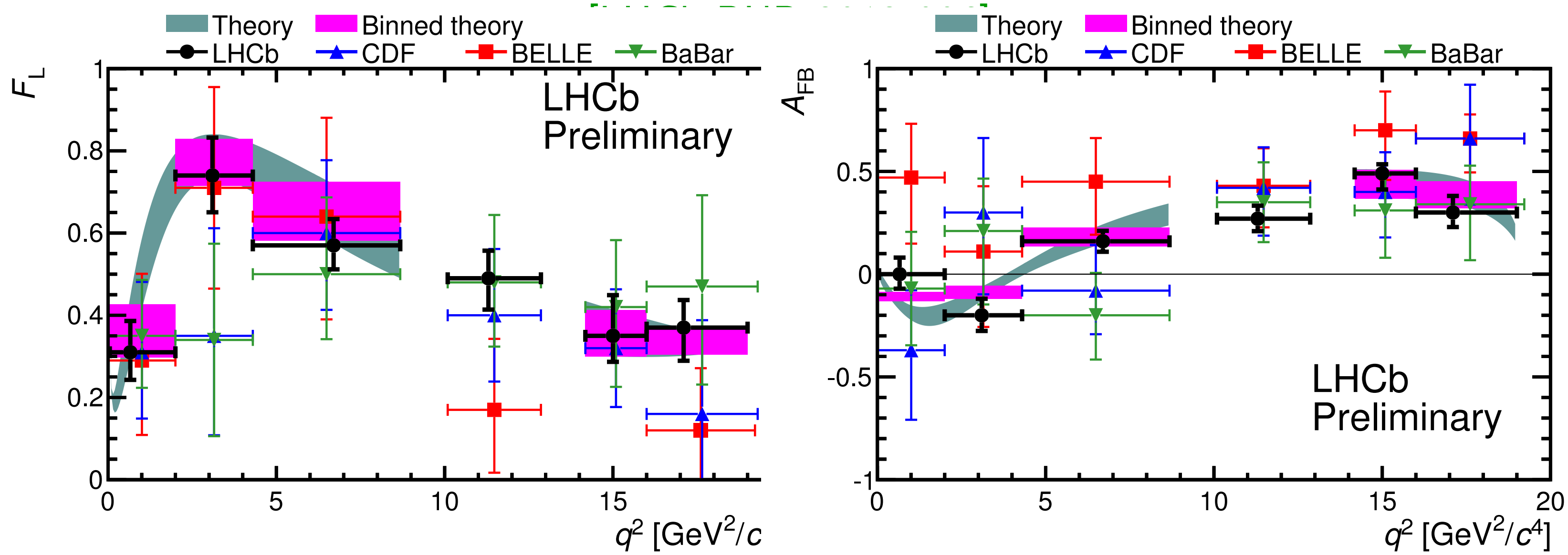
$q_0^2 = (4.39^{+0.38}_{-0.35})\ \mathrm{GeV}^2$ with $\Delta(q_0^2)_{\mathrm{scale}} = \pm 0.25\ \mathrm{GeV}^2$

Ahmed Ali $B \to K^* \ell^+ \ell^-$ decay in soft-collinear effective theory

Experiment: $q_0^2(B^0 \to K^{*0}\mu^+\mu^-) = 4.9^{+1.3}_{-1.1}$ GeV^2

- In agreement with the SM; however, current precision on q_0^2 is only 25%. Will improve at the upgraded LHCb and Super-B factories

Recent Measurements of Angular Observables in $B \to K^* \mu^+ \mu^-$



- Angular variables F_L and A_{FB} have been extracted from the decays $B \to K^*(\to K\pi)\mu^+\mu^-$ from the following expressions

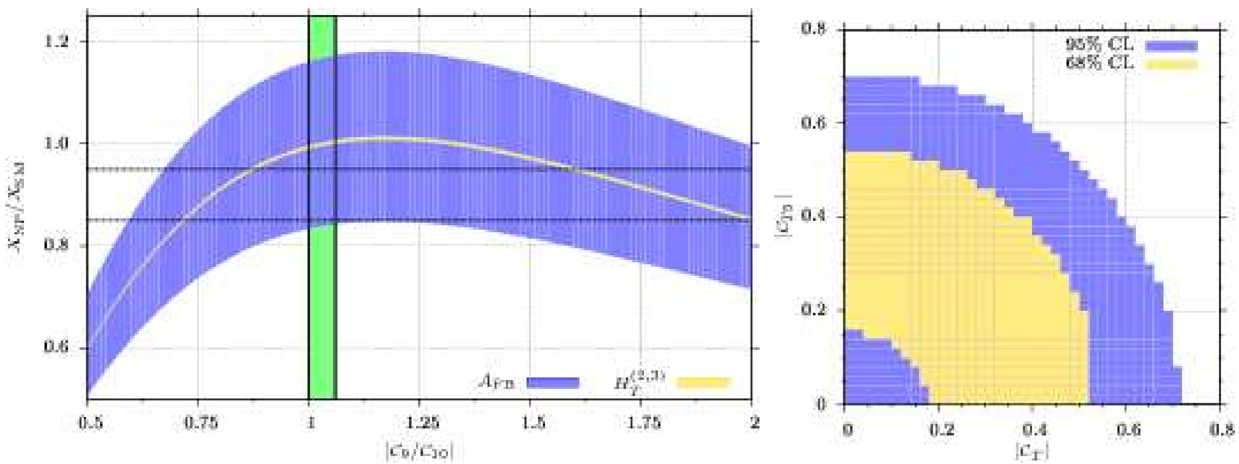
$$\frac{d\Gamma'}{d\theta_K} = \frac{3\Gamma'}{4} \sin\theta_K \left(2F_L \cos^2\theta_K + (1 - F_L)\sin^2\theta_K\right)$$

$$\frac{d\Gamma'}{d\theta_\ell} = \Gamma' \left(\frac{3}{4} F_L \sin^2_{\theta_\ell} + \frac{3}{8}(1 - F_L)(1 + \cos^2_{\theta_\ell}) + A_{FB} \cos\theta_\ell\right) \sin\theta_\ell$$

with $\Gamma' = \Gamma + \bar{\Gamma}$

- Their dependence on the Wilson Coeffs. and the FFs has been worked out in great detail and the measurements are in agreement with the SM

Analysis at Low Recoil of $B \to K^* \ell^+ \ell^-$; Bobeth, Hiller, van Dyk [1212.2321]



- More angular variables can be extracted from the four-fold differential decay rate

$$\frac{d^4\Gamma(B \to K^* \ell^+ \ell^-)}{dq^2 d\cos\theta_\ell d\cos\theta_K d\phi} \propto J_i(q^2)$$

- In general, 12 angular coefficients $J_i(q^2)$; many are 0 in the SM and require extensions of the SM operator basis to include scalar, pseudoscalar, tensor and pseudotensor operators

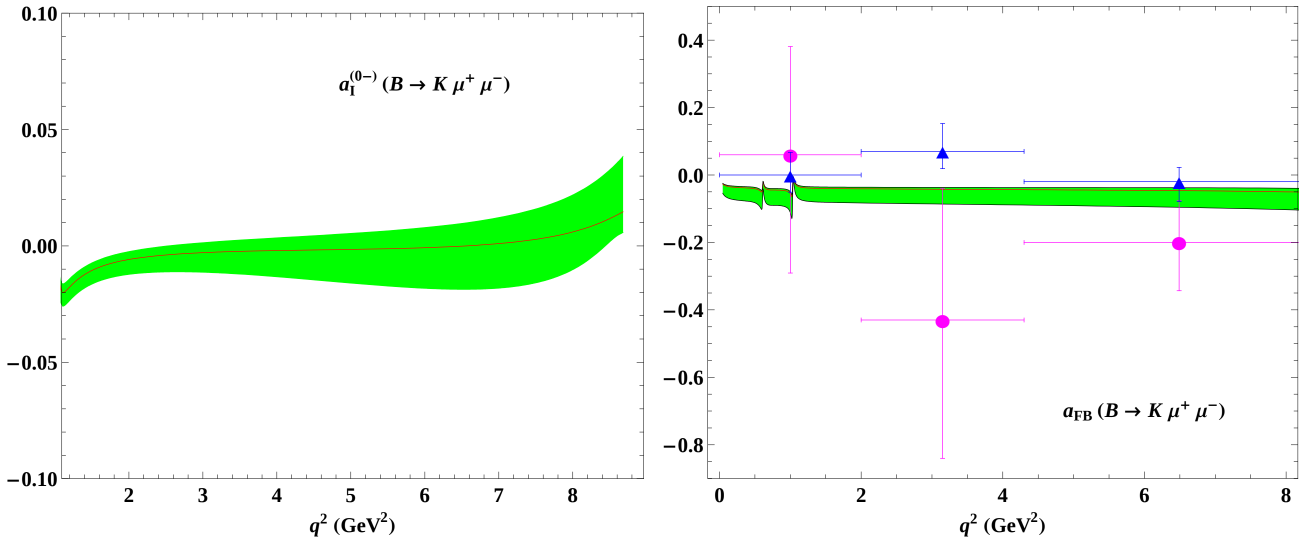
- Several short- and long-distance-free ratios can be formed from the ratios of $J_i(q^2)$; 2 interesting ones are $H_T^{(2,3)} = 2\rho_2/\rho_1 \sim r/(1+r^2); r = C_9/C_{10}$

- Measurement of $H_T^{(2,3)}$ and A_{FB} constrain the ratio $|C_9/C_{10}|$

- Current constraints on the tensor and pseudotensor Coeffs. $|C_T|^2 + |C_{T5}|^2 \leq 0.5$

Analysis at Large Recoil of $B \to K\ell^+\ell^-$

Khodjamirian, Mannel, Wang [1211.0234]



- Includes an estimate of the non-local contributions based on QCD sum rules and dispersion relations at large hadronic recoil; find these effects are modest
- Re-evaluated $d\Gamma(B \to K\ell^+\ell^-)/dq^2$, CP-averaged isospin asymmetry $a_I^{(0-)}(q^2)$, and forward-backward asymmetry $a_{\rm FB}(q^2) \Longrightarrow$ improved theoretical estimates at large recoil
- Both $a_I^{(0-)}(q^2)$ and $a_{\rm FB}(q^2)$ are small in the SM
- $a_{\rm FB}(q^2)$ in agreement with SM, but current data hints at significantly larger isospin asymmetry $a_I^{(0-)}(q^2)$

Isospin Asymmetries (Current Experimental Summary)

[HFAG 2012]

- $\Delta_{0-}(K^*\gamma) = 0.052 \pm 0.026$
- $\Delta_{0-}(X_s\gamma) = -0.01 \pm 0.06$
- $\Delta_{0-}(\rho\gamma) = -0.46^{+0.17}_{-0.16}$
- $\Delta_{0-}(K\ell\ell) = -0.40^{+0.16}_{-0.15}$
- $\Delta_{0-}(K^*\ell\ell) = -0.44^{+0.13}_{-0.12}$

- Currently, there is no measurement of $\Delta_{0-}(X_d\gamma)$
- Others remain to be well measured; all will be undertaken at Belle II & LHCb
- More theoretical work needed to reduce the parametric uncertainties

$B_s \to \mu^+\mu^-$ in the SM

- Effective Hamiltonian

$$\mathcal{H}_{eff} = -\frac{G_F\alpha}{\sqrt{2}\pi} V_{ts}{}^* V_{tb} \sum_i \left[C_i(\mu)\mathcal{O}_i(\mu) + C_i'(\mu)\mathcal{O}_i'(\mu)\right]$$

$$\begin{aligned}
\mathcal{O}_{10} &= (\bar{s}_\alpha \gamma^\mu P_L b_\alpha)\,(\bar{l}\gamma_\mu\gamma_5 l)\,, & \mathcal{O}_{10}' &= (\bar{s}_\alpha \gamma^\mu P_R b_\alpha)\,(\bar{l}\gamma_\mu\gamma_5 l)\\
\mathcal{O}_S &= m_b\,(\bar{s}_\alpha P_R b_\alpha)\,(\bar{l}l)\,, & \mathcal{O}_S' &= m_s\,(\bar{s}_\alpha P_L b_\alpha)\,(\bar{l}l)\\
\mathcal{O}_P &= m_b\,(\bar{s}_\alpha P_R b_\alpha)\,(\bar{l}\gamma_5 l)\,, & \mathcal{O}_P' &= m_s\,(\bar{s}_\alpha P_L b_\alpha)\,(\bar{l}\gamma_5 l)
\end{aligned}$$

$$\begin{aligned}
\mathrm{BR}\left(\bar{B}_s \to \mu^+\mu^-\right) = &\frac{G_F^2\alpha^2 m_{B_s}^2 f_{B_s}^2 \tau_{B_s}}{64\pi^3}|V_{ts}{}^* V_{tb}|^2\sqrt{1-4\hat{m}_\mu^2}\\
&\times\left[\left(1-4\hat{m}_\mu^2\right)|F_S|^2 + |F_P + 2\hat{m}_\mu^2 F_{10}|^2\right]
\end{aligned}$$

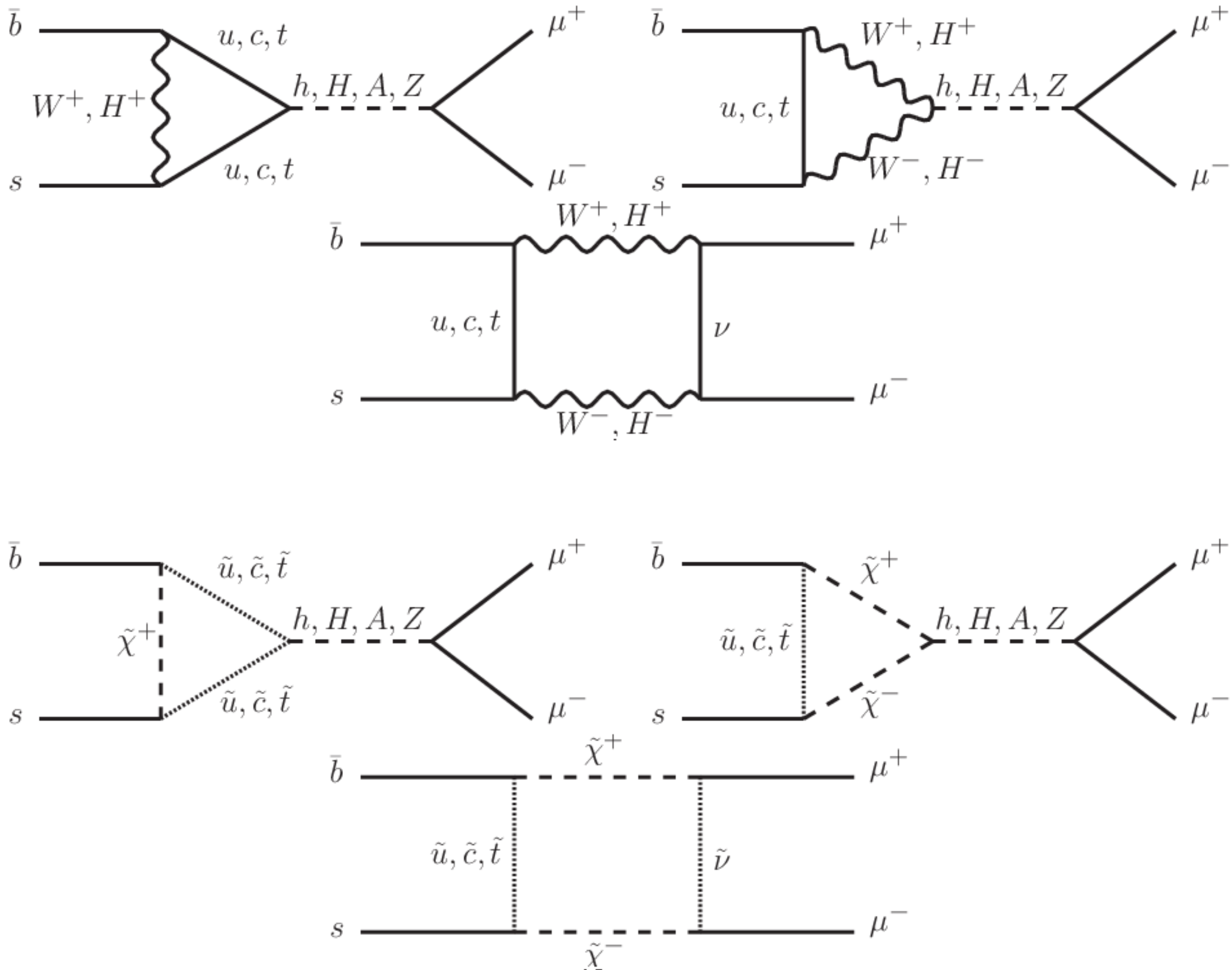
where $\hat{m}_\mu = m_\mu/m_{B_s}$ and

$$F_{S,P} = m_{B_s}\left[\frac{C_{S,P}m_b - C_{S,P}' m_s}{m_b + m_s}\right], \qquad F_{10} = C_{10} - C_{10}'$$

$$\mathrm{BR}\left(\bar{B}_s \to \mu^+\mu^-\right)_{\mathrm{SM}} = (3.23 \pm 0.27)\times 10^{-9}$$ [Buras et al.; arxiv:1208.09344]

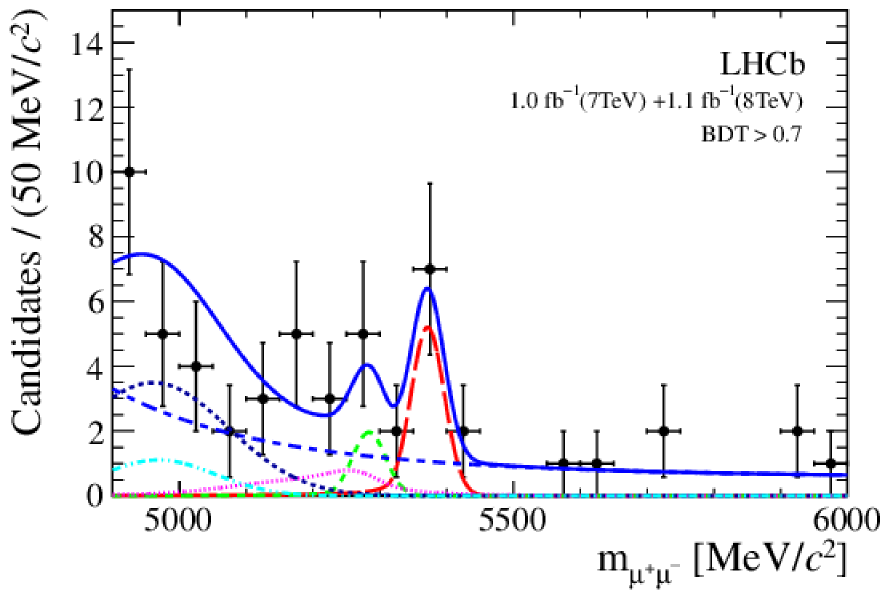
- Experimentally, the measured BR is time-averaged (TA), which differs from this value beacause of $y_S^{\mathrm{SM}} = \Delta\Gamma_s/\Gamma_s = 0.088 \pm 0.014$
  $BR(B_s^0 \to \mu^+\mu^-)_{\mathrm{TA}}^{\mathrm{SM}} = (3.54 \pm 0.30)\times 10^{-9}$; $= (3.2^{+1.5}_{-1.2})\times 10^{-9}$ (LHCb: PRL 110, 021801 (2013))

Leading diagrams for $B_s \to \mu^+\mu^-$ in SM, 2HDM & MSSM



First Evidence for the Decays $\mathcal{B}(B_s \to \mu^+\mu^-)$

[R. Aaij et al. (LHCb), PRL 110, 021801 (2013)]]

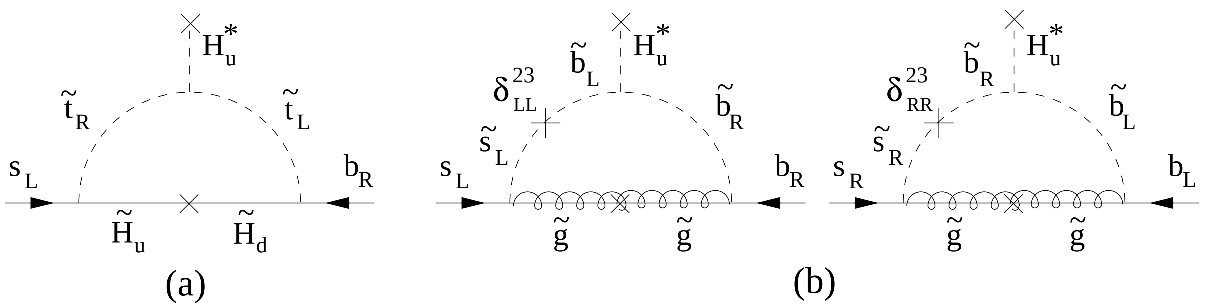


$B_s \to \mu^+\mu^-$ in Supersymmetric Models

- The decay $B_s \to \mu^+\mu^-$ probes essentially the Higgs sector of Supersymmetry, a type-II two-Higgs doublet model

$$\mathcal{L} = \overline{Q} Y_U U_R H_u + \overline{Q}_L Y_D D_R H_d$$

- Higgs-induced FCNC interactions are generated through loops



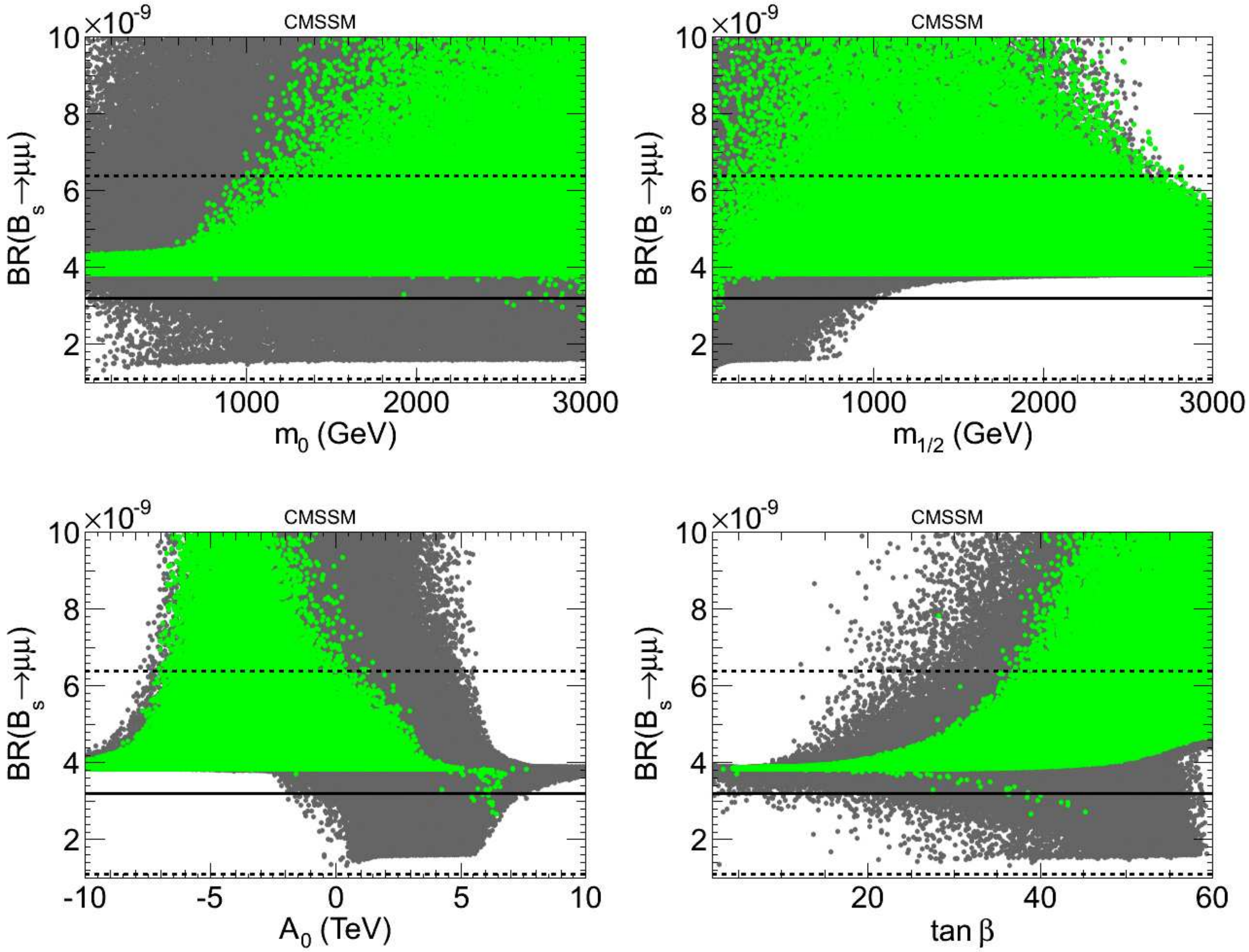
- As H_u gets a VEV (v_u), it contributes an off-diagonal piece to the down-type fermion mass matrix, mixing s_L and b_L by an angle θ

$$\sin\theta = y_b \epsilon v_u / m_b; \quad \text{as } m_b = y_b v_d, \ \sin\theta = \epsilon \tan\beta$$

- $\mathcal{A}(b\bar{s} \to \mu^+\mu^-) \simeq \sin\theta \mathcal{A}(b\bar{b} \to \mu^+\mu^-) \ \propto \tan\beta / \cos^2\beta \Longrightarrow \tan^3\beta$ for large-$\tan\beta$

$\mathcal{B}(B_s \to \mu^+\mu^-)$ vs. the CMSSM parameters

[Arbey et al.; arxiv:1212.4887]



Summary

- Thanks to dedicated experiments and progress in theoretical techniques (Pert. QCD, Lattice-QCD, QCD Sum Rules, Heavy quark Expansion, SCET) Rare B-Decays are under quantitative control, but the precision varies between (10 - 30)%

- From the CKM Phenomenology, there is added value in precisely measuring Rare B-Decays and in improving the SM theoretical accuracy, as this would overconstrain $|V_{ts}|$ and $|V_{td}|$

- Rare B-Decays provide invaluable constraints on Beyond-the-SM Physics; theoretical interest in their dedicated studies remains high and they may turn out to be the harbinger of BSM physics, as they probe very high mass scales

- A new chapter on precision B_s-meson physics has opened at the LHC, in particular, by the LHCb, resolving some open issues and testing SM at an unprecedented rate, of which $B_s^0 \to \mu^+\mu^-$ is a shining example

- We look forward to new data from the ongoing and planned experiments at the LHC and the Super-B factories